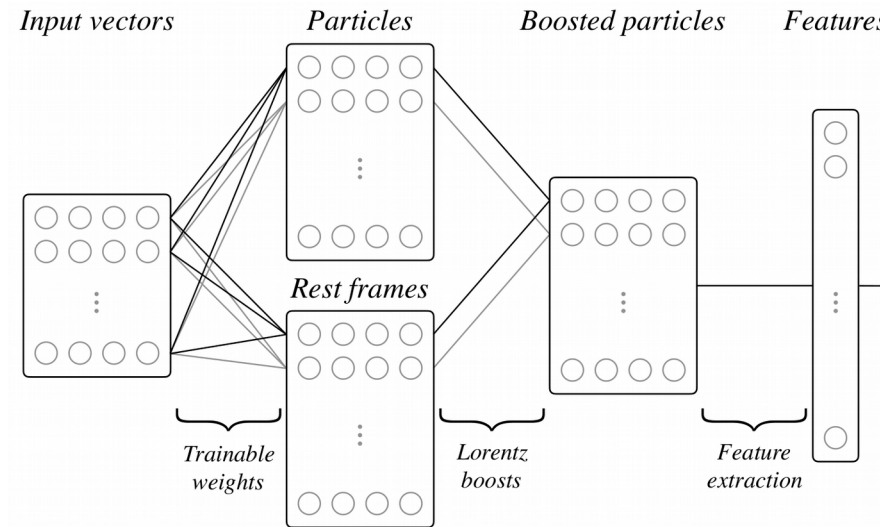


Autonomous Physics-Inspired Feature Engineering with Lorentz Boost Networks

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III. Physikalisches Institut A, RWTH Aachen University

12.03.19



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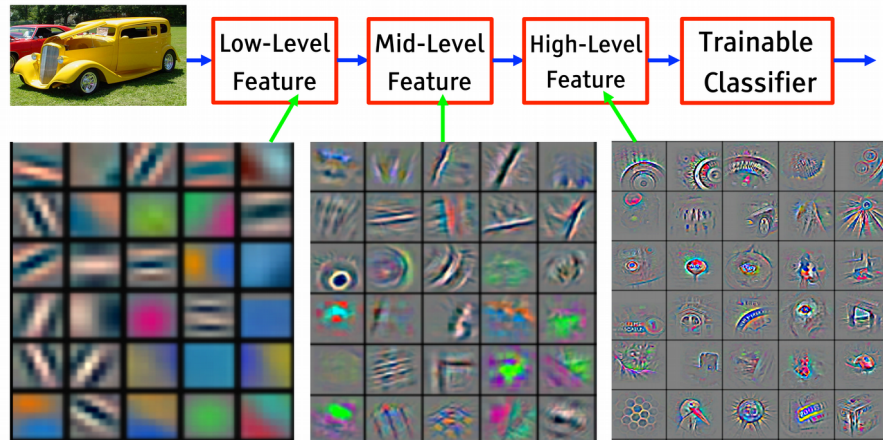
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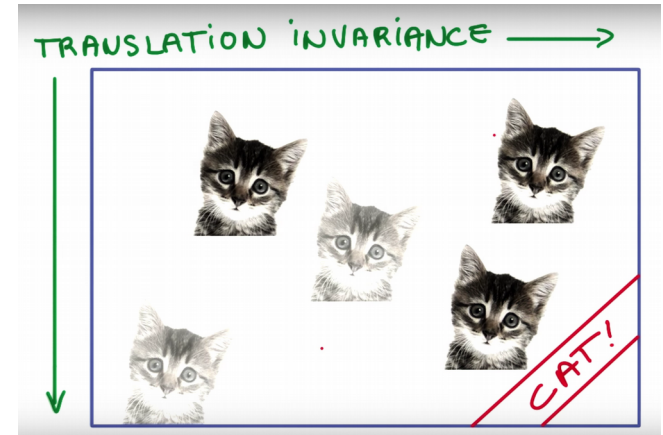
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- Wishlist:
 - Model requiring **only low level data** (e.g. particle four momenta)
 - **Interpretable model**
- Computer science: Dedicated architectures exploiting problem structure

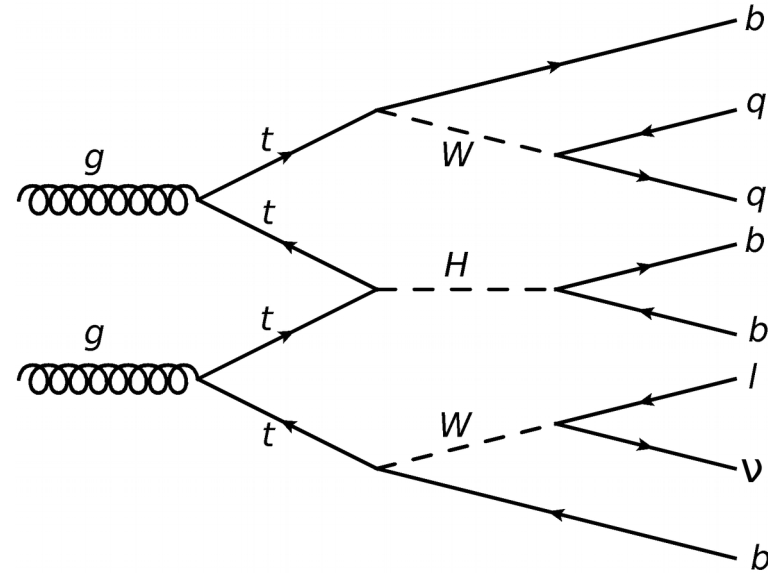
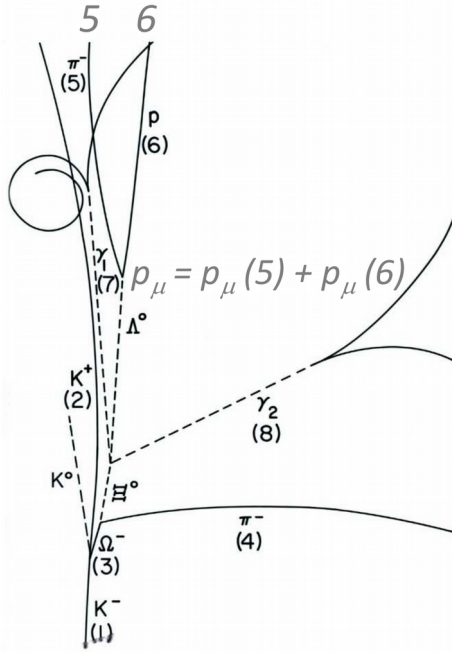


