Dark Matter Predictions in E_6 Inspired SUSY Models

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Based on:

P. Athron, D. Harries, R. Nevzorov, and A. G. Williams, Phys. Lett. **B760**, 19 (2016) [arXiv:1512.07040]

P. Athron, D. Harries, R. Nevzorov, and A. G. Williams, JHEP **12**, 128 (2016) [arXiv:1610.03374]

April 9, 2018

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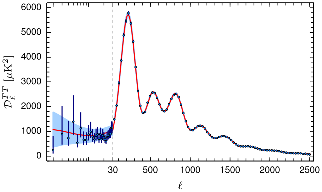
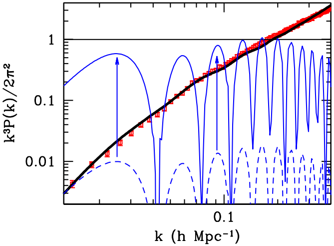
Evidence for DM



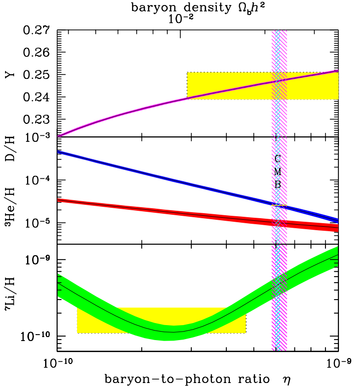
[arXiv:1502.02114]

CMB measurements



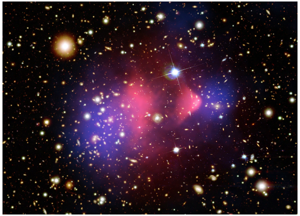
[arXiv:1112.1320]

Structure formation



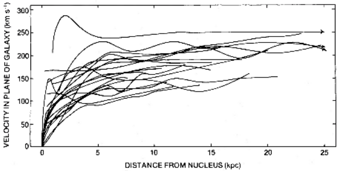
[PDG 2017]

BBN



[APOD/NASA]

Galaxy cluster mergers

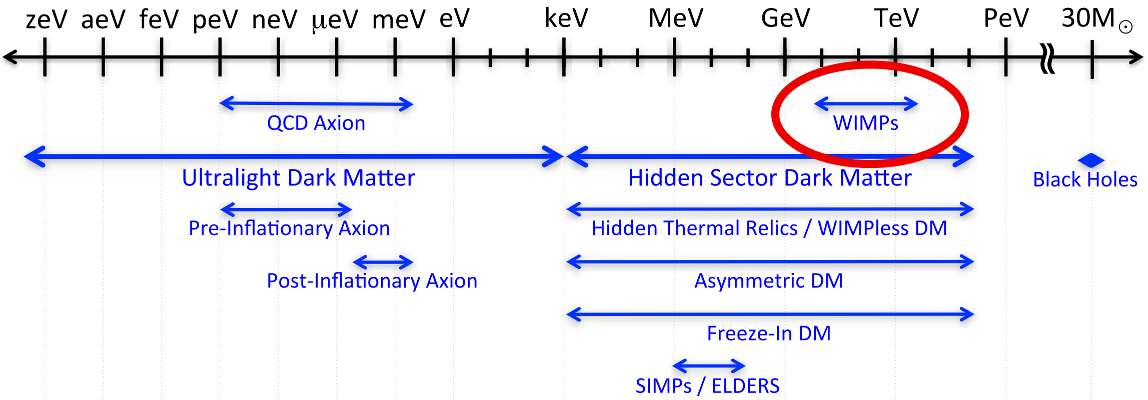


[V. C. Rubin, N. Thonnard, and W. K. Ford, Jr.,

Astrophys. J. **238** (1980) 471]

Galaxy rotation curves

DM Candidates



[arXiv:1707.04591]

WIMP DM: Relic Density

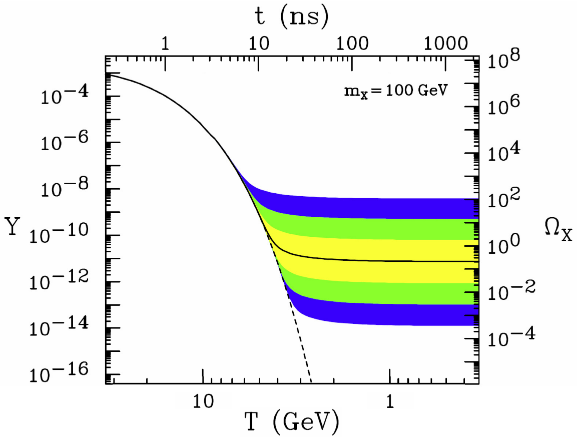
- Relic density of standard WIMPs results from freeze-out ($Y = n_\chi/s$),

$$\frac{dY}{dt} = -s\langle\sigma v\rangle(Y^2 - Y^2_{\text{eq.}}) \Rightarrow \Omega_\chi h^2 = \frac{m_\chi s_0 h^2}{\rho_c} Y_\infty$$

