Pseudo-Goldstone bosons in grand unified theories

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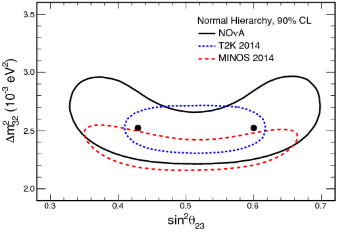
Outline

1. Motivation of BSM physics
2. The minimal renormalizable $SO(10)$ GUT
3. Calculation of one loop SM singlet mass
4. Consistency checks
5. Summary

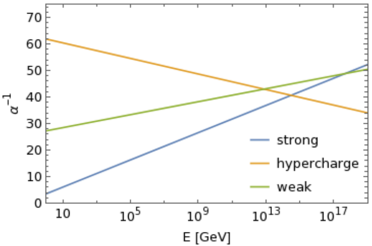
Motivation of BSM physics

Neutrino mass:

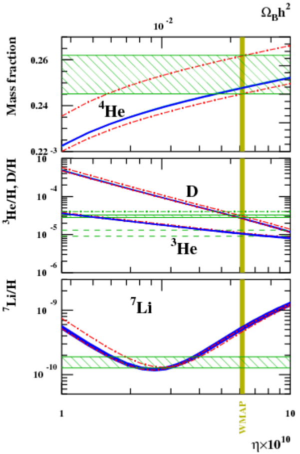


[Adamson,P. et al.: Phys.Rev. D93 (2016) no.5]

SM Running couplings:



Baryon-antibaryon asymmetry:



[Coc,A. et al.: Phys.Rev. D87 (2013) no.12]

$\longrightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \subset SO(10)$

The $SO(10)$ group

In defining 10-dimensional representation the special orthogonal group is defined as

$$SO(10) = \left\{ M \in \mathbb{R}^{10\times 10} : M^T M = \mathbb{I}_n, \det M = 1 \right\}.$$

and corresponding special orthogonal algebra

$$\mathfrak{so}(10) = \left\{ iT \in \mathbb{R}^{10\times 10} : T^T + T = 0, \mathrm{Tr}\, T = 0 \right\}.$$

- Rank $= 5 \Rightarrow$ more Cartan generators than SM
- Every finite dimensional representation is anomaly free and exhibits charge quantization

Gauge fields

$$45_G = G^b_\mu \oplus A^a_\mu \oplus B_\mu,\ Y_\mu \oplus X_\mu$$

$$G^b_\mu \to (8,1,0), \quad A^a_\mu \to (1,3,0), \quad B_\mu, Y_\mu \to (1,1,0) \oplus (1,1,0)$$

$$X_\mu \to (3,1,\tfrac{2}{3}) \oplus (3,2,-\tfrac{5}{6}) \oplus (3,2,\tfrac{1}{6}) \oplus (1,1,1) + h.c.$$

Matter fields

$$16_F = \begin{pmatrix} u_L^{R,G,B} \\ d_L^{R,G,B} \end{pmatrix} \oplus \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \oplus (\overline{u}_L^{R,G,B}) \oplus (\overline{d}_L^{R,G,B}) \oplus (\overline{N}_L) \oplus (e_L^+)$$