[Zeiler & Fergus 2013], adapted by Yann LeCun

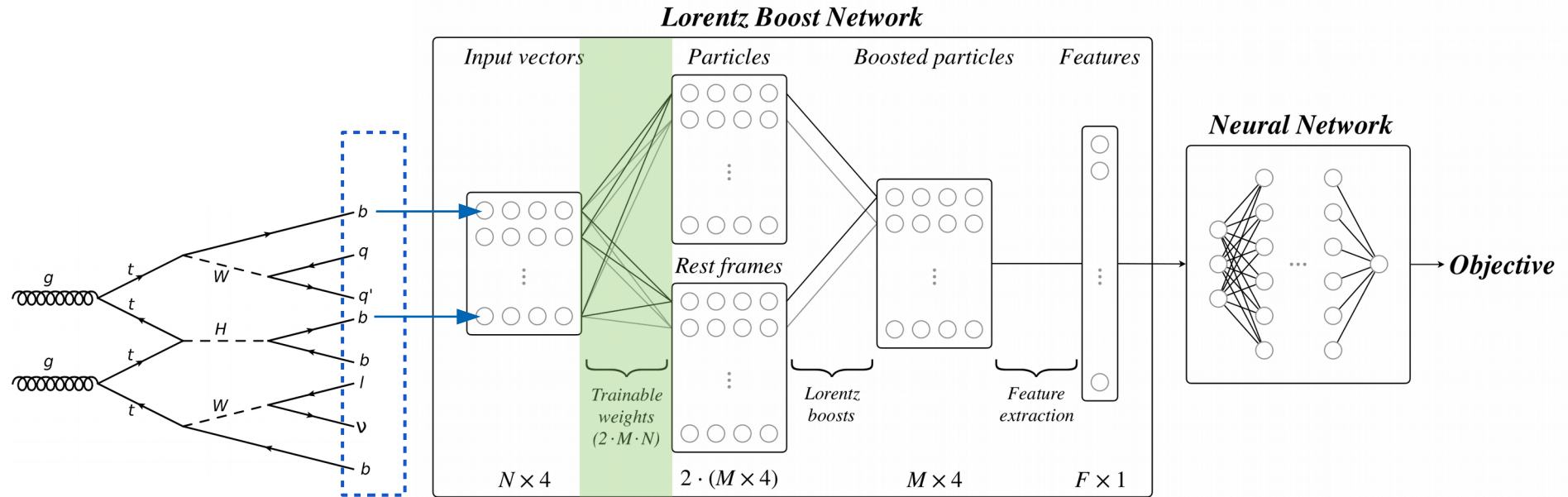


Udacity Course 730, Deep Learning

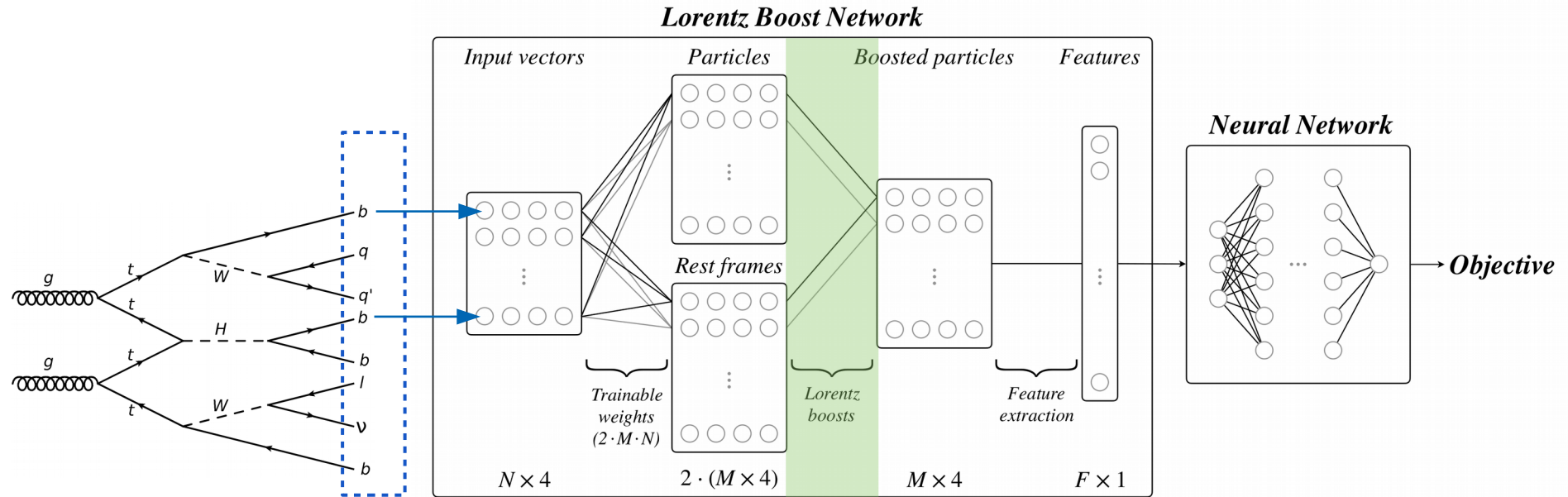
- Recently many similar developments in physics (e.g. [LoLa](#), [Recursive NN](#))
- Our model: Lorentz Boost Network (LBN)



