Cutoff Dependence of the Quark Mass Paramter in Angular Ordered Parton Showers

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arXiv:1805.xxxxx

LHC Top Quark WG Meeting, CERN, May 16, 2018

Aim of this Work

Solid field theoretic understanding of how the interpretation of the massive quark appearing in parton showers depends on the shower cut Q_0.

Parton shower: carries out perturbative calculations with a finite IR cutoff. → lattice QCD

- Want to understand mass of the top quark state (= top + gluons around) that is produced in the hard interaction (do not address issues related to decay)

- Do not address finite lifetime issues

  adopt narrow width approximation as used in state of the art MCs (factorization of production and decay)

- Questions we want to address:

  (A) Can a state of the art parton shower in principle describe the single top mass threshold (resonance) with NLO precision?

  (B) What is the impact of the shower cut Q_0 on the peak of jet masses?

  (C) Does the shower cut imply that the MC top mass is a short-distance mass?

Observable

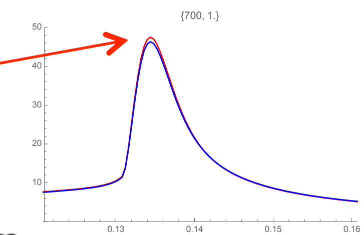
- Want to study the mass of the top state (i.e. top + gluons) produced in the hard interaction (quasi-coll. limit → boosted top quarks)

- Want an observable that is not sensitive to differential details of the top decay or further branching of daughter particles (e.g. $b \to b + g$)

- Here we look at the sum of the hemisphere masses (w.r. to the thrust axis) in e^+e^-

$$\tau \equiv \frac{M_1^2 + M_2^2}{Q^2}$$

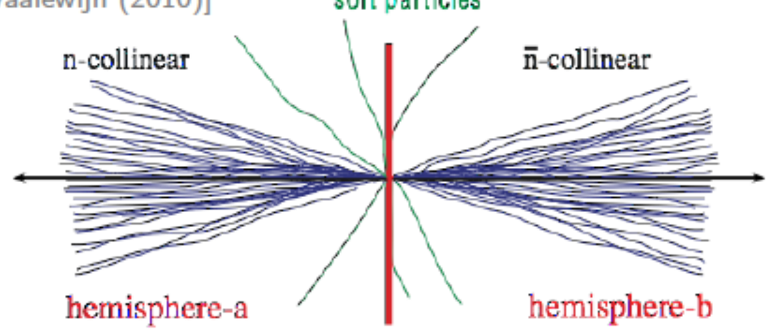
Peak position



$\approx$ thrust in the resonance region (2-jettiness in the massive case)

[Farhi (1977)] [Stewart, Tackmann, Waalewijn (2010)]

- Study e^+e^- for simplicity, but results are general



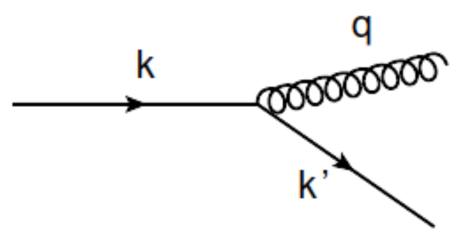
Coherent Branching

Dokshitzer, Fadin, Khoze (1982)
Bassetto, Ciafaloni, Marchesini (1983)
Catani, Marchesini, Webber (1991)

$\rightarrow$ default shower in Herwig

$$k'^\mu = zk^- \frac{n^\mu}{2} + \frac{k'^2 - q_\perp^2}{zk^-}\frac{\overline{n}^\mu}{2} - q_\perp^\mu$$

$$q^\mu = (1-z)k^- \frac{n^\mu}{2} + \frac{q^2 - q_\perp^2}{(1-z)k^-}\frac{\overline{n}^\mu}{2} + q_\perp^\mu$$



momentum conservation:

$$k^2 = \frac{k'^2}{z} + \frac{q^2}{1-z} + \frac{q_\perp^2}{z(1-z)}$$

evolution variable: $\tilde{q}^2 = \frac{q_\perp^2}{z^2(1-z)^2}$

color coherence of soft gluon emissions $\rightarrow$ angular ordering: $z_i^2 \tilde{q}_i^2 > \tilde{q}_{i+1}^2$

$\rightarrow$ jet mass distribution (inv. mass generated from CB from one hard quark)

$k^2 \approx$ hemisphere mass (does not account for out of cone radiation)

$$J(Q^2,k^2) = \delta(k^2) + \int_0^{Q^2}\frac{d\tilde{q}^2}{\tilde{q}^2}\int_0^1 dz\, P_{qq}\Big[\alpha_s\big(z(1-z)\tilde{q}\big), z\Big]$$
$$\times \left[\int_0^\infty dk'^2 \int_0^\infty dq^2 \delta\Big(k^2 - \frac{k'^2}{z} - \frac{q^2}{1-z} - z(1-z)\tilde{q}^2\Big) J(z^2\tilde{q}^2, k'^2) J_g((1-z)^2\tilde{q}^2, q^2) - J(\tilde{q}^2, k^2)\right]$$

Coherent Branching

Thrust (and jet mass) at resonance peak insensitive to gluon splitting

$$\rightarrow \quad J(Q^2,k^2) = \delta(k^2) + \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{qq}\Big[\alpha_s\big(z(1-z)\tilde{q}\big), z\Big] \Big(zJ(z^2\tilde{q}^2, zk^2 - z^2(1-z)\tilde{q}^2) - J(\tilde{q}^2, k^2) \Big)$$

Partonic cross section

$$\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau} = \int \mathrm{d}k^2\,\mathrm{d}k'^2\,\delta\big(\tau - \frac{k^2+k'^2}{Q^2}\big)\, J(Q^2,k^2) J(Q^2,k'^2)$$

Splitting function

$$P_{qq}\left[\alpha_s(\mu), z\right] = \frac{1}{(1-z)} \frac{\alpha_s(\mu)}{4\pi} \left(\Gamma_0^{\mathrm{cusp}} + \frac{\alpha_s}{4\pi} \Gamma_1^{\mathrm{cusp}}\right) - \frac{\alpha_s(\mu) C_F}{2\pi} (1+z)$$

CMW scheme for α_S: absorbs 2-loop cusp into strong coupling

$$P_{qq}\left[\alpha_s(\mu), z\right] = \frac{\alpha_s^{\mathrm{CMW}}(\mu) C_F}{2\pi} \frac{1+z^2}{1-z}$$

Renormalization scale of α_S

$$\mu^2 = q_\perp^2 = z^2(1-z)^2\tilde{q}^2$$

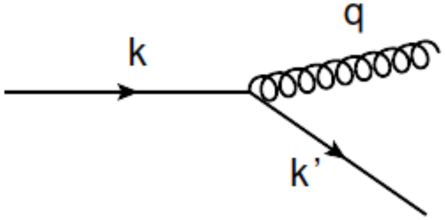
Coherent Branching for Massive Quarks

Gieseke, Stephens, Webber (2003)

→ as implemented in Herwig

momentum conservation:

$$k^2 - m^2 = \frac{k'^2 - m^2}{z} + \frac{q^2}{1-z} + \frac{q_\perp^2 + m^2(1-z)^2}{z(1-z)}$$



ordering variable:

$$\tilde{q}^2 = \frac{q_\perp^2 + m^2(1-z)^2}{z^2(1-z)^2}$$

massive splitting function:

$$P_{QQ}[\alpha_s, z, m] = P_{qq}[\alpha_s, z] - \frac{\alpha_s C_F}{\pi} \frac{m^2}{z(1-z)\tilde{q}^2}$$

scale in α_S: $z^2(1-z)^2\tilde{q}^2$ cutoff: $q_\perp > Q_0$

$$\rightarrow \quad J(Q^2, k^2 - m^2, m^2) = \delta(k^2 - m^2)$$
$$+ \int_{m^2}^{Q^2} \frac{d\tilde{q}^2}{\tilde{q}^2} \int_0^1 dz\, P_{QQ}[\alpha_s(z(1-z)\tilde{q}), z, m]\, \theta\Big(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\Big)$$
$$\times \Big[zJ\big(z^2\tilde{q}^2, z(k^2-m^2) - z^2(1-z)\tilde{q}^2\big) - J\big(\tilde{q}^2, k^2 - m^2\big)\Big]$$

