New relations for graviton-matter amplitudes

Jan Plefka

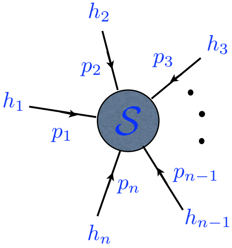
&

based on

- D. Nandan, JP, O. Schlotterer, C. Wen; arXiv:1607.05701
- D. Nandan, JP, G. Travaglini; arXiv:1803.08497, CERN-TH-2018-048
- JP, W. Wormsbecher; arXiv:1804.xxxxx, CERN-TH-2018-095

CERN TH Colloquium, April 2018

Scattering amplitudes

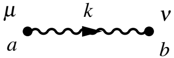


$$\mathcal{A}_n(\{p_i, h_i\}) = \text{probability amplitude for scattering process}$$

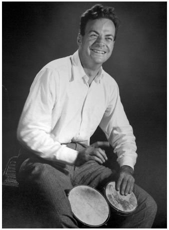
Central quantum field theory prediction for collider experiments

Computed via Feynman diagrams:

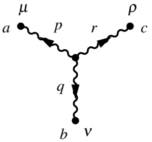
Propagator



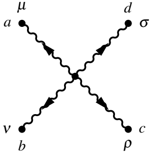
$$= \frac{\delta^{ab}\eta_{\mu\nu}}{k^2+i\epsilon} \qquad \text{(gluons)}$$



Vertices



$$= g\, f^{abc}\Big[(q-r)_\mu\, \eta_{\nu\rho} + (r-p)_\nu\, \eta_{\rho\mu} + (p-q)_\rho\, \eta_{\mu\nu}\Big]$$



$$= -ig^2\Big[\; f^{abe} f^{cde}\,(\eta_{\mu\rho}\,\eta_{\nu\sigma} - \eta_{\mu\sigma}\,\eta_{\nu\rho}) + f^{ace} f^{dbe}\,(\eta_{\mu\sigma}\,\eta_{\rho\nu} - \eta_{\mu\nu}\,\eta_{\rho\sigma}) + f^{ade} f^{bce}\,(\eta_{\mu\nu}\,\eta_{\sigma\rho} - \eta_{\mu\rho}\,\eta_{\sigma\nu})\Big]$$

"Most perfect microscopic objects in the universe" [Dixon]

Since 2005 revolutionary progress in our understanding of scattering amplitudes based on on-shell formulations bypassing Feynman diagramatic representations

- On-shell recursion relations: BCFW, ...
- Hidden symmetries: (Dual) conformal & Yangian symmetry
- Novel amplitude representations: Scattering eqs. & CHY formalism, Grassmannian picture
- Generalized unitarity: Novel loop techniques
- Duality between Gluons and Gravitons: Kinematical algebra
- String representation of field theory amplitudes
- Subleading factorization theorems
- ...

Developments with impact in both formal theory and phenomenology (and even mathematics).

Outline:

Here: New results in pure Einstein-Yang-Mills (EYM) and Einstein-QCD (EQCD) amplitudes w/o susy.

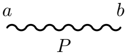
1. Brief review of Einstein $= (\mathrm{YM})^2$ double-copy construction
2. EYM from YM amplitudes @ tree-level
3. EQCD from QCD amplitudes @ tree-level
4. EYM @ 1-loop order

Einstein vs. Yang-Mills theory

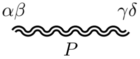
- On level of action gravity & Yang-Mills theory look rather different:

$$S_G = \frac{2}{\kappa^2}\int d^4x\,\sqrt{-g}\,R \qquad\qquad S_{YM} = -\frac{1}{2}\int d^4x\,\mathrm{tr}(F_{\mu\nu}F^{\mu\nu})$$

- Perturbative quantization: $g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}$, in Feynman/de Donder gauge



