

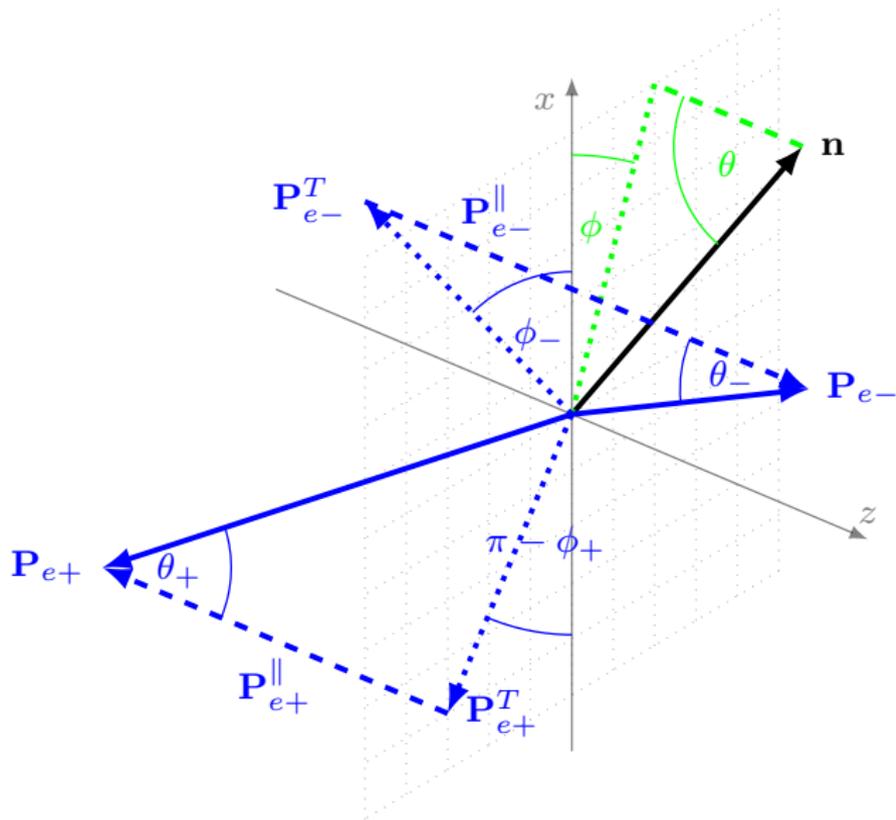
MCSAN_{C_{ee}} – Event Generator for polarized e^+e^- scattering at one-loop EW

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Decomposition of the e^\pm polarization vectors



Matrix element squared

$$\begin{aligned}
 |\mathcal{M}|^2 = & L_{e-}'' R_{e+}'' |\mathcal{H}_{-+}|^2 + R_{e-}'' L_{e+}'' |\mathcal{H}_{+-}|^2 + L_{e-}'' L_{e+}'' |\mathcal{H}_{--}|^2 + R_{e-}'' R_{e+}'' |\mathcal{H}_{++}|^2 \\
 & - \frac{1}{2} P_{e-}^\perp P_{e+}^\perp \operatorname{Re} \left[e^{i(\Phi_+ - \Phi_-)} \mathcal{H}_{++} \mathcal{H}_{--}^* + e^{i(\Phi_+ + \Phi_-)} \mathcal{H}_{+-} \mathcal{H}_{-+}^* \right] \\
 & + P_{e-}^\perp \operatorname{Re} \left[e^{i\Phi_-} \left(L_{e+}'' \mathcal{H}_{+-} \mathcal{H}_{--}^* + R_{e+}'' \mathcal{H}_{++} \mathcal{H}_{-+}^* \right) \right] \\
 & - P_{e+}^\perp \operatorname{Re} \left[e^{i\Phi_+} \left(L_{e-}'' \mathcal{H}_{-+} \mathcal{H}_{--}^* + R_{e-}'' \mathcal{H}_{++} \mathcal{H}_{+-}^* \right) \right],
 \end{aligned}$$

where

$$L_{e\pm}'' = \frac{1}{2}(1 - P_{e\pm}''), \quad R_{e\pm}'' = \frac{1}{2}(1 + P_{e\pm}''), \quad \Phi_\pm = \phi_\pm - \phi,$$

$\mathcal{H}_{--}, \mathcal{H}_{++}, \mathcal{H}_{-+}, \mathcal{H}_{+-}$ — helicity amplitudes.

Moortgat-Pick, G. et al. Phys.Rept. 460 (2008) 131-243

Cross-section structure

The cross-section of these processes at one-loop can be divided into four parts:

$$\sigma^{1\text{-loop}} = \sigma^{\text{Born}} + \sigma^{\text{virt}}(\lambda) + \sigma^{\text{soft}}(\lambda, \omega) + \sigma^{\text{hard}}(\omega),$$

Contributions due to:

σ^{Born} — Born level cross-section,

σ^{virt} — virtual(loop) corrections,

σ^{soft} — soft photon emission,

σ^{hard} — hard photon emission (with energy $E_\gamma > \omega$).

Auxiliary parameters λ ("photon mass") and ω cancel out after summation.

Helicity Amplitudes (HA) approach

We count all contributions through HA approach.

The cross section of the generic $2 \rightarrow 2(\gamma)$ process $e^+e^- \rightarrow X_3X_4(\gamma)$
($X_3X_4 = e^-e^+, \mu^-\mu^+, ZH, \gamma Z, \gamma\gamma, \dots$)

$$\sigma_{P_{e^-}P_{e^+}} = \frac{1}{4} \sum_{\chi_1, \chi_2} (1 + \chi_1 P_{e^-})(1 + \chi_2 P_{e^+}) \sigma_{\chi_1 \chi_2},$$

where $\chi_i = -1(+1)$ corresponds to lepton with left (right) helicity state.