- "WIMP miracle": for $m_\chi \sim$ GeV $-$ TeV, $g_\chi \sim g_{\text{weak}}$,

$$\Omega_\chi \sim \frac{m_\chi^2}{g_\chi^4} \sim \Omega_{\text{DM}}$$

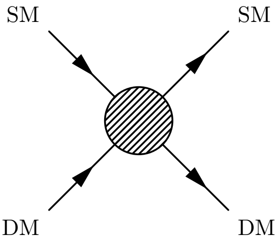
- $\Omega_\chi h^2 > (\Omega_{\text{DM}} h^2)_{\text{Planck}} \Rightarrow$ model ruled out
    - N.B. $\Omega_\chi h^2 < (\Omega_{\text{DM}} h^2)_{\text{Planck}}$ allowed
    - e.g., multiple DM candidates χ_i, $\sum_i \Omega_{\chi_i} = \Omega_{\text{DM}}$



[arXiv:1003.0904]

$$(\Omega_{\text{DM}} h^2)_{\text{Planck}} = 0.1188 \pm 0.0010$$

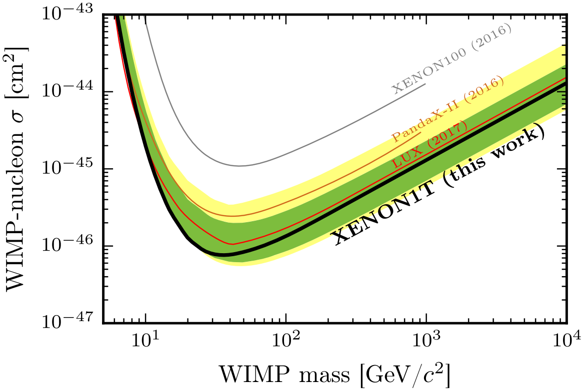
Constraining WIMP Models



- Multiple approaches: direct detection (→), indirect detection (↑), collider searches (↓)
- Direct detection: observe nuclear recoil in DM-nucleus interaction,

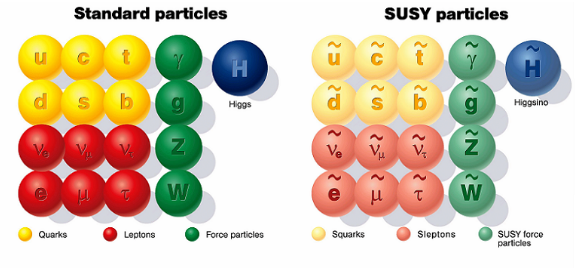
$$\frac{dR}{dE} = \frac{1}{2m_\chi m_r^2}\sigma(q)\rho_{\chi,\mathrm{local}}\eta(v_{\min}(E), t)$$

- LUX, XENON1T, ... ⇒ stringent limits on $\sigma^{(SI/SD)}$:



[arXiv:1705.06655]

The MSSM



[http://www.physics.gla.ac.uk/ppt/bsm.htm]

$\hat\Phi$	$s=0$	$s=\frac{1}{2}$	$SU(3)_C$	$SU(2)_L$	$\sqrt{\frac{5}{3}}Q_i^Y$
$\hat Q_i$	$\begin{pmatrix}\tilde u_L\\ \tilde d_L\end{pmatrix}_i$	$\begin{pmatrix}u_L\\ d_L\end{pmatrix}_i$	**3**	**2**	$\frac{1}{6}$
$\hat u_i^c$	$\tilde u_{iR}^*$	u_{iR}^c	$\bar{\mathbf{3}}$	**1**	$-\frac{2}{3}$
$\hat d_i^c$	$\tilde d_{iR}^*$	d_{iR}^c	$\bar{\mathbf{3}}$	**1**	$\frac{1}{3}$
$\hat L_i$	$\begin{pmatrix}\tilde\nu_L\\ \tilde e_L\end{pmatrix}_i$	$\begin{pmatrix}\nu_L\\ e_L\end{pmatrix}_i$	**1**	**2**	$-\frac{1}{2}$
$\hat e_i^c$	$\tilde e_{iR}^*$	e_{iR}^c	**1**	**1**	1
$\hat H_d$	$\begin{pmatrix}H_d^0\\ H_d^-\end{pmatrix}$	$\begin{pmatrix}\tilde H_d^0\\ \tilde H_d^-\end{pmatrix}$	**1**	**2**	$-\frac{1}{2}$
$\hat H_u$	$\begin{pmatrix}H_u^+\\ H_u^0\end{pmatrix}$	$\begin{pmatrix}\tilde H_u^+\\ \tilde H_u^0\end{pmatrix}$	**1**	**2**	$\frac{1}{2}$

$$W_{\text{MSSM}} = \mu(\hat H_d\cdot\hat H_u) + y_{ij}^e\hat e_i^c(\hat L_j\cdot\hat H_d) + y_{ij}^d\hat d_i^c(\hat Q_j\cdot\hat H_d) + y_{ij}^u\hat u_i^c(\hat H_u\cdot\hat Q_j)$$
$$-\epsilon_i\hat L_i\cdot\hat H_u + \frac{1}{2}\rho_{ijk}\hat L_i\cdot\hat L_j\hat e_k^c + \rho'_{ijk}\hat L_i\cdot\hat Q_j\hat d_k^c + \frac{1}{2}\rho''_{ijk}\hat u_i^c\hat d_j^c\hat d_k^c$$