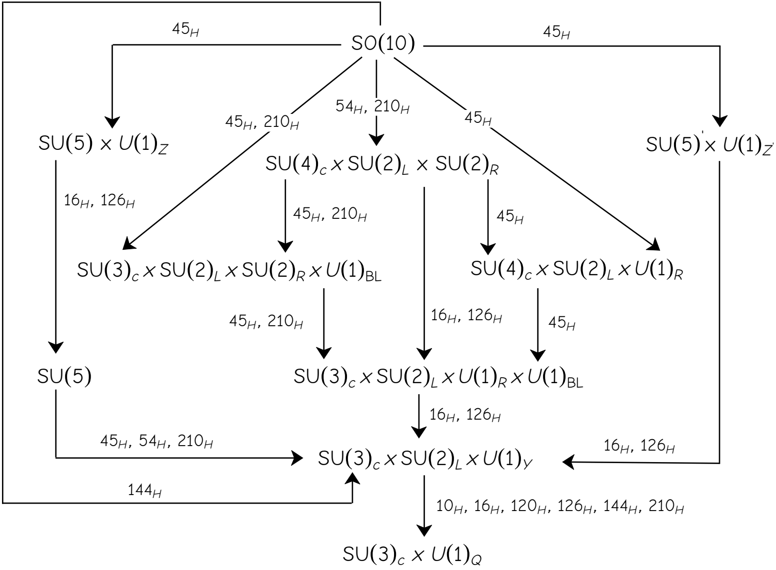
$$\overline{16}_F = \begin{pmatrix} \overline{d}_R^{R,G,B} \\ \overline{u}_R^{R,G,B} \end{pmatrix} \oplus \begin{pmatrix} e_R^+ \\ \overline{\nu}_R \end{pmatrix} \oplus (u_R^{R,G,B}) \oplus (d_R^{R,G,B}) \oplus (N_R) \oplus (e_R^-)$$

Scalar sector

- More freedom, distinguish between particular $SO(10)$ GUTs
- $SO(10) \xrightarrow{SSB} SU(3)_c \times SU(2)_L \times U(1)_Y$ by two stages:
  - $SO(10)$ SSB at the $10^{15-16} GeV$ scale
  - intermediate-scale SSB at the $10^{13-14} GeV$ scale

$\Rightarrow$ we need two SSB stages
$\Rightarrow$ two irreducible representations of scalar fields

Different symmetry breaking chains



Scalar sector of the minimal renormalizable model

GUT scale SSB by 45_H:

- The least dimensional real representation
- Preserves rank
- Suppression of eventual gravitational effects on GUT scale
- Two real full SM singlets (in $(SU(3)_c, SU(2)_L, U(1)_Y)_{SU(4)_c}$ notation):

$$\langle(1,1,0)_{15}\rangle = \sqrt{3}\omega_{BL} \quad \langle(1,1,0)_{1}\rangle = \sqrt{2}\omega_{R}$$

Seesaw scale SSB by 126_H:

- Representation lowering the rank
- Renormalizable Yukawa interaction with fermions $\leftarrow 16 \times 16 \supset 126$
- One complex full SM singlet

$$\langle(1,1,0)_{\overline{10}}\rangle = \sqrt{2}\sigma \text{ where } |\sigma| \ll \max(\omega_{BL}, \omega_R)$$

Tree level scalar spectrum

Scalar fields $45_H \oplus 126_H \leftrightarrow \Phi = (\phi_{ij}, \Sigma_{ijklm}, \Sigma^*_{ijklm})$ form the most general scalar potential

$$V_0(\phi, \Sigma, \Sigma^*) = V_\phi(\phi) + V_\Sigma(\Sigma, \Sigma^*) + V_{\phi,\Sigma}(\phi, \Sigma, \Sigma^*)$$