The virt(Born) cross section:

$$\frac{d\sigma_{\chi_1 \chi_2}^{\text{virt(Born)}}}{d \cos \theta} = \frac{\sqrt{\lambda(s, M_3^2, M_4^2)}}{2^5 \pi s^2} \left| \mathcal{H}_{\chi_1 \chi_2}^{\text{virt(Born)}} \right|^2,$$

where

$$\left| \mathcal{H}_{\chi_1 \chi_2}^{\text{virt(Born)}} \right|^2 = \sum_{\chi_3, \chi_4} \left| \mathcal{H}_{\chi_1 \chi_2 \chi_3 \chi_4}^{\text{virt(Born)}} \right|^2.$$

Helicity Amplitudes (HA) approach

The soft term is factorized to Born-level cross section:

$$\frac{d\sigma_{\chi_1\chi_2}^{\text{soft}}}{d\cos\theta} = \frac{d\sigma_{\chi_1\chi_2}^{\text{Born}}}{d\cos\theta} \cdot \frac{\alpha}{2\pi} K^{\text{soft}}(\omega, \lambda).$$

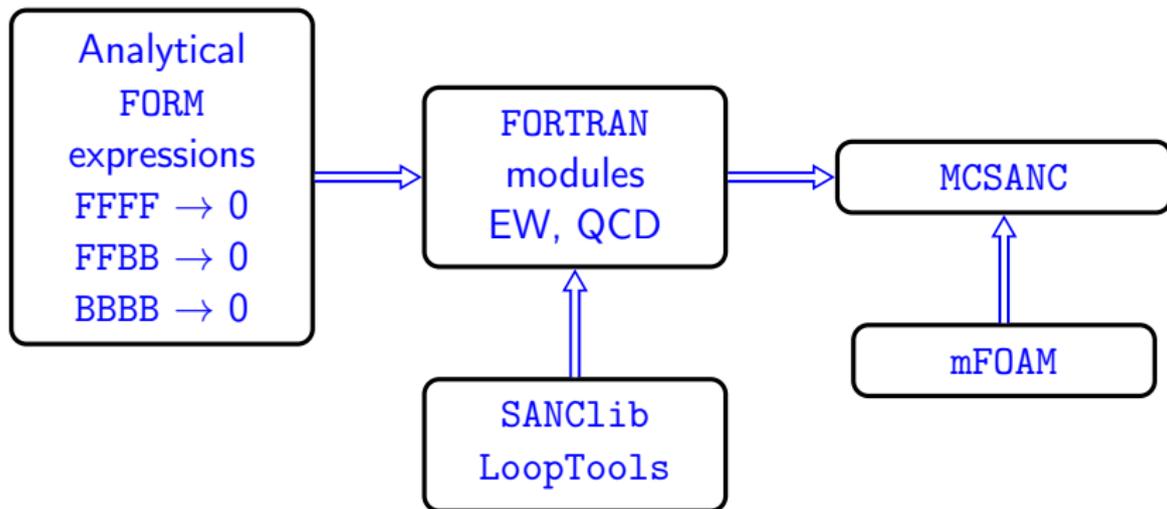
The cross section for hard Bremsstrahlung

$$\frac{d\sigma_{\chi_1\chi_2}^{\text{hard}}}{ds' d\cos\theta_4 d\phi_4 d\cos\theta_5} = \frac{s-s'}{8(4\pi)^4 ss'} \frac{\sqrt{\lambda(s', M_3^2, M_4^2)}}{\sqrt{\lambda(s, m_e^2, m_e^2)}} \left| \mathcal{H}_{\chi_1\chi_2}^{\text{hard}} \right|^2,$$

where $s = (p_1 + p_2)^2$, $s' = (p_3 + p_4)^2$ and

$$\left| \mathcal{H}_{\chi_1\chi_2}^{\text{hard}} \right|^2 = \sum_{\chi_3, \chi_4, \chi_5} \left| \mathcal{H}_{\chi_1\chi_2\chi_3\chi_4\chi_5}^{\text{hard}} \right|^2.$$

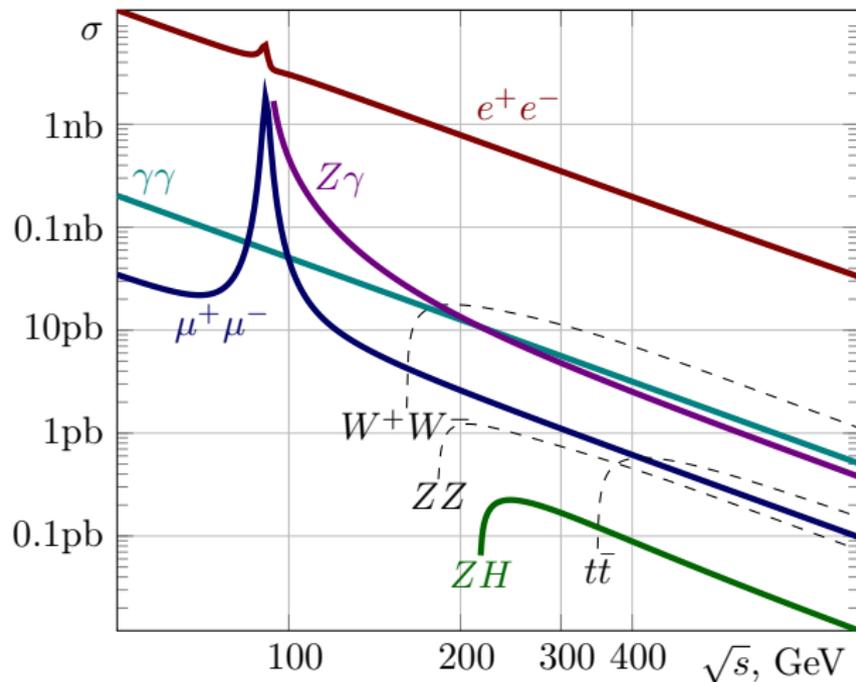
MCSANC_{ee} generator



We created Monte Carlo generator of unweighted events for the polarized e^+e^- scattering with complete one-loop EW corrections.

This generator uses adaptive Monte Carlo algorithm **mFOAM** [S. Jadach and P. Sawicki. CPC 177:441-458,2007] which is a part of **ROOT** [<https://root.cern.ch>] program.

