Geant4 Parameter Tuning using Professor

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Geant4 Physics Models and Parameters

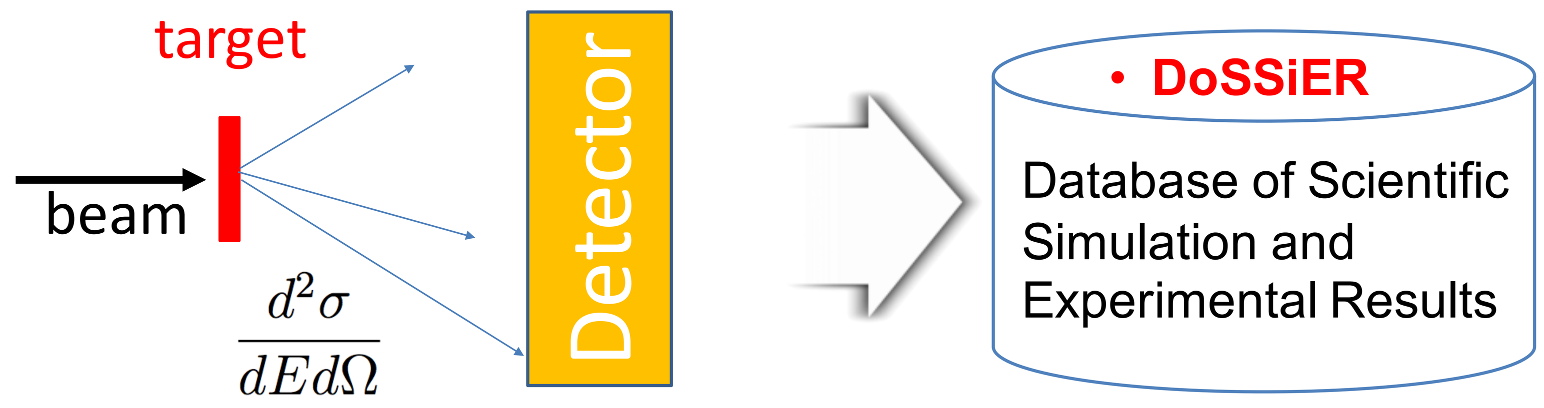
Geant4 uses underlying models with parameters to predict particles interactions kinematics. Since Geant4 (version) 10.4, an interfaces to configure some hadronic model parameters is provided to developers to study the impact of varying parameters with respect to datasets.

Number of configurable model parameters in the FTFP_BERT model

Models	Energy Range	Switches	Variables	Example Parameter
Precompound	0– 0.1 GeV	9	10	Level Density
Bertini Cascade	0.1 - 12 GeV	7	11	Fermi Energy
Fritiof (FTF)	3 GeV - 10 TeV	4	18	Baryon Average P_T^2

- Objectives of tuning:
  - What are values of model parameters that describe datasets best?
  - What are uncertainties of model parameters?
  - How to propagate uncertainties of parameters to physics measurements?

Thin Target Experiment and Observables



Experiment	Beam (GeV)	Target	Production	Observables
IAEA	0.8, 1.5, 3	C, Al, Fe, In, Pb	n	$d^2\sigma/dE_{KIN}d\theta$
HARP	5	C, Pb	π^+, π^-	$d^2\sigma/dpd\Omega$
ITEP	5	C, Pb	p, n	$d\sigma/dx_F$
NA61	31	C	$\pi^+, \pi^-, \pi^0, K, \Lambda, p$	$d^2\sigma/dpd\theta$
NA49	158	C	$\pi^+, \pi^-, p, pbar, n$	dN/dx_F, <PT> vs dx_F

Example of datasets used for tuning Geant4 hadronic models

Generating Tuning Samples

- Select data sets for the relevant energy range of a selected model or models
- Select N_p parameters that are sensitive to observables (χ^2 scan, asymmetry)
- Generate N_S samples by varying N_p parameters randomly within limits

$$\vec{p}_i = \vec{p}_{min} + (\vec{p}_{max} - \vec{p}_{min}) \cdot \vec{u}(0,1)$$