Resummation in CB

Catani, Trentadue, Turnock Webber (1993)

At NLL we can expand z→1 (not in splitting function)

$$\to J(Q^2,k^2) = \delta(k^2) + \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{qq}\Big[\alpha_s\big((1-z)\tilde{q}\big), z\Big] \Big(J(\tilde{q}^2, k^2 - (1-z)\tilde{q}^2) - J(\tilde{q}^2, k^2)\Big)$$

Go to Laplace space

$$\begin{aligned}\tilde{J}(Q^2,\nu) &= \int_0^\infty \mathrm{d}k^2\, \mathrm{e}^{-\frac{\nu k^2}{Q^2}} J(Q^2,k^2) \\ &= 1 + \int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{qq}\Big[\alpha_s\big((1-z)\tilde{q}\big), z\Big] \Big(\mathrm{e}^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}} - 1\Big) \tilde{J}(\tilde{q}^2,\nu)\end{aligned}$$

Jet mass function exponentiates

$$\tilde{J}(Q^2,\nu) = \exp\left[\int_0^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_0^1 \mathrm{d}z\, P_{qq}\Big[\alpha_s\big((1-z)\tilde{q}\big), z\Big] \Big(\mathrm{e}^{\frac{-\nu(1-z)\tilde{q}^2}{Q^2}} - 1\Big)\right]$$

Sum of hemisphere masses:

Convolution in k-space → product in Lapace space

$$\to \quad \big(\tilde{J}(Q^2,\nu)\big)^2$$

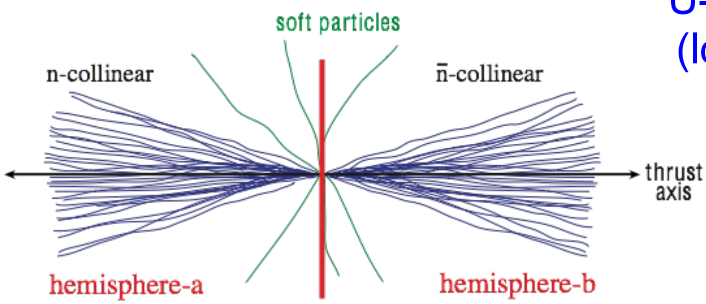
QCD Factorization (SCET)

→ **Allows for direct field theoretic considerations that are not possible in the coherent branching formalism**

Hard function

Jet function
(collinear radiation)

$$\frac{1}{\sigma_0}\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau}(\tau,Q) = Q\,H_Q(Q,\mu_H)\int_0^{Q^2\tau}\mathrm{d}s\int_0^{s}\mathrm{d}s'\,U_J(s-s',\mu_H,\mu_J)\,J^{(\tau)}(s',\mu_J)$$
$$\times\int_0^{Q\tau-s/Q}\mathrm{d}k\,U_S(k,\mu_H,\mu_S)\,S^{(\tau)}\Big(Q\tau-\frac{s}{Q}-k,\mu_S\Big)$$



U-Evolution function
(log resummations)

Soft function
(large angle soft radiation)

Korchemsky Sterman
Becher, Schwartz
Abbate, Hoang, Mateu, Stewart
Many more authors

BHQET/SCET: Massive Quarks

Fleming, Hoang, Mantry, Stewart 2007

Hard function

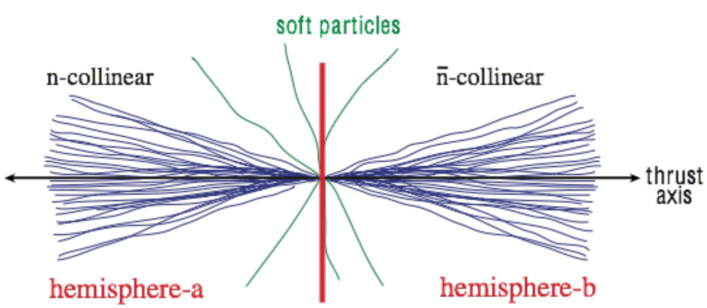
Mass mode function
(offshell massive quark)

$$\frac{d\hat{\sigma}_s}{d\tau}(\tau, Q, m^{\rm pole}) = \sigma_0\, Q\, H_Q(Q,\mu_H) U_H(Q,\mu_H,\mu_m) H_m(Q, m^{\rm pole},\mu_m) U_m\Big(\frac{Q}{m^{\rm pole}},\mu_m,\mu_H\Big)$$
$$\times \int_0^{Q^2\tau} d\hat{s} \int_0^{s} d\hat{s}'\, U_{J_B}(\hat{s}-\hat{s}',\mu_H,\mu_B)\, J_B^{(\tau)}(\hat{s}',\mu_B)$$
$$\times \int_0^{Q\tau - s/Q} dk\, U_S(k,\mu_H,\mu_S)\, S^{(\tau)}\Big(Q(\tau - \tau_{\rm min}^{\rm pole}) - \frac{m^{\rm pole}}{Q}\hat{s} - k, \mu_S\Big)$$

U-Evolution function
(log resummations)

Soft function
(large angle soft)
UNIVERSAL !

BHQET Jet function
(ultra-collinear radiation)

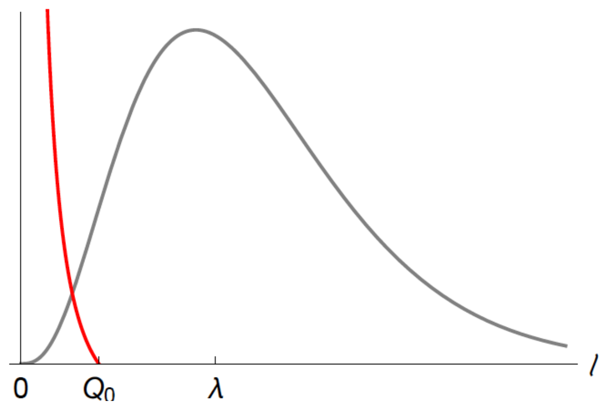


Soft radiation in the massive quark rest frame

MASS SCHEME FIXED HERE !

Partonic calculation → Hadron Level

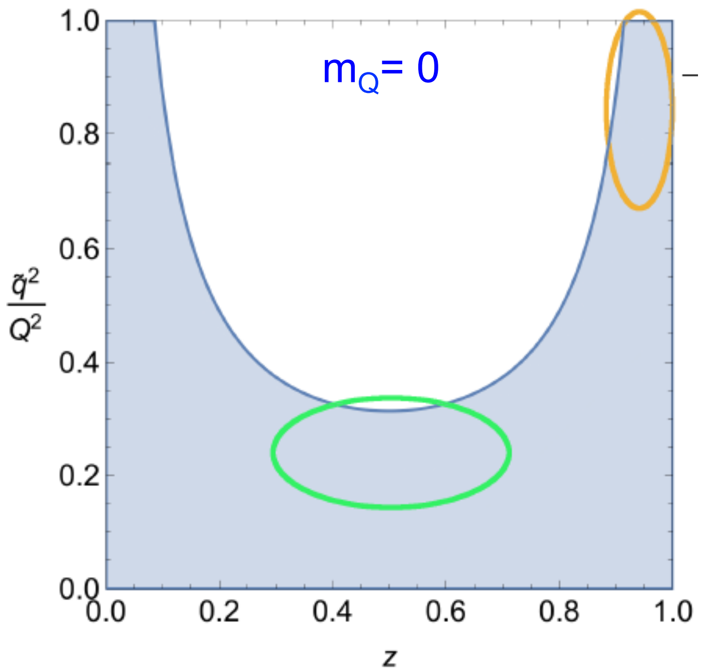
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\tau}(\tau, Q) = \int_0^{Q\tau} \mathrm{d}\ell \; \underbrace{\frac{\mathrm{d}\hat{\sigma}_s}{\mathrm{d}\tau}\left(\tau - \frac{\ell}{Q}, Q\right)}_{\text{Partonic cross section}} \; \underbrace{S_{\mathrm{mod}}(\ell)}_{\text{Non-perturbative shape function}}$$