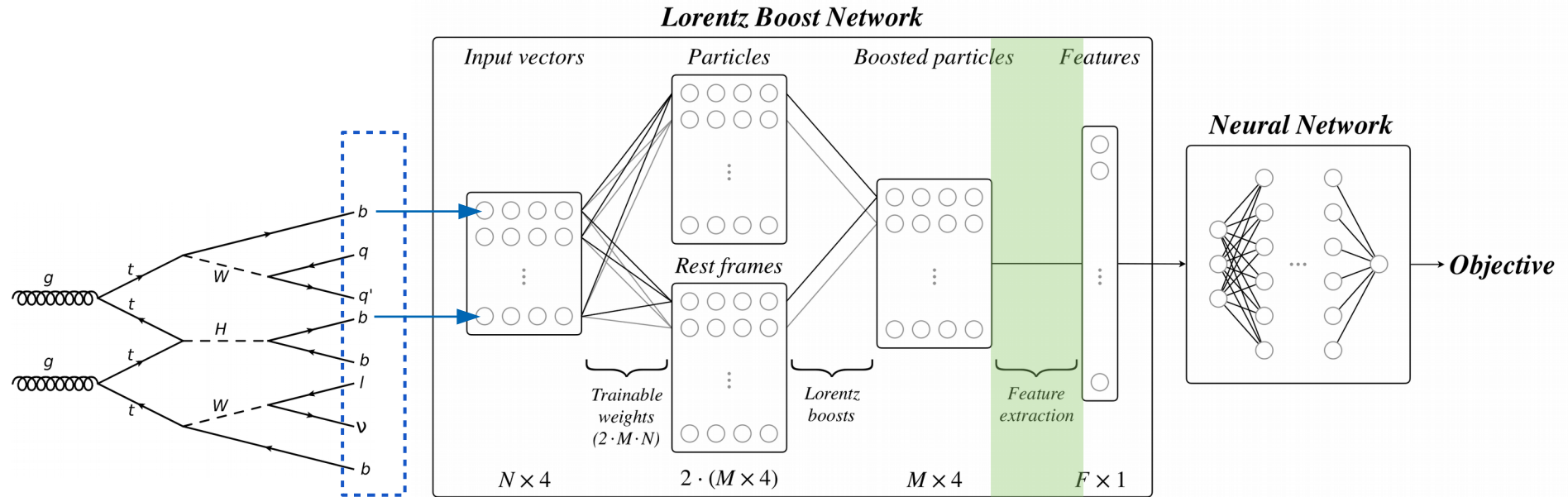
- Physicist: Combine four momenta of final state particles to build intermediate particles
 - Create characteristic variables
 - Analyse decays in rest frame of parent particle
- **Can we incorporate this into a network architecture?**



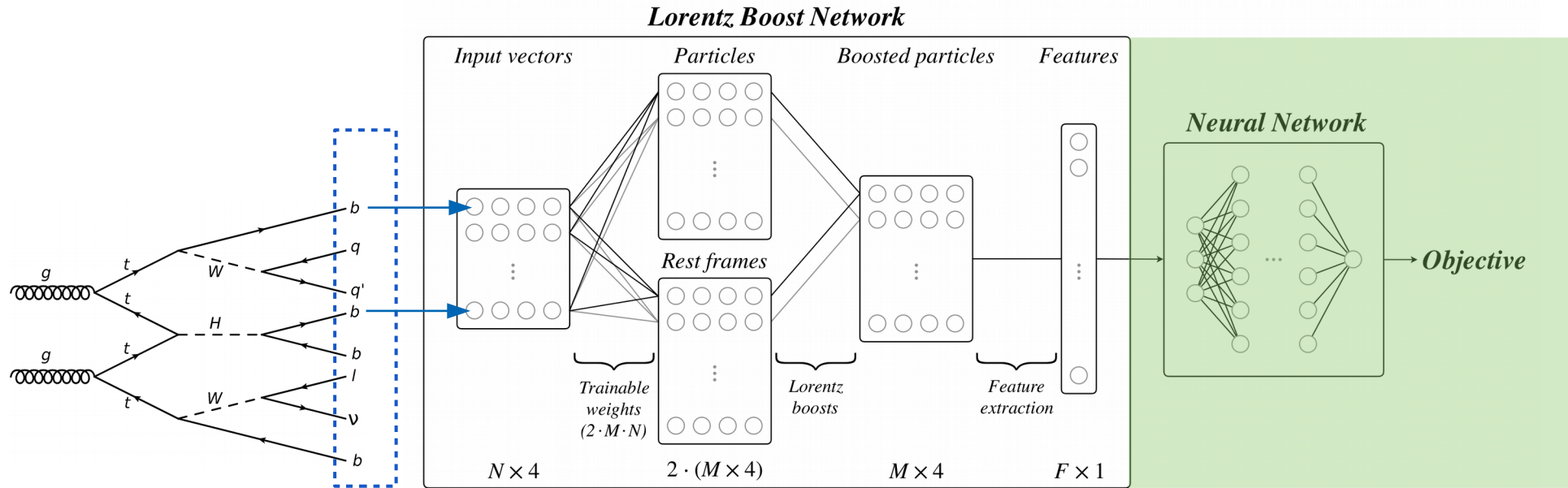
- Create composite particles and rest frames (trainable)



- Create composite particles and rest frames (trainable)
- Perform Lorentz transformation

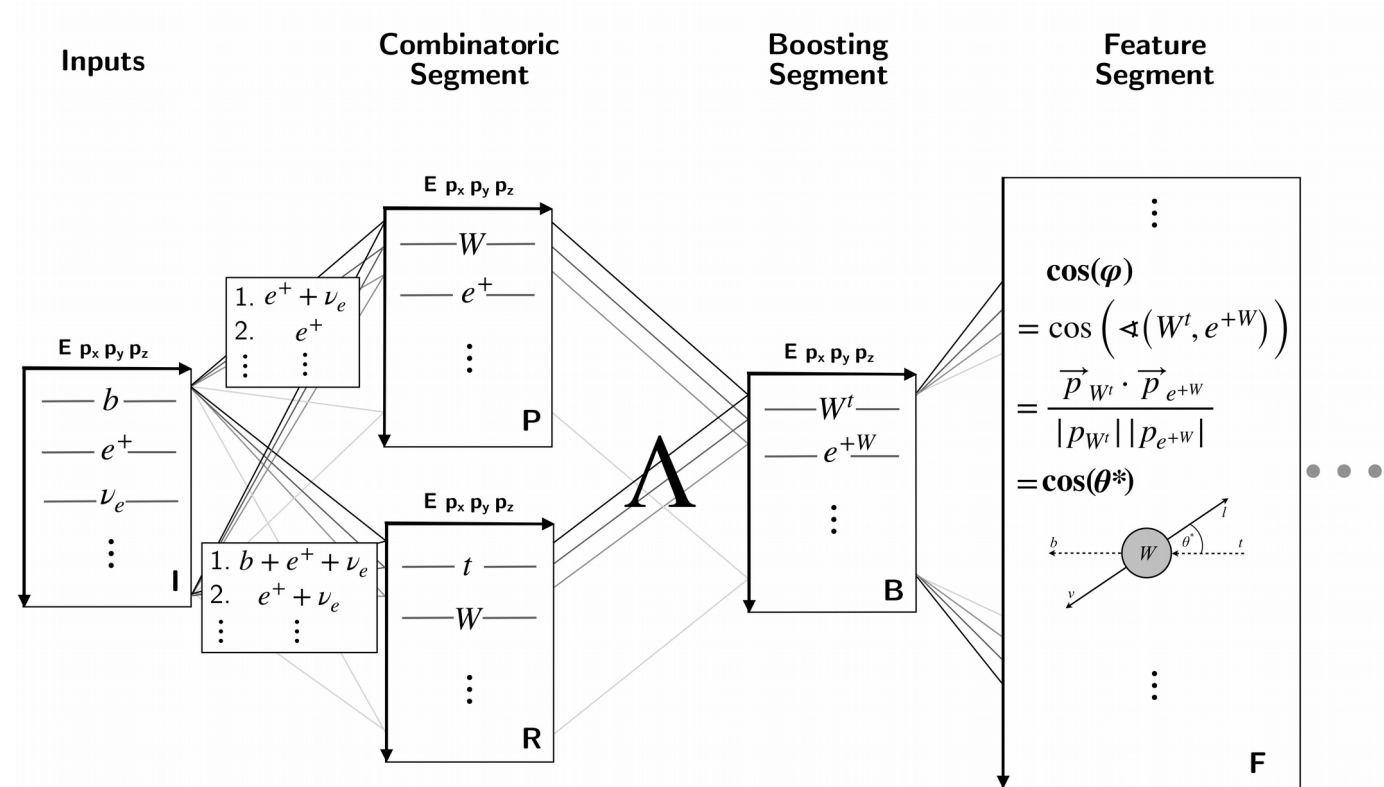
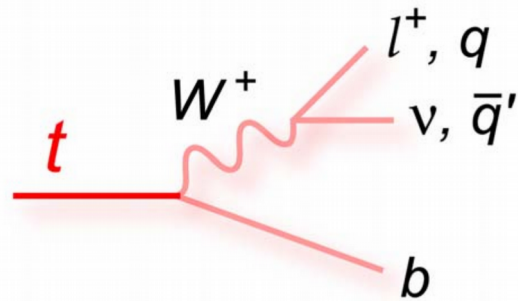


