From Higgs boson to graviton

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"Výjezdní seminář ÚČJF"

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Outline:

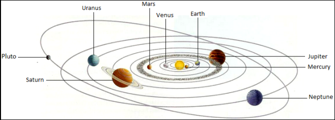
- Motivation – Dark universe
- Motivation – gluon amplitudes
- Effective field theories: pions
- Scalars
- Spin-1
- Spin-2
- Summary

Dark space

Our universe is composed of

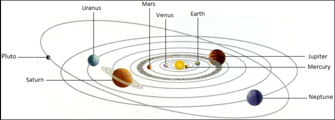
- 5% normal matter
- 25% dark matter
- 70% dark energy

Dark space: dark matter



1933: Fritz Zwicky: “dunkle (kalte) materie”

Dark space: dark matter



1933: Fritz Zwicky: “dunkle (kalte) materie”

Stay tuned: Galaxy NGC 1052DF2 seems to be without dark matter [arXiv:1803.10237]

Dark space: dark energy

We know that space is expanding and natural question: how this is changing, or naively how this is decreasing

big surprise in 1998: space is accelerating (!)
$\rightarrow$ nobel priset 2011

Dark space: dark energy

General relativity:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{m_P}T_{\mu\nu}$$

Dark space: dark energy

General relativity:

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after 1998

Dark space: dark energy

General relativity:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} \neq \frac{1}{m_P}T_{\mu\nu}$$

after 1998

Two possible solutions

1. "dark energy"

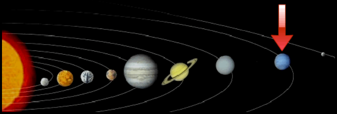
$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{m_P}(T_{\mu\nu} - m_P^2\Lambda g_{\mu\nu})$$

2. "modification of GR"

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{m_P}T_{\mu\nu}$$

Two examples from history

1. 1846: Le Verrier's discovery of Neptune in order to explain discrepancies with Uranus's orbit



"with the top of his pen"

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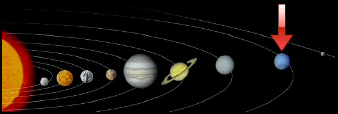
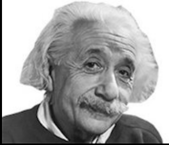


"with the top of his pen"

2. thesame Le Verrier tried to explain the precession of Mercury

new planet Vulcan closer to Sun?



No, the answer is: GR!

Amplitudes

Motivation

Objective of amplitude community:

look at familiar objects from different perspective

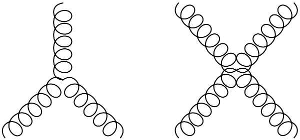
why? $\rightarrow$ two possible answers:

1. technical: we can obtain easier some results
2. conceptual: having completely equivalent reformulation of a given theory can lead to
   - new property discoveries (invisible in traditional formulation)
   - natural description, framework for something new
     /example: principle of least action for Newton's laws vs. quantum revolution/

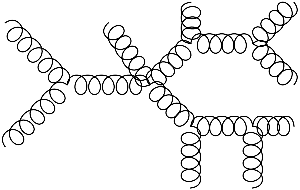
Example: gluon amplitudes

standard method of calculating n-gluon scattering processes:

- dominated by pure-gluon interactions in QCD
- elementary 3pt and 4pt vertices



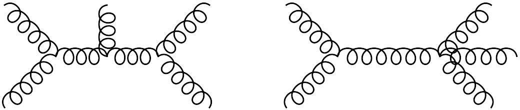
- construct all possible Feynman diagrams, e.g.:



- complicated already for tree level diagrams even for small number of external legs

History: gluon amplitude, tree-level

- 3pt: 1 diagram, on-shell = 0
- 4pt: 4 diagrams, can be calculated by hand, differential cross section nice (but intermediate steps complicated)
- 5pt: calculated in '80, calculation blows up on several pages



structure schematically the numerator:

$$\text{single-propagator: } (p_k \cdot \epsilon)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon),$$
$$\text{double-propagator: } (p_i \cdot p_j)(p_k \cdot \epsilon)(\epsilon \cdot \epsilon)(\epsilon \cdot \epsilon),$$

- 6pt: impossible by standard method, but...

History: gluon amplitude, tree-level, 6pt

SSC approved in 1983 (to be cancelled 10 years later) motivated the following work

THE CROSS SECTION FOR FOUR-GLUON PRODUCTION BY GLUON–GLUON FUSION

Stephen J. PARKE and T.R. TAYLOR

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510 USA

Received 13 September 1985

The cross section for two-gluon to four-gluon scattering is given in a form suitable for fast numerical calculations.

Theoretical predictions for four-jet production at hadron colliders allow detailed tests of QCD. Moreover, at SSC energies, four jets become a serious background to many interesting processes which probe new physics, e.g. pair production of electroweak bosons [1]. Hence a detailed knowledge of four-jet event characteristics is crucial for good background rejection. Although some individual contributions to four-jet production have already been analysed (see e.g. ref. [2]), the two-gluon to four-gluon scattering, which is the dominant contribution for a wide range of subprocess energies, has remained beyond the scope of previous computational techniques. Here we outline our calculation of the cross section for this process, in the tree approximation of perturbative QCD. The final cross section is presented in a form suitable for fast numerical calculations.