Basic processes of SM for e^+e^- annihilation



The cross sections are given for polar angles between $10^\circ < \theta < 170^\circ$ in the final state

Setup for tuned comparison

Input parameters:

$$\alpha^{-1}(0) = 137.03599976,$$

$$M_W = 80.4514958 \text{ GeV}, \quad M_Z = 91.1876 \text{ GeV}, \quad \Gamma_Z = 2.49977 \text{ GeV},$$

$$m_e = 0.51099907 \text{ MeV}, \quad m_\mu = 0.105658389 \text{ GeV}, \quad m_\tau = 1.77705 \text{ GeV},$$

$$m_d = 0.083 \text{ GeV}, \quad m_s = 0.215 \text{ GeV}, \quad m_b = 4.7 \text{ GeV},$$

$$m_u = 0.062 \text{ GeV}, \quad m_c = 1.5 \text{ GeV}, \quad m_t = 173.8 \text{ GeV}.$$

Cuts:

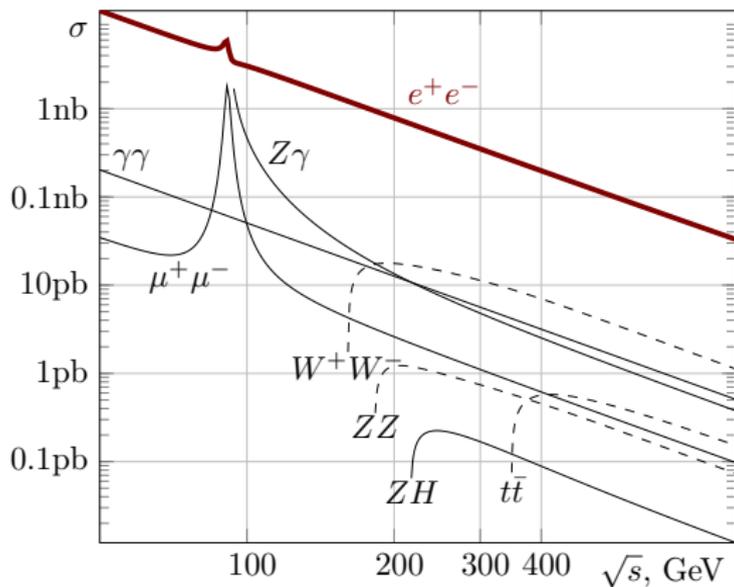
$$|\cos \theta| < 0.9,$$

$$E_\gamma > 1 \text{ GeV} \quad (\text{for comparison of hard Bremsstrahlung}).$$

Tuned comparison of our results for polarized Born and hard Bremsstrahlung with the results **WHIZARD** [Eur.Phys.J.C71 (2011) 1742] and **CalcHEP** [CPC 184(2013) 1729-1769] programs shows agreement within statistical errors.

Unpolarized Soft + virtual contribution agree with the results of **a1TALC** [CPC 174 (2006) 71-82] (for $e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \tau^+\tau^-$) and **Grace-Loop** [Phys.Rept. 430 (2006) 117-209] (for $e^+e^- \rightarrow ZH$)

$$e^+e^- \rightarrow e^+e^-$$



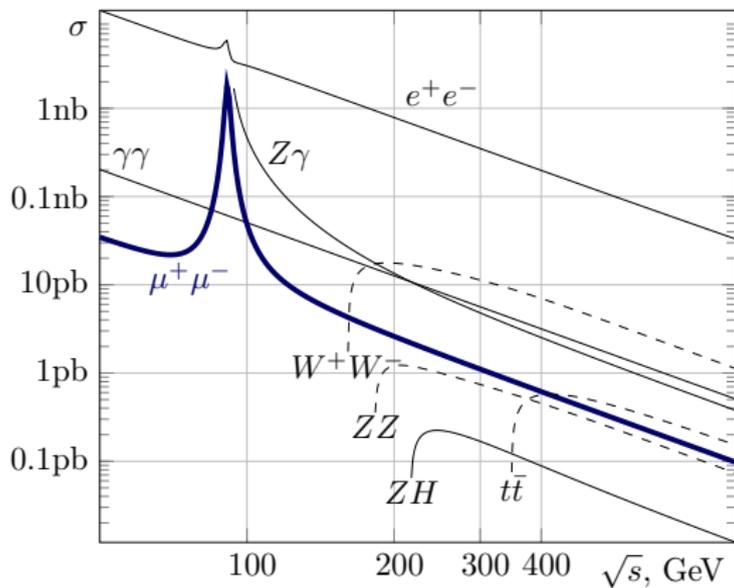
For details, see the talk by [Andrej Arbuzov](#)

$e^+e^- \rightarrow e^+e^-$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	56.676(1)	57.774(1)	56.273(1)	59.275(1)
$\sigma_{e^+e^-}^{\text{1-loop}}$, pb	61.73(1)	62.59(1)	61.88(1)	63.29(1)
δ , %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	14.379(1)	15.031(1)	12.706(1)	17.355(1)
$\sigma_{e^+e^-}^{\text{1-loop}}$, pb	15.47(1)	15.87(1)	13.86(1)	17.88(1)
δ , %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{e^+e^-}^{\text{Born}}$, pb	3.6792(1)	3.9057(1)	3.0358(1)	4.7756(1)
$\sigma_{e^+e^-}^{\text{1-loop}}$, pb	3.864(1)	3.945(1)	3.233(1)	4.654(1)
δ , %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

$$e^+e^- \rightarrow \mu^-\mu^+$$

$$e^+e^- \rightarrow \tau^-\tau^+$$



$e^+e^- \rightarrow \mu^+\mu^-$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{ pb}$	1.417(1)	1.546(1)	0.7690(2)	2.323(1)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{ pb}$	2.397(1)	2.614(1)	1.301(1)	3.927(1)
$\delta, \%$	69.1(1)	69.1(1)	69.2(1)	69.1(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{ pb}$	0.3436(1)	0.3716(1)	0.1857(1)	0.5575(1)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{ pb}$	0.4696(1)	0.4953(1)	0.2506(1)	0.7399(1)
$\delta, \%$	36.7(1)	33.3(1)	35.0(1)	32.7(1)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{\mu^+\mu^-}^{\text{Born}}, \text{ pb}$	0.08535(1)	0.09213(1)	0.04608(1)	0.1382(1)
$\sigma_{\mu^+\mu^-}^{1\text{-loop}}, \text{ pb}$	0.1163(1)	0.1212(1)	0.06169(1)	0.1807(1)
$\delta, \%$	36.2(1)	31.6(1)	33.9(1)	30.8(1)