- Create composite particles and rest frames (trainable)
- Perform Lorentz transformation
- Project generic set of features, e.g. energies, pairwise angles



- Create composite particles and rest frames (trainable)
- Perform Lorentz transformation
- Project generic set of features, e.g. energies, pairwise angles
- Use learned features to solve physics problem

- Example of feature construction: helicity angle $\cos(\theta^*)$

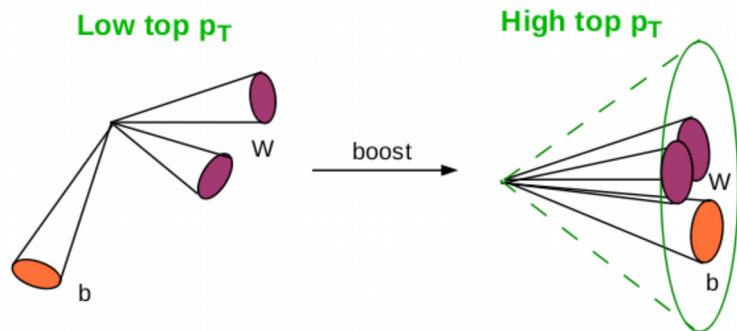


Application

- Currently studying two applications

Top Tagging

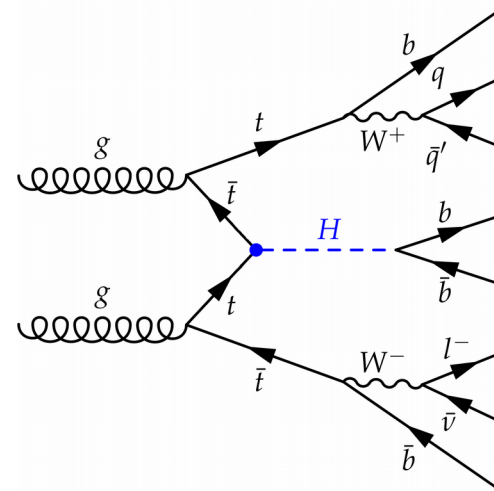
- Distinguish hadronically decaying, highly boosted top quarks from QCD background



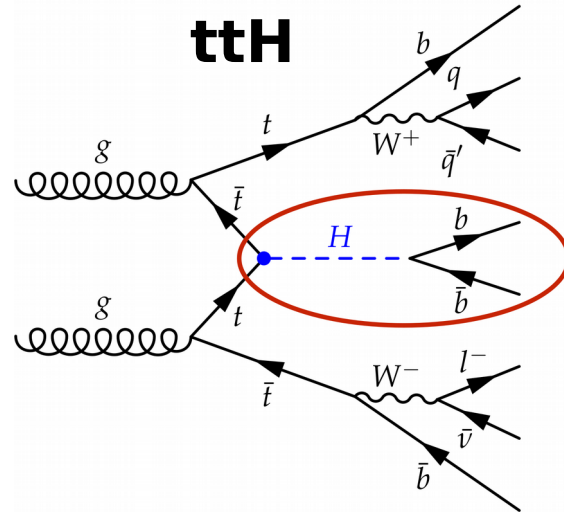
[Method comparison paper:
<https://arxiv.org/abs/1902.09914>]

ttH vs tt+bb

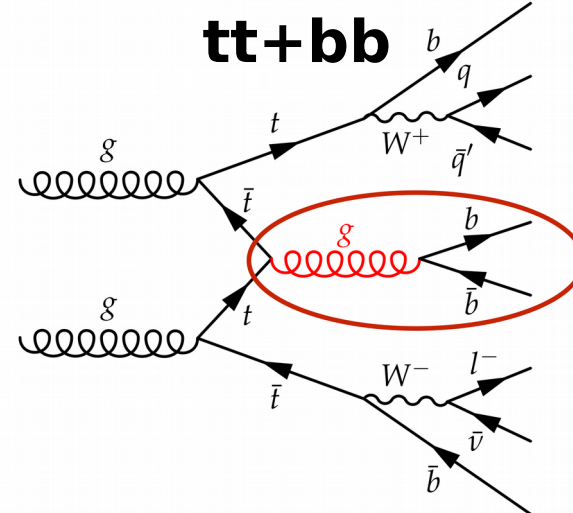
- Separate ttH(H→bb) events from tt+bb background



[<https://arxiv.org/abs/1812.09722>]



VS.