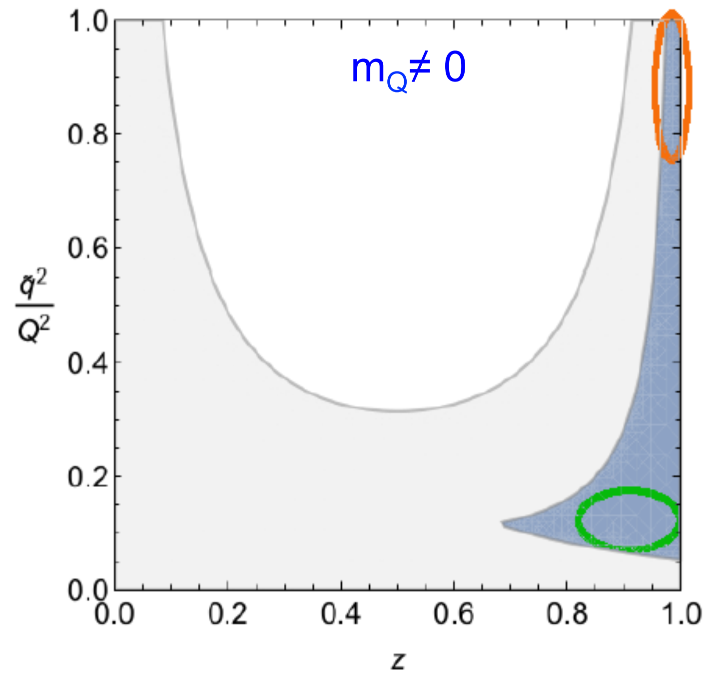


SMEARING of partonic cross section by amount λ/Q

→ same for CB and QCD factorization / SCET / BHQET

Phase Space and Power Counting





phase space regions for $\tau_\text{peak} \sim \frac{\Lambda}{Q} \ll 1,\ m = 0$		
	coherent branching	QCD factorization
n-coll.	$z \sim (1-z) \sim 1$ $\tilde q \sim (Q\Lambda)^{\frac{1}{2}}$ $q_\perp \sim (Q\Lambda)^{\frac{1}{2}}$	$q^\mu \sim (\Lambda, Q, (Q\Lambda)^{\frac{1}{2}})$
soft	$1 - z \sim \frac{\Lambda}{Q},\ z \sim 1$ $\tilde q \sim Q$ $q_\perp \sim \Lambda$	$q^\mu \sim (\Lambda, \Lambda, \Lambda)$

phase space regions for $\tau_\text{peak} - \tau_\text{min} \sim \frac{\Lambda}{Q} \ll 1,\ m \neq 0$		
	coherent branching	QCD factorization
u. coll.	$1 - z \sim \frac{Q\Lambda}{m^2},\ z \sim 1$ $\tilde q \sim m$ $q_\perp \sim \frac{Q}{m}\Lambda$	$q^\mu \sim (\Lambda, \frac{Q^2}{m^2}\Lambda, \frac{Q}{m}\Lambda)$
soft	$1 - z \sim \frac{\Lambda}{Q},\ z \sim 1$ $\tilde q \sim Q$ $q_\perp \sim \Lambda$	$q^\mu \sim (\Lambda, \Lambda, \Lambda)$

CB and SCET are equivalent at NLL (m_Q=0)

Resummation: CB vs SCET (pQCD)

CB:

neglecting higher logarithmic terms, the resummed jet mass function can be written in the form

$$2\ln\tilde{J}(Q^2,\nu) \approx 8\int_{\alpha_s(\mu_J)}^{\alpha_s(\mu_H)} \frac{\mathrm{d}\alpha_s}{\beta[\alpha_s]}\left(\frac{\alpha_s}{4\pi}\Gamma_0^{\text{cusp}} + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma_1^{\text{cusp}}\right)\int_{\alpha_s(\mu_J)}^{\alpha_s}\frac{\mathrm{d}\alpha_s'}{\beta[\alpha_s']} + 12C_F\int_{\alpha_s(\mu_J)}^{\alpha_s(\mu_H)}\frac{\mathrm{d}\alpha_s}{\beta[\alpha_s]}\frac{\alpha_s}{4\pi}$$

$$-4\int_{\alpha_s(\mu_S)}^{\alpha_s(\mu_H)} \frac{\mathrm{d}\alpha_s}{\beta[\alpha_s]}\left(\frac{\alpha_s}{4\pi}\Gamma_0^{\text{cusp}} + \left(\frac{\alpha_s}{4\pi}\right)^2\Gamma_1^{\text{cusp}}\right)\int_{\alpha_s(\mu_S)}^{\alpha_s}\frac{\mathrm{d}\alpha_s'}{\beta[\alpha_s']}$$

$$= \ln\left(\tilde{U}_J\left(\frac{\nu}{Q^2},\mu_H^2,\mu_J^2\right)\tilde{U}_S\left(\frac{\nu}{Q},\mu_H^2,\mu_S^2\right)\right)$$

with $\mu_H^2 = Q^2$ $\quad \mu_J^2 = \dfrac{Q^2 e^{-\gamma_E}}{\nu}$ $\quad \mu_S^2 = \dfrac{Q^2 e^{-2\gamma_E}}{\nu^2}$ $\quad$ for $\tau \sim \dfrac{1}{\nu}$ corresponds to $\mu_J^2 \sim Q^2\tau$

$\mu_S^2 \sim Q^2\tau^2$

SCET:

$$Q^3 H(Q^2,\mu^2) \times \int \mathrm{d}\tau' J(Q^2(\tau-\tau'),\mu^2) S(Q\tau',\mu^2) \xRightarrow[\text{L-space}]{\text{NLL}} \tilde{U}_J\left(\frac{\nu}{Q^2},\mu_H^2,\mu_J^2\right)\tilde{U}_S\left(\frac{\nu}{Q},\mu_H^2,\mu_S^2\right)$$

evolve everything to μ_H $\quad$ Korchemsky, Sterman (1994)

→ CB and SCET equivalent at NLL: parton showers based on CB are NLL precise for τ

CB and SCET are equivalent at NLL ($m_Q \neq 0$)

CB: $$\ln \tilde{J}(Q^2,\nu,m^2) = 2\int_{m^2}^{Q^2}\frac{d\tilde{q}^2}{\tilde{q}^2}\int_{\frac{m}{\tilde{q}}}^{1} dz\, P_{QQ}\Big[\alpha_s\big(z(1-z)\tilde{q}\big),z,m\Big]\Big(e^{\frac{-\nu z(1-z)\tilde{q}^2}{Q^2}}-1\Big)$$

$$\approx 4\int_{\alpha_s(\mu_m)}^{\alpha_s(\mu_H)}\frac{d\alpha_s}{\beta[\alpha_s]}\Big(\frac{\alpha_s}{4\pi}\Gamma_0^{\rm cusp}+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Gamma_1^{\rm cusp}\Big)\int_{\alpha_s(\mu_m)}^{\alpha_s}\frac{d\alpha_s'}{\beta[\alpha_s']}+4C_F\int_{\alpha_s(\mu_m)}^{\alpha_s(\mu_H)}\frac{d\alpha_s}{\beta[\alpha_s]}\frac{\alpha_s}{4\pi}$$

$$+4\int_{\alpha_s(\mu_B)}^{\alpha_s(\mu_H)}\frac{d\alpha_s}{\beta[\alpha_s]}\Big(\frac{\alpha_s}{4\pi}\Gamma_0^{\rm cusp}+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Gamma_1^{\rm cusp}\Big)\int_{\alpha_s(\mu_B)}^{\alpha_s}\frac{d\alpha_s'}{\beta[\alpha_s']}+8C_F\int_{\alpha_s(\mu_B)}^{\alpha_s(\mu_H)}\frac{d\alpha_s}{\beta[\alpha_s]}\frac{\alpha_s}{4\pi}$$

$$-4\int_{\alpha_s(\mu_S)}^{\alpha_s(\mu_H)}\frac{d\alpha_s}{\beta[\alpha_s]}\Big(\frac{\alpha_s}{4\pi}\Gamma_0^{\rm cusp}+\Big(\frac{\alpha_s}{4\pi}\Big)^2\Gamma_1^{\rm cusp}\Big)\int_{\alpha_s(\mu_S)}^{\alpha_s}\frac{d\alpha_s'}{\beta[\alpha_s']}$$

