QED and electroweak radiative corrections to polarized Bhabha scattering

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Abstract. Complete one-loop electroweak radiative corrections to polarized Bhabha scattering are presented. Numerical results are shown for the conditions of future circular and linear electron-positron colliders with polarized beams. A new Monte Carlo event generator for simulation of Bhabha scattering is created. Higher order QED collinear radiation factors are evaluated in the next-to-leading logarithmic approximation.

1. Introduction

Electron-positron (Bhabha) scattering is a basic process in particle physics. It is widely used for the luminosity monitoring at e^+e^- colliders. Bhabha scattering with unpolarized initial particles has been studied for many years since Ref. [1]. Radiative corrections (RC) for the case of polarized e^+e^- beams were considered in Refs. [2, 3]. There are several Monte Carlo event generators for unpolarized Bhabha scattering, see e.g. [4]. In the `SANC` computer system framework, the complete one-loop electroweak (EW) RC to Bhabha scattering were recently calculated [5] and expressed in terms of helicity amplitudes (HA) and form factors (FF).

In comparison with processes being studied at modern hing-energy hadron colliders, e^+e^- interactions have a clean initial state, a lower multiplicity in the final state, and therefore provide the possibility to perform much more precise measurements in most cases. The substantially higher energy range of the future colliders also demands re-estimation of various effects from both experimental and theoretical points of view. Precise measurements with polarized beams at future e^+e^- colliders like FCC-ee, CEPC, ILC and CLIC definitely require an advanced theoretical support [6, 7, 8, 9, 10]. In particular, physical programs of the future e^+e^- linear colliders [11, 12] always demonstrated a great interest to the effects related to the beam polarization. These facts motivate us to work on high-precision theoretical predictions for processes which will be studied at future e^+e^- colliders. In order to match experimental requirements, we have to implement our results into Monte Carlo event generators.

In Sect. 2 we present and discuss the results [13] on the complete one-loop electroweak radiative corrections to polarized Bhabha scattering. Tuned comparisons with results of alternative calculations partially available in the literature were performed. In Sect. 3 we show preliminary results on QED collinear radiative factors in the $\mathcal{O}(\alpha^3 L^2)$ order.

2. One-loop corrections to polarized Bhabha scattering

Consider scattering of longitudinally polarized positrons and electrons $e^+ + e^- \longrightarrow e^- + e^+ + (\gamma)$. At large energies we will neglect contributions proportional to the ratio of the electron mass to the beam energy.

The complete one-loop covariant amplitude comes out from the straightforward calculation by means of the `SANC` computer system [5]. The amplitude is parameterized by a certain number of form factors (FFs) $\mathcal{F} = 1 + k\tilde{\mathcal{F}}$, where "1" stands for the Born level and the term $\tilde{\mathcal{F}}$ with the factor $k \equiv g^2/(16\pi^2)$ is the one-loop contribution. After squaring the amplitude we neglect terms proportional to k^2 in order to get the pure one-loop approximation without any admixture of higher-order terms which can be added later if required.

The covariant amplitude for high-energy Bhabha scattering can be written in terms of the electromagnetic running coupling constant and four FFs with permuted arguments s and t as:

$$
\begin{aligned}
\mathcal{A} &= \mathcal{A}_\gamma(s) + \mathcal{A}_Z(s) - [\mathcal{A}_\gamma(t) + \mathcal{A}_Z(t)] \qquad (1)\\
&= i\,e^2 \Biggl\{ \biggl[\gamma_\mu \otimes \gamma_\mu \frac{\mathcal{F}_\gamma(s)}{s} - \gamma_\mu \otimes \gamma_\mu \frac{\mathcal{F}_\gamma(t)}{t} \biggr] + \frac{\chi_Z(s)}{s} \biggl\{ \Bigl(I_e^{(3)}\Bigr)^2 \gamma_\mu\gamma_6 \otimes \gamma_\mu\gamma_6 \mathcal{F}_{LL}(s,t,u) \\
&+ 2\delta_e I_e^{(3)} \gamma_\mu \otimes \gamma_\mu\gamma_6 \mathcal{F}_{QL}(s,t,u) + \delta_e^2 \gamma_\mu \otimes \gamma_\mu\, \mathcal{F}_{QQ}(s,t,u) \biggr\} - \frac{\chi_Z(t)}{t} \biggl\{ s \leftrightarrow t \biggr\},
\end{aligned}
$$

see Ref. [13] for the notation details.