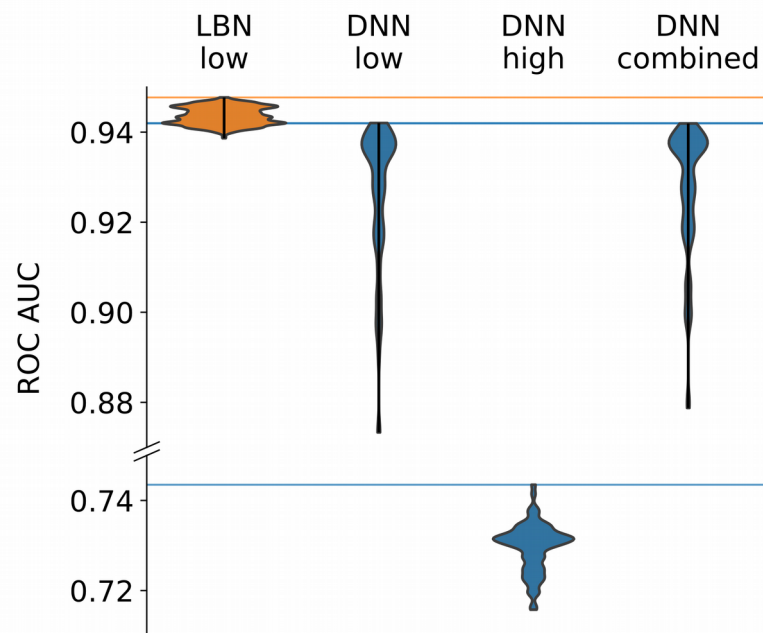


- Final state: 6 jets, 1 charged lepton, 1 neutrino (missing transverse energy)
→ 8 four vectors as input
- Compare LBN against fully connected neural network
 - Trained with same input, with dedicated high-level variables, or both

- Hyperparameter scan with $O(100)$ trainings for each configuration

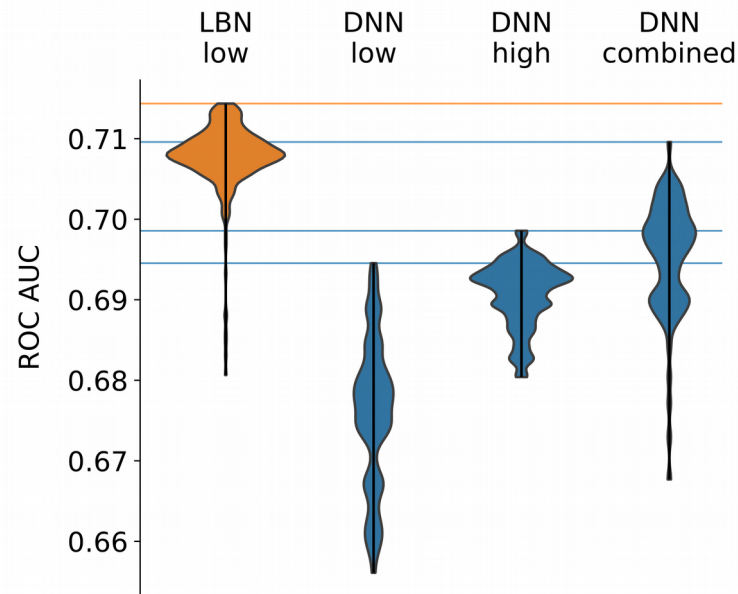
Generator (truth) ordering

→ Isolated LBN performance study



Jet p_T sorting

→ Realistic application



What is learned by the network?

- Study particle combination weights (generator sorting) to gain insights into what the network is learning

		Combined Particle												
		0	1	2	3	4	5	6	7	8	9	10	11	12
Input Particle	b_1	39	0	8	51	7	90	0	0	40	0	10	51	47
	b_2	61	0	1	21	8	0	32	0	3	0	84	18	16
	b_{had}	0	68	14	0	22	7	12	1	17	11	1	2	0
	q_1	0	17	13	0	27	0	9	1	22	12	1	12	1
	q_2	0	15	16	2	27	2	11	0	10	21	1	1	9
	b_{lep}	0	0	13	3	2	0	11	68	0	18	1	13	4
	lep	0	0	16	8	0	0	14	25	8	38	1	3	13
	ν	0	0	19	15	5	0	10	6	0	0	1	1	9

H^1

t_{had}

t_{lep}

boost

		Combined Restframe												
		0	1	2	3	4	5	6	7	8	9	10	11	12
Input Particle	b_1	7	5	0	7	11	15	99	5	14	25	47	17	5
	b_2	6	33	100	5	8	32	0	56	45	0	8	58	41
	b_{had}	11	30	0	0	7	15	0	4	10	3	8	2	5
	q_1	36	15	0	0	10	8	0	3	11	5	6	12	5
	q_2	24	13	0	3	9	6	0	2	5	9	8	1	17
	b_{lep}	2	1	0	30	19	7	0	7	2	11	8	9	6
	lep	4	1	0	29	21	12	0	13	8	11	9	1	7
	ν	9	3	0	26	16	4	0	9	3	35	8	0	14

H^1

t_{had}

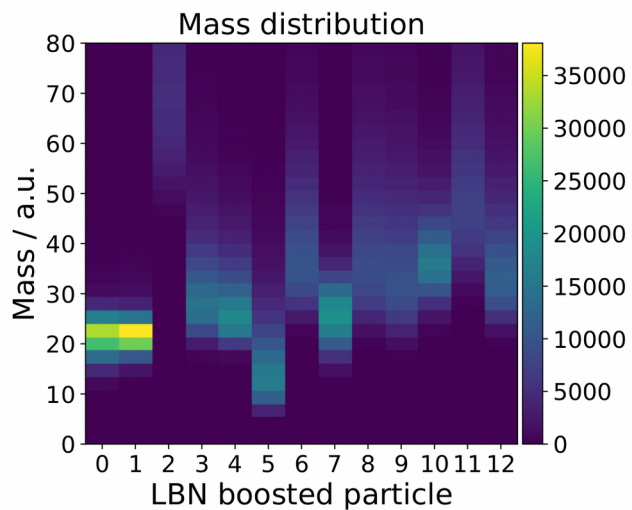
t_{lep}

- Approximate construction of intermediate particles by LBN
- Focus on Higgs boson (difference between processes)
- Which features relevant for the separation?**

