Neutrina

Rupert Leitner

Neutrino **flavor eigenstates** $|\nu_f\rangle \quad f = e, \mu, \tau$

produced in weak interactions are different from

mass eigenstates $|\nu_i\rangle, \; i = 1,2,3$

They are related by the unitary
mixing matrix: $U_{fi} \equiv \langle \nu_f | \nu_i \rangle$

$$|\nu_f\rangle = \left(|\nu_1\rangle\langle\nu_1| + |\nu_2\rangle\langle\nu_2| + |\nu_3\rangle\langle\nu_3| \right) |\nu_f\rangle$$

$$|\nu_f\rangle = U^*_{f1}|\nu_1\rangle + U^*_{f2}|\nu_2\rangle + U^*_{f3}|\nu_3\rangle$$

If the neutrino of a given flavor f is produced

$$\left|\nu_f\right\rangle = \left(\sum_i \left|\nu_i\right\rangle\left\langle\nu_i\right|\right)\left|\nu_f\right\rangle = U^*_{f1}\left|\nu_1\right\rangle + U^*_{f2}\left|\nu_2\right\rangle + U^*_{f3}\left|\nu_3\right\rangle$$

Then at a distance L it is in the state:

$$\left|\nu_f(L)\right\rangle = \left(\sum_i e^{-i\frac{m_i^2}{2\hbar c}\frac{L}{E}}\left|\nu_i\right\rangle\left\langle\nu_i\right|\right)\left|\nu_f\right\rangle = U^*_{f1}e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}\left|\nu_1\right\rangle + U^*_{f2}e^{-i\frac{m_2^2}{2\hbar c}\frac{L}{E}}\left|\nu_2\right\rangle + U^*_{f3}e^{-i\frac{m_3^2}{2\hbar c}\frac{L}{E}}\left|\nu_3\right\rangle$$

If the phase factors are different (different masses m_i) then other flavors will appear

$$\left|\nu_f(L)\right\rangle = \left(\sum_g \left|\nu_g\right\rangle\left\langle\nu_g\right|\right)\left|\nu_f(L)\right\rangle = \left(\sum_g \left|\nu_g\right\rangle\left\langle\nu_g\right|\right)\left(\sum_i e^{i\frac{m_i^2}{2\hbar c}\frac{L}{E}}\left|\nu_i\right\rangle\left\langle\nu_i\right|\right)\left|\nu_f\right\rangle = \sum_g \left|\nu_g\right\rangle \sum_i e^{i\frac{m_i^2}{2\hbar c}\frac{L}{E}}\left\langle\nu_g\|\nu_i\right\rangle\left\langle\nu_i\|\nu_f\right\rangle = \sum_g A_{\nu_f\to\nu_g}(L)\left|\nu_g\right\rangle$$

With the amplitudes

$$A_{\nu_f\to\nu_g}(L) = \sum_i e^{-i\frac{m_i^2}{2\hbar c}\frac{L}{E}}\left\langle\nu_g|\nu_i\right\rangle\left\langle\nu_i|\nu_f\right\rangle = e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}U_{g1}U^*_{f1} + e^{-i\frac{m_2^2}{2\hbar c}\frac{L}{E}}U_{g2}U^*_{f2} + e^{-i\frac{m_3^2}{2\hbar c}\frac{L}{E}}U_{g3}U^*_{f3}$$

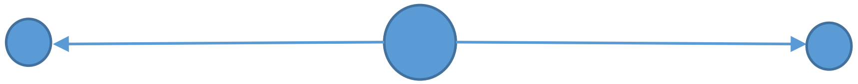
$$P_{\nu_f\to\nu_g}(L) = \left|A_{\nu_f\to\nu_g}(L)\right|^2$$

$$\left|\nu_f(L)\right\rangle = \left(\sum_i e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}\left|\nu_i\right\rangle\left\langle\nu_i\right|\right)\left|\nu_f\right\rangle = U_{f1}^* e^{-i\frac{m_1^2}{2\hbar c}\frac{L}{E}}\left|\nu_1\right\rangle + U_{f2}^* e^{-i\frac{m_2^2}{2\hbar c}\frac{L}{E}}\left|\nu_2\right\rangle + U_{f3}^* e^{-i\frac{m_3^2}{2\hbar c}\frac{L}{E}}\left|\nu_3\right\rangle$$

$$e^{-\frac{i}{\hbar c}(Ect-PL)} \rightarrow e^{-\frac{i}{\hbar c}(E-P)L} \rightarrow e^{-\frac{i}{\hbar c}\frac{m_i^2}{2E}}$$

Mají neutrina stejné hybnosti a rozdílné energie nebo stejné energie a různé hybnosti, …?

Neutrinos have both different energies and momenta. Momenta usually differ (much) more.



Neutrinos from pion decay at rest

$$E_\nu = \frac{m_\pi}{2} + \frac{m_\nu^2 - m_\mu^2}{2m_\pi} = E_0 + \frac{m_\nu^2}{2m_\pi} \cong 30MeV + \frac{m_\nu^2}{280MeV}$$

$$P_\nu = \sqrt{E_\nu^2 - m_\nu^2} = \sqrt{\left(E_0 + \frac{m_\nu^2}{2m_\pi}\right)^2 - m_\nu^2} \cong E_0 + \frac{m_\nu^2}{2m_\pi} - \frac{m_\nu^2}{2E_0} = 30MeV + \frac{m_\nu^2}{280MeV} - \frac{m_\nu^2}{60MeV}$$

$$E_\nu - P_\nu = \frac{m_\nu^2}{2E_0}$$

Reactor neutrinos of 4 MeV from decays of ~100 GeV heavy nuclei

$$X \to Y + e + \nu$$

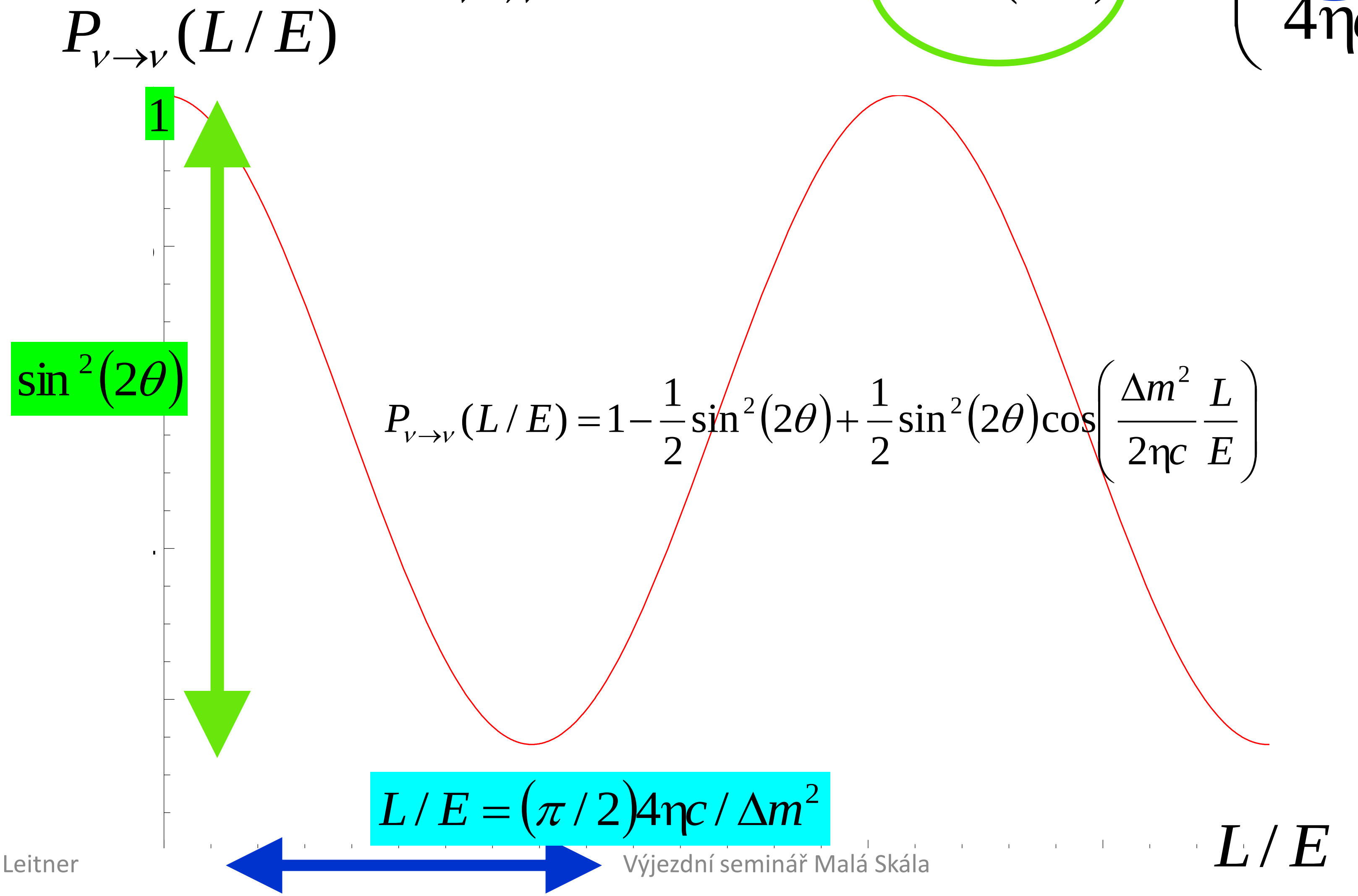
$$E_\nu = \frac{m_X}{2} + \frac{m_\nu^2 - m_{Ye}^2}{2m_X} = E_0 + \frac{m_\nu^2}{2m_{Ye}} \cong 4MeV + \frac{m_\nu^2}{100GeV}$$

Energies of different mass eigenstates are almost the same, momenta differs much more

$$P_\nu = \sqrt{E_\nu^2 - m_\nu^2} = \sqrt{\left(E_0 + \frac{m_\nu^2}{2m_\pi}\right)^2 - m_\nu^2} \cong E_0 + \frac{m_\nu^2}{2m_X} - \frac{m_\nu^2}{2E_0} = 4MeV + \frac{m_\nu^2}{100GeV} - \frac{m_\nu^2}{4MeV}$$