Neutralino DM

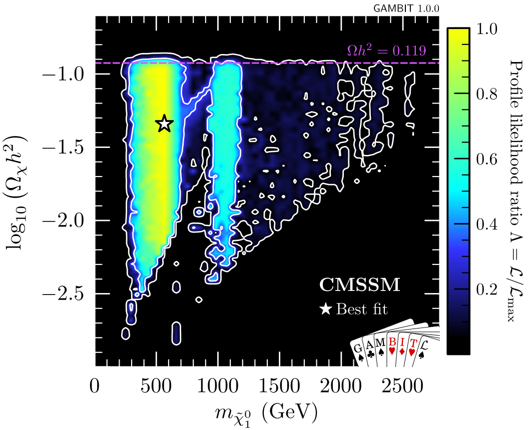
- Forbid B, L violating interactions $\Rightarrow$ impose R-parity $\Leftrightarrow$ matter parity

$$Z_2^R = (-1)^{3(B-L)+2s}, \quad Z_2^M = (-1)^{3(B-L)}$$

- $\Rightarrow$ lightest R-parity odd state (LSP) is natural WIMP DM candidate (provided $Q_{LSP} = 0$)
- Standard MSSM candidate is

$$\tilde{\chi}_1^0 = N_{11}\tilde{H}_d^0 + N_{12}\tilde{H}_u^0 + N_{13}\tilde{W}_3 + N_{14}\tilde{B}$$

- Properties dependent on $m_{\tilde{\chi}_1^0}$, N_{ij} (i.e., μ, soft breaking M_1, M_2)



[arXiv:1705.07935]

E_6 Inspired Models

- Motivated by MSSM shortcomings, e.g., tree-level $m_{h_1}^2 \leq M_Z^2 \cos^2 2\beta$ ("little hierarchy problem"), μ-problem, ν masses, ...
- Lead to $U(1)$ extended models at low-energies:

$$\begin{aligned} E_6 &\longrightarrow SO(10) \times U(1)_\psi \\ &\longrightarrow SU(5) \times U(1)_\psi \times U(1)_\chi \\ &\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi \\ &\longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \end{aligned}$$

- Resulting charges $Q' = Q_\chi \cos\theta_{E_6} + Q_\psi \sin\theta_{E_6}$
- Matter content fills complete **27** representations (anomaly cancellation)
  - $\Rightarrow$ additional exotic states
- Extra D- and F-terms $\Rightarrow$ larger m_{h_1}
- Break $U(1)'$ with singlet $\Rightarrow$ dynamically generate μ term, massive Z'

The E_6SSM

- $\tan\theta_{E_6} = \sqrt{15} \Rightarrow U(1)_N$ under which right-handed neutrinos are uncharged
  - allows ν masses via see-saw and successful baryogenesis [1]
- Extra $\hat{L}_4$, $\hat{\overline{L}}_4$ from incomplete $\mathbf{27}'$, $\overline{\mathbf{27}}'$ for gauge unification
- Low-energy matter content from $\mathbf{27}$-plet:

$$(\hat{Q}_i,\, \hat{u}^c_i,\, \hat{d}^c_i,\, \hat{L}_i,\, \hat{e}^c_i) + (\hat{D}_i,\, \hat{\overline{D}}_i) + (\hat{S}_i) + (\hat{H}^u_i) + (\hat{H}^d_i)$$

- Higgs doublets $\hat{H}^d_3$, $\hat{H}^u_3$ and one singlet $\hat{S}_3$ get VEVs ($\Rightarrow$ EWSB and break $U(1)_N$)

	$SU(3)_C$	$SU(2)_L$	$\sqrt{\frac{5}{3}}Q^Y_i$	$\sqrt{40}Q^N_i$
$\hat{Q}_i$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{6}$	1
$\hat{u}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$-\frac{2}{3}$	1
$\hat{d}^c_i$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	2
$\hat{L}_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\hat{e}^c_i$	$\mathbf{1}$	$\mathbf{1}$	1	1
$\hat{S}_i$	$\mathbf{1}$	$\mathbf{1}$	0	5
$\hat{H}^u_i$	$\mathbf{1}$	$\mathbf{2}$	$\frac{1}{2}$	-2
$\hat{H}^d_i$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	-3
$\hat{D}$	$\mathbf{3}$	$\mathbf{1}$	$-\frac{1}{3}$	-2
$\hat{\overline{D}}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	$\frac{1}{3}$	-3
$\hat{L}_4$	$\mathbf{1}$	$\mathbf{2}$	$-\frac{1}{2}$	2
$\hat{\overline{L}}_4$	$\mathbf{1}$	$\overline{\mathbf{2}}$	$\frac{1}{2}$	-2

$$W_{E_6SSM} \approx y_\tau \hat{L}_3 \cdot \hat{H}^d_3 \hat{e}^c_3 + y_b \hat{Q}_3 \cdot \hat{H}^d_3 \hat{d}^c_3 + y_t \hat{H}^u_3 \cdot \hat{Q}_3 \hat{u}^c_3 + \lambda_i \hat{S}_3 \hat{H}^d_i \cdot \hat{H}^u_i + \kappa_i \hat{S}_3 \hat{D}_i \hat{\overline{D}}_i + \mu_L \hat{L}_4 \cdot \hat{\overline{L}}_4$$