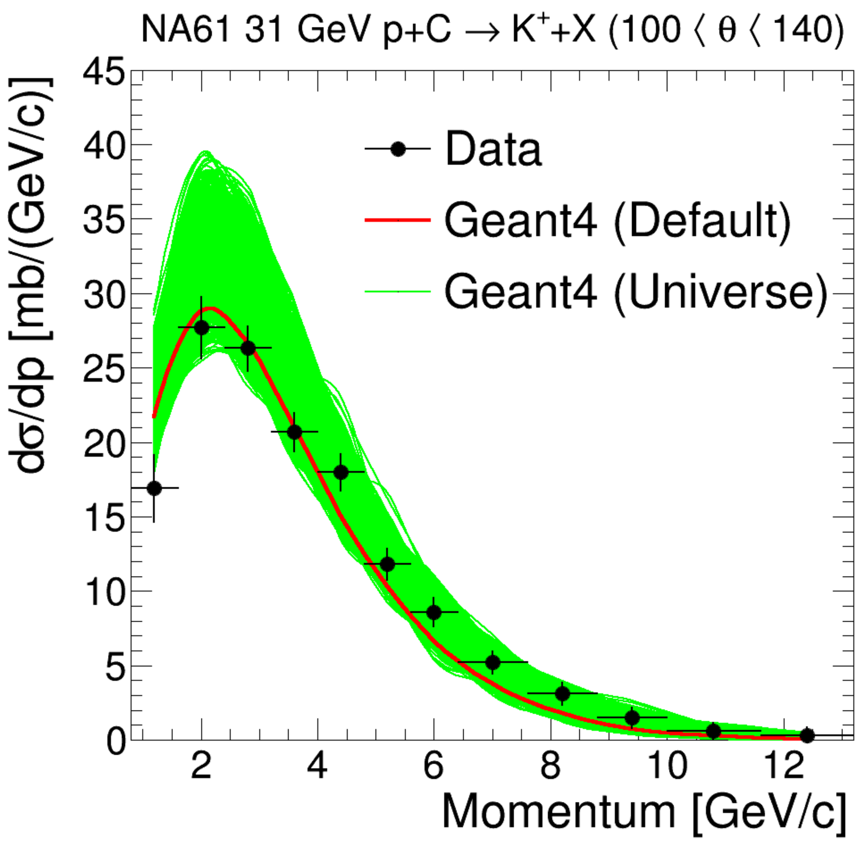
  - Statistical uncertainty of simulation << data error (set the minimum number of events to simulate per sample)
  - The minimum number of samples (N_S) for the n^{th} order polynomial interpolation of N_p number of parameters

$$N_S > N^{(n)}(N_p) = 1 + \sum_{i=1}^{n} \frac{1}{i!} \prod_{j=0}^{i-1} (N_p + j)$$

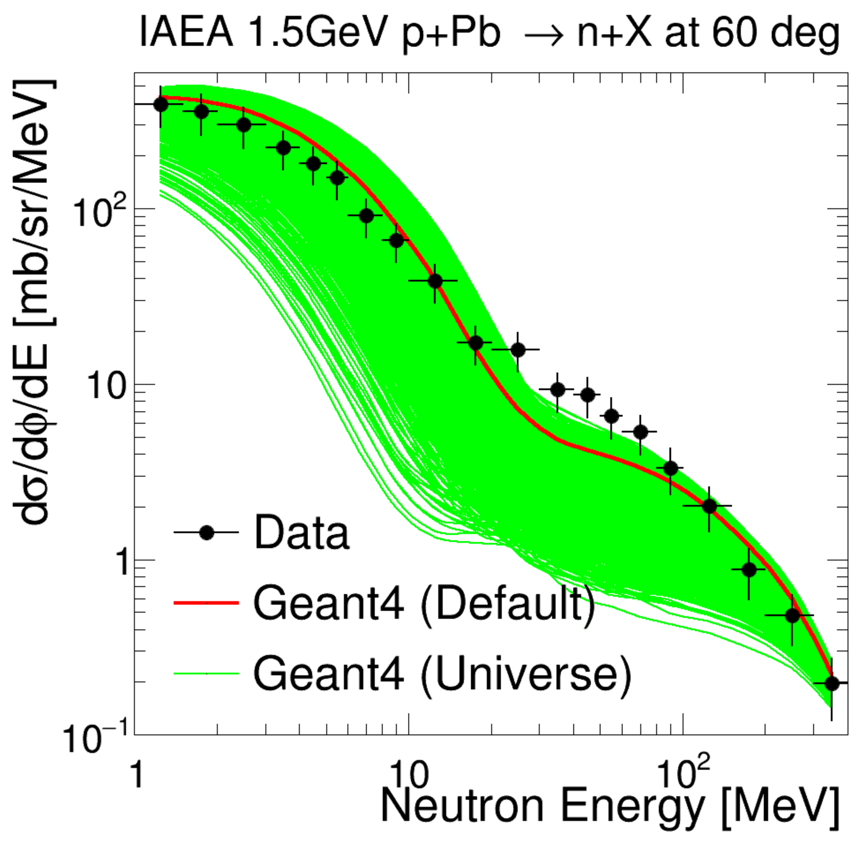
N_p	$n = 2$	$n = 3$
4	15	35
8	45	165
16	153	969