2x2 Mixing Amplitude of oscillations = $\sin^2(2\theta)$, oscillation length is inversely proportional to Δm^2

$$P_{\nu\to\nu}(L/E) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4\hbar c}\frac{L}{E}\right)$$



$$P_{\nu\to\nu}(L/E) = 1 - \frac{1}{2}\sin^2(2\theta) + \frac{1}{2}\sin^2(2\theta)\cos\left(\frac{\Delta m^2}{2\hbar c}\frac{L}{E}\right)$$

Currently there are no high energy electron neutrinos available

$P_{\nu e\to\nu e}$	$P_{\nu e\to\nu\mu}$	$P_{\nu e\to\nu\tau}$
$P_{\nu\mu\to\nu e}$	$P_{\nu\mu\to\nu\mu}$	$P_{\nu\mu\to\nu\tau}$
$P_{\nu\tau\to\nu e}$	$P_{\nu\tau\to\nu\mu}$	$P_{\nu\tau\to\nu\tau}$

Few events seen in OPERA

$P_{\bar{\nu} e\to\bar{\nu} e}$	$P_{\bar{\nu} e\to\bar{\nu}\mu}$	$P_{\bar{\nu} e\to\bar{\nu}\tau}$
$P_{\bar{\nu}\mu\to\bar{\nu} e}$	$P_{\bar{\nu}\mu\to\bar{\nu}\mu}$	$P_{\bar{\nu}\mu\to\bar{\nu}\tau}$
$P_{\bar{\nu}\tau\to\bar{\nu} e}$	$P_{\bar{\nu}\tau\to\bar{\nu}\mu}$	$P_{\bar{\nu}\tau\to\bar{\nu}\tau}$

No tau neutrino sources

Jaké oscilační experimenty můžeme dělat s elektronovými (anti)neutriny

$\bar{\nu}_e$ $\quad {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y + e^- + \bar{\nu}_e$

Zdroj antineutrin(např. reaktor)

Po cestě k detektoru elektronová antineutrina oscilují na ostatní typy $\quad \bar{\nu}_e \rightarrow \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau$

Můžeme proto měřit pouze mizení a znovuobjevovení elektronových antineutrin. Tento typ experimentů nazýváme **DISAPPEARANCE EXPERIMENT**

Dnes nemáme dostatečně intenzivní zdroje elektronových neutrin ani antineutrin s energiemi vyššími než ~10 MeV

V detektoru můžu rozpoznat typ antineutrina

$$E_\nu > 1.8 MeV \qquad \bar{\nu}_e + p \rightarrow n + e^+$$

$$E_\nu >\approx 100 MeV \qquad \bar{\nu}_\mu + p \rightarrow n + \mu^+$$

$$E_\nu > 3500 MeV \qquad \bar{\nu}_\tau + p \rightarrow n + \tau^+$$

Jaké oscilační experimenty můžeme dělat s mionovými (anti)neutriny

ν_μ $\pi^+ \rightarrow \mu^+ + \nu_\mu$

Zdroj mionových neutrin (urychlovače)

Po cestě k miononová neutrina oscilují na ostatní typy

$$\nu_\mu \rightarrow \nu_\mu, \nu_e, \nu_\tau$$

V detektoru můžu rozpoznat typ antineutrina

$E_\nu >\approx 100 MeV$ $\quad \nu_\mu + n \rightarrow p + \mu^-$

$E_\nu > 0 MeV$ $\quad \nu_e + n \rightarrow p + e^-$

$E_\nu > 3500 MeV$ $\quad \nu_\tau + n \rightarrow p + \tau^-$

Můžeme proto měřit nejen mizení a znovuobjevovení mionových (anti)neutrin.
DISAPPEARANCE EXPERIMENT

Ale také objevení neutrin elektronových a pokud mají dostatečnou energii, tak také tauonových neutrin
APPEARANCE EXPERIMENT

Častou otázkou je, zda se při oscilacích neutrin zachovává energie. Odpověď je samozřejmě ano.

$$E_e(L=0)=\langle \nu_e|H|\nu_e\rangle=\langle \nu_1\cos\vartheta+\nu_2\sin\vartheta|H|\nu_1\cos\vartheta+\nu_2\sin\vartheta\rangle$$

$$=\langle \nu_1|H|\nu_1\rangle\cos^2\vartheta+\langle \nu_2|H|\nu_2\rangle\sin^2\vartheta=E_1\cos^2\vartheta+E_2\sin^2\vartheta$$

$$E_e(L)=\langle \nu_e(L)|H|\nu_e(L)\rangle=\left\langle \nu_1\left(e^{-i\frac{M_1^2L}{2E}}\right)^*\cos\vartheta+\nu_2\left(e^{-i\frac{M_2^2L}{2E}}\right)^*\sin\vartheta\middle|H\middle|\nu_1e^{-i\frac{M_1^2L}{2E}}\cos\vartheta+\nu_2e^{-i\frac{{}_2^2L}{2E}}\sin\vartheta\right\rangle$$

$$=E_1\cos^2\vartheta+E_2\sin^2\vartheta$$

Co když se elektronové neutrino úplně změní na mionové neutrino?

$$E_\mu=\langle \nu_\mu|H|\nu_\mu\rangle=\langle -\nu_1\sin\vartheta+\nu_2\cos\vartheta|H|-\nu_1\sin\vartheta+\nu_2\cos\vartheta\rangle=E_1\sin^2\vartheta+E_2\cos^2\vartheta$$

$$E_\mu\neq E_e$$

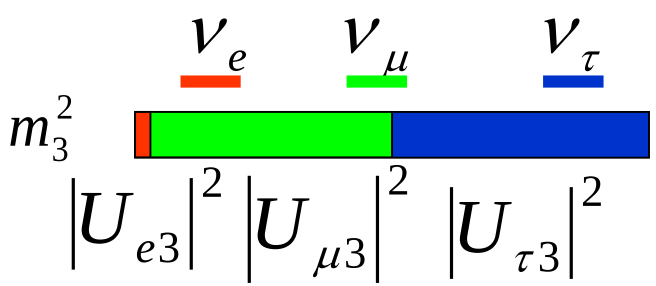
Ale úplná přeměna na jiný typ neutrina vyžaduje maximální směšovací úhel

$$\vartheta=45^o\Rightarrow\sin^2\vartheta=\cos^2\vartheta\Rightarrow E_1\sin^2\vartheta+E_2\cos^2\vartheta=E_1\cos^2\vartheta+E_2\sin^2\vartheta=(E_1+E_2)/2$$

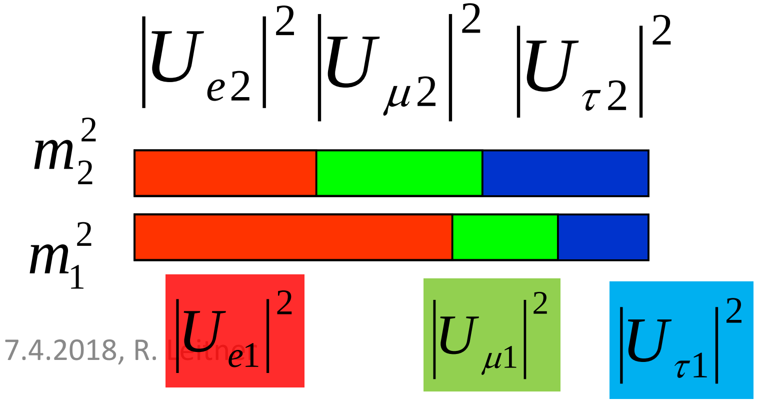
a energie se také samozřejmě zachovává

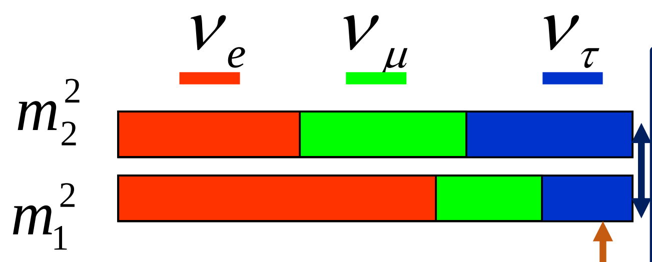
OSCILLATION PARAMETERS

Two mass splits differ by a factor of app 30



NORMAL MASS HIERARCHY (NH)





$\Delta m_{21}^2 \cong 7.54\times10^{-5}\,eV^2 \cong (8.7\,meV)^2$

INVERSE MASS HIERARCHY (IH)

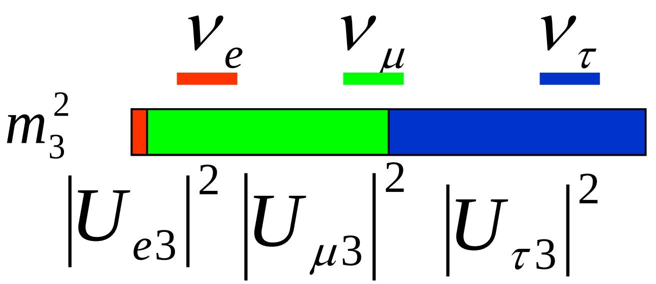
$$\begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_\mu \\ \bar{\nu}_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$|\Delta m_{31}^2| \cong 2.43\times10^{-3}\,eV^2 \cong (49\,meV)^2$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23}) \end{pmatrix} \cdot$$

$\theta_{23} \cong 45^o$

Half of both muon and tauon neutrinos in m3



$$\begin{pmatrix} \cos(\theta_{13}) & 0 & \sin(\theta_{13}) \cdot e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin(\theta_{13}) \cdot e^{i\delta} & 0 & \cos(\theta_{13}) \end{pmatrix} \cdot$$