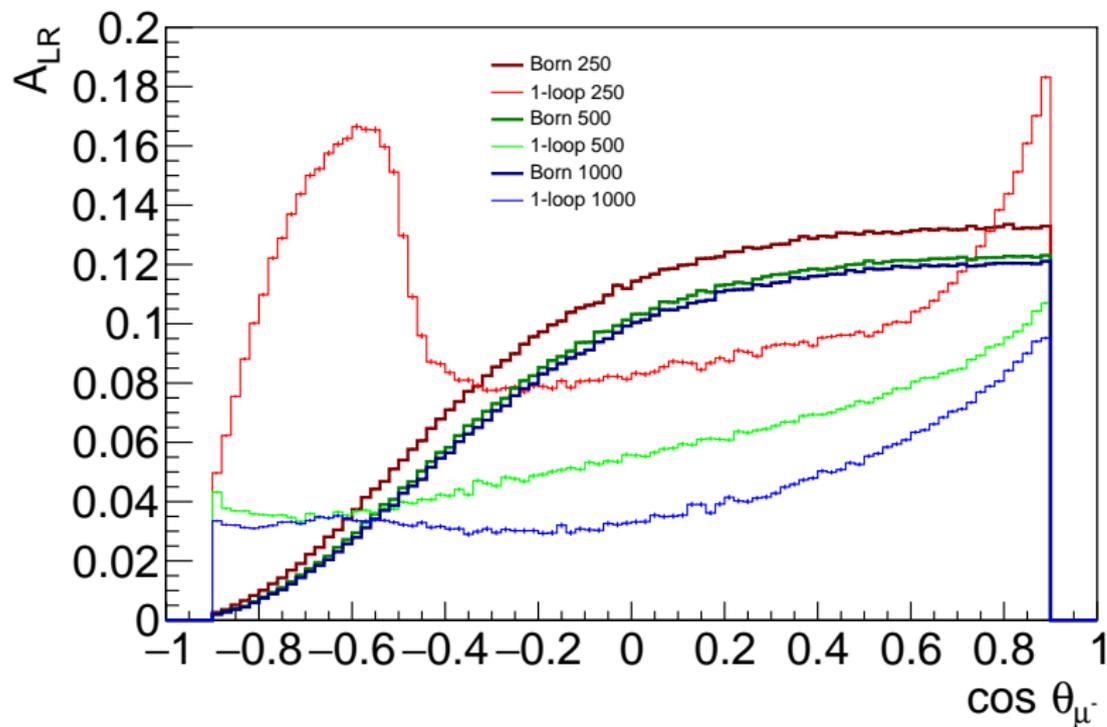
$e^+e^- \rightarrow \tau^+\tau^-$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma_{\tau^+\tau^-}^{\text{Born}}, \text{ pb}$	1.417(1)	1.546(1)	0.7692(1)	2.324(1)
$\sigma_{\tau^+\tau^-}^{1\text{-loop}}, \text{ pb}$	2.360(1)	2.575(1)	1.298(1)	3.850(1)
$\delta, \%$	66.5(1)	66.5(1)	68.8(1)	65.7(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma_{\tau^+\tau^-}^{\text{Born}}, \text{ pb}$	0.3436(1)	0.3715(1)	0.1857(1)	0.5575(1)
$\sigma_{\tau^+\tau^-}^{1\text{-loop}}, \text{ pb}$	0.4606(1)	0.4861(1)	0.2466(1)	0.7257(1)
$\delta, \%$	34.0(3)	30.8(1)	32.8(1)	30.1(1)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma_{\tau^+\tau^-}^{\text{Born}}, \text{ pb}$	0.08534(2)	0.09213(1)	0.04608(1)	0.1382(1)
$\sigma_{\tau^+\tau^-}^{1\text{-loop}}, \text{ pb}$	0.1134(1)	0.11885(2)	0.06067(1)	0.1770(1)
$\delta, \%$	33.6(1)	29.0(1)	31.7(1)	28.1(1)

$e^+e^- \rightarrow \mu^+\mu^-$: A_{LR} distributions in $\cos\theta$

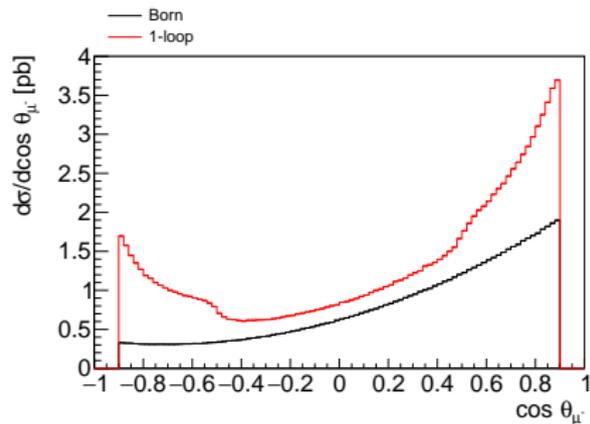
$$A_{LR} = \frac{\sigma_{LR} - \sigma_{RL}}{\sigma_{LR} + \sigma_{RL}}$$