Our calculation makes use of techniques developed in ref. [3], based on the application of extended supersymmetry. We adopt the convention that all particles

History: gluon amplitude, tree-level, 6pt

Parke and Taylor finished the article with:

> Details of the calculation, together with a full exposition of our techniques, will be given in a forthcoming article. Furthermore, we hope to obtain a simple analytic form for the answer, making our result not only an experimentalist's, but also a theorist's delight.

History: gluon amplitude, tree-level, 6pt

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Indeed it was given a year later [Parke, Taylor '86]:

$$A_n(- - + \ldots +) = \frac{\langle 12 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle}$$

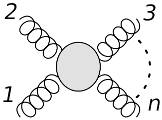
One line formula!

The so-called spinor-helicity formalism was introduced (reasonable variables for massless particles) cf. [Mangano,Parke,Xu '87]

$$\langle ij \rangle = \sqrt{|2p_i \cdot p_j|} \mathrm{e}^{i\phi_{ij}}$$

Is there some better way to calculate?

Example: gluon amplitudes



At tree level:

- colour ordering $\rightarrow$ stripped amplitude

$$M^{a_1 \ldots a_n}(p_1, \ldots p_n) = \sum_{\sigma / Z_n} \mathrm{Tr}(t^{a_{\sigma(1)}} \ldots t^{a_{\sigma(n)}}) M_\sigma(p_1, \ldots, p_n)$$

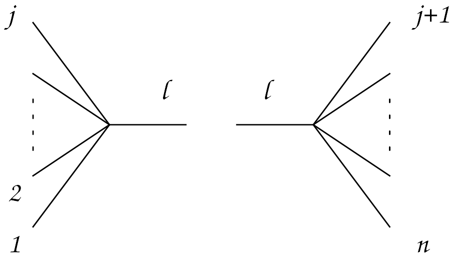
- $M_\sigma(p_{\sigma(1)}, \ldots, p_{\sigma(n)}) = M(p_1, \ldots, p_n) \equiv M(1, 2, \ldots n)$
- propagators $\Rightarrow$ the only poles of M_σ
- thanks to ordering the only possible poles are:

$$P_{ij}^2 = (p_i + p_{i+1} + \ldots + p_{j-1} + p_j)^2$$

Pole structure

Weinberg's theorem (one particle unitarity): on the factorization channel

$$\lim_{P_{1j}^2 \to 0} M(1,2,\ldots n) = \sum_{h_l} M_L(1,2\ldots j,l) \times \frac{i}{P_{1j}^2} \times M_R(l,j+1,\ldots n)$$



BCFW relations, preliminaries

[Britto, Cachazo, Feng, Witten '05]

Reconstruct the amplitude from its poles (in complex plane)

- shift in two external momenta

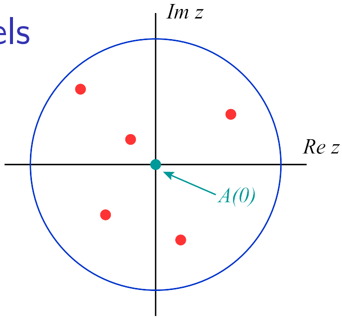
$$p_i \to p_i + zq, \qquad p_j \to p_j - zq$$

- keep p_i and p_j on-shell, i.e.

$$q^2 = q \cdot p_i = q \cdot p_j = 0$$

- amplitude becomes a meromorphic function $A(z)$
- only simple poles coming from propagators $P_{ab}(z)$
- original function is $A(0)$

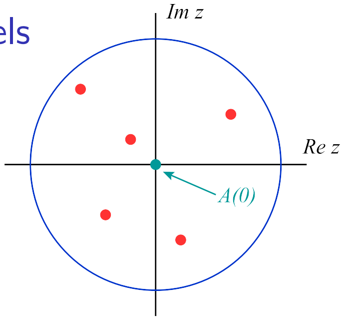
BCFW relations: factorization channels



Cauchy's theorem

$$\frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations: factorization channels



Cauchy's theorem

$$\color{red}{0 =} \frac{1}{2\pi \mathrm{i}} \int \frac{dz}{z} A(z) = A(0) + \sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

If $A(z)$ vanishes for $z \to \infty$

$$A = A(0) = -\sum_k \frac{\mathrm{Res}\,(A, z_k)}{z_k}$$

BCFW relations

$P_{ab}^2(z) = 0$ if one and only one i (or j) in $(a, a+1, \ldots, b)$.