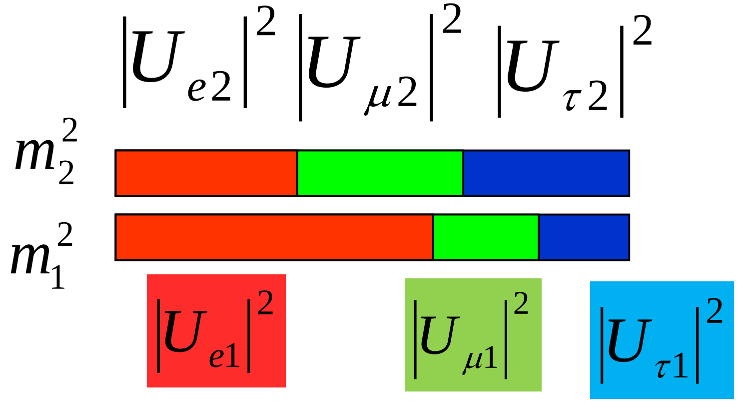
$\theta_{13} \cong 8.5^o$

Very small fraction of electron neutrinos in m3

$$\begin{pmatrix} \cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\ -\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$\theta_{12} \cong 34^o$

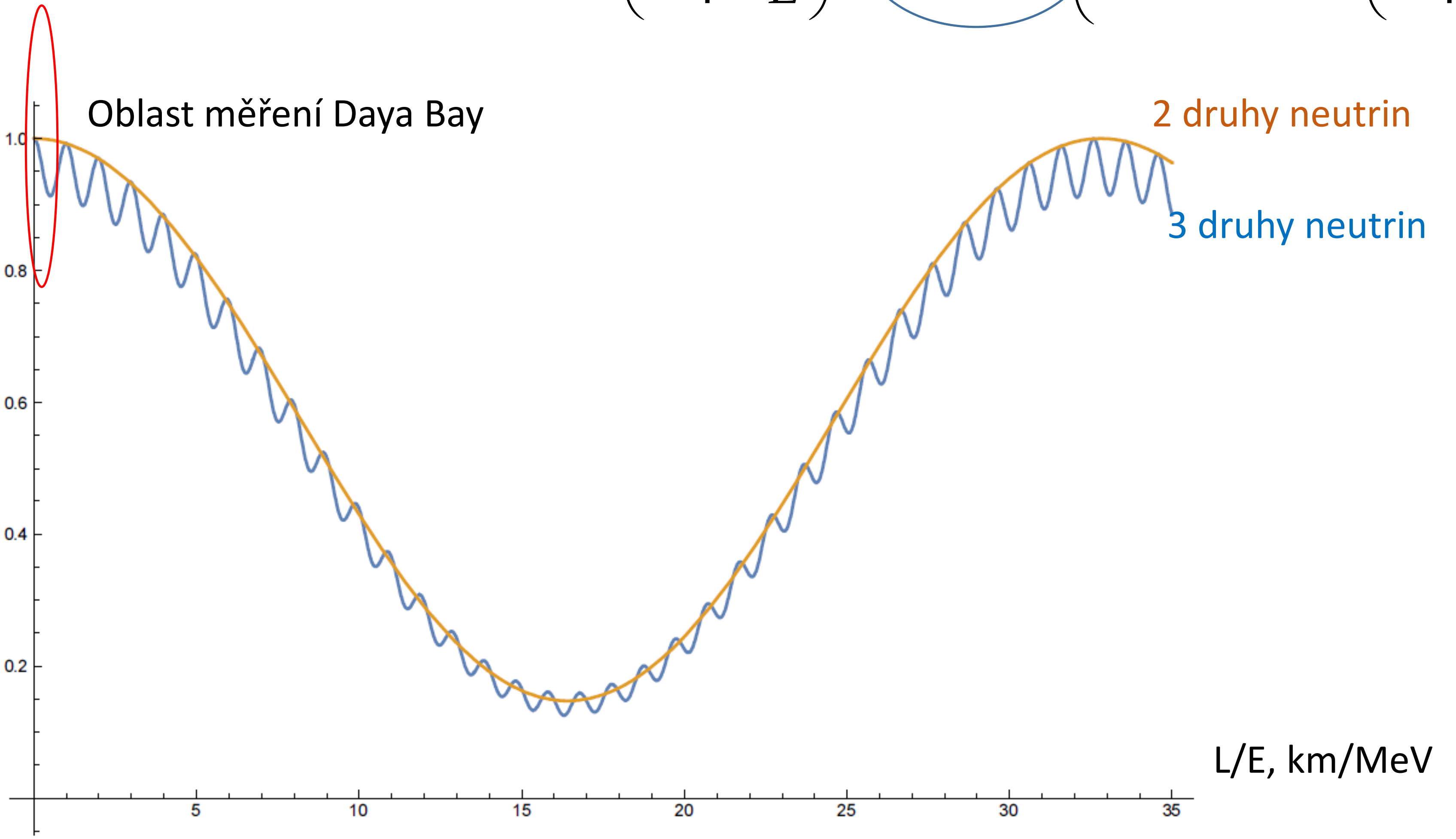
2/3 of electron neutrinos in m1 and 1/3 in m2



Pravděpodobnost oscilací pro elektronová (anti)neutrina pro 2 druhy a 3 druhy neutrin

$$P^{2x2}_{ve \to ve} = 1 - \sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21}}{4}\frac{L}{E}\right)$$

$$P^{3x3}_{ve \to ve} = 1 - \cos^4(\theta_{13})\sin^2(2\theta_{12})\sin^2\left(\frac{\Delta m^2_{21}}{4}\frac{L}{E}\right) - \sin^2(2\theta_{13})\left(\cos^2(\theta_{12})\sin^2\left(\frac{\Delta m^2_{31}}{4}\frac{L}{E}\right) + \sin^2(\theta_{12})\sin^2\left(\frac{\Delta m^2_{32}}{4}\frac{L}{E}\right)\right)$$

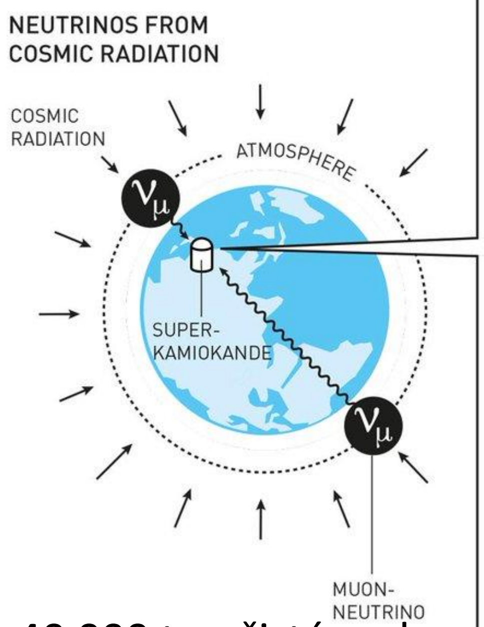


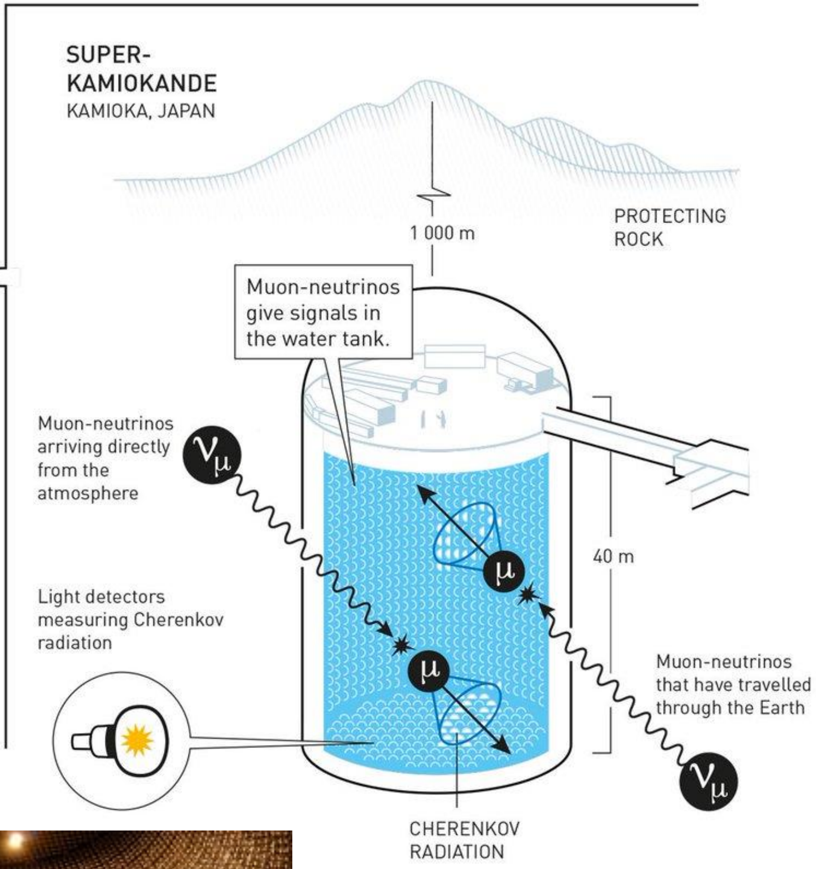
Neutrinos from extended source
Poor resolution in the measurement of E/L
Decoherence

Due to all these effects we measure distorted oscillation curve, ultimately only the mean value between maximum and minimum of oscillations

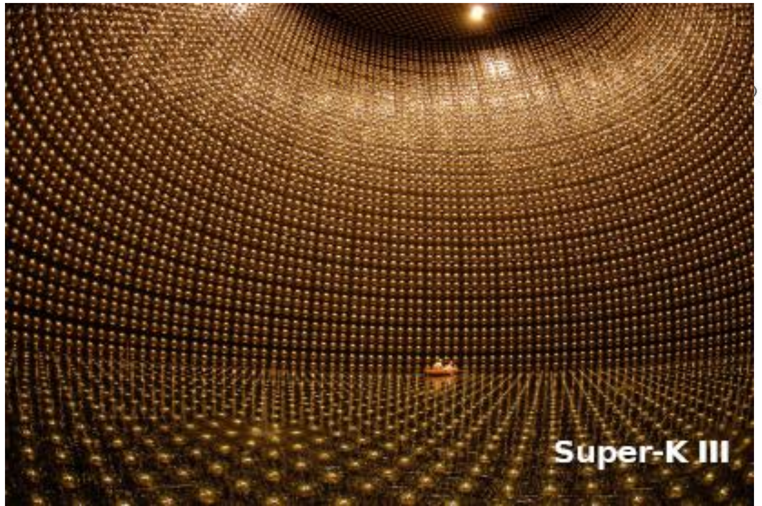
Výsledek ze SuperKamiokande – deficit mionových neutrin