NEW !

$$=\ln\Big(\big(\tilde{U}_H(Q^2,\mu_m^2,\mu_H^2)\big)^{-1}\tilde{U}_{H_m}\Big(\frac{Q}{m},\mu_m^2,\mu_H^2\Big)\tilde{U}_B\Big(\frac{\nu m}{Q^2},\mu_H^2,\mu_B^2\Big)\tilde{U}_S\Big(\frac{\nu}{Q},\mu_H^2,\mu_S^2\Big)\Big)$$

with $\mu_H^2=Q^2 \quad \mu_m^2=m^2 \quad \mu_B^2=\frac{Q^4e^{-2\gamma_E}}{\nu^2m^2} \quad \mu_S^2=\frac{Q^2e^{-2\gamma_E}}{\nu^2} \Rightarrow \mu_B^2\sim\frac{Q^4\tau^2}{m^2} \quad \mu_S^2\sim Q^2\tau^2$

bHQET: $$Q^3\,H(Q^2,\mu^2)\times H_m(m^2,\mu^2)\times\int d\tau'\,B\Big(\frac{Q^2(\tau-\tau_{\rm min}-\tau')}{m},\mu^2\Big)S(Q\tau',\mu^2)$$

$$\xRightarrow[\text{L-space}]{\text{NLL}}\big(\tilde{U}_H(Q^2,\mu_m^2,\mu_H^2)\big)^{-1}\tilde{U}_{H_m}\Big(\frac{Q}{m},\mu_m^2,\mu_H^2\Big)\tilde{U}_B\Big(\frac{\nu m}{Q^2},\mu_H^2,\mu_B^2\Big)\tilde{U}_S\Big(\frac{\nu}{Q},\mu_H^2,\mu_S^2\Big)$$

→ CB and bHQET equivalent at NLL: parton showers based on CB are NLL precise for τ

Question A: Can NLL describe peak position at NLO?

Peak position at NLL:

Partonic cross section in SCET at NLO

$$\frac{d\hat{\sigma}}{d\tau} = \delta(\tau) + \frac{\alpha_s C_F}{4\pi}\left\{ \underbrace{-8\left[\frac{\ln\tau}{\tau}\right]_+}_{\text{LL}} \underbrace{-6\left[\frac{1}{\tau}\right]_+}_{\text{NLL}} + \underbrace{\delta(\tau)\left(\frac{2\pi^2}{3}-2\right)}_{\text{N}^2\text{LL}} \right\} + \mathcal{O}(\alpha_s^2)$$

Use that NNLL@NLO part is proportional to LO $\qquad f(\tau) = \frac{d\hat{\sigma}}{d\tau} \otimes S_{\text{mod}}$

$$f_{\text{NLO}}(\tau) = \tilde{f}_{\text{NLO}}(\tau) + \alpha f_{\text{LO}}(\tau)$$

$$f(\tau) = f_{\text{LO}}(\tau) + f_{\text{NLO}}(\tau) = (1+\alpha) f_{\text{LO}}(\tau) + \tilde{f}_{\text{NLO}}(\tau) + \mathcal{O}(\alpha_s^2)$$

LO peak position

$$f'_{\text{LO}}(\tau_0) = 0$$

Same for massive quarks !

NLO peak position $\qquad \delta\tau \sim \alpha_s$

$$\begin{aligned} f'(\tau_0 + \delta\tau) &\stackrel{!}{=} 0 + \mathcal{O}(\alpha_s^2) \\ &= (1+\alpha) f'_{\text{LO}}(\tau_0+\delta\tau) + \tilde{f}'_{\text{NLO}}(\tau_0+\delta\tau) + \mathcal{O}(\alpha_s^2) \\ &= (1+\alpha)\underbrace{f'_{\text{LO}}(\tau_0)}_{=0} + \delta\tau f''_{\text{LO}}(\tau_0) + \tilde{f}'_{\text{NLO}}(\tau_0) + \mathcal{O}(\alpha_s^2) \end{aligned}$$

$$\rightarrow \quad \delta\tau = \frac{-\tilde{f}'_{\text{NLO}}(\tau_0)}{f''_{\text{LO}}(\tau_0)} + \mathcal{O}(\alpha_s^2)$$

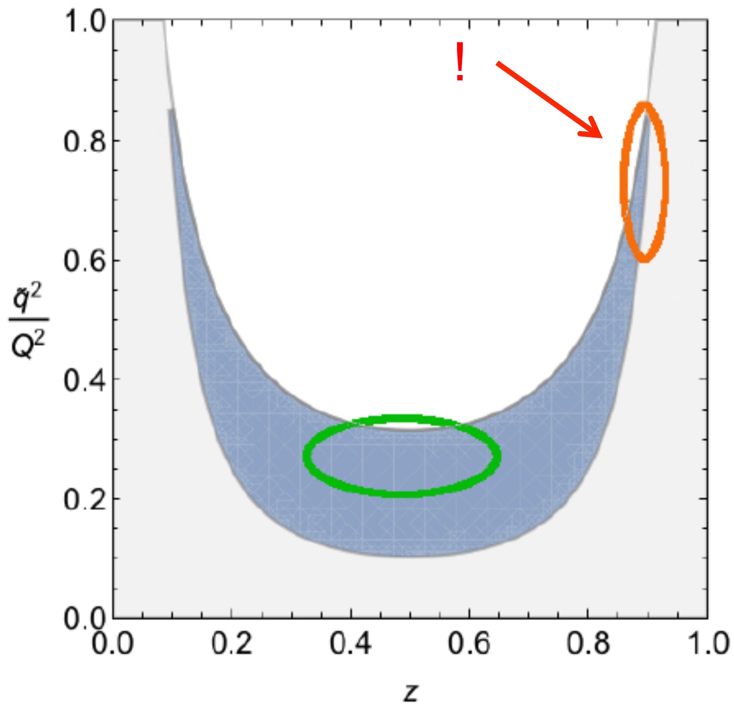
→ NLL resummation has full NLO information on the peak position

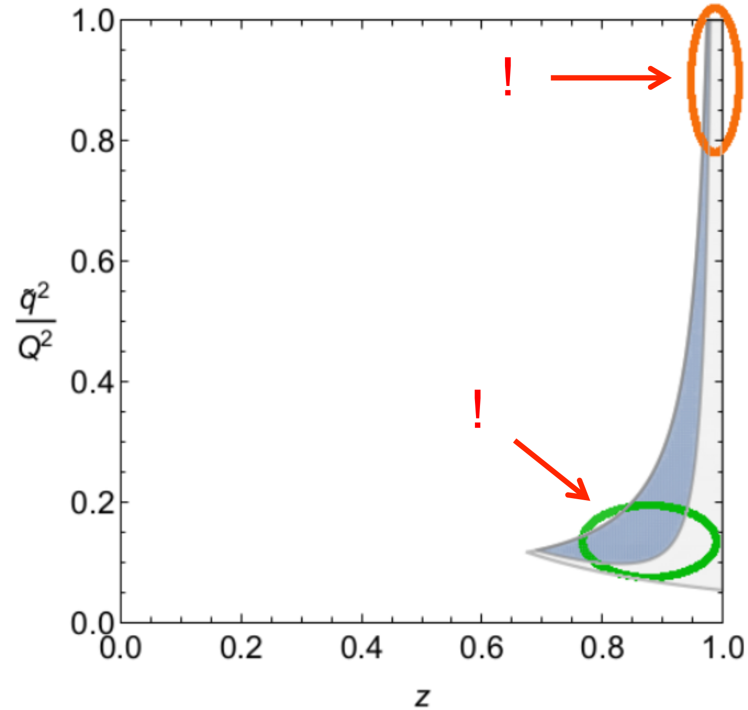
Question A: Can NLL describe peak position at NLO?