Suppose $i \in (a, \ldots, b) \not\ni j$

$$P_{ab}^2(z) = (p_a + \ldots + p_{i-1} + p_i + zq + p_{i+1} + \ldots + p_b)^2 =$$
$$= P_{ab}^2 + 2q \cdot P_{ab} z = 0$$

solution

$$z_{ab} = -\frac{P_{ab}^2}{2(q \cdot P_{ab})} \qquad \Rightarrow \qquad P_{ab}^2(z) = -\frac{P_{ab}^2}{z_{ab}}(z - z_{ab})$$

Thus

$$\text{Res}(A, z_{ab}) = \sum_s A_L^{-s}(z_{ab}) \times i\frac{-z_{ab}}{P_{ab}^2} \times A_R^s(z_{ab})$$

and for allowed helicities it factorizes into two subamplitudes

BCFW relations

Using Cauchy's formula, we have finally as a result

$$A = \sum_{k,s} A_L^{-s_k}(z_k) \frac{i}{P_k^2} A_R^{s_k}(z_k)$$

- based on two-line shift (convenient choice: adjacent i,j)
- recursive formula (down to 3-pt amplitudes)
- number of terms small = number of factorization channels
- all ingredients are on shell

BCFW Example: gluon amplitudes

od diagrams for n-body gluon scatterings at tree level

n	3	4	5	6	7	8
# diagrams (inc.crossing)	1	4	25	220	2485	34300
# diagrams (col.ordered)	1	3	10	38	154	654
# BCFW terms	–	1	2	3	6	20

BCFW recursion relations: problems

We have assumed that

$$A(z) \to 0, \qquad \text{for} \qquad z \to \infty$$

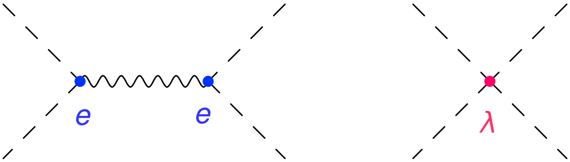
More generally we have to include a boundary term in Cauchy's theorem.

This is intuitively clear: we can formally use the derived BCFW recursion relations to obtain any higher n amplitude starting with the leading interaction. But this does not have to be the correct answer.

BCFW recursion relations: problems

example: scalar-QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - |D\phi|^2 - \frac{1}{4}\lambda|\phi|^4$$



Due to the power-counting the boundary term is proportional to

$$B \sim 2e^2 - \lambda$$

In order to eliminate the boundary term we have to set $\lambda = 2e^2$, then the original BCFW works.

I.e. we needed some further information (e.g. supersymmetry) to determine the λ piece.

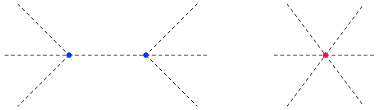
Effective field theories

Effective field theories: homogeneity

Now we have infinitely many unfixed "λ" terms. Schematically

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \lambda_4(\partial\phi)^4 + \lambda_6(\partial\phi)^6 + \dots$$

Example: 6pt scattering, Feynman diagrams



Corresponding amplitude:

$$\mathcal{M}_6 = \sum_{I=poles} \lambda_4^2 \frac{\cdots}{P_I} + \lambda_6(\dots)$$

λ_6 part: not fixed by the pole behaviour.

Task: to find a condition in order to link these two terms

Non-linear sigma model

[KK, Novotny,Trnka '12 and '13]

Leading order Lagrangian

- assume general simple compact Lie group G
- we will build a chiral non-linear sigma model, which will correspond to the spontaneous symmetry breaking ($G_L \simeq G_R \simeq G_V \simeq G$)

$$G_L \times G_R \to G_V$$

- consequence of the symmetry breaking: Goldstone bosons ($\equiv \phi$)

$$U = \exp\left(\sqrt{2}\frac{i}{F}\phi\right)$$

- transformation of U:

$$U \to V_R U V_L^{-1}$$

- their dynamics given by a Lagrangian (at leading order)

$$\mathcal{L} = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^{-1} \rangle$$

where $\langle \ldots \rangle$ stands for a trace

Leading order Lagrangian

- note we are still general, i.e. the group can be

$$G = SU(N), \quad SO(N), \quad Sp(N), \quad \ldots$$

- generators t^i ($\phi = \phi^i t^i$)

$$\langle t^a t^b \rangle = \delta^{ab}, \quad [t^a, t^b] = i\sqrt{2} f^{abc} t^c$$

- "group structure" in structure constants, we will define

$$D_\phi^{ab} \equiv -i f^{abc} \phi^c$$

- and rewrite

$$\mathcal{L} = \frac{F^2}{4} \langle \partial_\mu U \partial^\mu U^{-1} \rangle = -\frac{F^2}{4} \langle (U^{-1} \partial_\mu U)(U^{-1} \partial^\mu U) \rangle$$

Leading order Lagrangian

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- "group structure" in structure constants, we will define

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- and rewrite after some algebra

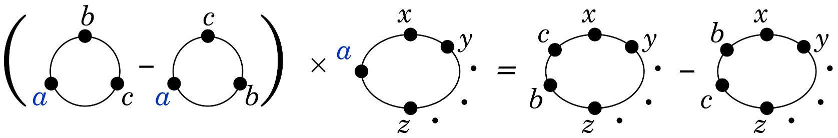
$$\mathcal{L} = -\partial\phi^T \cdot \left(\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \left(\frac{2}{F}\right)^{2n-2} D_\phi^{2n-2} \right) \cdot \partial\phi$$