The complete result for $\mathcal{O}(\alpha)$ corrections can be separated into the virtual (loop) contribution, the part due to the soft photon emission, and the one due to the real hard photon Bremsstrahlung. It is more convenient to calculate a cross-section by squaring non-interfering helicity amplitudes (HA). Complete analytic results were obtained for HA of Bhabha scattering by means of the SANC system. There are six non-zero HA, however, since for Bhabha scattering $\mathcal{F}^Z_{LQ} = \mathcal{F}^Z_{QL}$, the number of independent HA is reduced to four.

We obtained the compact expression for the Born ($\mathcal{F}_{QL,LL,QQ} = 1$) and the virtual (loop) part within the HA approach in the form

$$
\begin{aligned}
\mathcal{H}_{++++} &= \mathcal{H}_{----} = -2e^2\, \frac{s}{t} \Bigl[\mathcal{F}^{(\gamma,Z)}_{QQ}(t,s,u) - \chi_Z(t)\delta_e \mathcal{F}^Z_{QL}(t,s,u) \Bigr], \\
\mathcal{H}_{+-+-} &= \mathcal{H}_{-+-+} = -e^2\, c_- \Bigl[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) - \chi_Z(s)\delta_e \mathcal{F}^Z_{LQ}(s,t,u) \Bigr], \\
\mathcal{H}_{+--+} &= -e^2\, c_+ \Bigl(\Bigl[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) + \chi_Z(s) \left(\mathcal{F}^Z_{LL}(s,t,u) - 2\delta_e \mathcal{F}^Z_{LQ}(s,t,u) \right) \Bigr] + \frac{s}{t} \Bigl[s \leftrightarrow t \Bigr] \Bigr), \\
\mathcal{H}_{-++-} &= -e^2\, c_+ \Bigl(\Bigl[\mathcal{F}^{(\gamma,Z)}_{QQ}(s,t,u) \Bigr] + \frac{s}{t} \left[s \leftrightarrow t \right] \Bigr), \qquad (2)
\end{aligned}
$$

where $c_\pm = \cos\vartheta \pm 1$ and $\mathcal{F}^{(\gamma,Z)}_{QQ}(a,b,c) = \mathcal{F}^{(\gamma)}_{QQ}(a,b,c) + \chi_Z(a)\delta_e^2 \mathcal{F}^{(Z)}_{QQ}(a,b,c)$.

The Bremsstrahlung module of the `SANC` system computes the contributions due to the soft and inclusive hard photon emission. The soft photon contribution contains infrared divergences which are compensated by the corresponding divergences in the one-loop virtual QED (EW) corrections. For a crosscheck, the contribution of the hard photon emission was obtained both by direct squaring of the matrix element and by the helicity amplitude approach.

To study the case of longitudinal polarization, we generated helicity amplitudes and applied the formalism described in [14]. In our notation the Bhabha scattering cross-section with longitudinally polarized initial particles can be expressed as

$$\frac{d\sigma(P_{e^-},P_{e^+})}{d\cos\vartheta} = \frac{1}{128\pi s}\Big[(1-P_{e^-})(1-P_{e^+})\sum_{ij}|\mathcal{H}_{++ij}|^2 + (1-P_{e^-})(1+P_{e^+})\sum_{ij}|\mathcal{H}_{+-ij}|^2 + (1+P_{e^-})(1-P_{e^+})\sum_{ij}|\mathcal{H}_{-+ij}|^2 + (1+P_{e^-})(1+P_{e^+})\sum_{ij}|\mathcal{H}_{--ij}|^2\Big]. \qquad (3)$$

For a crosscheck we got an analytic zero for the difference between the square of the covariant amplitude (we introduced the spin density matrix into our procedures) and Eq. (3).

The left-right asymmetry A_{LR} and the relative correction δ are defined as

$$A_{LR} = \frac{d\sigma(-1,1)-d\sigma(1,-1)}{d\sigma(-1,1)+d\sigma(1,-1)}, \qquad \delta = \frac{d\sigma^{\text{1-loop}}(P_{e^-},P_{e^+})}{d\sigma^{\text{Born}}(P_{e^-},P_{e^+})} - 1, \qquad (4)$$

where we dropped $d\cos\vartheta$ for the sake of brevity.

Here we present some numerical results for EW RC to Bhabha scattering obtained by means of a new Monte Carlo event generator created by the `SANC` group which was presented at this conference in the talk by R. Sadykov. To crosscheck our results, we performed comparisons with the ones obtained by means of other modern packages. But actually, it was possible to compare results only partially since there is no any other code describing high-energy polarized Bhabha scattering with one-loop precision.