Message 1:

The quark mass parameter in coherent branching at NLL in perturbation theory and without imposing any infrared cutoff is the POLE MASS.

Question B: Impact of Shower Cut Q_0





- Not required in pure perturbation theory (fixed renormalization scale)
- Needed in parton shower evolution: Landau singularities + ∞ multiplicities

scale in α_S: $z^2(1-z)^2\tilde{q}^2$ cutoff: $q_\perp > Q_0$

Question: Can the cutoff dependence be controlled theoretically and what does it imply?

Question B: Impact of Shower Cut Q_0

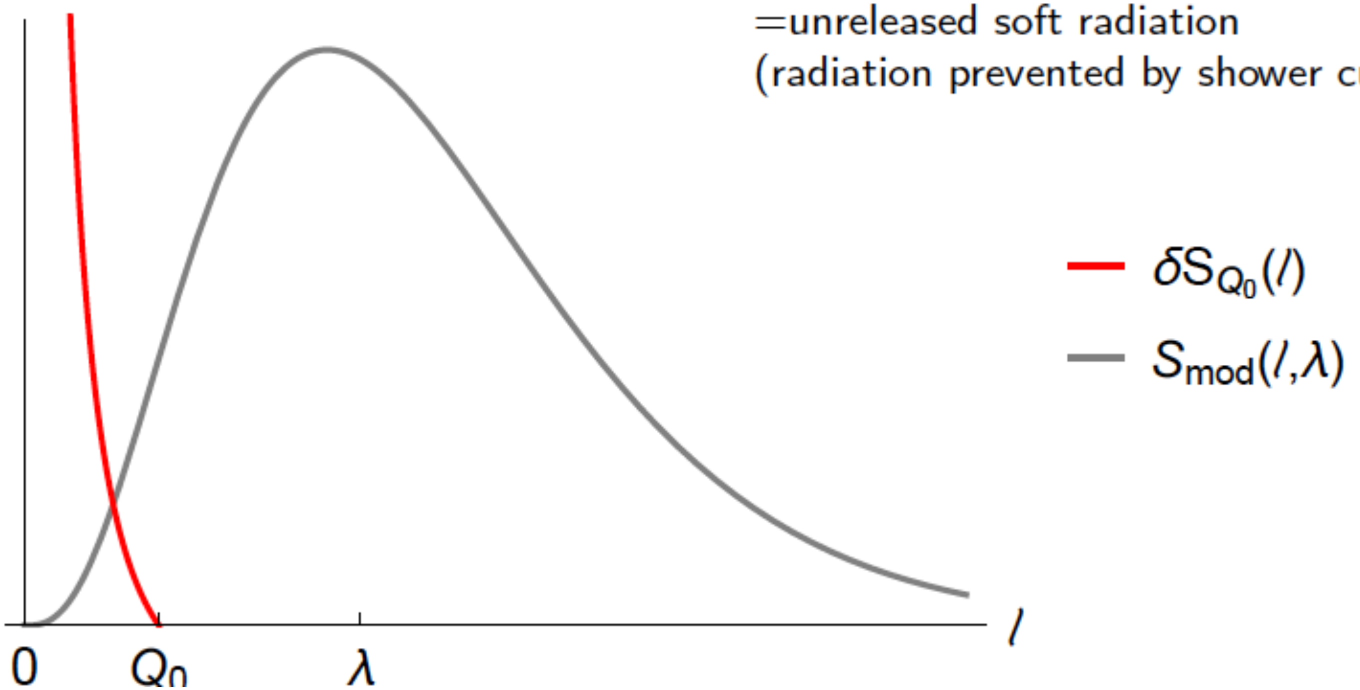
SCET Soft Function with $q_\perp$ cutoff

Introduce $q_\perp$ cutoff in partonic soft function:

$$\hat{S}_{Q_0}(\ell) = \delta(\ell) + \hat{S}^{(1)}(\ell) - \delta\hat{S}^{(1)}_{Q_0}(\ell) + \mathcal{O}(\alpha_s^2)$$

$$\delta\hat{S}^{(1)}_{Q_0}(\ell) = \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\theta(Q_0 - q_\perp)\,\ldots\; = -\theta(Q_0-\ell)\,\frac{\alpha_s C_F}{4\pi}\,16\left[\frac{\ln\frac{\ell}{Q_0}}{\ell}\right]_+$$

=unreleased soft radiation
(radiation prevented by shower cut)



Question B: Impact of Shower Cut Q_0

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=unreleased soft radiation
(radiation prevented by shower cut)

Cutoff Q_0 unresolved if $Q_0 < \lambda$:

→ multipole expansion (~ expansion in Q_0/λ, with λ ~ width of the model function):

$$\delta\hat{S}_{Q_0}(\ell) \approx -\delta'(\ell)\int_0^\infty \mathrm{d}\ell'\,\ell'\,\delta\hat{S}_{Q_0}(\ell') + \ldots = \delta'(\ell)\,\underbrace{(-Q_0)\frac{\alpha_s(Q_0)}{4\pi}16C_F}_{=\Delta^{(1)}(Q_0)} + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow \hat{S}_{Q_0}(\ell) \approx \delta(\ell) + \hat{S}^{(1)}(\ell) - \Delta^{(1)}(Q_0)\times\delta'(\ell) + \mathcal{O}(\alpha_s^2)$$

Question B: Impact of Shower Cut Q_0

Soft Function Gap

$$S_{Q_0}(\ell) = \left(\hat{S}_{Q_0} \otimes S_{\rm mod}\right)(\ell)$$
$$\approx \left(\hat{S} \otimes S_{\rm mod}\right)(\ell) - \Delta^{(1)}(Q_0) \times S'_{\rm mod}(\ell) + \mathcal{O}(\alpha_s^2)$$
$$= S\big(\ell - \Delta^{(1)}(Q_0)\big) + \mathcal{O}(\alpha_s^2)$$

$$\Rightarrow S_{\mathrm{mod},Q_0}(\ell) = S_{\mathrm{mod}}\big(\ell - \underbrace{\Delta^{(1)}(Q_0)}_{\text{soft function gap}}\big)$$

[Hoang, Stewart (2008)]

Q_0 dependence of gap:

$$\frac{\mathrm{d}\Delta^{(1)}(Q_0)}{\mathrm{d}Q_0} = -16 C_F \frac{\alpha_s(Q_0)}{4\pi} + \mathcal{O}(\alpha_s^2)$$

$$\Delta^{(1)}(Q_0) = \Delta^{(1)}(Q_0') - 16 C_F \int_{Q_0'}^{Q_0} \mathrm{d}R\, \frac{\alpha_s(R)}{4\pi}$$

R-evolution equation

→ Q_0 dependence of peak position:

[Hoang, Jain, Scimemi, Stewart (2008); Hoang, Kluth (2008)]

$$\boxed{\tau_{\max}(Q_0) = \tau_{\max}(Q_0') - \frac{1}{Q} 16 C_F \int_{Q_0'}^{Q_0} \mathrm{d}R\, \frac{\alpha_s(R)}{4\pi}}$$

Question B: Impact of Shower Cut Q_0

Resummation in CB with Cutoff

Recall resummed thrust distribution in Laplace space:

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q) = \exp\left[2\int_0^{Q^2}\frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2}\int_0^1 \mathrm{d}z\,\frac{\alpha_s\big(z(1-z)\tilde{q}\big)C_F}{2\pi}\,\frac{1+z^2}{1-z}\left(\mathrm{e}^{\frac{-\nu z(1-z)\tilde{q}^2}{Q^2}}-1\right)\right]$$

introduce cutoff: $\theta\big(z(1-z)\tilde{q}-Q_0\big) = 1-\theta\big(Q_0 - z(1-z)\tilde{q}\big)$

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}_{Q_0}}{\mathrm{d}\tau}\right)(\nu, Q) = \mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q)\times \mathrm{e}^{\mathcal{I}(\nu,Q,Q_0)}$$

Expand $\mathcal{I}$ in ν keeping only linear term:

$$\mathcal{I}(\nu,Q,Q_0) \approx -\nu\frac{1}{Q}\underbrace{Q_0\frac{\alpha_s(Q_0)}{4\pi}16C_F}_{=\Delta^{(1)}(Q_0)} + \mathcal{O}\left(\frac{\nu^2 Q_0^2}{Q^2}\right)$$

Term linear in ν in the exponential gives shift in τ-space:

$$\frac{\mathrm{d}\hat{\sigma}_{Q_0}}{\mathrm{d}\tau}(\tau, Q) = \frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\left(\tau - \frac{1}{Q}\Delta(Q_0), Q\right)$$

→ CB reproduces the SCET results for the peak shift induced by the cutoff Q_0

Question B: Impact of Shower Cut Q_0

Message 2:

As far as large angle soft radiation is concerned the dependence on the shower cut Q_0 has to be interpreted as a modification of the hadronization effects.

A change of Q_0 needs to be compensated by a retuning of the hadronization model attached to the parton shower.

bHQET Jet Function with Shower Cutoff Q_0

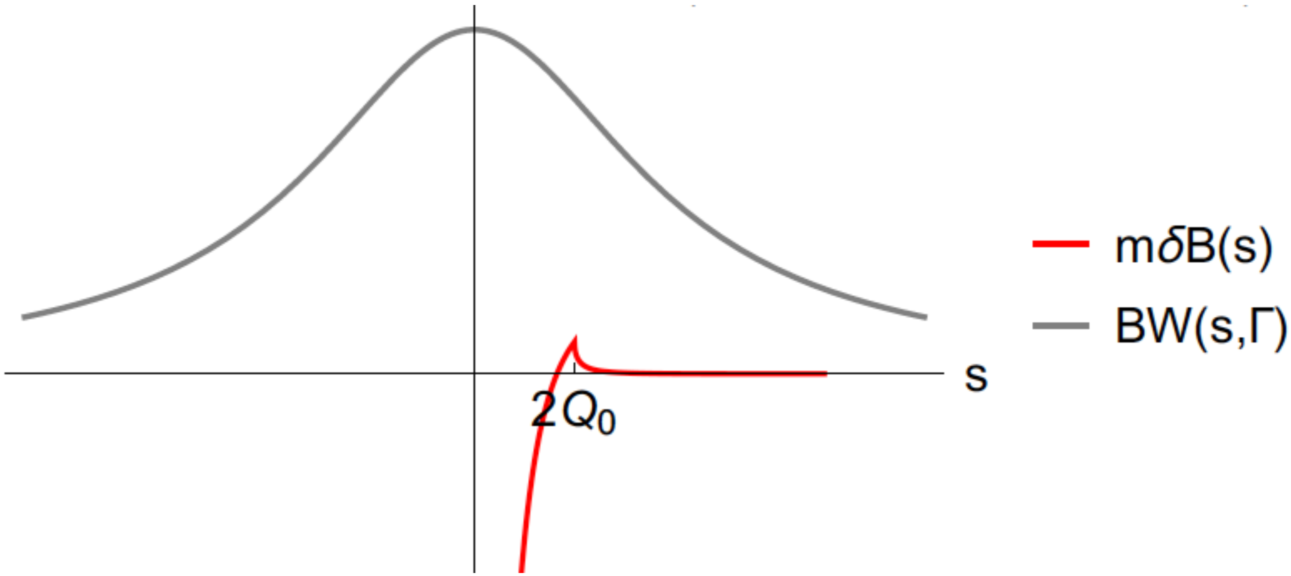
introduce $q_\perp$ cutoff in bHQET jet function:

$$\hat{s} = \frac{q^2 - m^2}{m}$$

$$m B_{Q_0}(\hat{s}) = \delta(\hat{s}) + m B^{(1)}(\hat{s}) - m\, \delta B^{(1)}_{Q_0}(\hat{s}) + \mathcal{O}(\alpha_s^2)$$

$$m\, \delta B^{(1)}_{Q_0}(\hat{s}) = \int \frac{\mathrm{d}^d q}{(2\pi)^d}\, \theta(Q_0 - q_\perp) \,\ldots\, = \frac{\alpha_s C_F}{4\pi} f(\hat{s})$$

=unreleased u.-collinear radiation
(radiation prevented by shower cut)



bHQET Jet Function with Shower Cutoff Q_0

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$$\hat{s} = \frac{q^2 - m^2}{m}$$

$$m B_{Q_0}(\hat{s}) = \delta(\hat{s}) + m B^{(1)}(\hat{s}) - m\,\delta B^{(1)}_{Q_0}(\hat{s}) + \mathcal{O}(\alpha_s^2)$$

$$m\,\delta B^{(1)}_{Q_0}(\hat{s}) = \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\theta(Q_0 - q_\perp)\,\ldots\; = \frac{\alpha_s C_F}{4\pi} f(\hat{s})$$

=unreleased u.-collinear radiation
(radiation prevented by shower cut)

Multipole expansion ($\Gamma > Q_0$):

$$m\,\delta B_{Q_0}(\hat{s}) \approx -\delta'(\hat{s}) \int_0^\infty \mathrm{d}\hat{s}'\,\hat{s}'\,m\,\delta B_{Q_0}(\hat{s}')\ldots = \delta'(\hat{s})\,\underbrace{(-Q_0)\frac{\alpha_s(Q_0)}{4\pi}\,c_m^{(1)}}_{=\Delta_m^{(1)}(Q_0)} + \mathcal{O}(\alpha_s^2)$$

Convolute with Breit-Wigner:

$$\begin{aligned} m B_{\Gamma,Q_0}(\hat{s}) &= \Big(m B_{Q_0} \otimes BW\Big)(\hat{s}) \\ &\approx \Big(m B \otimes BW\Big)(\hat{s}) - \Delta_m^{(1)}(Q_0) \times BW'(\hat{s}) + \mathcal{O}(\alpha_s^2) \\ &= m B_\Gamma\big(\hat{s} - \Delta_m^{(1)}(Q_0)\big) + \mathcal{O}(\alpha_s^2) \end{aligned}$$

→ leading cutoff effect is (mass !!) shift in bHQET jet function by $\Delta_m(Q_0) \sim \alpha_S\, Q_0$

Resummation in CB with Mass and Cutoff

Recall resummed thrust distribution in Laplace space:

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q, m) = \exp\left[2\int_{m^2}^{Q^2} \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \int_{\frac{m}{\tilde{q}}}^{1} \mathrm{d}z\, P_{QQ}\left[\alpha_s\left(z(1-z)\tilde{q}\right), z, m\right]\left(\mathrm{e}^{\frac{-\nu z(1-z)\tilde{q}^2}{Q^2}} - 1\right)\right]$$

introduce cutoff: $\theta\left(\tilde{q}^2 - \frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2}\right) = 1 - \theta\left(\frac{Q_0^2 + m^2(1-z)^2}{z^2(1-z)^2} - \tilde{q}^2\right)$

$$\mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}_{Q_0}}{\mathrm{d}\tau}\right)(\nu, Q, m) = \mathcal{L}\left(\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}\tau}\right)(\nu, Q, m) \times \mathrm{e}^{\mathcal{I}(\nu, Q, Q_0, m)}$$

Expand $\mathcal{I}$ in ν keeping only linear term:

$$\mathcal{I} \approx -\nu \frac{Q_0}{Q} \frac{\alpha_s(Q_0)}{4\pi}\left(16 C_F + \frac{m}{Q} c_m^{(1)}\right) + \ldots$$

Term linear in ν gives shift in $\boldsymbol{\tau}$-space:

→ CB reproduces the bHQET results for the peak shift induced by the cutoff Q_0

Peak Shift for massive quarks

Total shift due to cutoff Q_0 arises from restricting large angle soft and ultra-collinear radiation.