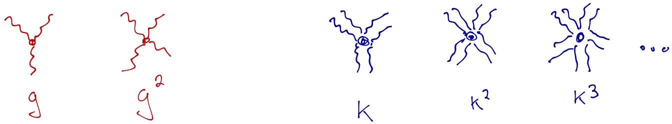
$$= \frac{-i\eta_{\mu_a\mu_b}\delta^{ab}}{P^2}$$



$$= \frac{i\,P_{\alpha\beta\gamma\delta}}{P^2}$$

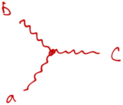
$$P_{\alpha\beta\gamma\delta} = \eta_{\alpha(\beta}\eta_{\gamma)\delta} - \frac{1}{d-2}\eta_{\alpha\beta}\eta\gamma\delta$$

- Vertices:



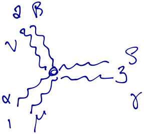
- Yet: Intricate connection betweeen S-matrices exist!

Gravity vs. Yang-Mills: 3pt Vertices

- YM: 

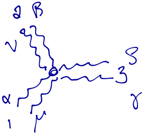
$= -g f^{abc} \left(\eta_{\mu_a \mu_b}(k_a - k_b)_{\mu_c} + \text{(a,b,c) cyclic}\right)$

- GR:

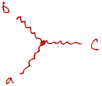


$$= i\kappa\,\text{sym}\Big[-\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\beta}\eta_{\sigma\gamma})-\tfrac{1}{2}P_6(k_{1\nu}k_{1\beta}\eta_{\mu\alpha}\eta_{\sigma\gamma})+\tfrac{1}{2}P_3(k_1\cdot k_2\eta_{\mu\nu}\eta_{\alpha\beta}\eta_{\sigma\gamma})$$
$$+P_6(k_1\cdot k_2\eta_{\mu\alpha}\eta_{\nu\sigma}\eta_{\beta\gamma})+2P_3(k_{1\nu}k_{1\gamma}\eta_{\mu\alpha}\eta_{\beta\sigma})-P_3(k_{1\beta}k_{2\mu}\eta_{\alpha\nu}\eta_{\sigma\gamma})$$
$$+P_3(k_{1\sigma}k_{2\gamma}\eta_{\mu\nu}\eta_{\alpha\beta})+P_6(k_{1\sigma}k_{1\gamma}\eta_{\mu\nu}\eta_{\alpha\beta})+2P_6(k_{1\nu}k_{2\gamma}\eta_{\beta\mu}\eta_{\alpha\sigma})$$
$$+2P_3(k_{1\nu}k_{2\mu}\eta_{\beta\sigma}\eta_{\gamma\alpha})-2P_3(k_1\cdot k_2\eta_{\alpha\nu}\eta_{\beta\sigma}\eta_{\gamma\mu})\Big]\,,$$

- Put graviton legs on-shell: Write $\epsilon_{\mu\nu} = \epsilon_\mu \epsilon_\nu$ and take $\epsilon \cdot p = 0 = p \cdot p$

$\Rightarrow$ 

$= i\kappa \left(\eta_{\mu\nu}(k_1 - k_2)_\rho + \text{cyclic}\right)\left(\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic}\right)$

$=$ 

$\Big|_{f^{abc} \to -i\frac{\kappa}{g}\left(\eta_{\alpha\beta}(k_1 - k_2)_\gamma + \text{cyclic}\right)}$

$\Rightarrow$ In this sense Gravity = $(\text{YM})^2$!

Color-Kinematic Duality

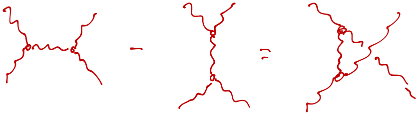
- Pure gluon amplitudes are controlled by hidden kinematical algebra: [Bern, Carrasco Johansson]

$$\mathcal{A}_n^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{L\cdot D}}{(2\pi)^{L\cdot D}} \frac{1}{S_i} \frac{n_i\, c_i}{\prod_\alpha D_\alpha}$$

n_i: kinematical numerators
c_i: Color factors
D_α: inverse propagators
S_i: Symmetry factor