Stripping down

Now we want to build up the interaction vertex
We need to learn how to connect structure constants together

$$\langle t^a t^b \rangle = \delta^{ab} \qquad\qquad f^{abc} = \frac{-i}{\sqrt{2}}\langle [t^a, t^b] t^c \rangle$$
$$\Rightarrow$$
$$[t^a, t^b] = i\sqrt{2} f^{abc} t^c \qquad\qquad f^{abc} t^c = \frac{-i}{\sqrt{2}} [t^a, t^b]$$

i.e. we can combine f^{abc}s into one trace, schematically



and thus the Feynman rule for the interaction vertices can be written as

$$V_n^{a_1 a_2 \ldots a_n}(p_1, p_2, \ldots, p_n) = \sum_{\sigma \in S_n / Z_n} \langle t^{a_{\sigma(1)}} t^{a_{\sigma(2)}} \ldots t^{a_{\sigma(n)}} \rangle V_n(p_1, \ldots, p_n)$$

Stripping and ordering

Up to now general group: we didn't need any property of f^{abc} or t^i.
From now on: we will simplify the problem setting $G = SU(N)$.
Simplification due to the completeness relation:

$$\sum_{a=1}^{N^2-1} \langle X t^a \rangle \langle t^a Y \rangle = \langle XY \rangle - \frac{1}{N} \langle X \rangle \langle Y \rangle$$

- double trace has to cancel out
- two vertices are connected via a propagator (δ^{ab})
- ordering of t^{a_i} in the final single trace is conserved

The tree graphs built form the stripped vertices and propagators are decorated with cyclically ordered external momenta.

$G = SU(N) \to U(N)$

- stripped amplitudes and vertices are unique
- $\mathcal{M}(p_1, \ldots, p_n)$ are thus "physical"
- we can study different parametrizations [Cronin'67],.., [Bijnens,KK,Lanz '12]
- already mentioned exponential parametrization
- there we can simply enlarge G to $U(N)$ group

$$U = \exp\left(\frac{\mathrm{i}}{F}\sqrt{\frac{2}{N}}\phi^0\right)\widehat{U}, \qquad \widehat{U} \in SU(N)$$

- ϕ^0 however decouples:

$$\mathcal{L}^{(2)} = \frac{1}{2}\partial\phi^0 \cdot \partial\phi^0 + \frac{F^2}{4}\langle\partial_\mu\widehat{U}\partial^\mu\widehat{U}^{-1}\rangle$$

- results from less restricted $U(N)$ equal to $SU(N)$

$G = U(N)$ – different parametrizations[1]

General form of the parametrization $U(\phi) \to f(x)$

$$f(x) = \sum_{k=0}^{\infty} a_k x^k, \qquad f(-x)f(x) = 1$$

- "exponential": $f_{\mathsf{exp}} = \mathsf{e}^x$
- "minimal": $f_{\mathsf{min}} = x + \sqrt{1 + x^2}$
- "Cayley" $f_{\mathsf{Caley}} = \frac{1+x/2}{1-x/2}$

[1] For details see Appendix A in KK,Novotny,Trnka '13. For original literature see Gursey60,Cronin'67, Ellis,Renner'70

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- "exponential": $f_{\text{exp}} = \mathrm{e}^x$ $\quad \to w_{k,n} = \frac{(-1)^k}{1+\delta_{kn}} \frac{1}{(2n+2)!} \binom{2n+2}{k+1}$
- "minimal": $f_{\text{min}} = x + \sqrt{1+x^2}$ $\quad \to w_{2k+1,n} = \frac{(-1)^n}{1+\delta_{2k+1,n}} \binom{k-\frac{1}{2}}{k+1} \binom{n-k-\frac{3}{2}}{n-k}$
- "Cayley" $f_{\text{Caley}} = \frac{1+x/2}{1-x/2}$ $\quad \to w_{k,n} = \frac{(-1)^k}{1+\delta_{kn}} \frac{1}{2^{2n}}$

The stripped Feynman rules can be written

$$V_{2n+2}(s_{i,j}) = (-1)^n \left(\frac{2}{F^2}\right)^n \sum_{k=0}^{n} w_{k,n} \sum_{i=1}^{2n+2} s_{i,i+k}$$

where $s_{i,j} \equiv P(i,j)^2$.

[1] For details see Appendix A in KK,Novotny,Trnka '13. For original literature see Gursey60,Cronin'67, Ellis,Renner'70

Explicit example: stripped 4pt amplitude

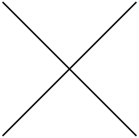
Natural parametrization for diagrammatic calculations: minimal

$$w^{\mathsf{min}}_{2k,n} = 0$$

Thus off-shell and on-shell stripped vertices are equal.