[1] S. F. King, S. Moretti, and R. Nevzorov, Phys. Rev. D **73**, 035009 (2006) (hep-ph/0510419)

Discrete Symmetries and DM in the E_6SSM

- Neutralino sector extended by "inert" $\tilde{S}_\alpha$, $\tilde{H}^{d,u}_\alpha$ $\Rightarrow$ DM candidate not MSSM-like in general
- General superpotential

$$W \supset g^D_{ijk}\hat{D}_i\hat{Q}_j \cdot \hat{Q}_k + \tilde{g}^E_{ijk}\hat{e}^c_i\hat{D}_j\hat{u}^c_k + y^U_{ijk}\hat{u}^c_i\hat{H}_{uj} \cdot \hat{Q}_k + y^D_{ijk}\hat{d}^c_i\hat{Q}_j \cdot \hat{H}_{dk} + \dots$$

$\Rightarrow$ impose exact $Z_2^{B/L}$ and approximate Z_2^H (compare single R-parity in MSSM)

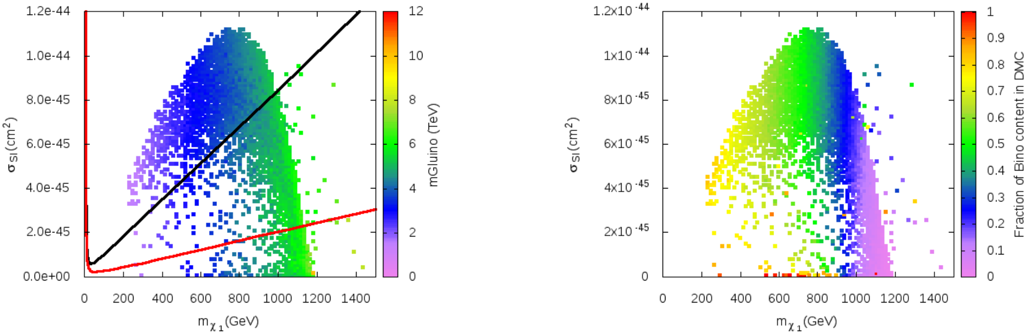
- Resulting Higgs, singlet couplings

$$W \supset \lambda\hat{S}\hat{H}_{d3} \cdot \hat{H}_{u3} + \lambda_{\alpha\beta}\hat{S}\hat{H}_{d\alpha} \cdot \hat{H}_{u\beta} + \tilde{f}_{\alpha\beta}\hat{S}_\alpha\hat{H}_{d\beta} \cdot \hat{H}_{u3} + f_{\alpha\beta}\hat{S}_\alpha\hat{H}_{d3} \cdot \hat{H}_{u\beta}(+\not{Z}_2^H \text{ terms})$$

- Yukawa hierarchy $\Rightarrow$ LSP, NSLP is "inert" neutralino
- But $m_{\tilde{\chi}_1^{I0}} \sim 60 - 65$ GeV $\Rightarrow$ ruled-out

Example: DM in the EZSSM

- Simplest viable models impose *another* exact Z_2^S, e.g., EZSSM [2]



[arXiv:1611.05966]

- Note: none of these Z_2 symmetries commute with E_6

[2] J. P. Hall and S. F. King, JHEP **1106**, 006 (20011) (arXiv:1104.2259)

The SE$_6$SSM

- E_6 inspired model arising from 5D or 6D orbifold GUT [3]
- Complete **27**-plets supplemented by components of extra **27**′-, $\overline{\mathbf{27}}'$-plets
- Stabilise Higgs potential $\Rightarrow$ pure singlet $\hat{\phi}$
- $U(1)_\psi \times U(1)_\chi \to U(1)_N \times Z_2^M$ at intermediate scale $\Rightarrow$ automatically conserved Z_2^M
- $Z_2^{B/L}$, Z_2^H superseded by single exact $\tilde{Z}_2^H$

$$\begin{aligned} W_{\mathrm{SE_6SSM}} = & \lambda \hat{S}(\hat{H}_d \cdot \hat{H}_u) - \sigma \hat{\phi}\hat{S}\hat{\overline{S}} + \frac{\kappa}{3}\hat{\phi}^3 + \frac{\mu}{2}\hat{\phi}^2 + \Lambda_F \hat{\phi} + \lambda_{\alpha\beta}\hat{S}(\hat{H}^d_\alpha \cdot \hat{H}^u_\beta) \\ & + \kappa_{ij}\hat{S}\hat{D}_i\hat{\overline{D}}_j + \tilde{f}_{i\alpha}\hat{S}_i(\hat{H}_u \cdot \hat{H}^d_\alpha) + f_{i\alpha}\hat{S}_i(\hat{H}^u_\alpha \cdot \hat{H}_d) + g^D_{ij}(\hat{Q}_i \cdot \hat{L}_4)\hat{\overline{D}}_j \\ & + h^E_{i\alpha}\hat{e}^c_i(\hat{H}^d_\alpha \cdot \hat{L}_4) + \mu_L(\hat{L}_4 \cdot \hat{\overline{L}}_4) + \tilde{\sigma}\hat{\phi}(\hat{L}_4 \cdot \hat{\overline{L}}_4) + W_{\mathrm{MSSM}}(\mu = 0) \end{aligned}$$