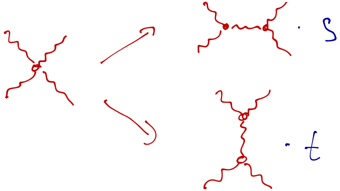
- Color factors and kinematical mumerators satisfy identical relations:

$$f^{dac}\, f^{dbc} - f^{dbc}\, f^{dac} = f^{abc}\, f^{dce} \quad \Rightarrow \quad c_i - c_j = c_k$$



Can find representation of $\mathcal{A}_n^{(L)}$ such that also $n_i - n_j = n_k$.

- Four gluon vertex absorbed into cubic graphs:



- Ambiguity of doing so is "generalized gauge transformation" $n_i \to n_i + \Delta_i$

Gravity is Double Copy of Yang-Mills [Bern, Carrasco Johansson]

$$\boxed{\mathcal{A}_n^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{L\cdot D}}{(2\pi)^{L\cdot D}} \frac{1}{S_i} \frac{n_i\, c_i}{\prod_\alpha D_\alpha} \quad \Rightarrow \quad \mathcal{M}_n^{(L)} = \sum_{i\in\Gamma_3} \int \frac{d^{L\cdot D}}{(2\pi)^{L\cdot D}} \frac{1}{S_i} \frac{n_i\, \tilde{n}_i}{\prod_\alpha D_\alpha}}$$

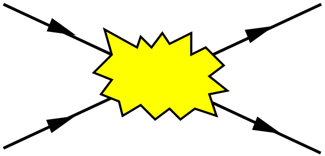
- Numerators n_i and $\tilde{n}_i$ may come from different color-kinematic duality respecting theories
- Examples

$$\begin{aligned} (\text{pure YM}) \otimes (\text{pure YM}) \quad &= \text{Einstein gravity} + \text{dilaton} + \text{axion} \\ (\mathcal{N}=4 \text{ SYM}) \otimes (\mathcal{N}=4 \text{ SYM}) &= \mathcal{N}=8 \text{ SUGRA} \end{aligned}$$

- What about Einstein-Matter amplitudes w/o susy?

  Up to 2016 not much known beyond partial results for Einstein-Yang-Mills (EYM) amplitudes of maximally helicity violating (MHV) type [Selivanov][Bern, Freitas Wong]

Tree level Einstein-Yang-Mills



with D. Nandan, O. Schlotterer, C. Wen

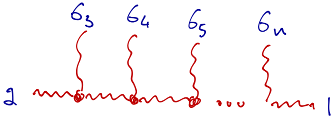
Color decomposition for gluons

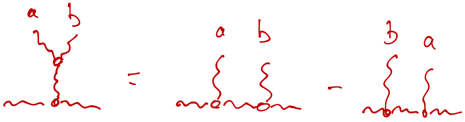
- Express amplitude $\mathcal{A}_n^{\text{tree}}$ in $(n-2)!$-sized color basis via partial or color ordered amplitudes A^{YM}

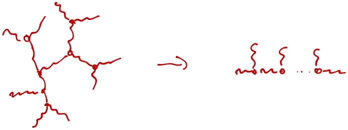
$$\mathcal{A}_{n;0}^{\text{tree}}(\{p_i, \epsilon_i, a_i\}) = \sum_{\sigma \in S_{n-2}(\{3,\ldots,n\})} C(1,2,\sigma)\, A^{\text{YM}}(1,2,\sigma)\,.$$

with Del Duca, Dixon, Maltoni (DDM) color factors

$$C(1,2,\sigma_3,\ldots,\sigma_n) := f^{a_2 a_{\sigma(3)} b_1} f^{b_1 a_{\sigma(4)} b_2} \ldots f^{b_{n-3} a_{\sigma(n)} a_1}$$

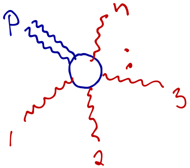
"half-ladders" 