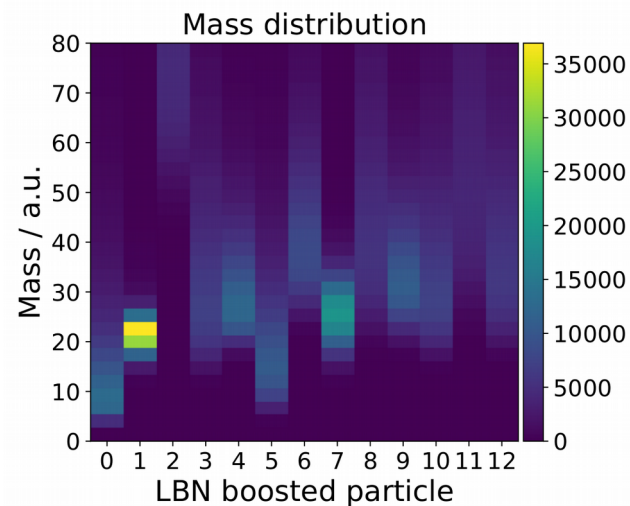
10 Feature Example (Mass)



ttH



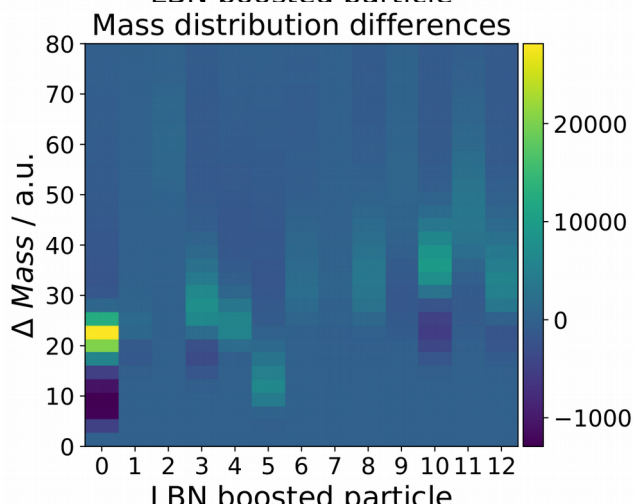
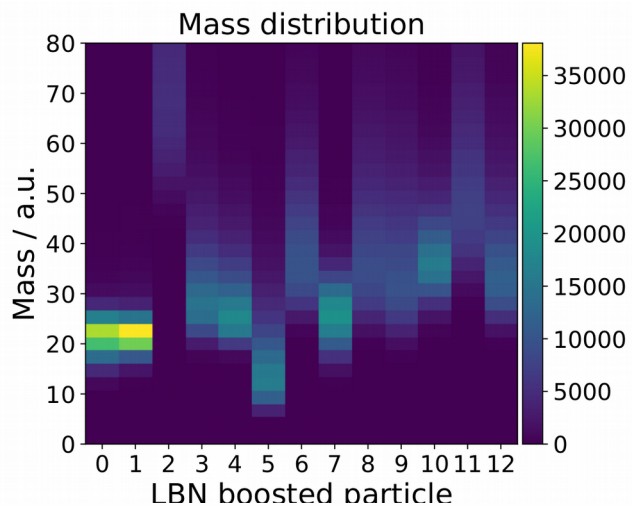
tt+bb



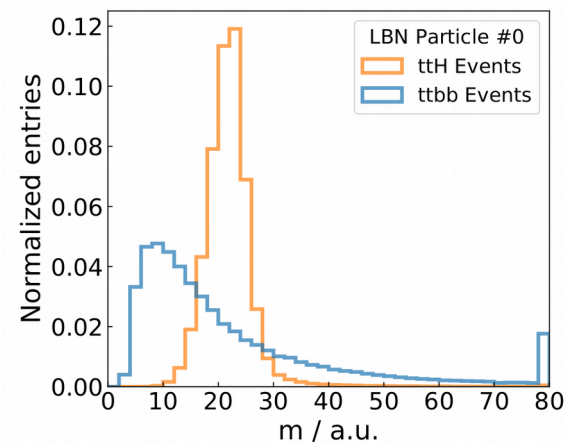
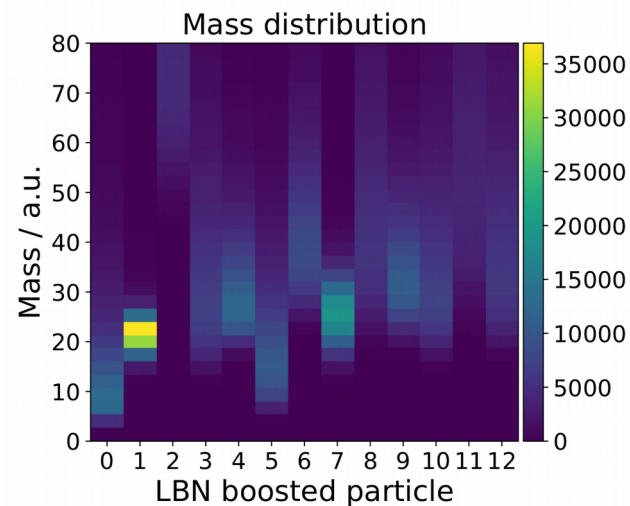
	0	1	2	3	4	5	6	7	8	9	10	11	12	
b_1	39	0	8	51	7	90	0	0	40	0	10	51	47	H^1
b_2	61	0	1	21	8	0	32	0	3	0	84	18	16	
b_{had}	0	68	14	0	22	7	12	1	17	11	1	2	0	t_{had}
q_1	0	17	13	0	27	0	9	1	22	12	1	12	1	
q_2	0	15	16	2	27	2	11	0	10	21	1	1	9	
b_{lep}	0	0	13	3	2	0	11	68	0	18	1	13	4	t_{lep}
lep	0	0	16	8	0	0	14	25	8	38	1	3	13	
ν	0	0	19	15	5	0	10	6	0	0	1	1	9	