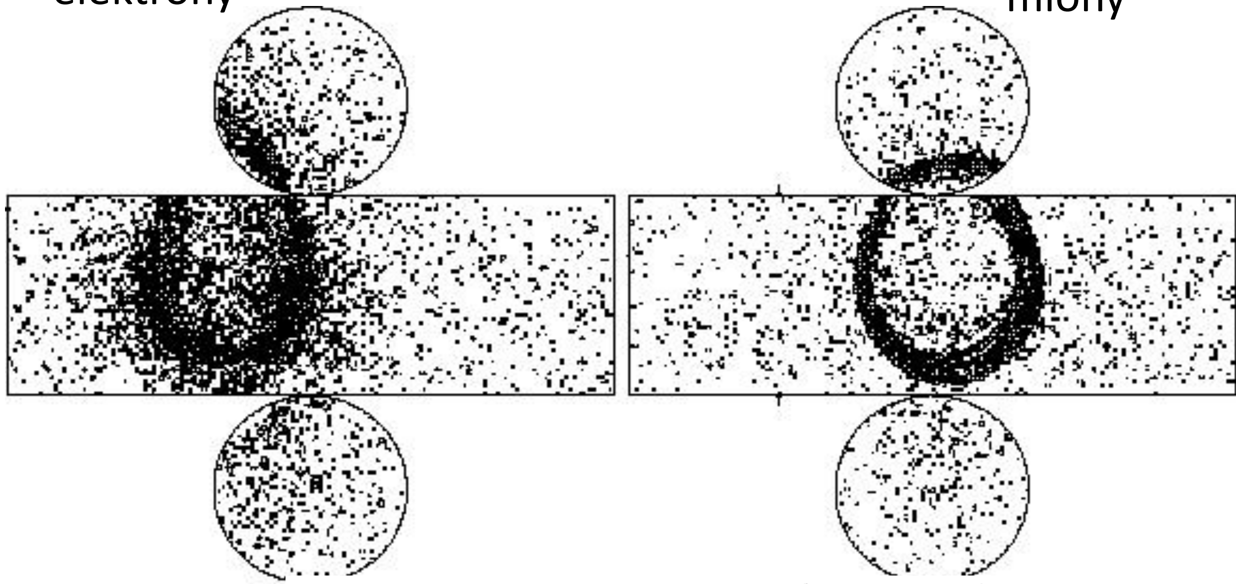
T. Kajita



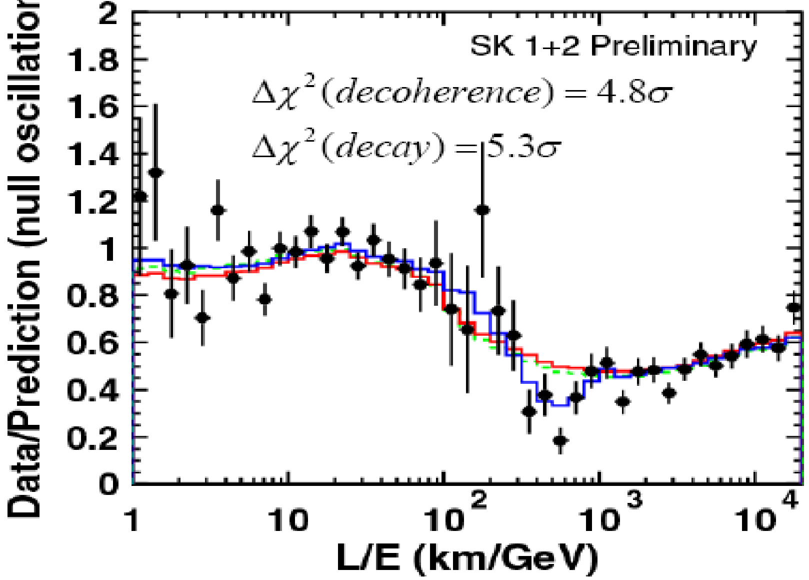


40 000 tun čisté vody a 11 000 20 palcových fotonásobičů



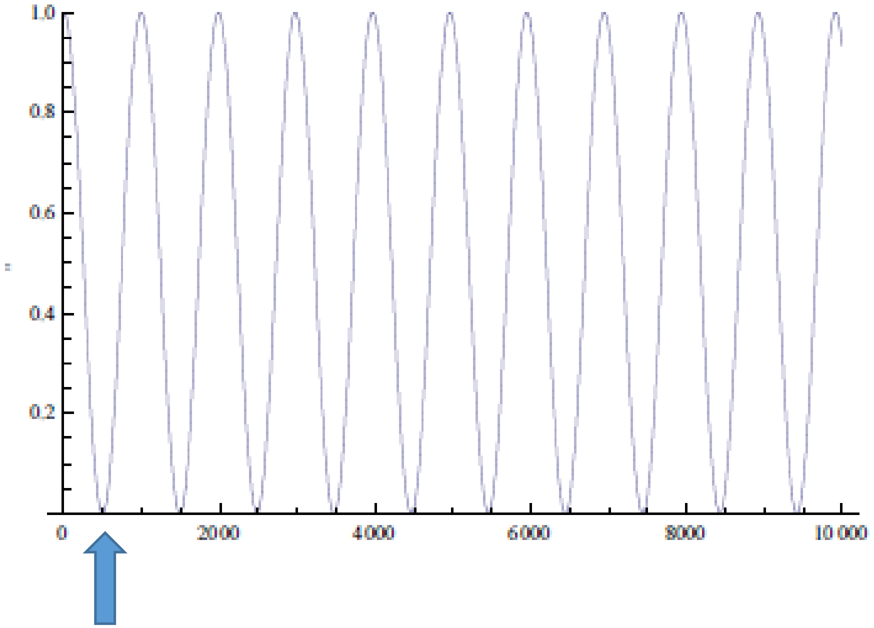


$$P_{\mu\tau} = \sin^2 2\theta \sin^2\left(1.27\,\Delta m^2 \frac{L}{E}\right)$$



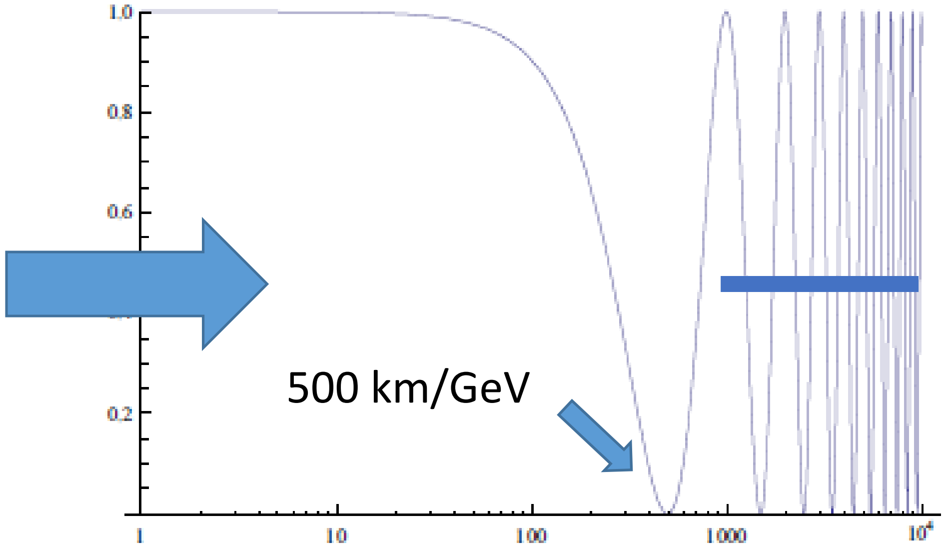
Experiment SuperKamiokande byl původně určen pro hledání rozpadu protonu. Očekávalo se několik rozpadů za rok a interakce neutrin byly hlavním pozadím.

P



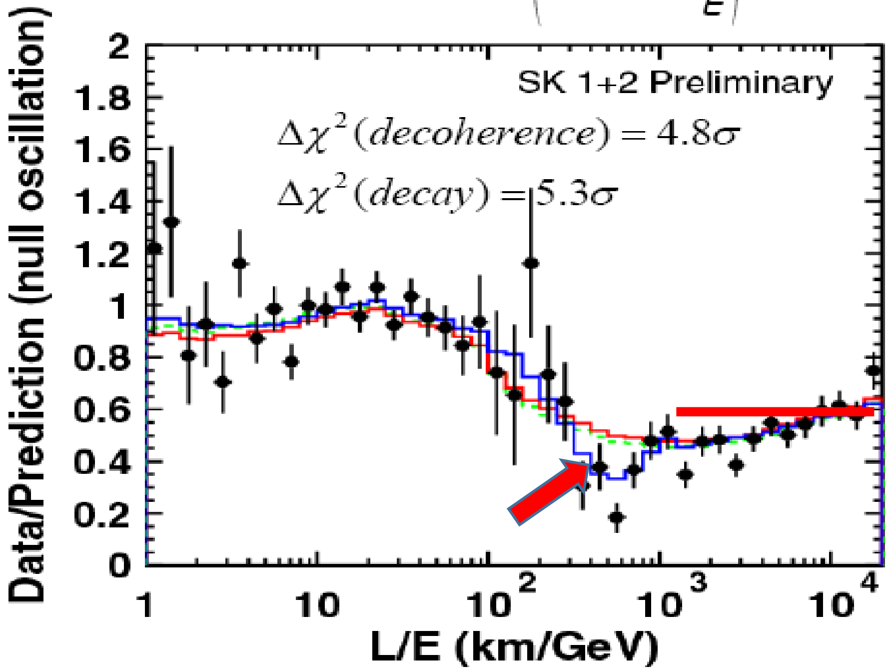
500 km/GeV

L/E [km/GeV]