Idea: 

then 

Einstein-Yang-Mills amplitudes

- Graviton - n-gluon scattering with color single trace (=leading order in κ) may also be decomposed in DDM basis



$$= \mathcal{A}^{\text{tree}}_{n;1} = \sum_{\sigma \in S_{n-2}(\{3,\ldots,n\})} C(1,2,\sigma)\, A^{\text{EYM}}(1,2,\sigma;p)$$

- Intriguing relation: EYM-amplitude as linear combination of gluon trees [Stieberger Taylor]

$$\boxed{A^{\text{EYM}}(1,2,\ldots,n;p) = \frac{\kappa}{g}\sum_{i=2}^{n}(\epsilon_p \cdot X_i)\, A^{\text{YM}}(1,2,\ldots,i,p,i+1,\ldots)}$$

with $X_i = k_{23\ldots i} = \sum_{j=2}^{i} k_j$ ("region momenta") and polarization $\epsilon_p^{\mu\nu} = \epsilon_p^{\mu}\epsilon_p^{\nu}$

  - First derived from string theory construction
  - Field theory proof [JP,Nandan Schlotterer,Wen] [de la Cruz Kniss,Weinzierl] using scattering eq. formalism [Cachazo,He Yuan]

New EYM to YM relations

- Two gravitons & n gluons from CHY-formalism [Nandan,JP Schlotterer,Wen]

$$
\begin{aligned}
A^{\mathsf{EYM}}(1,2,\ldots,n;{\color{blue}p},{\color{blue}q}) \; = \\
\frac{\kappa^2}{g^2}\Big[\sum_{1=i\le j}^{n-1}(\epsilon_p\cdot x_i)\,(\epsilon_q\cdot x_j)\,A(1,\ldots,i,{\color{blue}p},i{+}1,\ldots,j,{\color{blue}q},j{+}1,\ldots,n) \\
-(\epsilon_q\cdot p)\sum_{j=1}^{n-1}(\epsilon_p\cdot x_j)\sum_{i=1}^{j+1}A(1,2,\ldots,i{-}1,{\color{blue}q},i,\ldots,j,{\color{blue}p},j{+}1,\ldots,n) \\
-\frac{(\epsilon_p\cdot\epsilon_q)}{2}\sum_{l=1}^{n-1}(p\cdot k_l)\sum_{1=i\le j}^{l}A(1,2,\ldots,i{-}1,{\color{blue}q},i,\ldots,j{-}1,{\color{blue}p},j,\ldots,n) \\
+({\color{blue}p}\leftrightarrow{\color{blue}q})\Big]
\end{aligned}
$$

- Also up to 4 gravitons single-trace and double trace up to one [Nandan,JP Schlotterer,Wen]
- Double copy picture [Chiodaroli, Gunaydin Johansson, Roiban]: EYM = YM $\otimes$ (YM + ϕ^3) (up to 5 gravitons and higher traces).
- All multipicity problem [Fu,Du Huang,Feng] and multi-trace case [Teng Feng] solved recursively

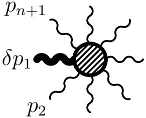
Back of envelope proof of single graviton result

- Take as an ansatz with undetermined X_i (respects factorization properties)

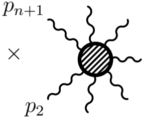
$$A^{\mathsf{EYM}}(1,2,\ldots,n;p) = \sum_{i=2}^{n} (\epsilon_p \cdot X_i)\, A^{\mathsf{YM}}(1,2,\ldots,i,p,i+1,\ldots)$$

- Exploit universal factorization when p (as gluon or graviton) goes soft

[Low 1958][Weinberg 1964]:



$$\xrightarrow{\delta \to 0} \quad S^{[j]}(\delta\, p_1, \{p_a\}) \times$$



Graviton: $$A^{\mathsf{EYM}}(1,\ldots,n;p) \overset{p\to 0}{=} \sum_{i=1}^{n} \frac{(\epsilon_p \cdot k_i)^2}{k_i \cdot p} A^{\mathsf{YM}}(1,\ldots,n)$$