- art (an event framework) is used for Geant4 simulation and analysis

Example of the spread of 1000 samples (green) - universe



Variation of 10 FTF parameters



Variation of 4 Bertini parameters

Professor (Procedure for estimating Systematic Errors)

- Interpolation: fits observables of the universe to the n-degree of polynomial to produce the response functions (f) that predicts data distribution for any set of parameters

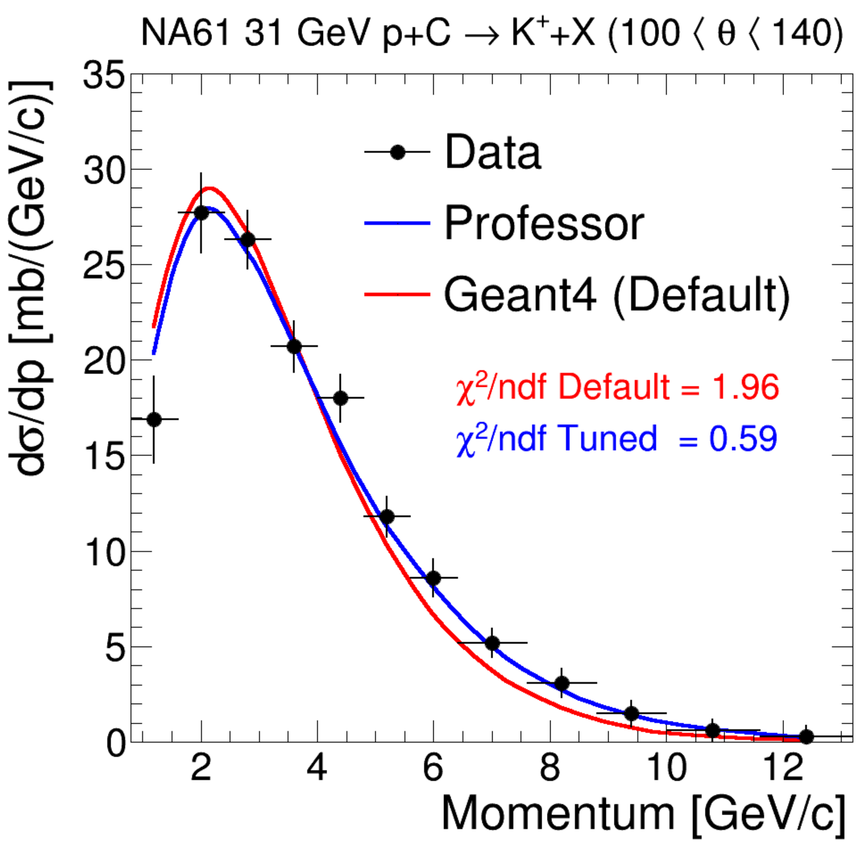
$$\text{parameters } (\vec{p}) \overset{f}{\Longleftrightarrow} \text{observables } (\mathcal{O}_i)$$

- Tuning: use response functions to obtain parameters which minimize the goodness of fit (ex. χ^2) with respect to datasets