4pt amplitude

$$2F^2\mathcal{M}(1,2,3,4) = -(s_{1,2} + s_{2,3})$$

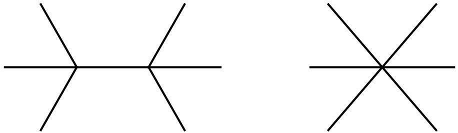


Explicit example: stripped 6pt amplitude

$$4F^4\mathcal{M}(1,2,3,4,5,6) = \\ = \frac{(s_{1,2}+s_{2,3})(s_{1,4}+s_{4,5})}{s_{1,3}} + \frac{(s_{1,4}+s_{2,5})(s_{2,3}+s_{3,4})}{s_{2,4}} \\ + \frac{(s_{1,2}+s_{2,5})(s_{3,4}+s_{4,5})}{s_{3,5}} - (s_{1,2}+s_{1,4}+s_{2,3}+s_{2,5}+s_{3,4}+s_{4,5})$$

This can be rewritten as

$$4F^4\mathcal{M}(1,2,3,4,5,6) = \frac{1}{2}\frac{(s_{1,2}+s_{2,3})(s_{1,4}+s_{4,5})}{s_{1,3}} - s_{1,2} + \text{cycl}\,,$$

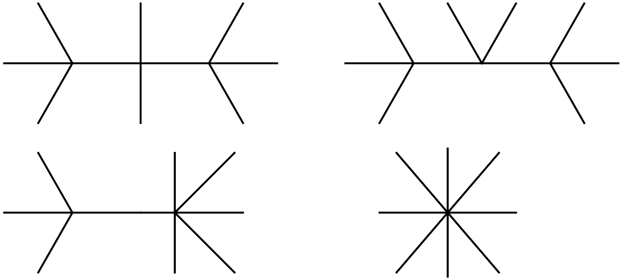


Explicit example: stripped 8pt amplitude

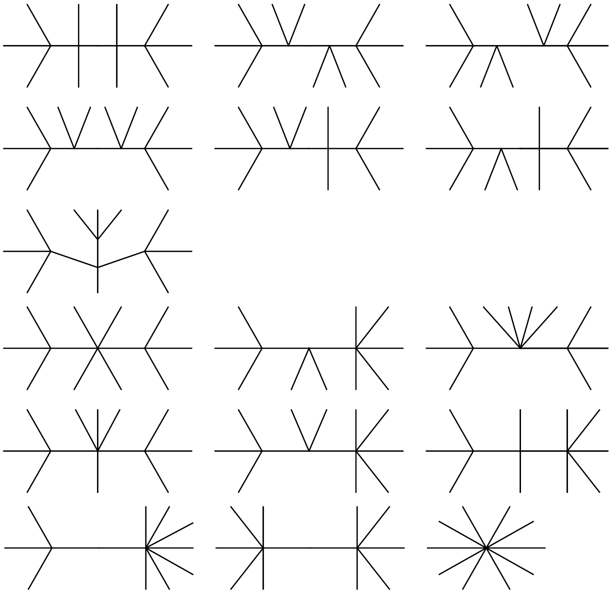
$$8F^6\mathcal{M}(1,2,3,4,5,6,7) =$$

$$= -\frac{1}{2}\frac{(s_{1,2}+s_{2,3})(s_{1,4}+s_{4,7})(s_{5,6}+s_{6,7})}{s_{1,3}s_{5,7}} - \frac{(s_{1,2}+s_{2,3})(s_{1,4}+s_{4,5})(s_{6,7}+s_{7,8})}{s_{1,3}s_{6,8}}$$

$$+ \frac{(s_{1,2}+s_{2,3})(s_{4,5}+s_{4,7}+s_{5,6}+s_{5,8}+s_{6,7}+s_{7,8})}{s_{1,3}} - 2s_{1,2} - \frac{1}{2}s_{1,4} + \text{cycl}$$



Explicit example: stripped 10pt amplitude



Generalization of reconstruction formula: subtractions

[Benincasa, Conde '11] [Feng et al.'11] [KK,Novotny,Trnka '12]

In introduction: $A(z) \to 0$ for $z \to \infty$
Suppose some deformation of the external momenta $p_k \to p_k(z)$ so that

$$A(z) \sim z^k \qquad \text{for} \quad z \to \infty$$

we have to generalize and use the $(k+1)$-times subtracted Cauchy formula

$$A(z) = \sum_{i=1}^{n} \frac{\mathrm{Res}\,(A; z_i)}{z - z_i} \prod_{j=1}^{k+1} \frac{z - a_j}{z_i - a_j} + \sum_{j=1}^{k+1} A(a_j) \prod_{l=1, l \neq j}^{k+1} \frac{z - a_l}{a_j - a_l}$$

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Suppose some deformation of the external momenta $p_k \to p_k(z)$ so that

$$A(z) \sim z^k \qquad \text{for} \quad z \to \infty$$

we have to generalize and use the $(k+1)$-times subtracted Cauchy formula

$$A(z) = \sum_{i=1}^{n} \frac{\mathrm{Res}\,(A; z_i)}{z - z_i} \prod_{j=1}^{k+1} \frac{z - a_j}{z_i - a_j} \qquad \text{if for } a_i \text{ we have } A(a_i) = 0$$