[3] R. Nevzorov, Phys. Rev. **D** 87, 015029 (2013) (arXiv:1205.5967)

DM Candidates?

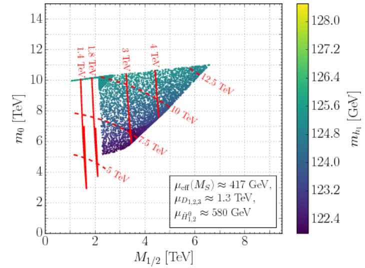
- Conserved Z_2^M, $\tilde{Z}_2^H \Rightarrow$ *two* (distinct) DM candidates
- "Exotics" $\equiv Z_2^E$ odd states, where $\tilde{Z}_2^H = Z_2^M \times Z_2^E$
- Z_2^E conserved $\Rightarrow$ lightest exotic is stable
- Limits from exotic Higgs decays, DM direct detection $\Rightarrow$ inert singlinos $\tilde{S}_i$ form subdominant hot DM, $m_{\tilde{S}_i} \ll 1$ eV
- $M_{Z'} \gg M_S \gg M_Z \Rightarrow$ singlet dominated $\tilde{\chi}^0$'s decouple
- $\Rightarrow$ allowed scenarios have MSSM-like $\tilde{\chi}_1^0$ as LSP
  - Permits interesting comparison with MSSM
  - Explore parameter space of constrained models

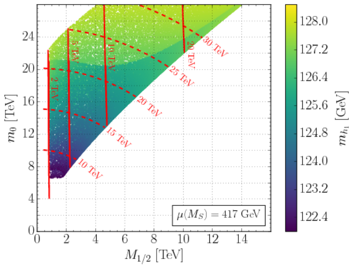
	$\tilde{Z}_2^H$	Z_2^M	Z_2^E
$\hat{Q}_i, \hat{u}_i^c, \hat{d}_i^c, \hat{L}_i, \hat{e}_i^c, \hat{N}_i^c$	$-$	$-$	$+$
$\hat{H}_\alpha^u, \hat{H}_\alpha^d, \hat{S}_i, \hat{D}_i, \hat{\bar{D}}_i$	$-$	$+$	$-$
$\hat{H}_u, \hat{H}_d$	$+$	$+$	$+$
$\hat{S}, \hat{\bar{S}}$	$+$	$+$	$+$
$\hat{L}_4, \hat{\bar{L}}_4$	$+$	$-$	$-$

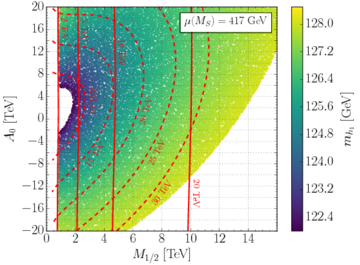
Enlarged (8×8) $\tilde{\chi}^0$ sector:

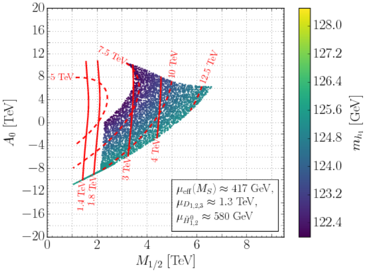
$$M_{\tilde{\chi}^0} = \begin{pmatrix} A & C^T \\ C & B \end{pmatrix}$$

Parameter Space Restrictions





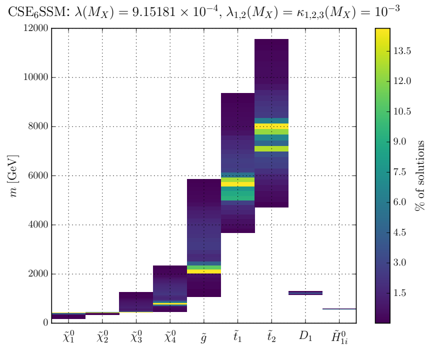




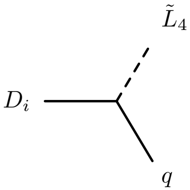
[arXiv:1610.03374]