Gluon: $$A^{\mathsf{YM}}(\ldots,i,p,i+1,\ldots) \overset{p\to 0}{=} \left(\frac{\epsilon_p \cdot k_{i+1}}{k_{i+1} \cdot p} - \frac{\epsilon_p \cdot k_i}{k_i \cdot p}\right) A^{\mathsf{YM}}(1,\ldots,n)$$

$$\sum_i \frac{(\epsilon_p \cdot k_i)^2}{k_i \cdot p} \overset{!}{=} \sum_i (\epsilon_p \cdot X_i) \left(\frac{\epsilon_p \cdot k_{i+1}}{k_{i+1} \cdot p} - \frac{\epsilon_p \cdot k_i}{k_i \cdot p}\right) \Rightarrow \quad k_i = X_i - X_{i-1} \quad \Rightarrow \boxed{X_i = -\sum_{j<i} k_j}$$

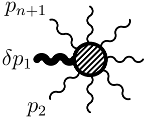
Back of envelope proof of single graviton result

- Take as an ansatz with undetermined X_i (respects factorization properties)

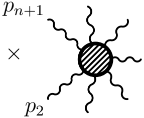
$$A^{\mathsf{EYM}}(1,2,\ldots,n;p) = \sum_{i=2}^{n} (\epsilon_p \cdot X_i)\, A^{\mathsf{YM}}(1,2,\ldots,i,p,i+1,\ldots)$$

- Exploit universal factorization when p (as gluon or graviton) goes soft

[Low 1958] [Weinberg 1964]:



$$\xrightarrow{\delta\to 0} \quad S^{[j]}(\delta\, p_1, \{p_a\}) \times$$

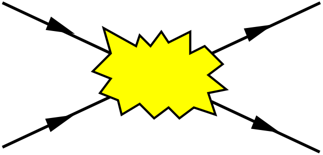


Graviton: $$A^{\mathsf{EYM}}(1,\ldots,n;p) \overset{p\to 0}{=} \sum_{i=1}^{n} \frac{(\epsilon_p \cdot k_i)^2}{k_i \cdot p} A^{\mathsf{YM}}(1,\ldots,n)$$

Gluon: $$A^{\mathsf{YM}}(\ldots,i,p,i+1,\ldots) \overset{p\to 0}{=} \left(\frac{\epsilon_p \cdot k_{i+1}}{k_{i+1}\cdot p} - \frac{\epsilon_p \cdot k_i}{k_i \cdot p}\right) A^{\mathsf{YM}}(1,\ldots,n)$$

$$\sum_i \frac{(\epsilon_p \cdot k_i)^2}{k_i \cdot p} \overset{!}{=} \sum_i (\epsilon_p \cdot X_i) \left(\frac{\epsilon_p \cdot k_{i+1}}{k_{i+1}\cdot p} - \frac{\epsilon_p \cdot k_i}{k_i \cdot p}\right) \Rightarrow \quad k_i = X_i - X_{i-1} \quad \Rightarrow \quad \boxed{X_i = -\sum_{j<i} k_j}$$

Tree level Einstein-Matter



with W. Wormsbecher

General matter minimally coupled to gravity?

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{sQCD} + \mathcal{L}_{YM} + \mathcal{L}_{EH} + \mathcal{L}_{GF} + \mathcal{L}_{Ghosts}$$

$$\mathcal{L}_{YM} = -\frac{1}{4}\sqrt{-g}\, g^{\mu\rho}\, g^{\nu\sigma}\, F^a_{\mu\nu}\, F^a_{\rho\sigma} \qquad \mathcal{L}_{QCD} = \sqrt{-g}\, \bar{\psi}\left(i\not{D} - m_\psi\right)\psi$$

$$\mathcal{L}_{sQCD} = \sqrt{-g}\, \left(g^{\mu\nu}\,(D_\mu\,\phi)^\dagger\,(D_\nu\,\phi) - m_\phi\,\phi^\dagger\,\phi\right) \qquad \mathcal{L}_{EH} = \frac{2}{\kappa^2}\sqrt{-g}\, g^{\mu\nu}\, R_{\mu\nu}$$