$$\begin{aligned}
V_\phi &= -\frac{\mu^2}{4}(\phi\phi)_0 + \frac{a_0}{4}(\phi\phi)_0(\phi\phi)_0 + \frac{a_2}{4}(\phi\phi)_2(\phi\phi)_2\,, \\
V_\Sigma &= -\frac{\nu^2}{5!}(\Sigma\Sigma^*)_0 + \frac{\lambda_0}{(5!)^2}(\Sigma\Sigma^*)_0(\Sigma\Sigma^*)_0 + \frac{\lambda_2}{(4!)^2}(\Sigma\Sigma^*)_2(\Sigma\Sigma^*)_2 + \\
&\quad + \frac{\lambda_4}{(3!)^2(2!)^2}(\Sigma\Sigma^*)_4(\Sigma\Sigma^*)_4 + \frac{\lambda'_4}{(3!)^2}(\Sigma\Sigma^*)_{4'}(\Sigma\Sigma^*)_{4'} + \\
&\quad + \frac{\eta_2}{(4!)^2}(\Sigma\Sigma)_2(\Sigma\Sigma)_2 + \frac{\eta_2^*}{(4!)^2}(\Sigma^*\Sigma^*)_2(\Sigma^*\Sigma^*)_2\,, \\
V_{\phi,\Sigma} &= \frac{i\tau}{4!}(\phi)_2(\Sigma\Sigma^*)_2 + \frac{\alpha}{2\cdot 5!}(\phi\phi)_0(\Sigma\Sigma^*)_0 + \frac{\beta_4}{4\cdot 3!}(\phi\phi)_4(\Sigma\Sigma^*)_4 + \\
&\quad + \frac{\beta'_4}{3!}(\phi\phi)_{4'}(\Sigma\Sigma^*)_{4'} + \frac{\gamma_2}{4!}(\phi\phi)_2(\Sigma\Sigma)_2 + \frac{\gamma_2^*}{4!}(\phi\phi)_2(\Sigma^*\Sigma^*)_2\,.
\end{aligned}$$

Tree level mass matrix

$$\mathrm{M}_S^2 = \frac{\partial^2 V_0}{\partial\Phi\partial\Phi^*}\bigg|_{\Phi=\langle\Phi\rangle}$$

with VEVs and potential parameters satisfying stationary conditions

$$\frac{\partial V_0}{\partial\Phi}\bigg|_{\Phi=\langle\Phi\rangle} \equiv \left\langle \frac{\partial V_0}{\partial\Phi} \right\rangle = 0.$$

Particularly $(\omega^n[x_1, \ldots, x_{n+1}] := \sum_{j=0}^{n} \omega_{BL}^{n-j}\omega_R^{j+1})$

$$\underbrace{\mathrm{M}_S^2[(8,1,0)] = -2a_2\omega^2[-2,1,1] \quad \mathrm{M}_S^2[(1,3,0)] = -2a_2\omega^2[1,1,-2]}_{\substack{\geq 0 \text{ iff near } SU(5)'\times U(1)_{Z'} \text{ SSB chain} \\ (a_2>0,\, -2<\frac{\omega_{BL}}{\omega_R}<-\frac{1}{2})}}$$

- Potentially realistic scenarios involve tachyons in the tree level mass spectrum
- $M_S^2[(8,1,0)], M_S^2[(1,3,0)] \propto a_2 \leftarrow$ if $a_2 << 1$, loop corrections dominant

[Bertolini S., Luzio L., Malinský M.: Phys.Rev. D81 (2010)]

SM singlet mas submatrix involving only fields from 45_H neglecting σ-dependent mixing 45_H and 126_H:

$$\begin{pmatrix} -2a_2\ \omega^2[-2,1,1] + 24a_0\omega_{BL}^2 - 12\sigma^2\beta_4' & 4\sqrt{6}\left(2a_0\omega_{BL}\omega_R - \sigma^2\beta_4'\right) \\ 4\sqrt{6}\left(2a_0\omega_{BL}\omega_R - \sigma^2\beta_4'\right) & -2a_2\ \omega^2[1,1,-2] + 16a_0\omega_R^2 - 8\sigma^2\beta_4' \end{pmatrix}.$$

Its eigenvalues are

$$m_1^2 \equiv m_{PG}^2 = a_2\left(-\frac{45\omega_{BL}^4}{\omega^2[3,0,2]} + \omega^2[13,-2,-2]\right) + O\left(a_2^2\right) + O\left(\frac{\sigma^2}{\omega_{max}^2}\right),$$

$$m_2^2 = 8a_0\ \omega^2[3,0,2] + a_2\left(\frac{45\omega_{BL}^4}{\omega^2[3,0,2]} + \omega^2[-11,-2,4]\right) + O\left(a_2^2\right) + O\left(\frac{\sigma^2}{\omega_{max}^2}\right).$$