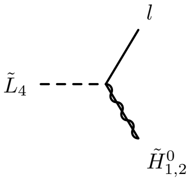
- Successful EWSB + $m_{h_1} \approx 125$ GeV $\Rightarrow$ large $m_0 > M_{1/2}, A_0$
- $m_{h_1} \approx 125$ GeV important constraint on range of variation of $M_{1/2}$, A_0
  - Additional constraints in CSE$_6$SSM from tachyonic CP-even and CP-odd Higgs states
- More generally: m_{h_1} constraint should not be ignored in BSM models
  - Expt. precision $\Rightarrow$ precise theoretical calculation required
  - E.g., on-going work on CNMSSM, CE$_6$SSM

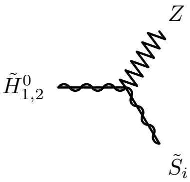
Sparticle Mass Spectrum

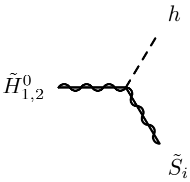


- Sfermions heavy, but gluino and EW-inos can be observable
- Extra matter content $\supset$ inert states + exotic leptons $\hat{L}_4$, $\hat{\overline{L}}_4$, spin-0, spin-1/2 leptoquarks $\hat{D}_i$, $\hat{\overline{D}}_i$
- Light exotic fermions can be observable
  - Exotic leptoquarks D_i: e.g., $p\,p \to t\,\bar{t}\,\tau^+\,\tau^- + E_T^{miss} + X$, $p\,p \to b\,\bar{b} + E_T^{miss} + X$
  - Charged, neutral inert Higgsinos: e.g., $p\,p \to W\,W/Z\,Z/W\,Z + E_T^{miss} + X$



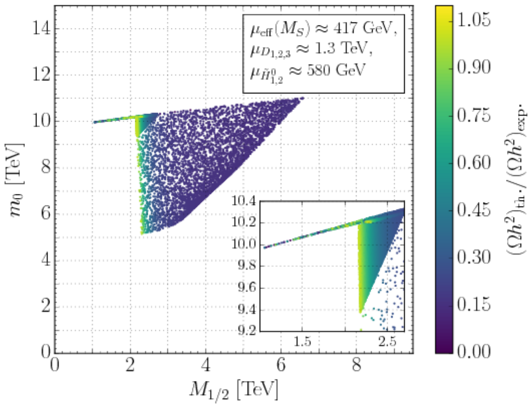


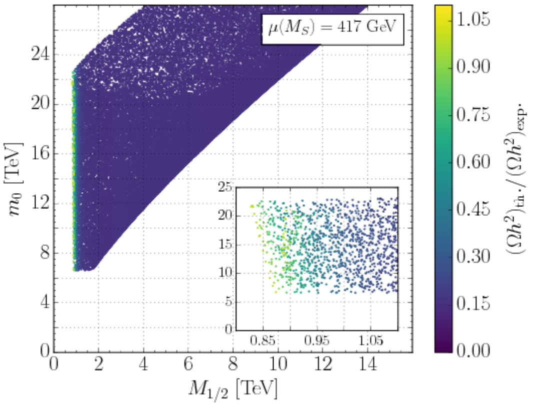




$\mu_{(\text{eff.})} \approx 400$ GeV: Relic Density

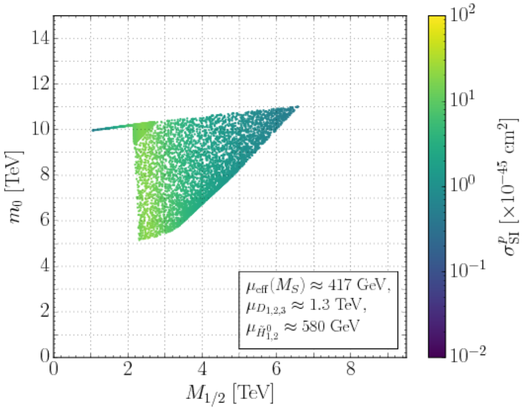
[arXiv:1610.03374]



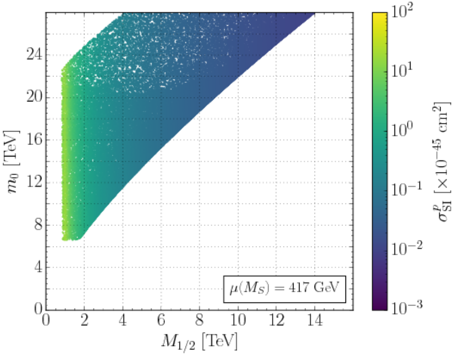


- $\Omega h^2 \approx 0.1187 \Rightarrow$ "well-tempered" bino-Higgsino $\tilde{\chi}^0$ ($\mu_{\text{eff.}} \sim M_1$)
- Pair annihilations $\tilde{\chi}^0_1 \tilde{\chi}^0_1 \to \bar{f} f$
- MSSM-like nature of neutralino sector clear – almost identical behaviour
- Existence of A-funnel at $\tan\beta = 10$ notable difference in CSE$_6$SSM

$\mu_{(\text{eff.})} \approx 400$ GeV: Direct Detection Cross Section



- Well-tempered $\tilde{\chi}^0_1 \Rightarrow$ large mixing, σ_{SI} exceeds, e.g., 90% LUX limits
- $M_1 \gg \mu_{\text{eff.}}$ both mixing and number density suppressed