BCFW-like reconstruction of non-linear sigma model

- for the first time we have used the amplitude methods for EFT
- number of diagrams much lower
- can be used for a proof of double Adler zero [conjectured in N. Arkani-Hamed, F. Cachazo and J. Kaplan,: *What is the Simplest Quantum Field Theory? '08*]

drawback:

- tailor-made for one special model, in special limit (two derivatives, massless case, tree-level)
- done using the 'off-shell object': this is against the amplitude idea

Lesson learnt:

- low energy limit, the Adler zero, plays a role of gauge symmetry

Scalar effective field theories

Natural classification

Soft limit of one external leg of the tree-level amplitude

$$A(tp_1, p_2, \ldots, p_n) = \mathcal{O}(t^{\sigma}), \qquad \text{as} \quad tp_1 \to 0$$

Interaction term

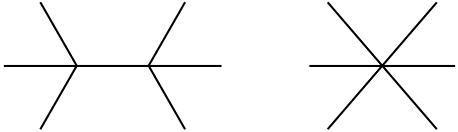
$$\mathcal{L} = \partial^m \phi^n$$

Then another natural parameter is (counts the homogeneity)

$$\rho = \frac{m-2}{n-2}$$

e.g.

$$\mathcal{L} = \partial^m \phi^4 + \partial^{\widetilde{m}} \phi^6$$



so these two diagrams can mix: $p^{2m-2} \sim p^{\widetilde{m}} \Rightarrow \rho = \widetilde{\rho}$, i.e. rho is fixed

We want to focus on a non-trivial case

$$\text{for: } \mathcal{L} = \partial^m \phi^n : \qquad m < \sigma n$$

or

$$\sigma > \frac{(n-2)\rho + 2}{n}$$

First case: $\rho = 0$ (i.e. two derivatives)

Schematically for single scalar case

$$\mathcal{L} = \tfrac{1}{2}(\partial\phi)^2 + \sum_i \lambda_4^i(\partial^2\phi^4) + \sum_i \lambda_6^i(\partial^2\phi^6) + \ldots$$

similarly for multi-flavour (ϕ_i: $\phi_1, \phi_2, \ldots$).
non-trivial case

$$\sigma = 1$$

Outcome:

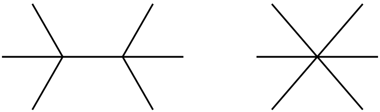
- single scalar: free theory
- multiple scalars (with flavour-ordering): non-linear sigma model

n.b. it represents a generalization of [Susskind, Frye '70], [Ellis, Renner '70]

Second case: $\rho = 1$, $\sigma = 2$ (double soft limit)

1. focus on the lowest combination and fix the form:

$$\mathcal{L}_{int} = c_2(\partial\phi \cdot \partial\phi)^2 + c_3(\partial\phi \cdot \partial\phi)^3 \qquad \text{condition: } c_3 = 4c_2^4$$



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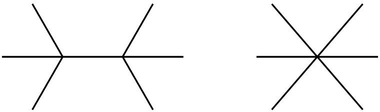

2. find the symmetry

$$\phi \to \phi - b_\rho x^\rho + b_\rho \partial^\rho \phi \, \phi \quad \text{(again up to 6pt so far)}$$

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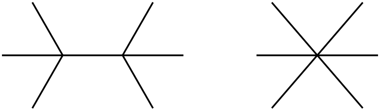

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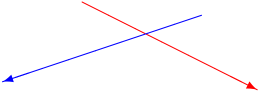
3. ansatz of the form



$$c_n(\partial\phi\cdot\partial\phi)^n + c_{n+1}(\partial\phi\cdot\partial\phi)^n\partial\phi\cdot\partial\phi$$

4. in order to cancel: $2(n+1)c_{n+1} = (2n-1)c_n$
   i.e. $c_1 = \frac{1}{2} \Rightarrow c_2 = \frac{1}{8}, c_3 = \frac{1}{16}, c_4 = \frac{5}{128}, \ldots$

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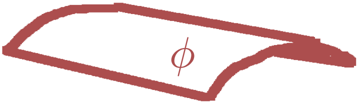
solution:

$$\mathcal{L} = -\sqrt{1 - (\partial\phi \cdot \partial\phi)}$$

This theory known as a scalar part of the Dirac-Born-Infeld [1934] – DBI action

Scalar field can be seen as a fluctuation of a 4-dim brane in five-dim Minkowski space



Remark: soft limit and symmetry are "equivalent"

Third case: $\rho = 2$, $\sigma = 2$ (double soft limit)

Similarly to previous case we will arrive to a unique solution: the Galileon Lagrangian

$$\mathcal{L} = \sum_{n=1}^{d+1} d_n \phi \mathcal{L}_{n-1}^{\mathrm{der}}$$

$$\mathcal{L}_n^{\mathrm{der}} = \varepsilon^{\mu_1 \ldots \mu_d} \varepsilon^{\nu_1 \ldots \nu_d} \prod_{i=1}^{n} \partial_{\mu_i} \partial_{\nu_i} \phi \prod_{j=n+1}^{d} \eta_{\mu_j \nu_j} = -(d-n)! \det \left\{ \partial^{\nu_i} \partial_{\nu_j} \phi \right\}_{i,j=1}^{n} .$$