Quark mass reduces Q_0 dependence from soft function: $c_m^{(0)} < 0$ (→ dead cone effects)

$$\boxed{\tau_{\max}(Q_0) = \tau_{\max}(Q_0') - \frac{1}{Q}\left(16C_F + \frac{m}{Q}c_m^{(1)}\right)\int_{Q_0'}^{Q_0} \mathrm{d}R\,\frac{\alpha_s(R)}{4\pi}}$$

Soft function gap

additional contribution only for massive quarks (dead cone)

$$\boxed{c_m^{(1)} = -8\pi C_F}$$

Comparison to Herwig (m_Q=0)

- compare our predictions for peak shift with real parton shower
- used Herwig 7 with angular ordered shower for $e^+e^- \to 2\,\text{jets}$
  [Bahr et al. (2008); Bellm et al. (2015)]
- modifications:
  - only light quarks + gluons, set all constituent masses to zero
  - unrestricted kinematics in evolution of CB
  - switched off: QED radiation, hadronization
- only partonic distribution from Herwig

„hadronization“: convolution with model function → disentanble parton shower and hadronization model

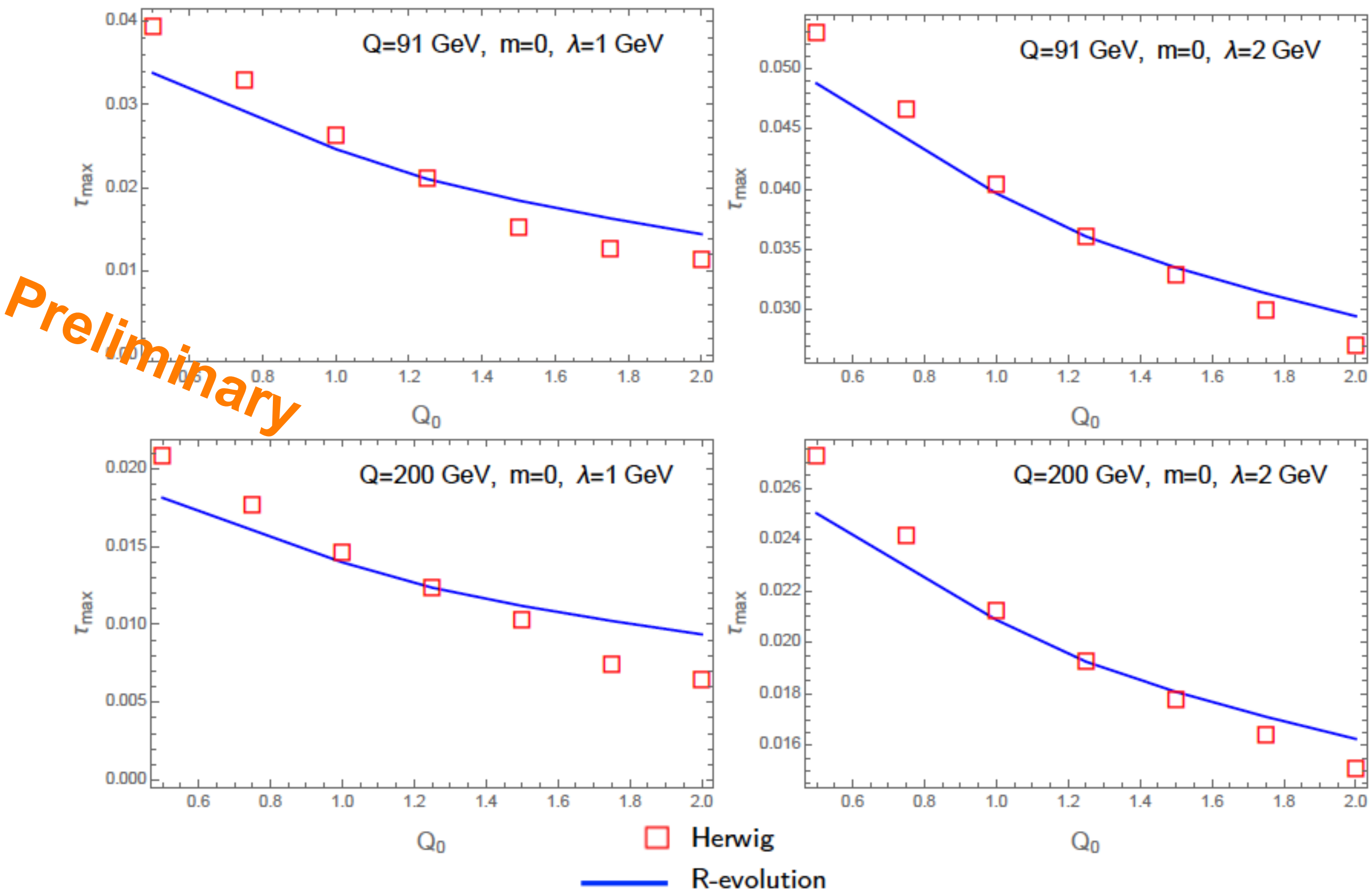
$$S_{\text{mod}}(k,\lambda) = \frac{128\,k^3 e^{-\frac{4k}{\lambda}}}{3\lambda^4}$$

- $30\,\text{GeV} \le Q \le 300\,\text{GeV}$

$0.5\,\text{GeV} \le Q_0 \le 2\,\text{GeV}$

$1 \le \lambda \le 3$

Comparison to Herwig (m_Q=0)



→ Good agreement between parton shower and SCET

Comparison to Herwig ($m_Q \neq 0$)

- run Herwig 7 for $e^+e^- \to t\bar{t}$

- same modifications on Herwig as in the massless case
  \+ on-shell top production
  \+ only leptonic W-decays