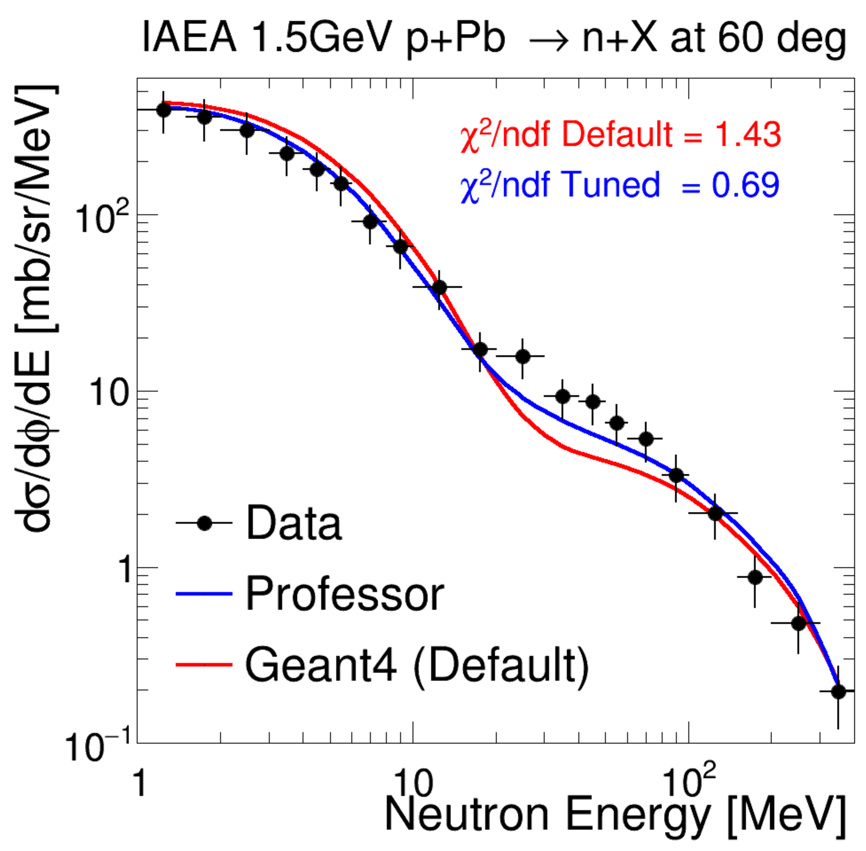
$$\frac{\partial}{\partial \vec{p}} \sum_{\mathcal{O}_i=1}^{datasets} w_{\mathcal{O}_i} \sum_{b \in \mathcal{O}_i}^{nbin} \frac{(f(\vec{p}) - v_d)^2}{\epsilon_b^2} = 0 \longrightarrow (\vec{p}_o, \sigma, \rho)$$

- v_d (ϵ_b) : value (error) of the reference bin
- $\sigma(\rho)$: error (correlation) of the tuned parameter vector $(\vec{p}_o)$

Example of Professor predictions tuned with thin target data



Best fit with a single observable



Global fit with multiple datasets

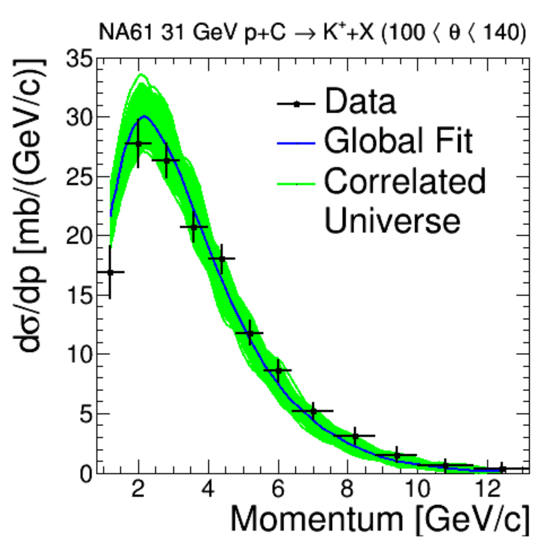
Estimation of Uncertainty

- Generate samples with a set of correlated parameters according to means (p_o), errors (σ) and their correlation (ρ) extracted from the Professor fit

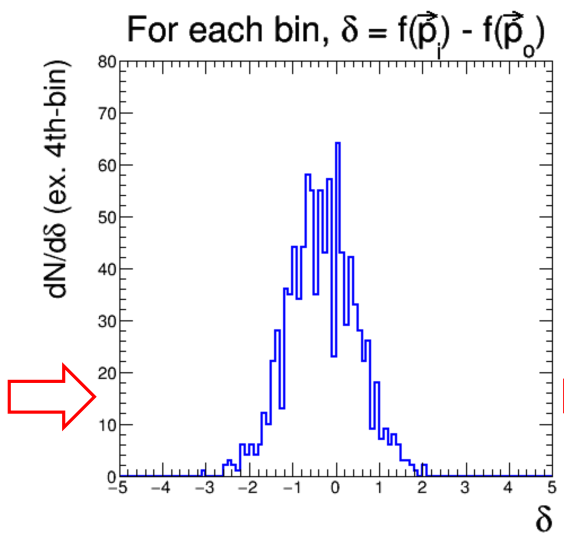
$$\vec{p}_i = \vec{p}_o + \sigma \mathcal{C} \vec{z}_i$$