It possesses the Galilean shift symmetry

$$\phi \to \phi + a + b_\mu x^\mu$$

(leads to EoM of second-order in field derivatives)

Surprise: $\rho = 2$, $\sigma = 3$ (enhanced soft limit)

- general galileon: three parameters (in 4D)
- only two relevant (due to dualities [de Rham, Keltner, Tolley '14] [KK, Novotny '14])

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- symmetry explanation: hidden symmetry [K. Hinterbichler and A. Joyce 1501.07600]

$$\phi \to \phi + s_{\mu\nu} x^\mu x^\nu - 12\lambda_4 s^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

New recursion for effective theories

[Cheung, KK, Novotny, Shen, Trnka 2015]

The high energy behaviour forbids a naive Cauchy formula

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The high energy behaviour forbids a naive Cauchy formula

$$A(z) \neq 0 \qquad \text{for} \quad z \to \infty$$

Can we instead use the soft limit directly? yes!
The standard BCFW not applicable, we propose a special shift:

$$p_i \to p_i(1 - z a_i) \qquad \text{on all external legs}$$

This leads to a modified Cauchy formula

$$\oint \frac{dz}{z} \frac{A(z)}{\Pi_i (1 - a_i z)^\sigma} = 0$$

note there are no poles at $z = 1/a_i$ (by construction).
Now we can continue in analogy with BCFW

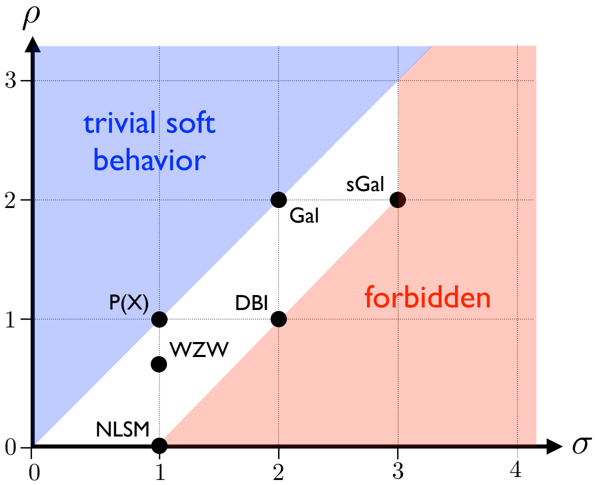
A Periodic Table of Effective Field Theories

[Cheung, KK, Novotny, Shen, Trnka 2016]

Classification of EFTs

We use the set of four parameters:

$$(\rho, \sigma, v, d)$$



Photon scatterings

[Cheung, KK, Novotny, Shen, Trnka, Wen '18]

Spin-1 sector

- we have followed same strategy as for spin-0
- good to have some model in mind
- there is one (from 1934): The Born-Infeld model

$$\mathcal{L}_{\mathrm{BI}} = 1 - \sqrt{(-1)^{D-1}\det(\eta_{\mu\nu} + F_{\mu\nu})},$$

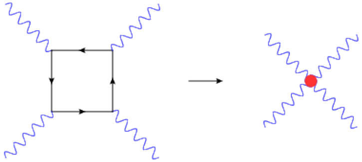
- represents a nonlinear extension of Maxwell theory
- a toy model of general gauge invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4}\langle FF\rangle + g_4^{(1)}\langle FFFF\rangle + g_4^{(2)}\langle FF\rangle^2 + g_6^{(1)}\langle FF\rangle^3 + g_6^{(2)}\langle FFFF\rangle\langle FF\rangle + g_6^{(3)}\langle FFFFFF\rangle + \dots,$$

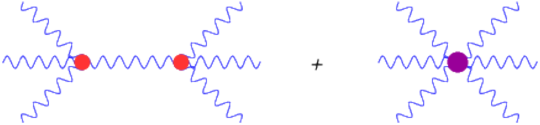
$\langle \dots \rangle$: traces over Lorentz indices

Spin-1 sector

Bottom-up strategy: we have to calculate n-point amplitudes
It reminds very well known problem: "Euler-Heisenberg", effective field theory of light-by-light scattering:



Though, now, the power-counting is different.
Eg. at 6-pt we have to deal with



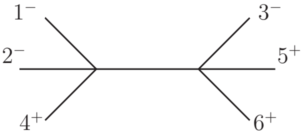
And so on for higher orders.