- $m_{PG}^2 \propto a_2$ up to $O\left(\frac{\sigma^2}{\omega_{max}^2}\right)$ corrections $\Rightarrow$ non-negligible one loop contributions
- Multiplets $(8,1,0),(1,3,0),(1,1,0) \leftrightarrow$ pseudo-Goldstone bosons of broken global symmetry $O(45)$ in the limit $\sigma \to 0, a_2 \to 0$

Calculation of one loop SM singlet mass

Effective potential

$$V \approx V_0 + V_1.$$

with one loop corrections ($\overline{\text{MS}}$ scheme, vanishing external momenta)

$$V_1 = \frac{1}{64\pi^2}\mathrm{Tr}\left[M_S^4(\Phi)\left(\log\frac{M_S^2(\Phi)}{\mu_R^2} - \frac{3}{2}\right)\right] + \frac{3}{64\pi^2}\mathrm{Tr}\left[M_G^4(\Phi)\left(\log\frac{M_G^2(\Phi)}{\mu_R^2} - \frac{5}{6}\right)\right]$$

where

$$M_S^2(\Phi)_{ij} = \frac{\partial^2 V_0}{\partial\Phi_i \partial\Phi_j^*}$$

and

$$M_G^2(\Phi)_{ab} = g^2\left(\hat{T}^a\Phi\right)^\dagger \hat{T}^b\Phi \quad \leftarrow \text{action } \hat{T}^a\Phi \text{ generators on } 45_H \oplus 126_H$$

[Coleman, S., Weinberg, E.: Phys.Rev. D7 (1973)]

- Field-dependent matrices are block diagonal in SM preserving vacuum

$$\langle V_1 \rangle = \frac{1}{64\pi^2} \sum_{\text{SM irreps } M} \left\{ \sum_{\text{eig } x_M} m_S^4[x_M] \left(\log \frac{m_S^2[x_M]}{\mu_R^2} - \frac{3}{2} \right) \right\} + \\ + \frac{3}{64\pi^2} \sum_{\text{SM irreps } M} \left\{ \sum_{\text{eig } x_M} m_G^4[x_M] \left(\log \frac{m_G^2[x_M]}{\mu_R^2} - \frac{5}{6} \right) \right\}$$

- In SM singlet mass matrix evaluation are derivatives of the fields with non-vanishing VEV

$$\left\langle \frac{\partial V_1(\Phi)}{\partial (1,1,0)_{15}} \right\rangle = \frac{1}{\sqrt{3}} \frac{\partial \langle V_1 \rangle (\omega_{BL}, \omega_R, \sigma)}{\partial \omega_{BL}}$$

$$\left\langle \frac{\partial V_1(\Phi)}{\partial (1,1,0)_{1}} \right\rangle = \frac{1}{\sqrt{2}} \frac{\partial \langle V_1 \rangle (\omega_{BL}, \omega_R, \sigma)}{\partial \omega_{R}}$$

$$\left\langle \frac{\partial V_1(\Phi)}{\partial (1,1,0)_{\overline{10}}} \right\rangle = \frac{1}{\sqrt{2}} \frac{\partial \langle V_1 \rangle (\omega_{BL}, \omega_R, \sigma)}{\partial \sigma}$$

One loop level singlet mass

- True one loop vacuum $v = v_0 + v_1$ is determined from modified stationary conditions

$$\frac{dV}{d\Phi_a}\bigg|_v = \frac{dV_0}{d\Phi_a}\bigg|_v + \frac{dV_1}{d\Phi_a}\bigg|_v = 0$$