Kvůli neurčitostem v určení energi a vzdálenosti měříme v této oblasti pouze střední hodnotu

$$P_{\mu\tau} = \sin^2 2\theta \sin^2\left(1.27\,\Delta m^2 \frac{L}{E}\right)$$



Z polohy minima 500 km/GeV plyne:

$$1{,}27 \cdot \Delta M^2[eV^2] \cdot 500 km/GeV = \pi/2$$

$$\Delta M^2[eV^2] = 2{,}5 \cdot 10^{-3} eV^2 = (50\, meV)^2$$

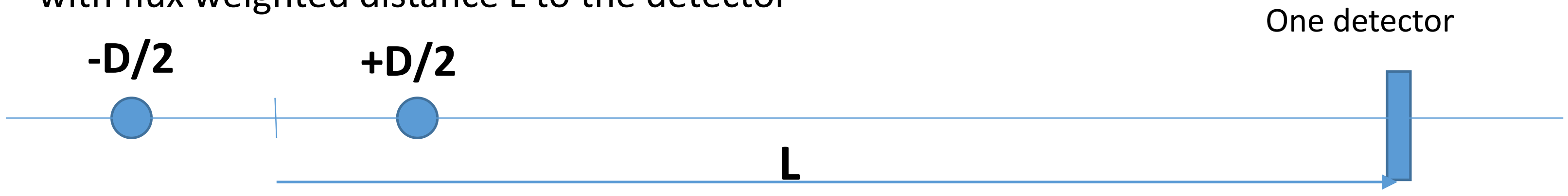
Z toho, že je střední hodnota mezi maximem a minimem oscilací rovna 0,5 plyne, že

$\theta \cong 45°$ Tyto hodnoty byly upřesněny v experimentech MINOS a T2K s neutriny z urychlovačů.

The oscillation curves in real experiment are often modified because of

- multiple sources
- extended neutrino sources or detectors
- the E and L are measured with limited precision

Simplest case – two equal sources
with flux weighted distance L to the detector

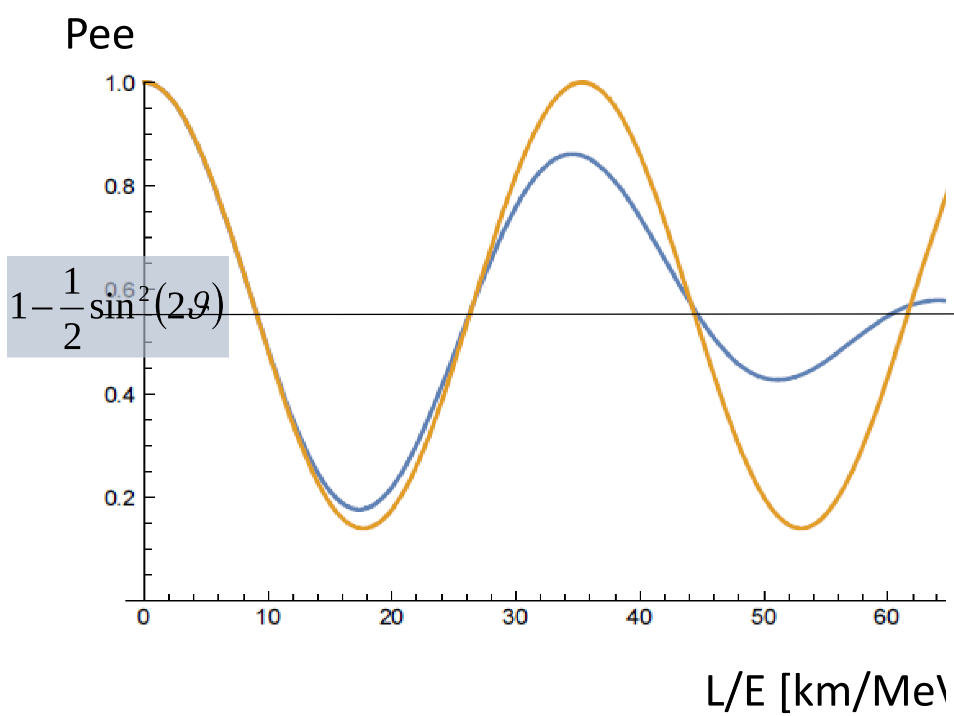


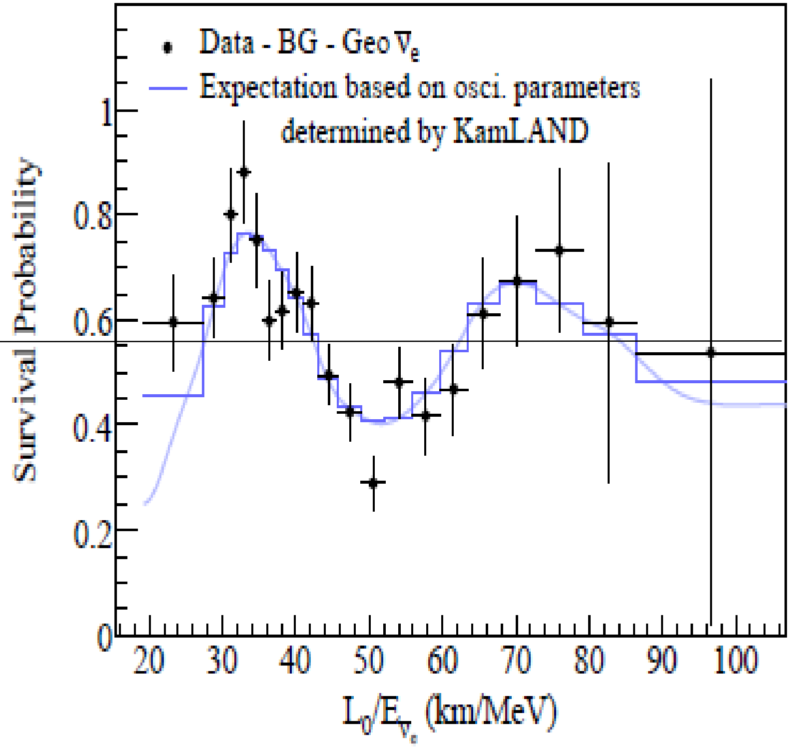
$$P_2ee\left(\frac{L}{E}\right)=\frac{1}{2}\left(Pee\left(\frac{L+D/2}{E}\right)+Pee\left(\frac{L-D/2}{E}\right)\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\frac{\cos\left(2\frac{\Delta m^2}{4\eta c}\frac{L+D/2}{E}\right)+\cos\left(2\frac{\Delta m^2}{4\eta c}\frac{L-D/2}{E}\right)}{2}=$$

$$P_2ee\left(\frac{L}{E}\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\eta c}\frac{L}{E}\right)\cos\left(\frac{\Delta m^2}{4\eta c}\frac{D}{L}\frac{L}{E}\right)\neq 1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\eta c}\frac{L}{E}\right)=P_1ee\left(\frac{L}{E}\right)$$

$$Pee\left(\frac{L}{E},\frac{D}{L}\right)=1-\frac{1}{2}\sin^2(2\vartheta)+\frac{1}{2}\sin^2(2\vartheta)\cos\left(\frac{\Delta m^2}{2\eta c}\frac{L}{E}\right)\cos\left(\frac{\Delta m^2}{4\eta c}\frac{D}{L}\frac{L}{E}\right)$$

Two sourreactors 170 and 190 km from the detector





Problem Calculate the disappearance electron neutrino probability Pee(L/E) for following cases. **A)** The source that extends from –D/2 to D/2 and has constant linear power density 1/D; **B)** One source, the variables L and E in are measured with a Gaussian resolutions σL, σE.

COHERENCE

DECOHERENCE

Problem A. Calculate differences in arrival times (ct) of ν1 ν2, ν1 ν3 for 4 MeV electron neutrinos at distances of 2 km, 150 mil. km, 150 k light-years.

B. Evaluate the disappearance Pee and appearance Pemu and Petau probabilities in the case of full decoherence. Check that the sum of probabilities is equal to 1.

$$\frac{\Delta x}{x} = \Delta\beta = \frac{\Delta m^2}{2E^2}$$

4 MeV neutrinos

1 GeV neutrinos

$$\Delta\beta_{31} = \frac{2.5\cdot 10^{-3} eV^2}{2(4\cdot 10^6 eV)^2} \cong 0.78\cdot 10^{-16} \qquad \Delta\beta_{31} = \frac{2.5\cdot 10^{-3} eV^2}{2(10^9 eV)^2} \cong 1.25\cdot 10^{-21}$$

$$\Delta\beta_{21} = \frac{7.5\cdot 10^{-5} eV^2}{2(4\cdot 10^6 eV)^2} \cong 0.23\cdot 10^{-17} \qquad \Delta\beta_{31} = \frac{7.5\cdot 10^{-5} eV^2}{2(10^9 eV)^2} \cong 3.75\cdot 10^{-23}$$

$$x = 2km \Rightarrow \Delta x = 0.23\cdot 10^{-17}\cdot 2\cdot 10^{18}\ fm = 4.6\ fm$$

2 km from reactor

$$x = 150000000km \Rightarrow \Delta x = 0.78\cdot 10^{-16}\cdot 1.5\cdot 10^{26}\ fm = 11.7\cdot 10^{9}\ fm = 11.7\,\mu m$$

Sun

$$x = 150000ly \Rightarrow \Delta x = 0.78\cdot 10^{-16}\cdot 1.5\cdot 10^{5}\cdot 3\cdot 10^{7}\cdot 3\cdot 10^{5}\,km = 105km$$

Supernova

Accelerator 1GeV nu 800 km

$$x = 800km \Rightarrow \Delta x = 3.75\cdot 10^{-23}\cdot 800\cdot 10^{18}\ fm = 0.03\ fm$$

Mass hierarchy

$$P_{\nu\to\nu}(L/E) = 1 - \sin^2(2\theta)\sin^2\left(\frac{\Delta m^2}{4\eta c}\frac{L}{E}\right)$$

Oscillation curve is not sensitive to the sign of Dm2

How we have learned that m2>m1?