Spin-1 sector - tree-level amplitudes

employing the spinor helicity formalism, the 4pt amplitudes:

$$A_{----} = c_{-}\langle 12\rangle^2\langle 34\rangle^2 + \text{perm}$$
$$A_{--++} = c_{+}\langle 12\rangle^2[34]^2$$

important simplification: helicity conservation, i.e. $c_{-} = 0$.
we can continue with higher orders
6pt:



$$A_{---+++} = \frac{\langle 12\rangle^2[56]^2\langle 3|1+2|4]^2}{s_{124}} + \text{perm.},$$

Spin-1 sector

We can go to very high orders (impossible by conventional methods) and study any sign of interesting behavior

<u>Result</u>

The studied amplitudes can be fixed by the following multi-chiral soft limit:

$$\lim_{\tilde{\lambda}_- \to \epsilon \text{ or } \lambda_+ \to \epsilon} A_{---+++} = \mathcal{O}(\epsilon)$$

Our conjecture: for any n-point amplitude we have this result, i.e.

$$A(1^- 2^- \dots n^- (n+1)^+ \dots (n+m)^+) = \mathcal{O}(\epsilon)$$

Spin-1 sector

- formally we have proved it using the supersymmetry (BI theory corresponds to the pure bosonic sector of the EFT describing spontaneous sym. breaking of $\mathcal{N}=2$ to $\mathcal{N}=1$ SUSY)
- BI can also be fixed uniquely by a combination of soft limits and dimensional reduction
- discussed methods can be used to vector "Galileon-like" theories

$$\mathcal{L} = F^2 + \partial^2 F^4 + \partial^4 F^6 + \partial^6 F^8 + \dots, \tag{1}$$

we can construct the theory recursively, starting with

$$A_{--++} = \langle 12\rangle^2 [34]^2 s_{12}$$

Spin-2 sector

[work in progress]

It looks like all this has nothing to do with gravity

Where is the gravity?

- all above theories appear also in the context of CHY-type formulation [Cachazo, He, Yuan '13]
- any new theory or property can be used via the so-called double-copy

$$\text{gravity} = (\text{YM}) \otimes (\text{YM})$$

- (in this context: would be interesting to have a non-abelian extension of our spin-1 study)
- galileon itself is a remarkable theory: can be connected with a local modification of gravity [Nicolis, Rattazzi, Trincherini '09]. Important also for cosmology (see e.g. [Chow,Khoury'09] or [de Rham and Heisenberg '11])
- direct way?

  footprint of modified gravity in scattering amplitudes $\rightarrow$

spin-2 scattering

Naively one can expect a toy model given by a variation of

$$\mathcal{L} \sim \sqrt{-\mathrm{det}(\eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i + F_{\mu\nu})}$$

to [Eddington '24], [Deser, Gibbons '98]

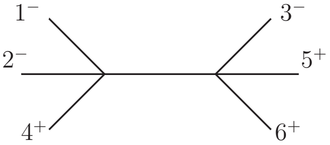
$$\mathcal{L} \sim \sqrt{-\mathrm{det}(\eta_{\mu\nu} + \partial_\mu \phi^i \partial_\nu \phi^i + F_{\mu\nu} + R_{\mu\nu})}$$

This wouldn't work - simply due to the homogeneity reasons

Still we can use the spinor-helicity formalism and the little-group scaling and start the systematic study → see next

spin-2 scattering

n.b. spin-1: $A_4 = \langle 12\rangle^2\langle 34\rangle^2$



$$A_6 = \frac{\langle 12\rangle^2[56]^2\langle 3|1+2|4]^2}{s_{124}}$$

note that $A_6^{\text{cont}} = 0$.

spin-2: the "Born-Infeld" gravity

$$A_4 = \langle 12\rangle^4\langle 34\rangle^4$$

$$A_6 = \frac{\langle 12\rangle^4[56]^4\langle 3|1+2|4]^4}{s_{124}}$$

It is possible to write a contact term

$$A_6^{cont} = (\langle 12\rangle\langle 23\rangle\langle 31\rangle)^2([45][56][61])^2 s_{ij}$$

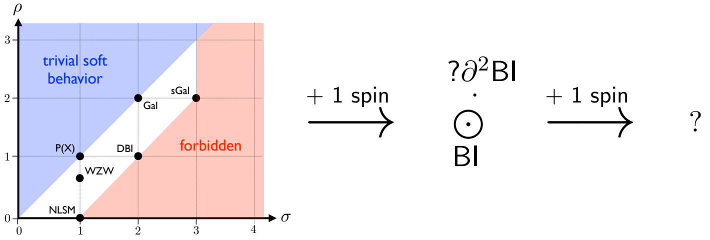
This is an ongoing study...

Summary

We have offered different approach to the subject of this workshop via systematic study of scattering amplitudes

Three-step process:

1. numerical study of all possible tree-level amplitudes (child's play)
2. if something interesting, try to understand it better and prove it
3. use it in phenomenology, GR, cosmology. . . (most difficult part)



(should and can be done for fermions as well, see e.g. [Elvang et al. '17])

possible modification of GR $\rightarrow$ massive graviton

$$V(r) \sim \frac{M}{m_P^2}\frac{1}{r}\mathrm{e}^{-mr}$$

massless graviton: two degrees of freedom
massive graviton: five degrees, one can see them as 1 graviton (massless), 1 photon and 1 scalar

We have to understand all possible theories of spin-1 and spin-0