We found an agreement between our analytic result for the hard photon Bremsstrahlung contribution with the one obtained by means of the `CalcHEP` computer system [15]. Numerical results for polarized Bhabha scattering with hard photon Bremsstrahlung were compared with the ones of the `WHIZARD` system [16]. A good agreement was observed as well. We obtained also a very good agreement (six significant digits) in the comparison of the `SANC` and `AItalc-1.4` [17] results for the unpolarized differential Born cross-section and for the sum of the virtual and the soft photon contributions. Tables with the numerical results of the mentioned above comparisons can be found in ref. [13].

Table 1. Born and 1-loop cross-sections of Bhabha scattering and the corresponding relative corrections δ for $\sqrt{s} = 250$, 500, and 1000 GeV.

P_{e^-}, P_{e^+}	0, 0	-0.8, 0	-0.8, -0.6	-0.8, 0.6
		$\sqrt{s} = 250$ GeV		
$\sigma^{\text{Born}}_{e^+e^-}$, pb	56.6763(1)	57.7738(1)	56.2725(4)	59.2753(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	61.731(6)	62.587(6)	61.878(6)	63.287(7)
δ, %	8.92(1)	8.33(1)	9.96(1)	6.77(1)
		$\sqrt{s} = 500$ GeV		
$\sigma^{\text{Born}}_{e^+e^-}$, pb	14.3789(1)	15.0305(1)	12.7061(1)	17.3550(2)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	15.465(2)	15.870(2)	13.861(1)	17.884(2)
δ, %	7.56(1)	5.59(1)	9.09(1)	3.05(1)
		$\sqrt{s} = 1000$ GeV		
$\sigma^{\text{Born}}_{e^+e^-}$, pb	3.67921(1)	3.90568(1)	3.03577(3)	4.77562(5)
$\sigma^{\text{1-loop}}_{e^+e^-}$, pb	3.8637(4)	3.9445(4)	3.2332(3)	4.6542(7)
δ, %	5.02(1)	0.99(1)	6.50(1)	-2.54(1)

In Figs. 1, we give an example of numerical results produced by the new `SANC` Monte Carlo event generator for the unpolarized differential cross-section of Bhabha scattering and the relative $\mathcal{O}(\alpha)$ correction δ (in percent) as a function of the electron scattering angle for

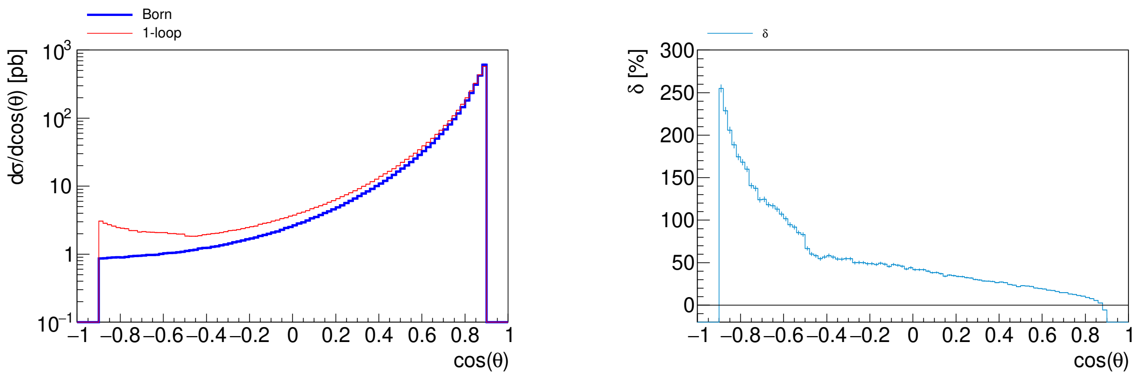

Figure 1. The differential cross-section (left) [in pb] and the relative correction δ (right) [in %] vs. the cosine of the electron scattering angle for $\sqrt{s} = 250$ GeV.