How we could determine mass hierarchy, i.e. m3>m1,m2 or m3<m1,m2

$$i\eta c\partial_x \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} = -\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix}$$

$$\begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} = \begin{pmatrix} e^{+i\Delta m_{21}^2 x/4\eta cE} & 0 \\ 0 & e^{-i\Delta m_{21}^2 x/4\eta cE} \end{pmatrix} \begin{pmatrix} \nu_1(0) \\ \nu_2(0) \end{pmatrix}$$

Oscillations in vacuum

$$i\eta c\partial_x \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix} = -\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_1(x) \\ \nu_2(x) \end{pmatrix}$$

$$i\eta c\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = -\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ -\sin 2\theta & -\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = +\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

Oscillations in matter with a constant electron density

$$i\eta c\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \left(+\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + V \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \right) \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

$$i\eta c\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \left(+\frac{\Delta m_{21}^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \frac{V}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right) \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

$$V = (\eta c)^3 \sqrt{2} G_F N_e$$

$$\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = +i\frac{\Delta m_{21}^2}{4\eta cE} \begin{pmatrix} \cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2} & \sin 2\theta \\ \sin 2\theta & -\left(\cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2}\right) \end{pmatrix} \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

$$\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = +i\frac{\Delta m_{21}^2\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2}\right)^2}}{4\eta cE\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2}\right)^2}} \begin{pmatrix} \left(\cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2}\right) & \sin 2\theta \\ \sin 2\theta & -\left(\cos 2\theta - \frac{4\eta cE}{\Delta m_{21}^2}\frac{V}{2}\right) \end{pmatrix} \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

$$\partial_x \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = +i\frac{\Delta M_{21}^2}{4\eta cE} \begin{pmatrix} \cos 2\Theta & \sin 2\Theta \\ \sin 2\Theta & -\cos 2\Theta \end{pmatrix} \begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix}$$

Oscillation parameters in matter with a constant electron density are sensitive to the sign of Dm12^2.

$$V = (\eta c)^3 \sqrt{2} G_F N_e$$

$$\cos 2\Theta = \frac{\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V}{2}\right)^2}} \qquad \sin 2\Theta = \frac{\sin 2\theta}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V}{2}\right)^2}}$$

$$\Delta M_{21}^2(x) = \Delta m_{21}^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V}{2}\right)^2}$$

Rotate mass eigenstates back to the flavor states at the detector

Transport the mass eigenstates to the detector

Rotate to the mass eigenstates at the source

Initial flavor state at the source

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{+i\Delta m_{21}^2 x/4\eta cE} & 0 \\ 0 & e^{-i\Delta m_{21}^2 x/4\eta cE} \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \left(\cos\left(\Delta m_{21}^2 x/4\eta cE\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - i\sin\left(\Delta m_{21}^2 x/4\eta cE\right) \begin{pmatrix} -\cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & \cos(2\theta) \end{pmatrix} \right) \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\left(\Delta m_{21}^2 x/4\eta cE\right) + i\sin\left(\Delta m_{21}^2 x/4\eta cE\right)\cos(2\theta) \\ -i\sin\left(\Delta m_{21}^2 x/4\eta cE\right)\sin(2\theta) \end{pmatrix}$$

$$P_{\nu_e \to \nu_e}(x) = \nu_e^*(x)\nu_e(x) = 1 - \sin^2(2\theta)\sin^2\left(\Delta m_{21}^2 x/4\eta cE\right)$$

$$P_{\nu_e \to \nu_{\mu\tau}}(x) = \nu_e^*(x)\nu_e(x) = \sin^2(2\theta)\sin^2\left(\Delta m_{21}^2 x/4\eta cE\right)$$

SOLUTIONS IN VACUUM OR MATTER WITH A CONSTANT DENSITY

Solution for variable matter density

$$\Delta M_{21}^2(x) = \Delta m_{21}^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2} \qquad \cos 2\Theta(x) = \frac{\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}} \qquad \sin 2\Theta(x) = \frac{\sin 2\theta}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}}$$

In so called adiabatic approximation, the solution is:

Rotate mass eigenstate to the flavor

Transport the mass eigenstates to the detector

Transform the initial state to mass eigenstates at the source

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\Theta(x) & \sin\Theta(x) \\ -\sin\Theta(x) & \cos\Theta(x) \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta M_{21}^2(y)dy/4\eta cE} & 0 \\ 0 & e^{-i\int_0^x \Delta M_{21}^2(y)dy/4\eta cE} \end{pmatrix} \begin{pmatrix} \cos\Theta(0) & -\sin\Theta(0) \\ \sin\Theta(0) & \cos\Theta(0) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

Compare to the solution at vacuum or constant mass density

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} e^{+i\Delta M_{21}^2 x/4\eta cE} & 0 \\ 0 & e^{-i\Delta M_{21}^2 x/4\eta cE} \end{pmatrix} \begin{pmatrix} \cos\Theta & -\sin\Theta \\ \sin\Theta & \cos\Theta \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\Theta(x) & \sin\Theta(x) \\ -\sin\Theta(x) & \cos\Theta(x) \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta M_{21}^2(y)dy/4\eta cE} & 0 \\ 0 & e^{-i\int_0^x \Delta M_{21}^2(y)dy/4\eta cE} \end{pmatrix} \begin{pmatrix} \cos\Theta(0) & -\sin\Theta(0) \\ \sin\Theta(0) & \cos\Theta(0) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\left(\int_0^x \Delta m_{21}^2(y)dy/4\eta cE\right) \begin{pmatrix} \cos(\Theta(x)-\Theta(0)) & \sin(\Theta(x)-\Theta(0)) \\ -\sin(\Theta(x)-\Theta(0)) & \cos(\Theta(x)-\Theta(0)) \end{pmatrix} \\ -i\sin\left(\int_0^x \Delta m_{21}^2(y)dy/4\eta cE\right) \begin{pmatrix} -\cos(\Theta(x)+\Theta(0)) & \sin(\Theta(x)-\Theta(0)) \\ \sin(\Theta(x)-\Theta(0)) & \cos(\Theta(x)-\Theta(0)) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

In vacuum or matter with constant density

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\left(\Delta M_{21}^2 x/4\eta cE\right) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ -i\sin\left(\Delta M_{21}^2 x/4\eta cE\right) \begin{pmatrix} -\cos(2\Theta) & \sin(2\Theta) \\ \sin(2\Theta) & \cos(2\Theta) \end{pmatrix} \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

How we have learned that m2>m1. Solar neutrinos

$$\cos 2\Theta(x) = \frac{\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}} \quad \sin 2\Theta(x) = \frac{\sin 2\theta}{\sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}}$$

$$\Delta M_{21}^2(x) = \Delta m_{21}^2 \sqrt{\sin^2 2\theta + \left(\cos 2\theta - \frac{4E\eta c}{\Delta m_{21}^2}\frac{V(x)}{2}\right)^2}$$

$$\left|\frac{4E\eta c}{\Delta m_{21}^2}\frac{V(0)}{2}\right| >> 1 \Rightarrow \cos 2\Theta(0) = -\mathrm{sgn}\left(\frac{4E\eta c}{\Delta m_{21}^2}\frac{V(0)}{2}\right) = \mu\, \mathrm{sgn}\, \Delta m_{21}^2 \Big|_{antineutrinos}^{neutrinos}$$

$$neutrinos:$$

$$\Delta m_{21}^2 > 0 \Rightarrow \cos 2\Theta(x) = -1 \Rightarrow \Theta(x) = \pi/2 \Rightarrow \nu_e = \nu_2$$

$$\Delta m_{21}^2 < 0 \Rightarrow \cos 2\Theta(x) = +1 \Rightarrow \Theta(x) = 0 \Rightarrow \nu_e = \nu_1$$

Electron neutrinos = heavier of the two mass eigenstates

Electron antineutrinos = lighter of the two mass eigenstates

How we have learned that m2>m1. Solar neutrinos

$$E \approx 1MeV \Rightarrow \left| \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2} \right| << 1$$

Low energies: oscillations like in vacuum, due to extended source of neutrinos we measure the mean value between oscillation maximum and minimum $1-\frac{1}{2}\sin^2(2\theta)$

$$E > 10MeV \Rightarrow \left| \frac{4E\hbar c}{\Delta m_{21}^2} \frac{V(x)}{2} \right| >> 1$$

$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\Theta(x) & \sin\Theta(x) \\ -\sin\Theta(x) & \cos\Theta(x) \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} & 0 \\ 0 & e^{-i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} \end{pmatrix} \begin{pmatrix} \cos\Theta(0) & -\sin\Theta(0) \\ \sin\Theta(0) & \cos\Theta(0) \end{pmatrix} \begin{pmatrix} \nu_e(0) \\ \nu_{\mu\tau}(0) \end{pmatrix}$$