$e^+e^- \rightarrow \mu^+\mu^-$

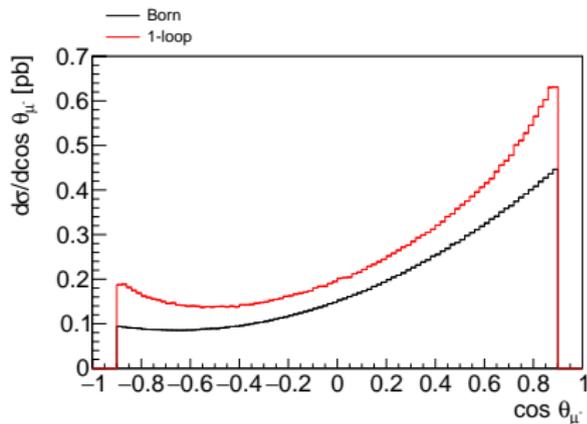


$e^+e^- \rightarrow \mu^+\mu^-$: distributions in $\cos\theta$

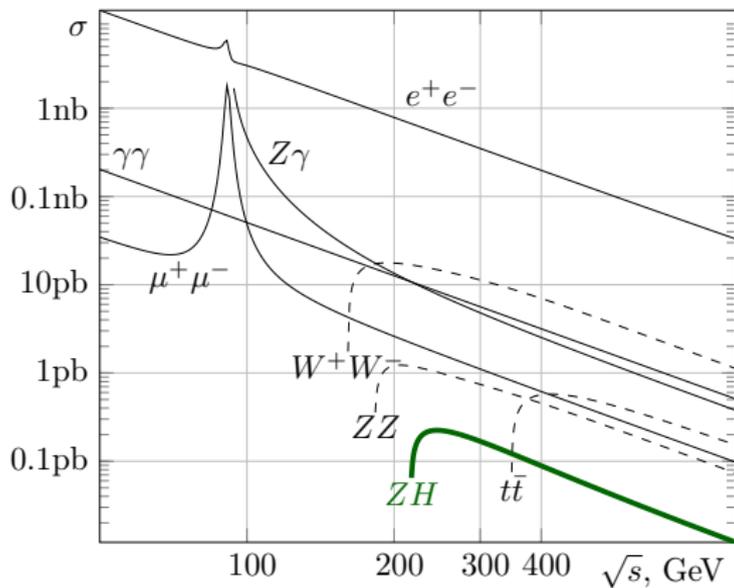
250 GeV:



500 GeV:



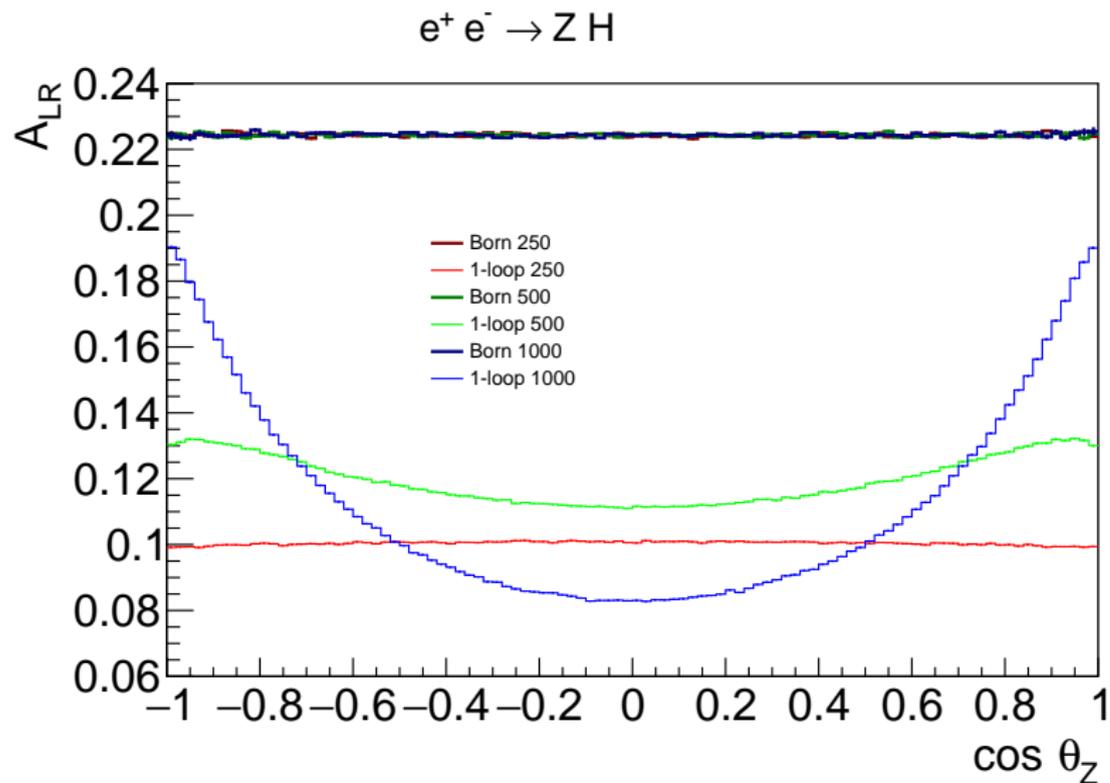
$$e^+ e^- \rightarrow ZH$$



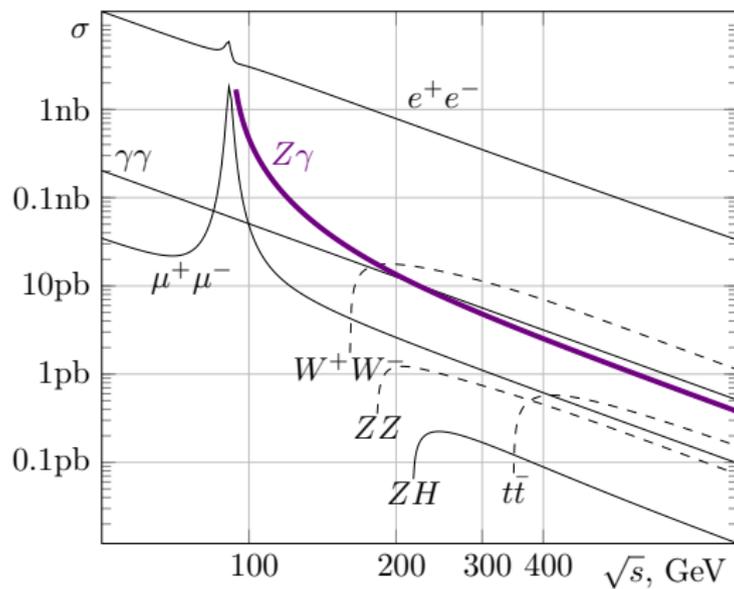
$e^+e^- \rightarrow ZH$: Born vs 1-loop

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma^{\text{Born}}, \text{fb}$	205.64(1)	242.55(1)	116.16(1)	368.93(1)
$\sigma^{1\text{-loop}}, \text{fb}$	186.6(1)	201.5(1)	100.8(1)	302.2(1)
$\delta, \%$	-9.24(1)	-16.94(1)	-13.25(1)	-18.10(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma^{\text{Born}}, \text{fb}$	51.447(1)	60.680(1)	29.061(1)	92.299(1)
$\sigma^{1\text{-loop}}, \text{fb}$	57.44(1)	62.71(1)	31.25(1)	94.17(2)
$\delta, \%$	11.65(1)	3.35(2)	7.54(1)	2.03(2)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma^{\text{Born}}, \text{fb}$	11.783(1)	13.898(1)	6.6559(1)	21.140(1)
$\sigma^{1\text{-loop}}, \text{fb}$	12.92(1)	13.91(1)	6.995(1)	20.83(1)
$\delta, \%$	9.68(1)	0.10(2)	5.09(2)	-1.47(2)