10 Feature Example (Mass)

ttH

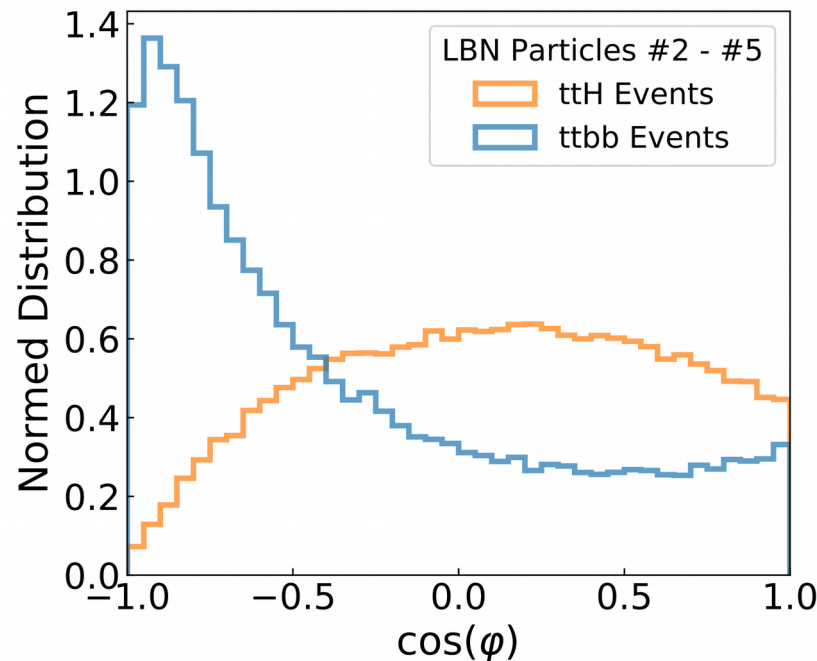
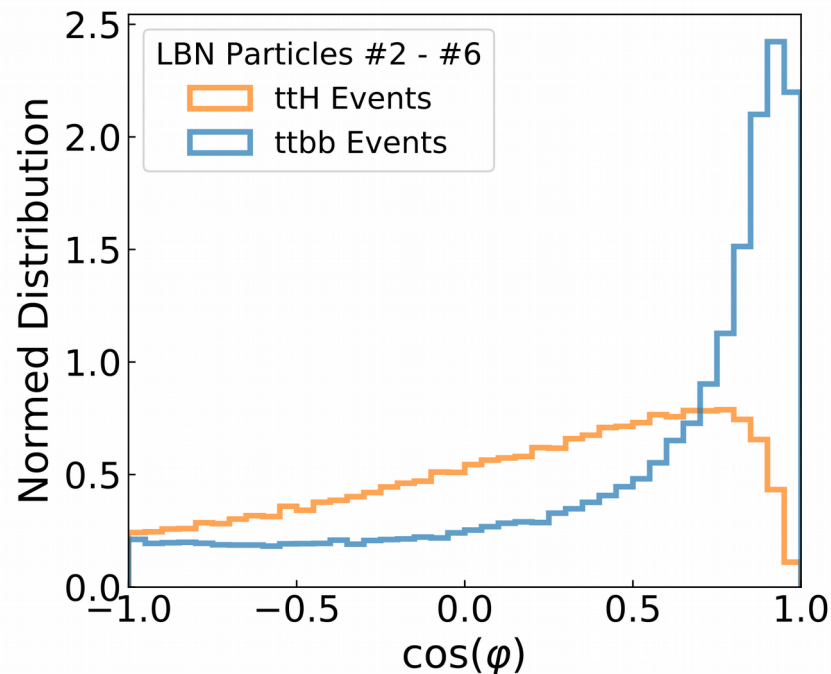


tt+bb



11 Feature Example (Pairwise Angle)

- Mass only dependent on combined particle
→ More complex example: Pairwise angles

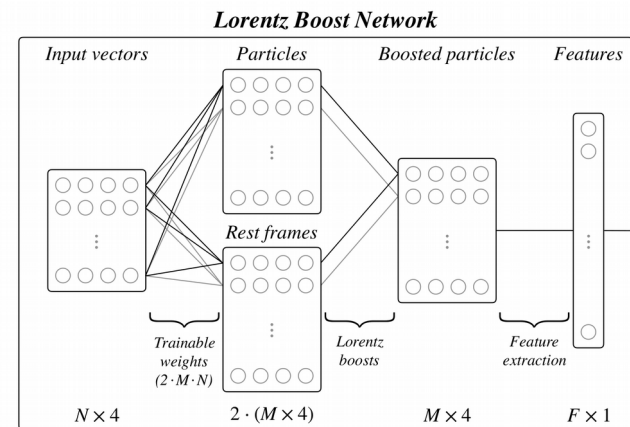


	2	5	6	
b_1	8	90	0	H^1
b_2	1	0	32	
b_{had}	14	7	12	
q_1	13	0	9	t_{had}
q_2	16	2	11	
b_{lep}	13	0	11	
lep	16	0	14	t_{lep}
ν	19	0	10	

Boosted
into

	2	5	6	
b_1	0	15	99	H^1
b_2	100	32	0	
b_{had}	0	15	0	
q_1	0	8	0	t_{had}
q_2	0	6	0	
b_{lep}	0	7	0	
lep	0	12	0	t_{lep}
ν	0	4	0	

- Lorentz Boost Network: Physics-motivated NN architecture
- Able to autonomously create characteristic features from particle four momenta
- Network components have direct physics interpretation
 - Finds meaningful particle relations, constructs intermediate particles



Input Particle

b_1	39	0	8	51	7	90	0	0	40	0	10	51	47	H^1
b_2	61	0	1	21	8	0	32	0	3	0	84	18	16	
b_{had}	0	68	14	0	22	7	12	1	17	11	1	2	0	
q_1	0	17	13	0	27	0	9	1	22	12	1	12	1	t_{had}
q_2	0	15	16	2	27	2	11	0	10	21	1	1	9	
b_{lep}	0	0	13	3	2	0	11	68	0	18	1	13	4	
lep	0	0	16	8	0	0	14	25	8	38	1	3	13	t_{lep}
ν	0	0	19	15	5	0	10	6	0	0	1	1	9	

...