- used same model function as for the massless case

- used a broader smearing to account for effects of finite top width

$$\lambda_{\text{eff}} = \lambda + \frac{4m_t\Gamma_t}{Q}$$

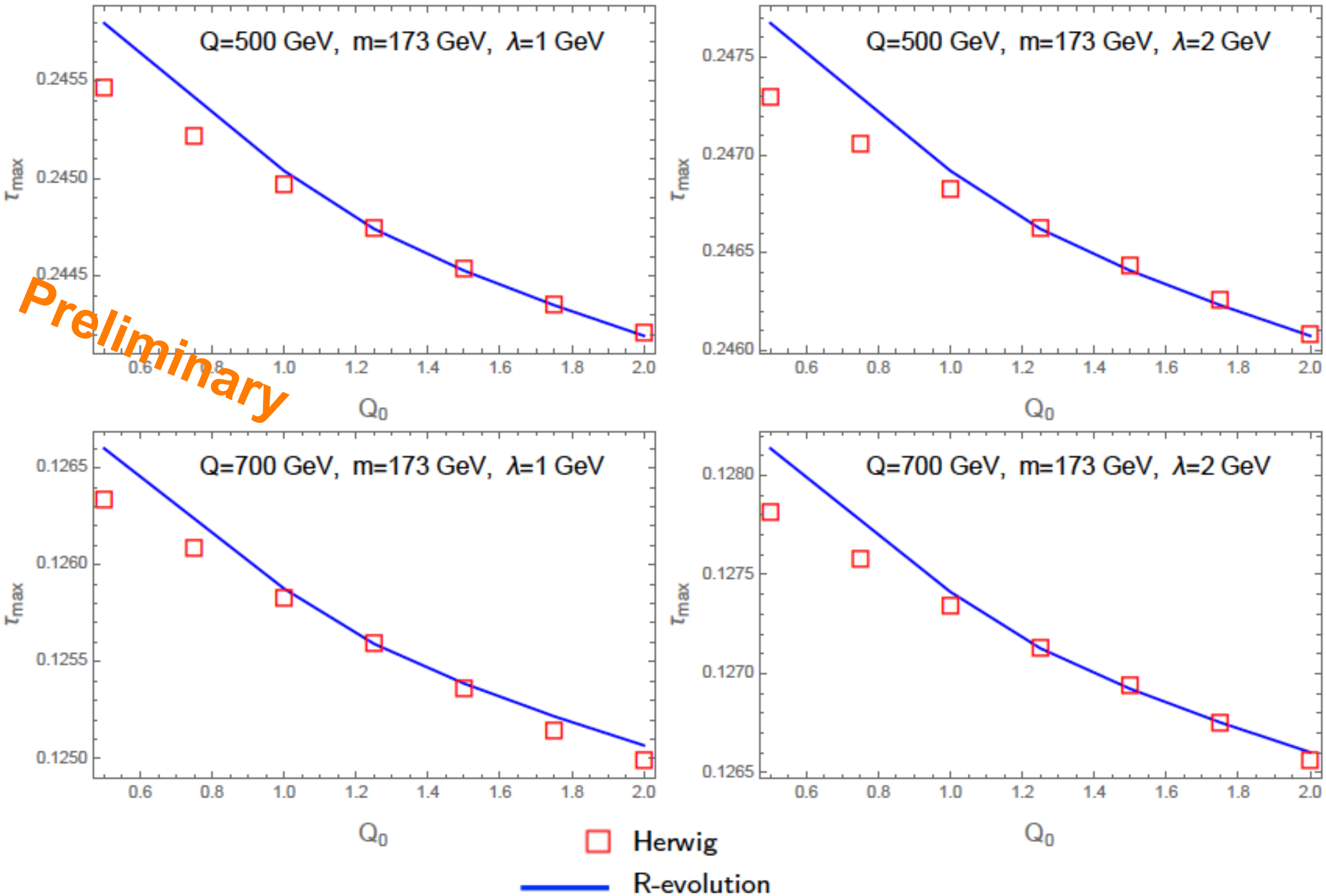
- $m_t = 173\,\text{GeV}$ $\qquad\qquad \Gamma_t = 1.5\,\text{GeV}$

- $500\,\text{GeV} \leq Q \leq 1\,\text{TeV}$
  $0.5\,\text{GeV} \leq Q_0 \leq 2\,\text{GeV}$
  $1 \leq \lambda \leq 3$

- compare cutoff dependence of peak position with R-evolution

$$\tau_{\max}(Q_0) = \tau_{\max}(Q_0') - \frac{1}{Q}\left(16C_F + \frac{m_t}{Q}c_m^{(1)}\right)\int_{Q_0'}^{Q_0} \mathrm{d}R\,\frac{\alpha_s^{\text{CMW}}(R)}{4\pi}$$

Comparison to Herwig ($m_Q \neq 0$)



→ Good agreement between parton shower and bHQET + SCET

Question C: What is the parton shower mass?

Message 3:

Shift linear in the mass and Q_0 implies change to a short-distance mass scheme different from m^{pole}:

$$\text{threshold:} \quad \frac{2(m^{\text{pole}})^2}{Q^2} = \frac{2(m+\delta m)^2}{Q^2} = \frac{2m^2}{Q^2} + \frac{4m\,\delta m}{Q^2} + \ldots$$

Using a $q_\perp$ cut for perturbative calculations (such as coherent branching) implies the use of a short-distance mass:

mass scheme of coherent branching algorithm with cutoff:

$$m^{\text{CB}}(Q_0) = m^{\text{pole}} + Q_0 \frac{\alpha_s(Q_0)}{4\pi} \frac{c_m^{(1)}}{4}$$

$$\boxed{Q_0 \frac{\alpha_s(Q_0)}{4\pi} \frac{c_m^{(1)}}{4} = Q_0\, \alpha_s(Q_0) \times 0.67}$$

$$\alpha_s^{\text{CMW}}(1\ \text{GeV}) = 0.98$$

$$\alpha_s^{\overline{\text{MS}}}(1\ \text{GeV}) = 0.46$$

Summary & Outlook

① Answers to our 3 initial questions:

A. For τ: Parton showers based on CB have NLO precision conc. peak mass of jet
B. Leading dependence of peak mass on Q_0 linear: Herwig and SCET agree at NLL
C. Cutoff in parton shower implies renormalon-free gap and mass scheme

② We have proven: $m^{CB}(Q_0 \neq 0) \neq m^{pole}$

③ Limitations:
- CB / parton showers for top quarks use narrow width approximation
- Practical MCs still have additional features that may affect m^{MC}
- Still a lot of work to be done to draw final conclusions to relate m^{MC} to a field theory mass from first principles.

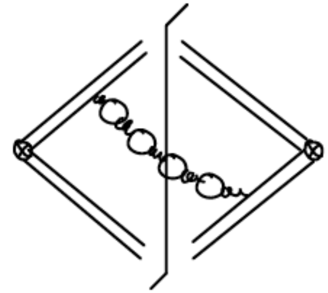
④ Outlook:
- Calibration analysis
- Implications for invariant mass reconstruction, kinematic distributions
- Cutoff dependence of MC hadronization models
- MCs other than Herwig, dipole shower, NLO matching, ...

Soft Function with Shower Cut Q_0: Renormalons

Bubble chain analysis:

$$\hat{S}_{Q_0}(\ell) = \delta(\ell) + \hat{S}^{(1)}(\ell) - \delta\hat{S}^{(1)}_{Q_0}(\ell) + \mathcal{O}(\alpha_s^2)$$

$$\delta\hat{S}^{(1)}_{Q_0}(\ell) = \int \frac{\mathrm{d}^d q}{(2\pi)^d}\,\theta(Q_0 - q_\perp)\,\ldots\; = -\theta(Q_0-\ell)\frac{\alpha_s C_F}{4\pi}16\left[\frac{\ln\frac{\ell}{Q_0}}{\ell}\right]_+$$



→ Borel transform: NO u=1/2 pole contained in $\hat{S}_{Q_0}(\ell)$

Implications:

Shower cut Q_0 results in a partonic soft function without Λ_{QCD} renormalon ambiguity. ✔

Shower cut Q_0 results in a gapped hadronization model. ✔

(negative shift in jet mass threshold)

Gap satisfies R-evolution equation in Q_0. ✔

bHQET Jet Function with Shower Cutoff Q_0: Renormalons

introduce $q_\perp$ cutoff in bHQET jet function:

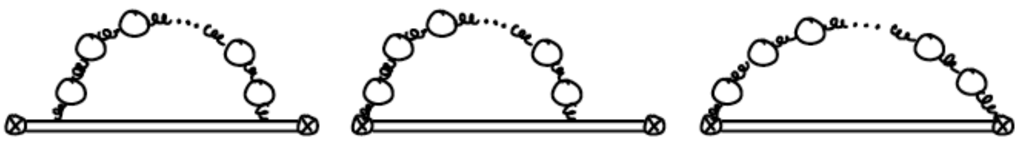
$$\hat{s} = \frac{q^2 - m^2}{m}$$

$$m B_{Q_0}(\hat{s}) = \delta(\hat{s}) + m B^{(1)}(\hat{s}) - m\, \delta B^{(1)}_{Q_0}(\hat{s}) + \mathcal{O}(\alpha_s^2)$$

$$m\, \delta B^{(1)}_{Q_0}(\hat{s}) = \int \frac{\mathrm{d}^d q}{(2\pi)^d}\, \theta(Q_0 - q_\perp)\, \ldots \; = \frac{\alpha_s C_F}{4\pi} f(\hat{s})$$

=unreleased u.-collinear radiation
(radiation prevented by shower cut)

Bubble chain analysis:



→ Borel transform: $B[B_{Q_0}(\hat{s})](u)$ has NO u=1/2 pole.

→ Cutoff implies that there is no $O(\Lambda_{QCD})$ renormalon related to the quark mass.