$e^+e^- \rightarrow ZH$: A_{LR} distributions in $\cos\theta$



$$e^+e^- \rightarrow Z\gamma$$

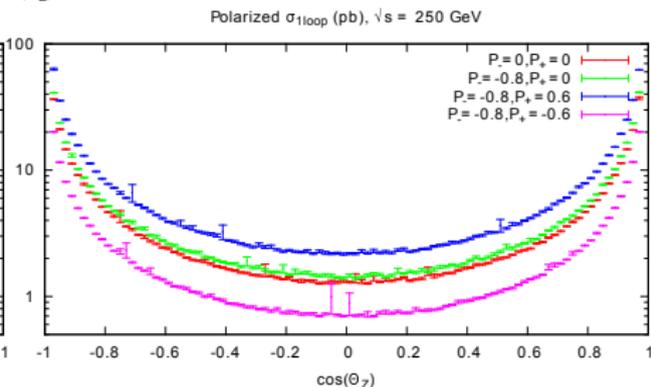
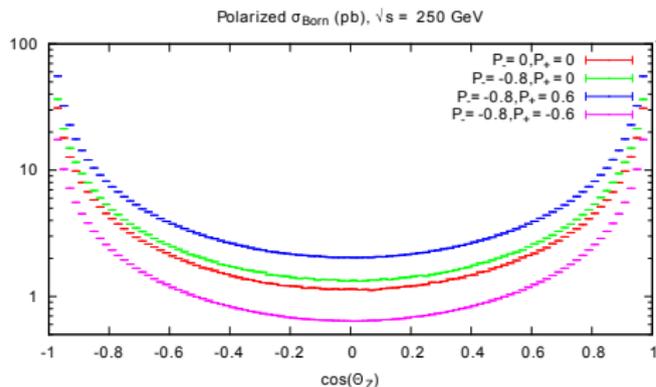
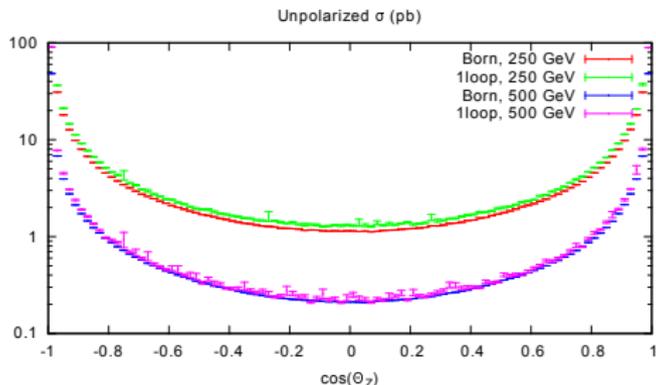


$e^+e^- \rightarrow Z\gamma$: Born vs 1-loop (preliminary)

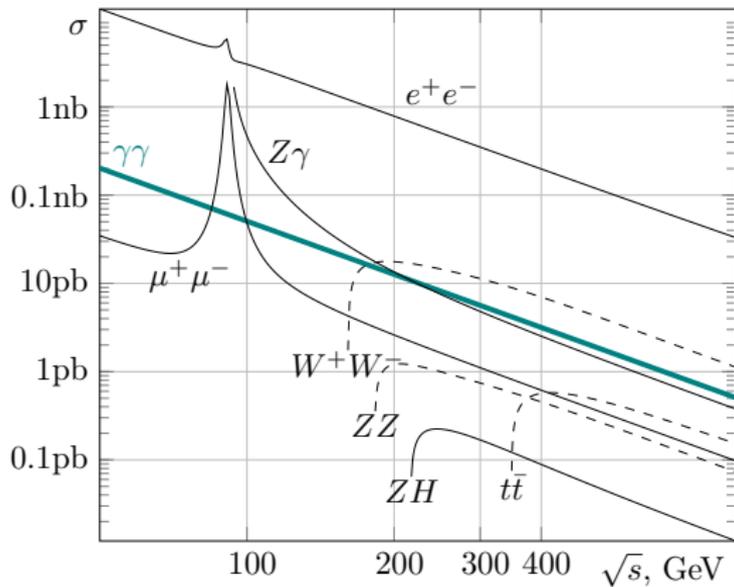
\sqrt{s} , GeV	250	500	1000
$\sigma_{Z\gamma}^{\text{Born}}$, pb	15.7038(6)	3.3858(3)	0.81958(3)
$\sigma_{Z\gamma}^{\text{1-loop}}$, pb	24.37(1)	5.23(6)	1.237(3)
δ , %	55.20(6)	54.4(2)	50.9(4)

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250$ GeV				
$\sigma_{Z\gamma}^{\text{Born}}$, pb	15.7038(6)	18.520(4)	8.870(3)	28.174(2)
$\sigma_{Z\gamma}^{\text{1-loop}}$, pb	24.37(1)	28.00(1)	13.53(1)	42.13(2)
δ , %	55.20(6)	51.11(8)	52.57(9)	49.55(7)