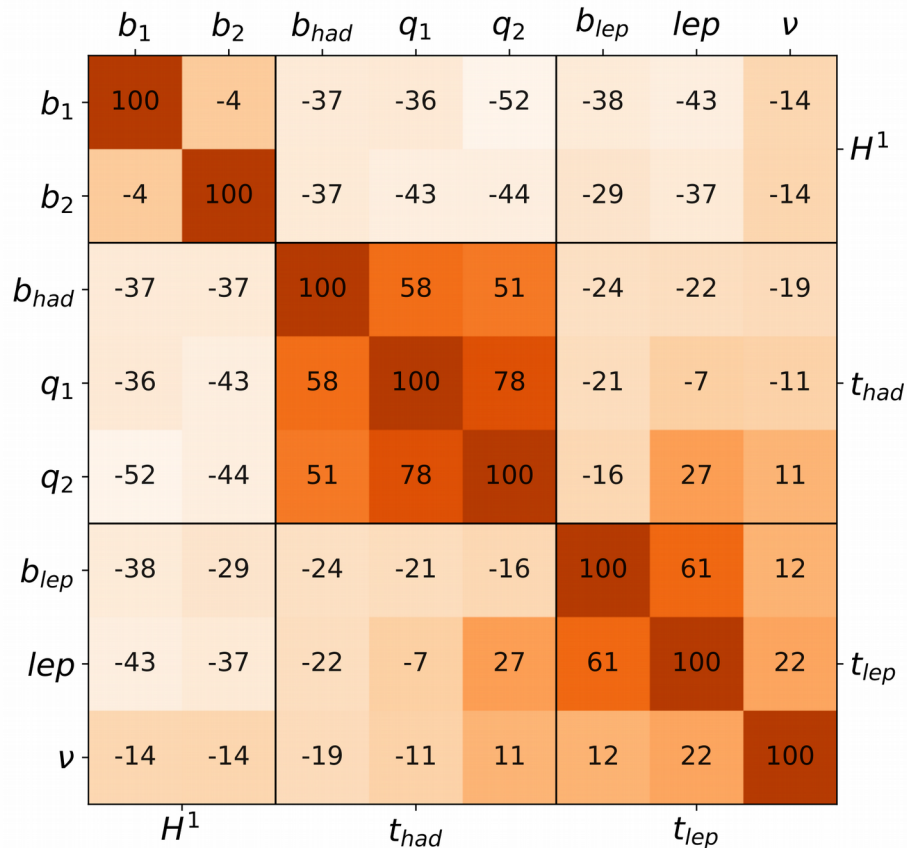
Combined Restframe

	0	1	2	3	4	5	6	7	8	9	10	11	12	
b_1	7	5	0	7	11	15	99	5	14	25	47	17	5	H^1
b_2	6	33	100	5	8	32	0	56	45	0	8	58	41	
b_{had}	11	30	0	0	7	15	0	4	10	3	8	2	5	
q_1	36	15	0	0	10	8	0	3	11	5	6	12	5	t_{had}
q_2	24	13	0	3	9	6	0	2	5	9	8	1	17	
b_{lep}	2	1	0	30	19	7	0	7	2	11	8	9	6	
lep	4	1	0	29	21	12	0	13	8	11	9	1	7	t_{lep}
ν	9	3	0	26	16	4	0	9	3	35	8	0	14	

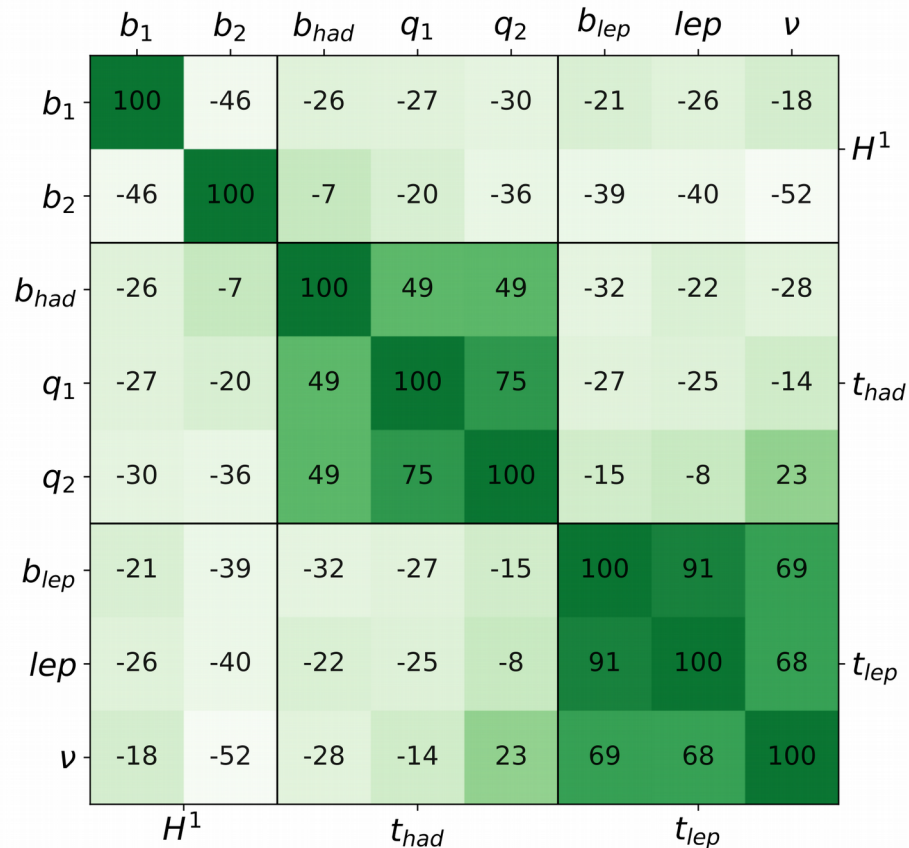
Backup

Correlations

Combined to Particles



Combined to Restframes



Lorentz Boost

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta n_x & -\gamma\beta n_y & -\gamma\beta n_z \\ -\gamma\beta n_x & 1 + (\gamma - 1)n_x^2 & (\gamma - 1)n_x n_y & (\gamma - 1)n_x n_z \\ -\gamma\beta n_y & (\gamma - 1)n_y n_x & 1 + (\gamma - 1)n_y^2 & (\gamma - 1)n_y n_z \\ -\gamma\beta n_z & (\gamma - 1)n_z n_x & (\gamma - 1)n_z n_y & 1 + (\gamma - 1)n_z^2 \end{bmatrix}$$

with $\vec{n} = \vec{\beta} / \beta$

- As 4D tensor (batch x particle x 4 x 4):

$$\Lambda = I + (U \oplus \gamma) \odot ((U \oplus 1) \cdot \beta - U) \odot (e \cdot e^T)$$

with

$$U = \begin{bmatrix} -1^{1 \times 1} & 0^{1 \times 3} \\ 0^{3 \times 1} & -1^{3 \times 3} \end{bmatrix} \quad e = \begin{bmatrix} 1^{1 \times 1} \\ -\vec{n}^{3 \times 1} \end{bmatrix}$$

Network Parameters

Network Variables Input Ordering	LBN+NN		DNN					
	low-level		low-level		high-level		combined	
	gen.	p _T	gen.	p _T	gen.	p _T	gen.	p _T
$M_{\text{part.,rest fr.}}$	13	16	–	–	–	–	–	–
n_{layers}	8	8	8	8	4	4	8	6
n_{nodes}	1024	1024	1024	1024	512	512	1024	1024