Rotated by vacuum mixing angle

Transported outside the Sun

Electron neutrinos at the centre of Sun

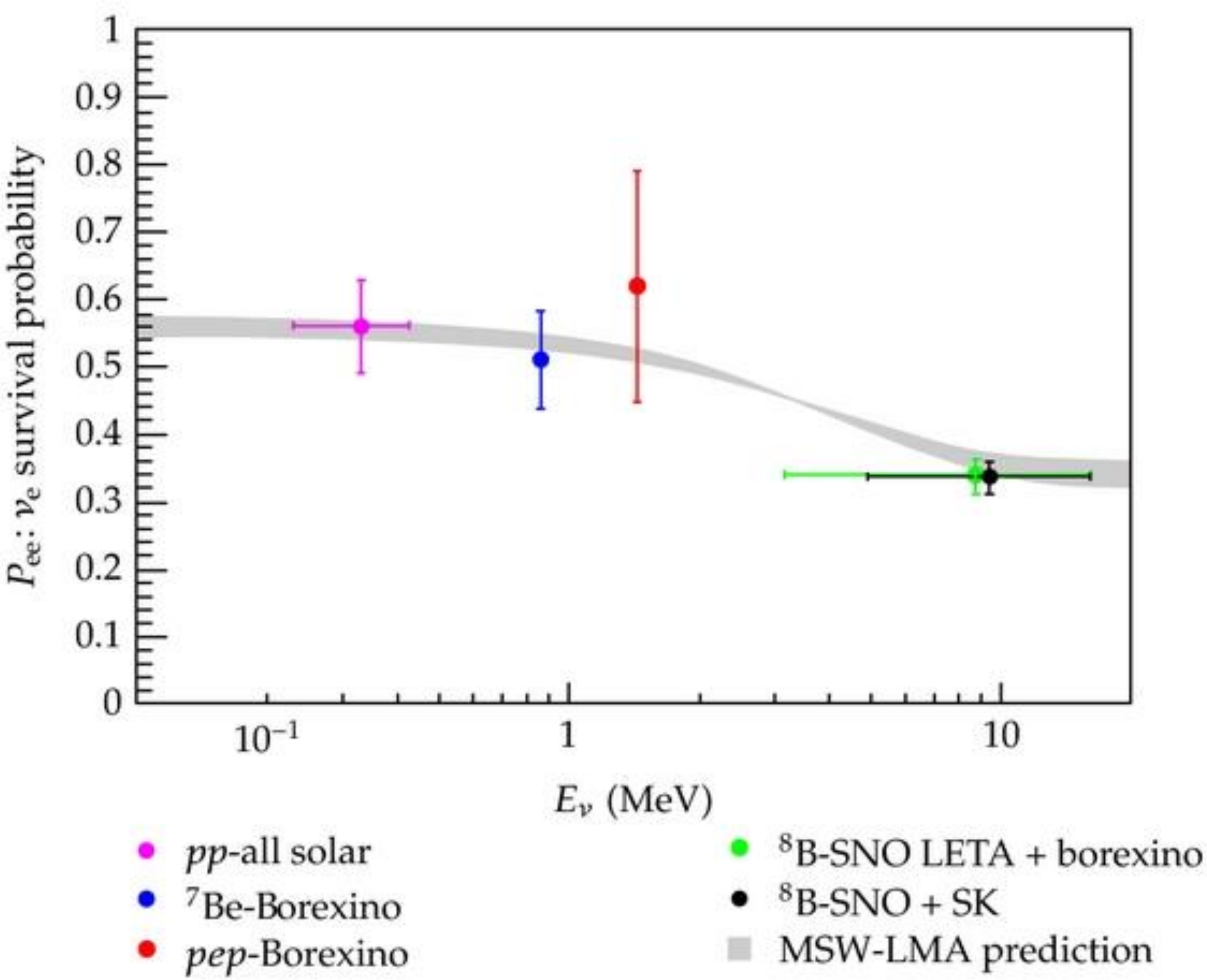
$$\begin{pmatrix} \nu_e(x) \\ \nu_{\mu\tau}(x) \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{+i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} & 0 \\ 0 & e^{-i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= e^{-i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = e^{-i\int_0^x \Delta m_{21}^2(y)dy/4\hbar cE} \begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix}$$

High energy neutrinos leave the Sun in mass eigenstate m2, we will measure $\sin^2(\theta)$

$$1-\frac{1}{2}\sin^2(2\theta)\cong 0.57$$

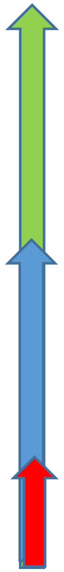
$$\Rightarrow \theta \cong 34.9^o$$

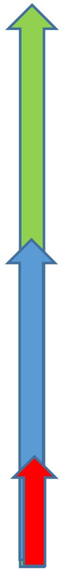


$$\sin^2(\theta)\cong 0.34$$

$$\Rightarrow \theta \cong 35.7^o$$

Problem XX: Why we will really get different values of theta12? Hint – assume 3x3 neutrino mixing.

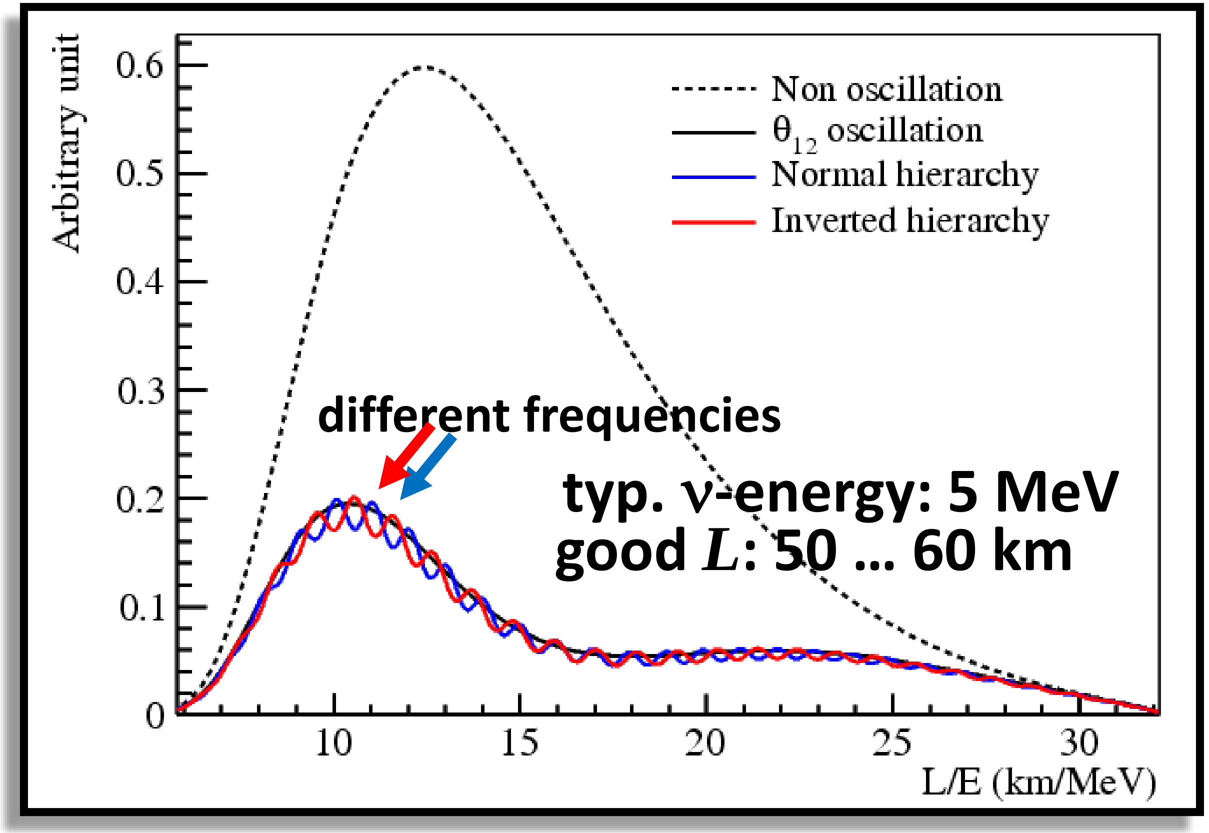




$$A_{\nu_e \to \nu_e}(x) = e^{-i\frac{m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e1}\right|^2 + e^{-i\frac{m_2^2}{2\hbar c}\frac{x}{E}}\left|U_{e2}\right|^2 + e^{-i\frac{m_3^2}{2\hbar c}\frac{x}{E}}\left|U_{e3}\right|^2$$

$$= e^{-i\frac{m_1^2}{2\hbar c}\frac{x}{E}}\left(\left|U_{e1}\right|^2 + e^{-i\frac{m_2^2-m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e2}\right|^2 + e^{-i\frac{m_3^2-m_1^2}{2\hbar c}\frac{x}{E}}\left|U_{e3}\right|^2\right)$$

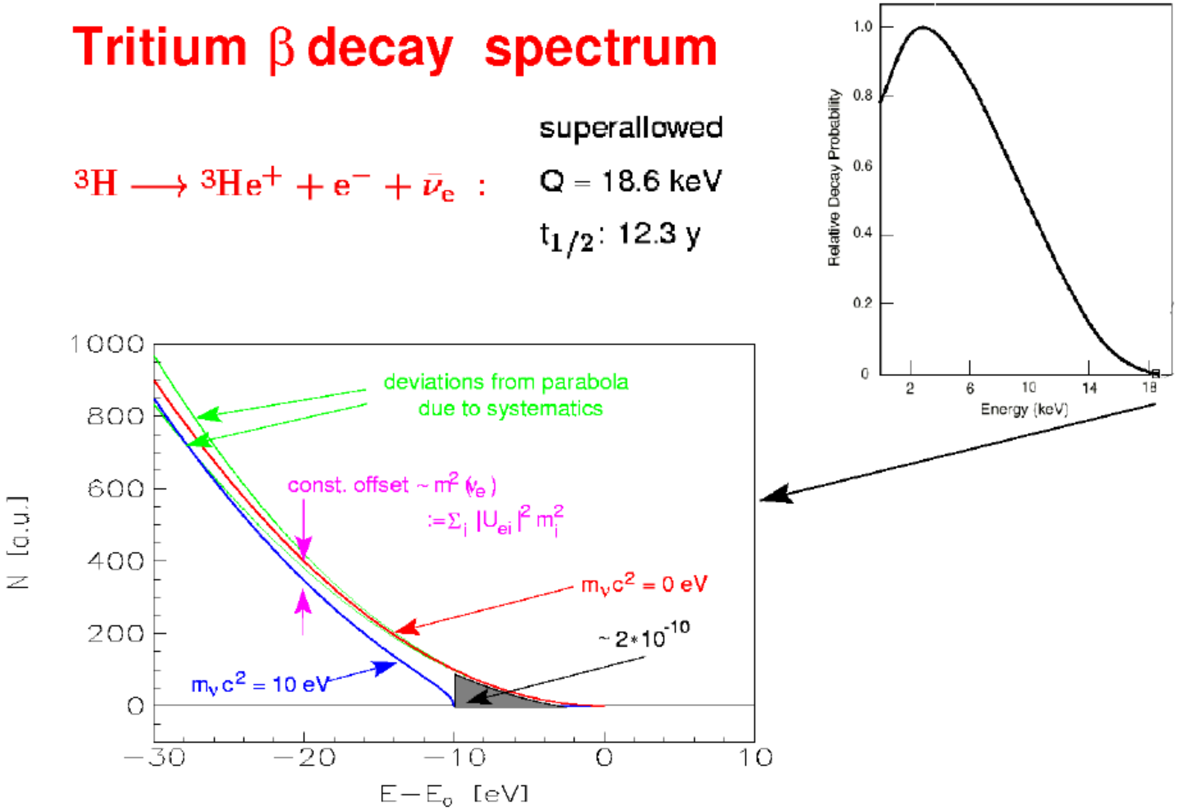
Expected spectrum in future JUNO experiment