- Our claim @ tree-level: "Upgrading gluons to gravitons" [JP Wormsbecher]

$$\boxed{A^{\text{EYM-X}}(1,2,\ldots,n;\underbrace{p}_{\text{graviton}}) = \sum_{i=2}^{n}(\epsilon_p\cdot k_{2\ldots i})\, A^{\text{YM-X}}(1,2,\ldots,i,\underbrace{p}_{\text{gluon}},i+1,\ldots)}$$

  Relation is universal! Other particles $\{1,2\ldots,n\}$ may be gluons, complex rep. fermions or scalars.

- For concreteness take Einstein-QCD. $\Rightarrow$ Needs to understand color decomposition with fermions (scalars) in representation $T^a_{i\bar{j}}$ of gauge group.

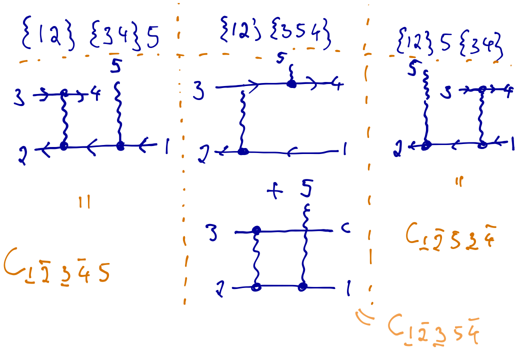
Color decomposition in QCD [Melia][Johansson Ochirov]

- Consider generic n-gluon, k-quark, k-anti-quark amplitude: MJO basis

$$\left\{ A(\underline{1}, \bar{2}, \sigma) \middle| \sigma \in \text{Dyck}_{k-1} \times \{\text{gluon insertions}\}_{n-2k} \right\}$$

- Dyck word is "well formed bracket" using: $q \to \{\underline{q}, \quad \bar{q} \to \bar{q}\}, \quad g \to g$

E.g. $n = 5,\ k = 2$



- QCD amplitude decomposes in MJO basis as

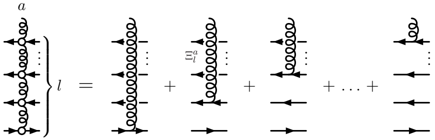
$$\mathcal{A}^{\text{tree}}_{n,k} = \sum_{\sigma \in \text{MJO basis}}^{\chi(n,k)} C(\underline{1}, \bar{2}, \sigma)\, A^{\text{QCD}}(\underline{1}, \bar{2}, \sigma) \qquad \chi(n,k) = (n-2)!/k!$$

MJO basis color factors

- Color factors read (l is level of nestedness)

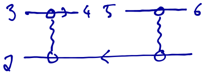
$$C(\underline{1},\bar{2},\sigma) = (-)^{k-1}\,\{2|\sigma|1\}\ \left|\begin{array}{l} q \to \{q|\,T^b \otimes \Xi^b_{l-1} \\ \quad \bar{q} \to |q\} \\ \quad\ g \to \Xi^{a_g}_l \end{array}\right.$$

$$\Xi^a_l = \sum_{s=1}^{l} \underbrace{1 \otimes \ldots \otimes 1 \otimes \overbrace{T^a \otimes 1 \otimes \ldots 1 \otimes \bar{1}}^{s}}_{l} =$$



- The Ξ^a_l obey Lie algebra of gauge group G: $[\Xi^a_l, \Xi^b_l] = f^{abc}\Xi^c_l$
- Example:

$C_{\underline{1}\bar{2}\underline{3}\bar{4}\underline{5}\bar{6}}$ =



$$= \{2|\,\{3|\,T^a \otimes \Xi^a_1 |4\}\ \{5|\,T^b \otimes \Xi^b_1 |6\} |1\} = \ldots = (T^b T^a)_{i_1 \bar{i}_2}\ T^a_{i_3 \bar{i}_4}\ T^b_{i_5 \bar{i}_6}$$