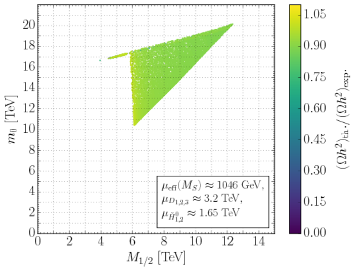


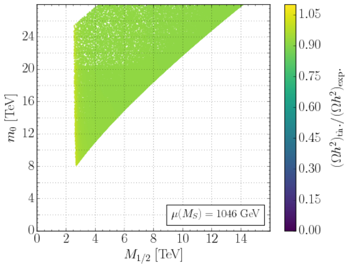
- σ^{SI} set by $g_{h_1\chi_1\chi_1}$ (t-channel h exchange):

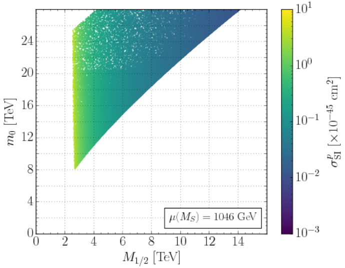
$$g_{h_1\chi_1\chi_1} \approx \frac{1}{2}\left(\sqrt{\frac{3}{5}}g_1 N_{14} - g_2 N_{13}\right)\left[N_{11}(U_h)_{11} - N_{12}(U_h)_{12}\right]$$

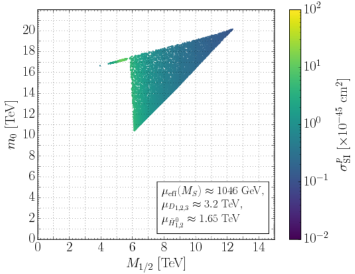
[arXiv:1610.03374]

$\mu_{(\text{eff.})} \approx 1$ TeV: Pure Higgsino DM Candidate







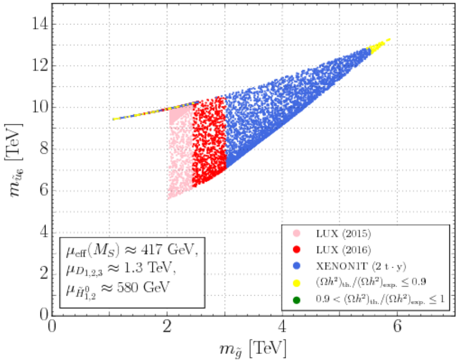


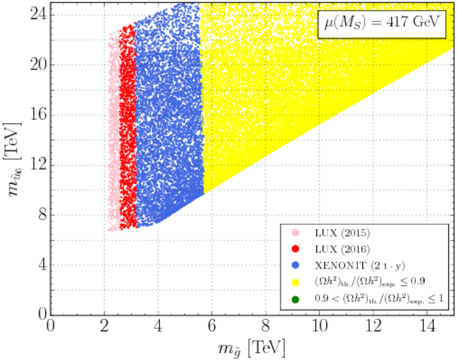
[arXiv:1610.03374]

- Similar parameter space constraints due to m_{h_1}, tachyonic states
- Suppress $\tilde{B}$ fraction $\Rightarrow$ large $M_{1/2}$, $m_{\tilde{g}} \gtrsim 4$ TeV, $m_{\tilde{q}} \gtrsim 10$ TeV
- Exotics not forced to be heavy
- σ^{SI} (mostly) acceptably small

Current and Future Limits

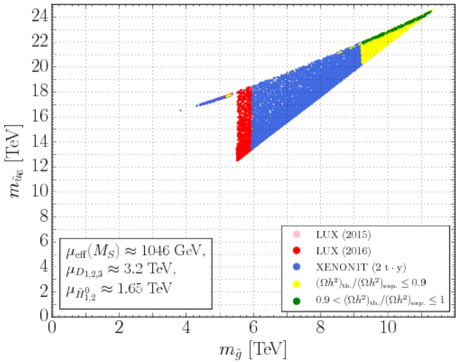
[arXiv:1610.03374]





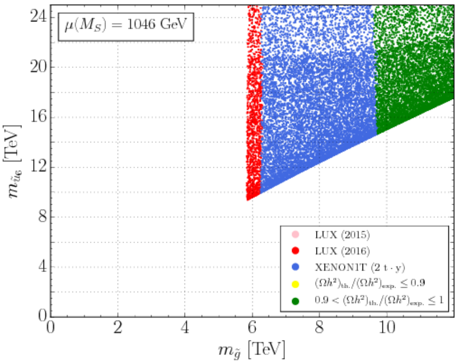
- LUX $\Rightarrow$ already stringent limits on highly mixed scenarios
- XENON1T expected to cover many of the remaining solutions
- SD limits (LUX, IceCube) can also be relevant
- *A*-funnel solutions can survive, but in reach of LHC run II $\Rightarrow$ complementarity of searches

Current and Future Limits



- Essentially no collider limits for $\mu_{\text{eff.}} \approx 1$ TeV (except possibly exotics)
- LUX now excludes mixed $\tilde{\chi}_1^0$ even when $m_{\tilde{\chi}_1^0} \approx 1$ TeV