- One loop level SM singlet mass matrix ($\overline{\text{MS}}$ scheme, loops with vanishing external momenta)

$$\mathcal{M}^2_{ab} = \frac{\partial^2 V_0}{\partial\Phi_a \partial\Phi_b^*}\bigg|_v + \frac{\partial^2 V_1}{\partial\Phi_a \partial\Phi_b^*}\bigg|_{v_0}$$

where $\Phi_a, \Phi_b \in \{(1,1,0)_1, (1,1,0)_{15}\} \;\Rightarrow$

$$\mathcal{M}^2_{\text{singlet}} = \begin{pmatrix} \mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\ \mathcal{M}^2_{12} & \mathcal{M}^2_{22} \end{pmatrix} \left\} \begin{array}{l} \tilde{m}_1^2 = \frac{1}{2}\left(\mathcal{M}^2_{11} + \mathcal{M}^2_{12} + \sqrt{(\mathcal{M}^2_{11} - \mathcal{M}^2_{22})^2 + 4\mathcal{M}^4_{12}}\right) \\ \tilde{m}_2^2 = \frac{1}{2}\left(\mathcal{M}^2_{11} + \mathcal{M}^2_{12} - \sqrt{(\mathcal{M}^2_{11} - \mathcal{M}^2_{22})^2 + 4\mathcal{M}^4_{12}}\right) \end{array} \right.$$

Example of the polynomial part of element $\mathcal{M}^2_{11}$:

$$\mathcal{M}^2_{\text{Tree,11poly}} = -2a_2\ \omega^2[-2,1,1] + 24a_0\omega^2_{BL} - 12\sigma^2\beta'_4$$

$$\mathcal{M}^2_{\text{Gauge,11poly}} = \frac{g^4}{16\pi^2}\left(\omega^2[40,1,13] - 6\sigma^2\right)$$

(scalar contribution in limit $|\sigma|/\max(\omega_{BL},\omega_R) \to 0, |a_2| \to 0, |\gamma_2| \to 0$)

$$\mathcal{M}^2_{\text{Scal,11poly}} = \frac{35\tau^2}{8\pi^2} + \frac{\omega^5[-5,19,-170,1222,-5,19]\omega_R}{16R^2\pi^2}\beta_4^2 +$$

$$+ \frac{15\omega[1,1]\omega_R}{4\pi^2}\beta_4\beta'_4 + \frac{5\omega_R\omega[-5,11]}{4\pi^2}\beta'^2_4$$

[Bertolini S., Luzio L., Malinský M.: Phys.Rev. D85 (2012)]
[Kolešová H., Malinský M.: Phys.Rev. D90 (2014)]

Consistency checks

Particular symmetry breaking chains

- $SU(5) \times U(1)_Z$ ($\sigma \to 0, \omega_R \to \omega_{BL}$):

$$(1,1,0)_{PG}, (8,1,0), (1,3,0) \subset (24,0)$$

- $SU(5)' \times U(1)_{Z'}$ ($\sigma \to 0, \omega_R \to -\omega_{BL}$):

$$(1,1,0)_{PG}, (8,1,0), (1,3,0) \subset (24,0)$$

- $SU(4)_c \times SU(2)_L \times U(1)_R$ ($\sigma \to 0, \omega_{BL} \to 0$):

$$(1,1,0)_{PG}, (8,1,0) \subset (15,1,0)$$

- $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{BL}$ ($\sigma \to 0, \omega_R \to 0$):

$$\text{D-parity}[(1,1,0)_{PG} \subset (1,3,1,0)] \;, (1,3,0) \subset (1,1,3,0)$$

$$\uparrow$$

$$\text{D-parity}[(x,y,z,u)] = (\overline{x}, z, y, -u)$$

[Gráf L., Malinský M., Mede T.,Susič V.:Phys.Rev. D95 (2017) no.7]

Summary

- The minimal renormalizable $SO(10)$ GUT field content consists of
  - Gauge bosons 45_G,
  - Matter fields $16_F, \oplus\overline{16}_F$,
  - Scalar sector $45_H \oplus 126_H$.
- All potentially realistic SSB scenarios involve tachyonic instabilities. This issue has been resolved by one loop corrections of masses of $(8, 1, 0), (1, 3, 0)$ multiplets.
- The scalar mass spectrum contains the third pseudo-Goldstone boson, full SM singlet, that initially escaped the attention.
- One loop corrections to the pseudo-Goldstone singlet mass were calculated in potentially realistic realization via effective potential approach. Resulting formulae in particular symmetry breaking chains were crosschecked with previous computations.