Einstein-QCD

- Single Graviton-$(n-2k)$-Gluon-k $q\bar{q}$ - amplitude color decomposes as (in single color trace sector/leading power in κ)

$$\mathcal{A}^{\text{tree}}_{n,k;1} = \sum_{\sigma \in \text{MJO basis}}^{\chi(n,k)} C(\underline{1}, \bar{2}, \sigma)\, A^{\text{EQCD}}(\underline{1}, \bar{2}, \sigma; p)$$

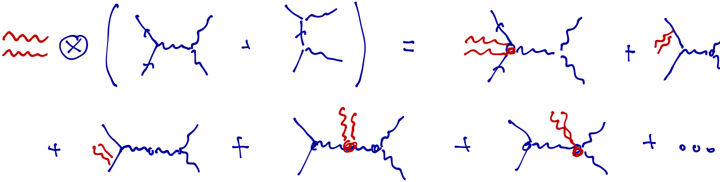
- Claim

$$A^{\text{EQCD}}(1,2,\ldots,n;p) = \frac{\kappa}{2g} \sum_{i=2}^{n} (\epsilon_p \cdot X_i) A^{\text{QCD}}(1,2,\ldots,i,p,i+1,\ldots,)\,.$$

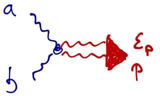
- Proof:
  - If one buys above as an ansatz (factorization properties for region momenta going on-shell are obeyed) $\Rightarrow X_i = k_{2\ldots i}$ direct consequence of universal soft argument as before
  - If not: Diagramatic analysis ...

Diagramatic Proof 1

- To obtain graviton dressed amplitude $\mathcal{A}^{\text{tree}}_{n,k;1}$ simply attach graviton leg to lower point QCD amplitude $\mathcal{A}^{\text{tree}}_{n,k;0}$ to all propagators and vertices:



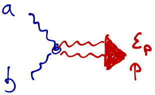
- Gluon-gluon-graviton vertex with graviton leg put on-shell:



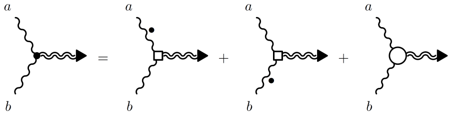
$$= i\kappa\delta^{ab}\left[\tfrac{1}{2}(p_a^2 + p_b^2)\epsilon_p^{\mu_a\mu_b} + (\epsilon_p \cdot p_a)\big(M^{\mu_a\mu_b} + W^{\mu_a\mu_b}\big)\right]$$

with $M^{\mu_a\mu_b} = (\epsilon_p \cdot p_a)\eta^{\mu_a\mu_b} + \epsilon_p^{\mu_a}p^{\mu_b} - \epsilon_p^{\mu_b}p^{\mu_a}$ and $W^{\mu_a\mu_b} = \epsilon_p^{\mu_a}p_b^{\mu_b} - (a \leftrightarrow b)$

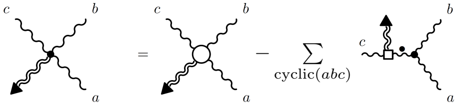
- W vanishes by Ward Identity in an amplitude, p_a^2 and p_b^2 are inverse propagators of gluon legs.



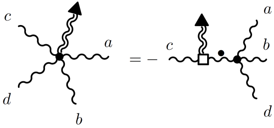
$$= i\kappa\delta^{ab}\left[\tfrac{1}{2}(p_a^2+p_b^2)\epsilon_p^{\mu_a\mu_b} + (\epsilon_p\cdot p_a)M^{\mu_a\mu_b}\right]$$

- Hence: 

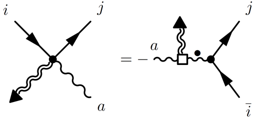
with eff. vertices

- Vertex corrections: 

$+(a, b, c$ cyclic)

and: 

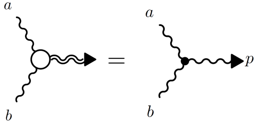
- $(a,\ b,\ d,\ c)$ cyclic

- Quarks: 

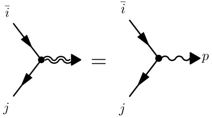
- Hence the box-vertices cancel out and one only has white circle-vertices.