- Non-mixed scenarios expected to be discoverable at XENON1T
- Larger $M_{1/2}$ and heavy exotics ⇒ only accessible at, e.g., LZ



[arXiv:1610.03374]

Summary

- E_6 inspired models are well-motivated extensions of the MSSM, addressing, e.g., little hierarchy problem, μ-problem, ...
- Novel DM scenarios involving inert states viable in simplest E_6 models, but require multiple discrete symmetries
- SE$_6$SSM is a well-motivated extension with an exact custodial symmetry
- DM relic density can be fitted by MSSM-like $\tilde{\chi}_1^0$, with e.g., $\tilde{\chi}_1^0$ bino-Higgsino or pure Higgsino
- Direct detection searches $\Rightarrow$ stringent limits on allowed mixing
- XENON1T expected to probe much of remainder space, complementary probe to LHC searches
- E_6 exotics not required to be heavy $\Rightarrow$ possible means of constraining/discovering model

Thank you for listening!

Additional Slides

The CSE_6SSM

- General model is complicated
  - $O(200)$ new parameters (assuming no new sources of CP-violation)
  - Many masses and mixings

- Consider constrained model (CSE_6SSM) inspired by gravity mediated SUSY breaking

- Universal soft breaking parameters: $M_{1/2}$, A_0, B_0, m_0

- Interested in mechanism decoupling Z' from EWSB conditions $\Rightarrow$ can have large $s = \sqrt{s_1^2 + s_2^2}$

- Higgsino mass set by $\mu_{\text{eff}} = \lambda s_1/\sqrt{2} \Rightarrow$ acceptable LSP mass ($\lesssim$ TeV) for small λ

- $\Rightarrow$ other exotic couplings must be small, otherwise exotic states are tachyonic

Parameter Space Scans

- Focus on heavy Z', $s = 650$ TeV, choose fixed μ_{eff}
- Achieve using semi-analytic solutions for soft parameters:

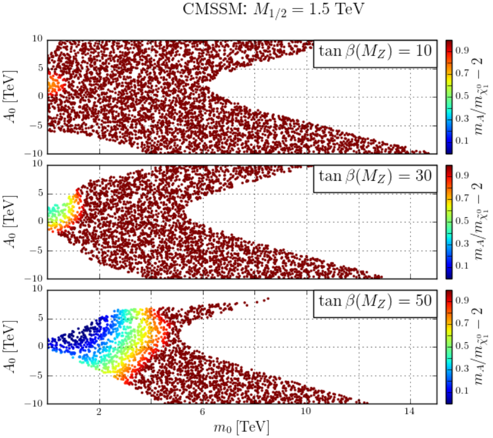
$$M_i(Q) = p_i(Q)M_{1/2} + q_i(Q)A_0\,, \quad A_i(Q) = e_i(Q)A_0 + f_i(Q)M_{1/2}\,,$$
$$m_i^2(Q) = a_i(Q)m_0^2 + b_i(Q)M_{1/2}^2 + c_i(Q)A_0M_{1/2} + d_i(Q)A_0^2\,, \ldots$$

- Fix m_0 from EWSB, $m_0^2 \sim -\frac{b_{H_u}}{a_{H_u}}M_{1/2}^2 - \ldots$
- Implemented in FlexibleSUSY for full 1-loop masses and 2-loop RGEs
  - Resulting "semi-analytic solver" forms part of FlexibleSUSY 2.0 [4], along with many other updates.
- Require $\Omega h^2 \leq 0.1187$ (micrOMEGAs) and $m_{h_1} = 125.09 \pm 3$ GeV
- Compare with CMSSM for $|\mu| \sim 400$ GeV and $|\mu| \sim 1$ TeV

[4] P. Athron, M. Bach, D. Harries, T. Kwasnitza, J.-h. Park, D. Stöckinger, A. Voigt, and J. Ziebell, arXiv:1710.03760

A-funnel in the CSE$_6$SSM

- Solutions at lower $M_{1/2}$ due to $m_{A_1} \sim 2m_{\tilde{\chi}_1^0}$
- CMSSM: A-funnel requires $\tan\beta \gtrsim 40$
- CSE$_6$SSM: tune A_0 for given $\tan\beta$, $M_{1/2}$ to that $m_{A_1} \to 0$, keeping $m_{\tilde{\chi}_1^0} \sim$ fixed
- Lightest state A_1 mixture of singlets for $s \gg M_S \gg v$ ($\tan\delta \approx \frac{s_1 s_2}{\varphi\sqrt{s_1^2+s_2^2}}$):



$$m_{A_1}^2 \approx \cos^2\delta \left(-2B\mu - 3\frac{\kappa A_\kappa}{\sqrt{2}}\varphi - \sqrt{2}\xi\frac{\Lambda}{\varphi} + \frac{9}{2}\sigma\kappa s_1 s_2 + 2\sqrt{2}\frac{\sigma\mu s_1 s_2}{\varphi} + \frac{\sigma s_1 s_2 \Lambda}{\varphi^2} \right)$$