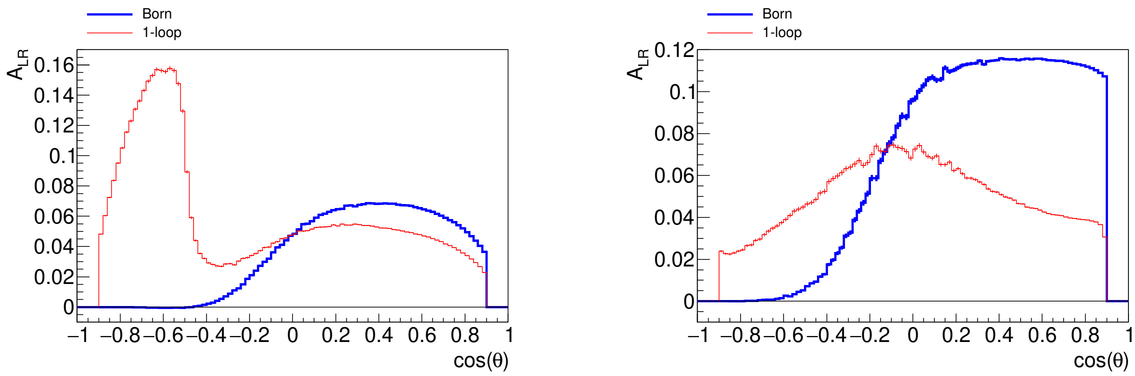

Figure 2. The left-right asymmetry A_{LR} vs the cosine of the electron scattering angle at $\sqrt{s} = 250$ GeV (left) and $\sqrt{s} = 1000$ GeV (right).

$|\cos\theta| < 0.9$ and $\sqrt{s} = 250$ GeV. The huge relative radiative corrections for the backward scattering angles are due to the smallness of the Born cross-section in this domain. That does not mean any problem with the perturbation theory. The integrated cross-section of the Bhabha scattering and the relative correction δ are given in the Table 1 for various energies and beam polarization degrees. The A_{LR} asymmetry at $\sqrt{s} = 250$ and 1000 GeV is shown in Fig. 2. One can see that the EW radiative corrections affect the asymmetry considerably.

3. Higher-order collinear NLO radiation functions

At high energies, QED radiative corrections enhanced by large logarithms $L = \ln(Q^2/m_e^2)$ are numerically very important. The so-called leading logarithmic (LO) approximation takes into account terms proportional to $\alpha^2 L^n$, $n = 1, 2, \ldots$ The next-to-leading logarithmic (NLO) approximation includes also terms of the order $\mathcal{O}(\alpha^2 L^{n-1}$. The factorization theorem well known in QCD allows us to write down the Bhabha scattering cross-section in the next-to-leading logarithmic approximation in the following form [18]:

$$d\sigma = \sum_{a,b,c,d=e,\bar{e},\gamma} \int_{\bar{z}_1}^{1} dz_1 \int_{\bar{z}_2}^{1} dz_2 \mathcal{D}^{\rm str}_{ae}(z_1)\mathcal{D}^{\rm str}_{b\bar{e}}(z_2)\Big[d\sigma^{(0)}_{ab\to cd}(z_1,z_2) + d\bar{\sigma}^{(1)}_{ab\to cd}(z_1,z_2)\Big]$$
$$\times \int_{\bar{y}_1}^{1} \frac{dy_1}{Y_1} \int_{\bar{y}_2}^{1} \frac{dy_2}{Y_2} \mathcal{D}^{\rm frg}_{ec}\left(\frac{y_1}{Y_1}\right) \mathcal{D}^{\rm frg}_{\bar{e}d}\left(\frac{y_2}{Y_2}\right) + \mathcal{O}(\alpha^n L^{n-2}),$$

where $\mathcal{D}^{\rm str(frg)}_{ba}(z)$ are QED structure (fragmentation) functions which provide the probability density to find parton (particle) b in particle (parton) a with the energy fraction z. Cross-sections $d\sigma^{(0)}_{ab\to cd}$ and correction to it $d\bar{\sigma}^{(1)}_{ab\to cd}$ are computed for massless partons a, b, c, d in the Born and one-loop approximations, respectively. Mass singularities are subtracted from the one-loop parton cross-section $d\bar{\sigma}^{(1)}$ in the MSbar scheme.

Explicit expression for the QED structure functions are well known in the leading logarithmic approximation [19, 20, 21]. The NLO contributions to for QED structure and fragmentation functions in the $\mathcal{O}(\alpha^2 L^1)$ order were specified in Refs. [22, 23]. Continuously increasing experimental precision makes it worth to include also the $\mathcal{O}(\alpha^3 L^2)$ order contributions. They can be found by means of iterative solutions of the QED DGLAP evolution equations in the NLO approximation. Here we present new results for the $\mathcal{O}(\alpha^3 L^2)$ contributions to non-singlet electron structure function. Taking into account the pure photonic contributions we get

$$\mathcal{D}^{(\gamma)}_{ee}(x,\mu_f,m_e) = \delta(1-x) + \frac{\alpha}{2\pi} d_1(x,\mu_0,m_e) + \frac{\alpha}{2\pi} L_f P^{(0)}_{ee}(x)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^2 \left(\frac{1}{2} L_f^2 P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + L_f P^{(0)}_{ee} \otimes d_1(x,\mu_0,m_e) + L_f P^{(1,\gamma)}_{ee}(x)\right)$$
$$+ \left(\frac{\alpha}{2\pi}\right)^3 \left(\frac{1}{6} L_f^3 P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes P^{(0)}_{ee}(x) + L_f^2 P^{(0)}_{ee} \otimes P^{(1,\gamma)}_{ee}(x)\right.$$
$$\left. + L_f^2 P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes d_1(x,\mu_0,m_e)\right) + \mathcal{O}(\alpha^3 L^1). \tag{5}$$