  - C: Cholesky decomposition of ρ
  - z_i: normal random numbers

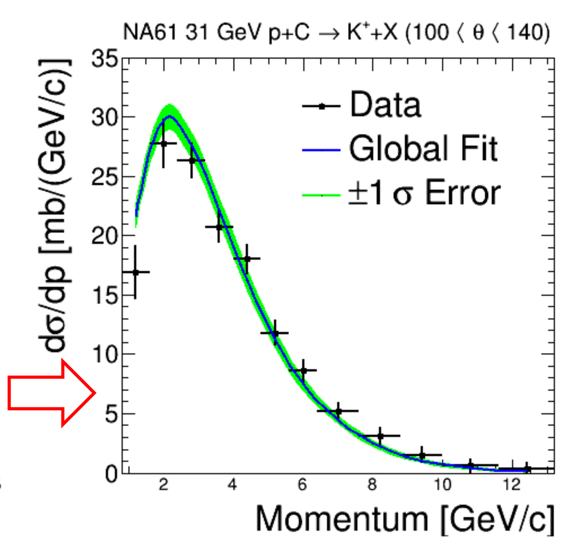
- Estimate RMS of universes for observables (ex.: bin values of cross sections)



Correlated universes



Estimate RMS for each bin

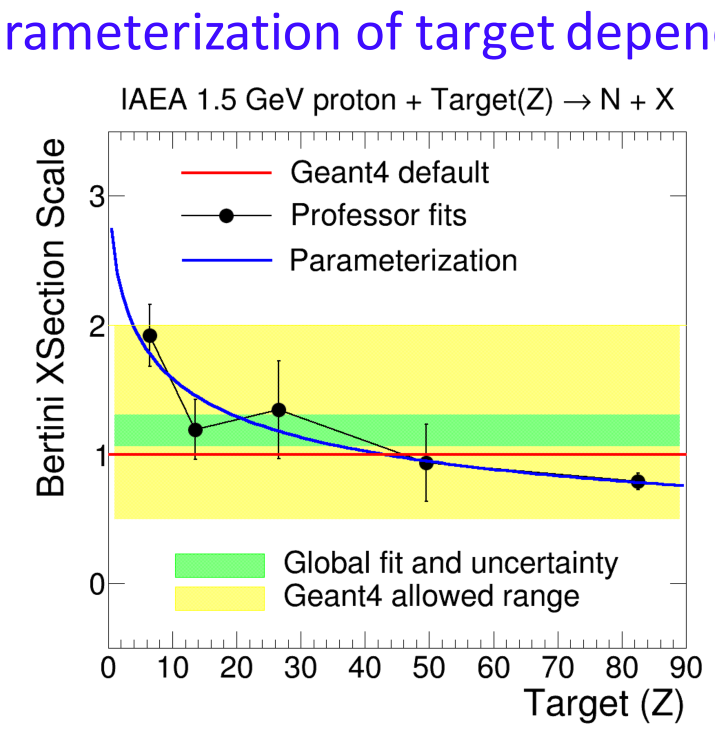


The error band

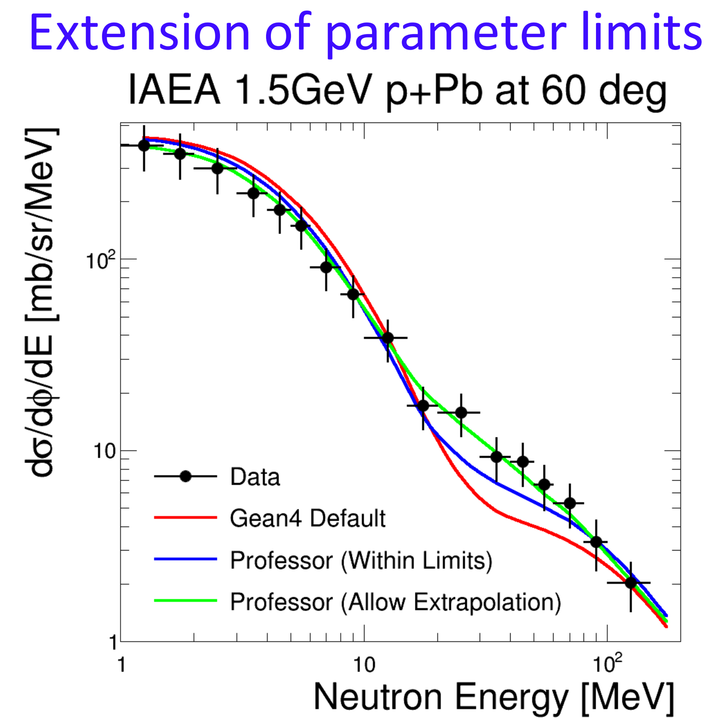
Ongoing and Future Work

- Ongoing work: understanding fit results: examples

Parameterization of target dependence



Extension of parameter limits



- Inclusion of cross sections and expansion to more Geant4 models
- Additional data sets and treatment of correlated errors in datasets
- Reweightable parameters