NEUTRINO MASSES

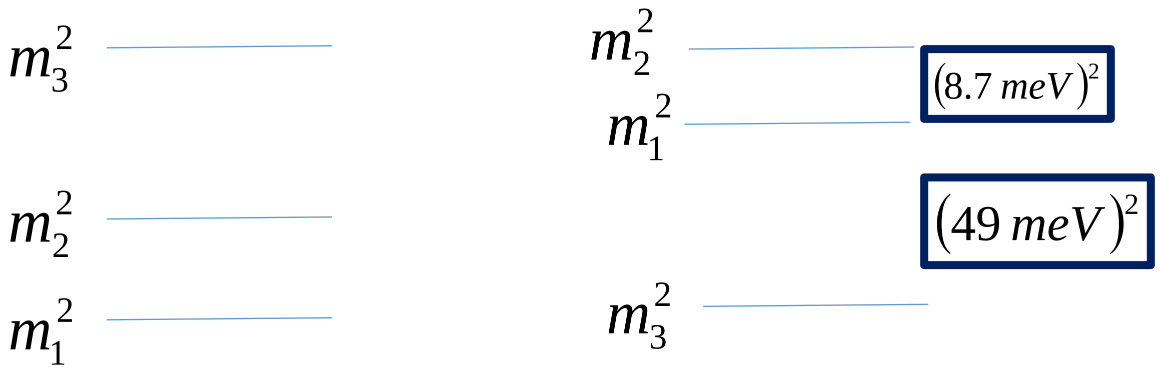
Co víme o velikosti hmot neutrin

Neutrina byla objevena před více než 60ti lety, ale dodnes neznáme jejich hmoty, máme pouze horní hranice.



Z přesného měření konce spektra elektronů z rozpadu tritia víme, že hmota elektronových antineutrin je menší než 2 eV, právě zahajovaný experiment KATRIN má ambici tuto hranici snížit na 0,2 eV

Výsledky oscilačních experimentů dávají dolní hranici na hmoty neutrin, konkrétně
m2 > 0,009 eV, m3>0,05 eV pro tzv. normální hierarchii hmot neutrin a
m1 i m2> 0,05 eV pro inverzní



$$X \to Y + e^- + \bar{\nu}_e$$

$$dN/dE \cong P_e \cdot E_e \cdot p_{\bar{\nu}} \cdot e_{\bar{\nu}} \qquad T_e + t_{\bar{\nu}} = Q \equiv M_X - M_Y + M_e + m_{\bar{\nu}}$$

$$dN/dE \cong \sqrt{(T_e + M_e)^2 - M_e^2} \cdot (T_e + M_e) \cdot \sqrt{(t_{\bar{\nu}} + m_{\bar{\nu}})^2 - m_{\bar{\nu}}^2} \cdot (t_{\bar{\nu}} + m_{\bar{\nu}})$$

dN/dT_e

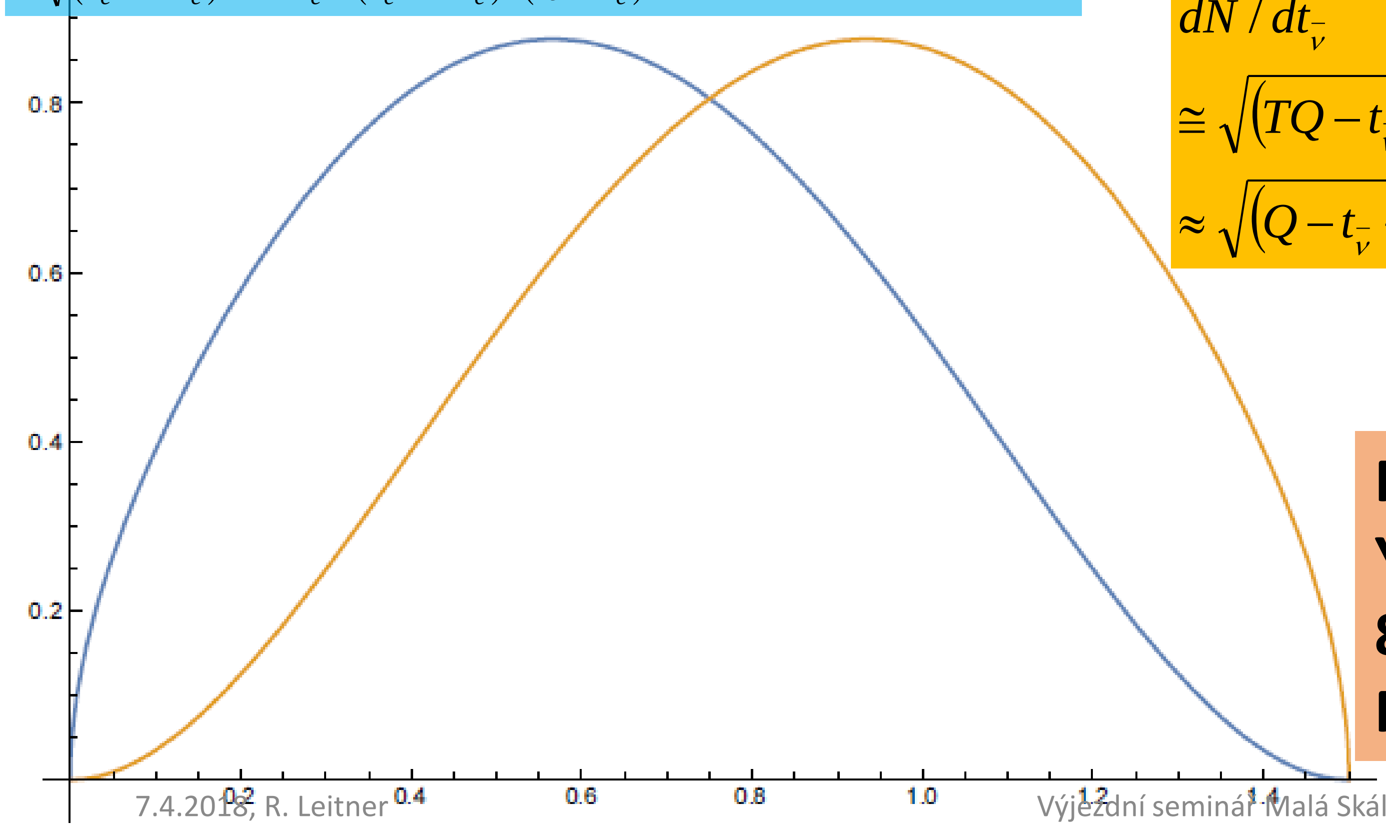
$$\approx \sqrt{(T_e + M_e)^2 - M_e^2} \cdot (T_e + M_e) \cdot \sqrt{(Q - T_e + m_{\bar{\nu}})^2 - m_{\bar{\nu}}^2} \cdot (Q - T_e + m_{\bar{\nu}})$$

$$\approx \sqrt{(T_e + M_e)^2 - M_e^2} \cdot (T_e + M_e) \cdot (Q - T_e)^2$$

$dN/dt_{\bar{\nu}}$

$$\cong \sqrt{(TQ - t_{\bar{\nu}} + M_e)^2 - M_e^2} \cdot (Q - t_{\bar{\nu}} + M_e) \cdot \sqrt{(t_{\bar{\nu}} + m_{\bar{\nu}})^2 - m_{\bar{\nu}}^2} \cdot (t_{\bar{\nu}} + m_{\bar{\nu}})$$

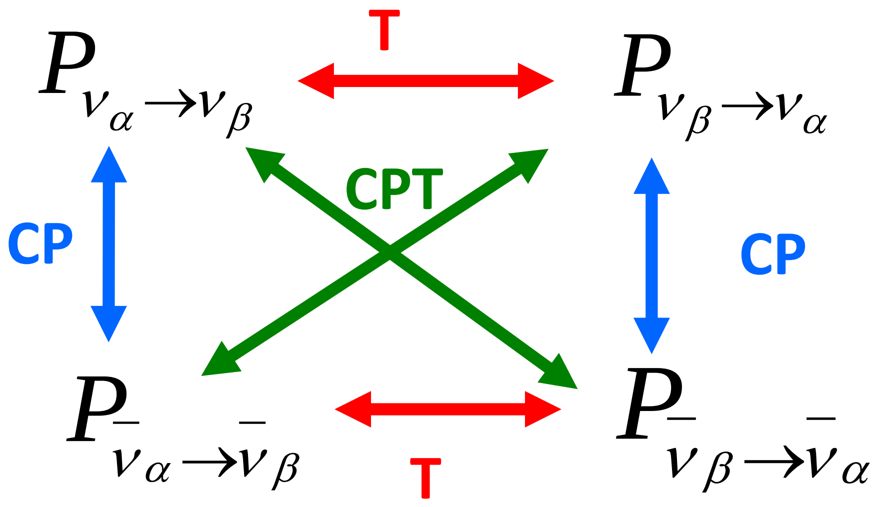
$$\approx \sqrt{(Q - t_{\bar{\nu}} + M_e)^2 - M_e^2} \cdot (Q - t_{\bar{\nu}} + M_e) \cdot t_{\bar{\nu}}^2$$



PROBLEM

You measured 45 events at Te=0,1 MeV, 80 ev. at Te=0,2 MeV, 120 ev. at Te=0,3 MeV. Determine the electron mass.

CP and T violation in neutrino oscillations



Combination of reactor and accelerator experiments.

Accelerator experiments measure appearance of electron neutrinos in muon neutrino beam

$$P_{\nu\mu\to\nu e} = f\left(\sin^2(2\theta)_{13}, \sin(\delta)\right) \Rightarrow \sin^2(2\theta)_{13} = g\left(P_{\nu\mu\to\nu e}, \sin(\delta)\right)$$

Reactor experiments measure theta13

Combination of both measurement is sensitive to CP violating phase delta