Here μ_f and μ_0 are the factorization and renormalization scales; P_{ee} are electron splitting functions; $d_1(x)$ is the initial condition for evolution equations, see details in Ref. [23]. Pair contributions to the structure function will be considered elsewhere. The $\mathcal{O}(\alpha^3 L^2)$ contributions are

$$P^{(0)}_{ee} \otimes P^{(1,\gamma)\rm str}_{ee}(x) = \left[\frac{1+x^2}{1-x}\Big(-4\mathrm{S}_{12}(1-x) + 4\mathrm{Li}_2(1-x)(\ln(1-x) - \ln(x))\right.$$
$$-4\ln(x)\ln^2(1-x) + 4\ln^2(x)\ln(1-x) - \frac{2}{3}\ln^3(x) + 3\mathrm{Li}_2(1-x) - 3\ln(x)\ln(1-x)$$
$$+\frac{3}{2}\ln^2(x) + 4\zeta(2)\ln(x) + 6\zeta(3) - 3\zeta(2) + \frac{3}{8}\Big) + 4(1+x)\mathrm{S}_{12}(x) + \frac{1+x}{2}\ln^3(x)$$
$$-2(1+x)\ln^2(x)\ln(1-x) + (6x-2)\mathrm{Li}_2(1-x) + (6-2x)\ln(x)\ln(1-x)$$
$$\left. + \left(\frac{11}{4}x - \frac{9}{4}\right)\ln^2(x) + (6-4x)\ln(1-x) + (5x-3)\ln(x) + 2x - \frac{1}{2}\right]_+, \tag{6}$$

where the standard plus prescription is applied. The second relevant new contribution is

$$P^{(0)}_{ee} \otimes P^{(0)}_{ee} \otimes d_1(x,m_e,m_e) = \left[\frac{1+x^2}{1-x}\Big(16\mathrm{S}_{12}(1-x) + 8\mathrm{Li}_2(1-x)\ln(x) - 16\ln^3(1-x)\right.$$
$$+20\ln(x)\ln^2(1-x) - 4\ln^2(x)\ln(1-x) + \zeta(2)(40\ln(1-x) - 16\ln(x)) - 32\zeta(3)$$

$$-30\ln^2(1-x) - 2\ln^2(x) + 24\ln(x)\ln(1-x) + 24\zeta(2) - \frac{17}{2}\ln(1-x) + 2\ln(x) + \frac{15}{4}\biggr)$$
$$+(1+x)\biggl(20\mathrm{Li}_3(1-x) - 10\mathrm{S}_{12}(1-x) - 20\mathrm{Li}_2(1-x)\ln(1-x) - 2\mathrm{Li}_2(1-x)\ln(x)$$
$$-10\ln(x)\ln^2(1-x) + 3\ln^2(x)\ln(1-x) + 8\zeta(2)\ln(x)\biggr) - 2(1+x)\mathrm{Li}_2(1-x)$$
$$+20(1-x)\ln^2(1-x) + \frac{3}{2}(1-x)\ln^2(x) - 18\ln(x)\ln(1-x) + 14x\ln(x)\ln(1-x)$$
$$+4(1-x)\ln(1-x) - 2\ln(x) - 16(1-x)\zeta(2) - 2(1-x)\biggr]_+ . \qquad (7)$$

Note that expressions (6) and (7) represent contributions to universal collinear QED radiation factors which can be useful for description of a wide class of processes at high energies.

4. Conclusions

The SANC computer system is upgraded to facilitate description of processes at electron-colliders with polarized beams. The results on complete electroweak one-loop radiative corrections to polarized Bhabha scattering are presented. A new Monte Carlo event generator to simulate this process is created.

The $\mathcal{O}(\alpha^3 L^2)$ contributions to the electron structure function will be implemented into the ZFITTER code [24] in order to perform realistic numerical estimates of the corresponding effect in measurable cross-sections. Further, these contributions will be used in Monte Carlo event generators being under development by the SANC group for future e^+e^- colliders like FCC-ee and CEPC.

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