$e^+e^- \rightarrow Z\gamma$: distributions in $\cos\theta_Z$ (preliminary)



$$e^+ e^- \rightarrow \gamma\gamma$$

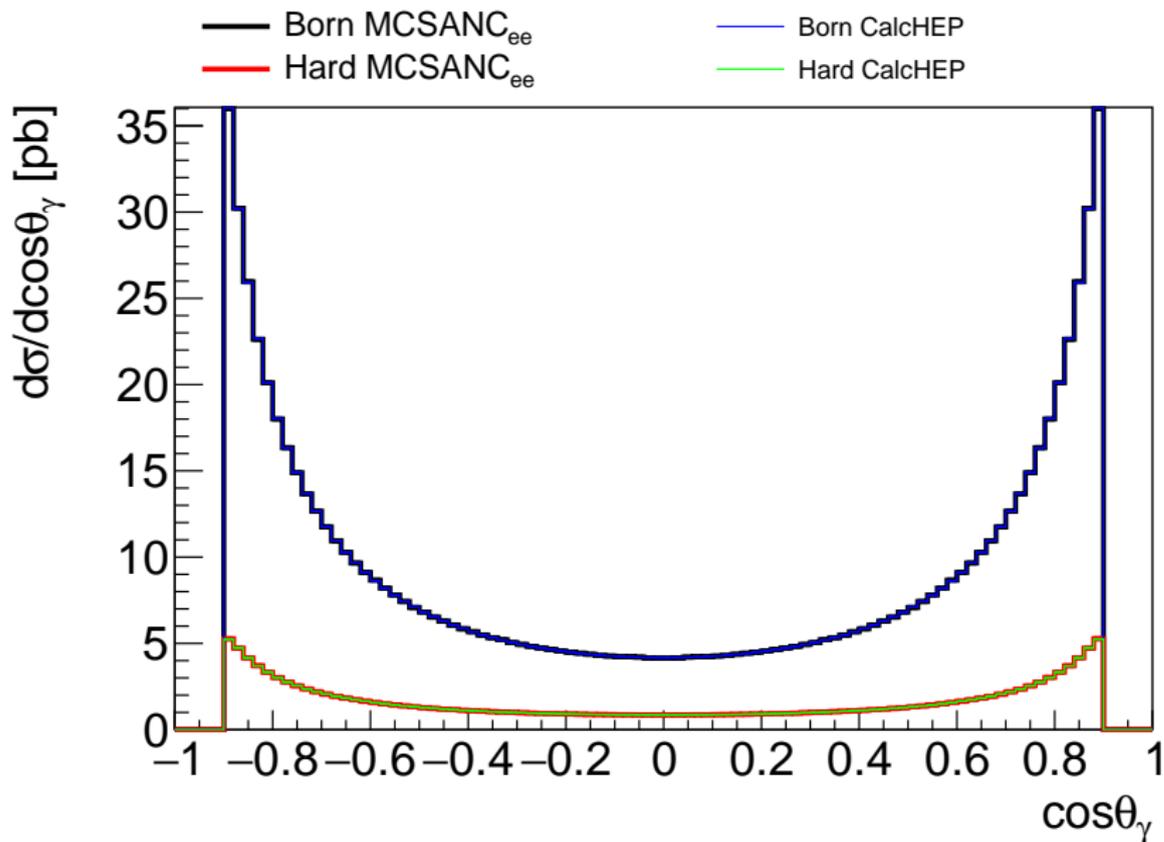


$e^+e^- \rightarrow \gamma\gamma$: WHIZARD vs CalcHEP vs MCSANee

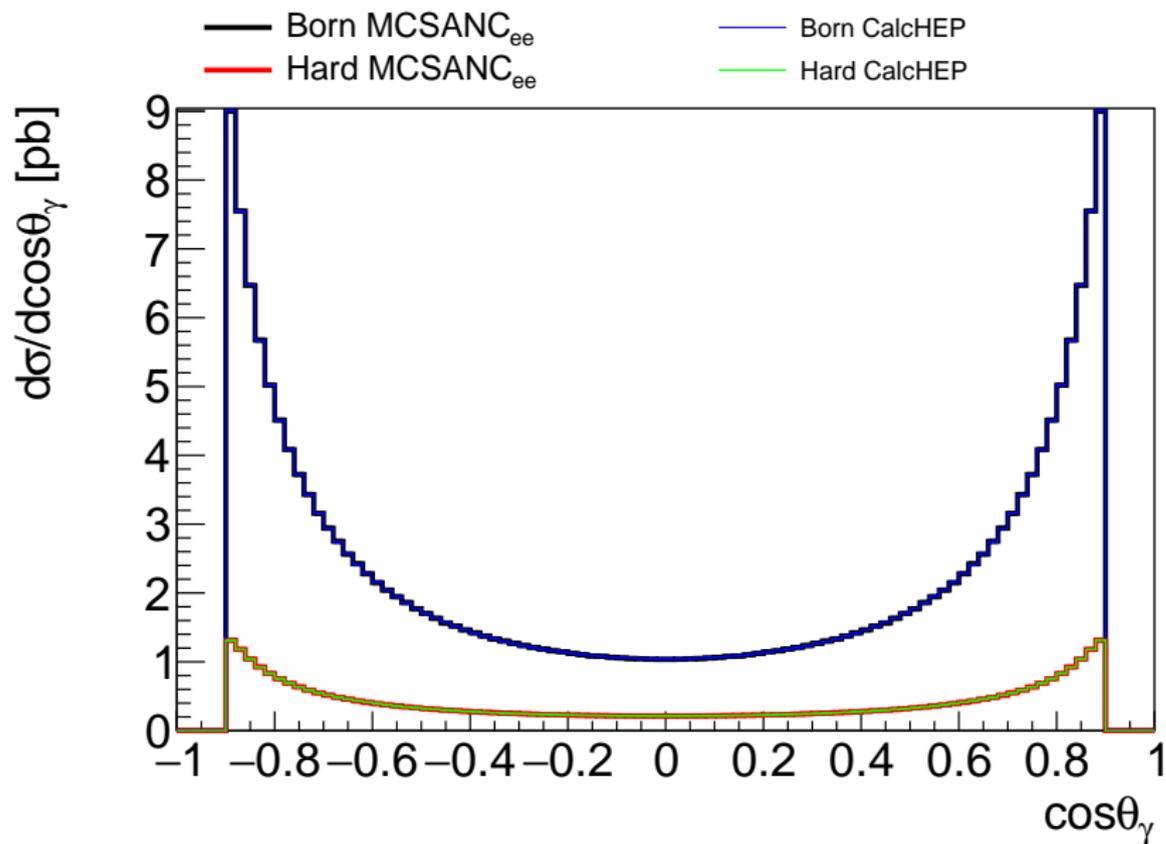
P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
$\sqrt{s} = 250 \text{ GeV}$				
$\sigma^{\text{hard}}, \text{fb}$	594.1(1)	594.1(1)	308.9(1)	879.2(1)
$\sigma^{\text{hard}}, \text{fb}$	594.0(1)	593.8(1)	308.8(1)	879.2(1)
$\sigma^{\text{hard}}, \text{fb}$	594.0(1)	594.2(1)	308.9(1)	879.1(1)
$\sqrt{s} = 500 \text{ GeV}$				
$\sigma^{\text{hard}}, \text{fb}$	158.6(1)	158.5(1)	82.50(1)	234.5(1)
$\sigma^{\text{hard}}, \text{fb}$	158.7(1)	158.6(1)	82.50(1)	234.8(1)
$\sigma^{\text{hard}}, \text{fb}$	158.7(1)	158.7(1)	82.50(1)	234.8(1)
$\sqrt{s} = 1000 \text{ GeV}$				
$\sigma^{\text{hard}}, \text{fb}$	42.18(1)	42.20(1)	21.95(1)	62.43(1)
$\sigma^{\text{hard}}, \text{fb}$	42.19(1)	42.18(1)	21.94(1)	62.45(1)
$\sigma^{\text{hard}}, \text{fb}$	42.17(1)	42.20(1)	21.94(1)	62.44(1)

Cuts: $|\cos\theta_\gamma| < 0.9$, $E_\gamma > 0.0025 \text{ GeV}$ for each of the final photons

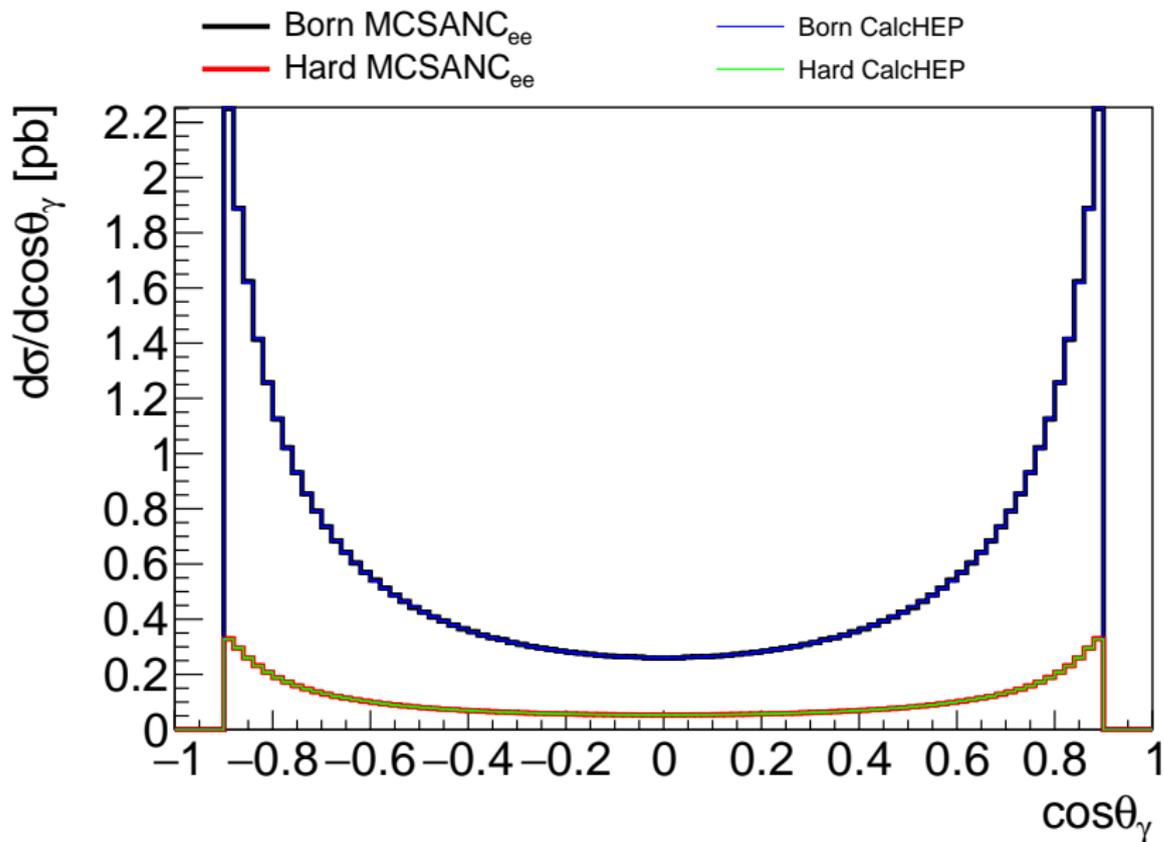
$e^+e^- \rightarrow \gamma\gamma$: distribution in $\cos\theta$, $\sqrt{s}=250$ GeV



$e^+e^- \rightarrow \gamma\gamma$: distribution in $\cos\theta$, $\sqrt{s}=500$ GeV



$e^+e^- \rightarrow \gamma\gamma$: distribution in $\cos\theta$, $\sqrt{s}=1000\text{GeV}$



Summary

- We created Monte Carlo generator of unweighted events MCSANC_{ee} for polarized e^+e^- scattering with complete one-loop EW corrections.
- Now it includes the following processes:
 - $e^+e^- \rightarrow e^+e^-$ ([Phys.Rev.D 98, 013001](#))
 - $e^+e^- \rightarrow ZH$ ([arXiv:1812.10965](#))
 - $e^+e^- \rightarrow \mu^+\mu^-$ (or $\tau^+\tau^-$) ([preliminary](#))
 - $e^+e^- \rightarrow Z\gamma$ ([preliminary](#))
 - $e^+e^- \rightarrow \gamma\gamma$ ([preliminary](#))
- Support of Les Houches format for events.

Plans

- to include in `MCSANCee` the following processes:
 - $e^+e^- \rightarrow t\bar{t}$
 - $e^+e^- \rightarrow ZZ$
 - $e^+e^- \rightarrow H\gamma$
 - $e^+e^- \rightarrow f\bar{f}\gamma$
 - $e^+e^- \rightarrow f\bar{f}H$
 - $\gamma\gamma \rightarrow \gamma\gamma, \gamma\gamma \rightarrow e^+e^-, \gamma\gamma \rightarrow HZ, \gamma\gamma \rightarrow Z\gamma, \gamma\gamma \rightarrow ZZ$
 - $e^\pm\gamma \rightarrow e^\pm\gamma$
- to extend the functionality of the generator with actual energy spectrum of initial particles, leading multi-loop corrections, and with transverse polarization