Effective vertices

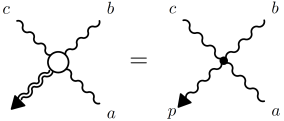
- Upgrading a gluon to a graviton through color-kinematic replacement



$f^{apb} \to -i\,\frac{\kappa}{2\,g}\,(\epsilon_p \cdot k_a)\,\delta_{ab}$



$(T^p)_{j\bar{i}} \to \frac{\kappa}{2\,g}\,(\epsilon_p \cdot k_j)\,\delta_{mn}$



$f^{mpn} \to -i\,\frac{\kappa}{2\,g}\,(\epsilon_p \cdot k_m)\,\delta_{mn}$

- It then follows that $\boxed{\mathcal{A}^{\text{EQCD}}_{n,k;1} = \mathcal{A}^{\text{QCD}}_{n+1,k}\big|_{\mathcal{R}_p}}$ with the color-kinematic replacement rule

$$\mathcal{R}_p := \{(T^p)_{j\bar{i}} \to (\epsilon_p \cdot k_j)\,\delta_{j\bar{i}}\,,\ f^{cpb} \to (\epsilon_p \cdot k_c)\,\delta_{cb}\}$$

- Moreover, one shows for the MJO color factors the key relation

$$C(\underline{1}, \bar{2}, \sigma_1, p, \sigma_2)\Big|_{\mathcal{R}_p} = (\epsilon_p \cdot k_{2\sigma_1})\,C(\underline{1}, \bar{2}, \sigma_1, \sigma_2)$$

This concludes the proof of central claim.

Summary

$$\boxed{A^{\text{EYM-X}}(1,2,\ldots,n;\underbrace{p}_{\text{graviton}}) = \frac{\kappa}{2g}\sum_{i=2}^{n}(\epsilon_p\cdot k_{2\ldots i})\,A^{\text{YM-X}}(1,2,\ldots,i,\underbrace{p}_{\text{gluon}},i+1,\ldots)}$$

- Result generalizes to YM + N_f massive fermions or scalars in D-dimensions.
- [Fu,Du; Huang,Feng] results for single trace EYM amplitudes should directly generalize as they are soley built on gauge invariance and BCJ relations.
- $\Rightarrow$ Complete tree-level S-Matrix of Einstein-Matter (single trace) has been reduced to pure matter amplitudes.

Claim

- E.g. we claim that for two gravitons coupled to matter:

$$A^{\text{EYM-X}}(1,2,\ldots,n;\ \underbrace{p,q}_{\text{gravitons}}\ ) =$$

$$\frac{\kappa^2}{g^2}\Big[\sum_{1=i\leq j}^{n-1}(\epsilon_p\cdot x_i)\,(\epsilon_q\cdot x_j)\,A^{\text{YM-X}}(1,\ldots,i,\underbrace{p}_{\text{gluon}},i{+}1,\ldots,j,\underbrace{q}_{\text{gluon}},j{+}1,\ldots,n)$$

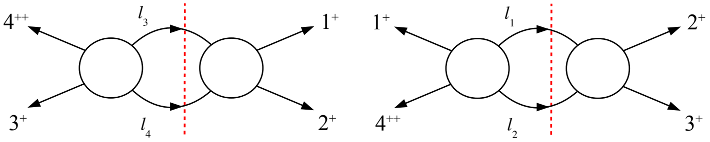
$$-\,(\epsilon_q\cdot p)\sum_{j=1}^{n-1}(\epsilon_p\cdot x_j)\sum_{i=1}^{j+1}A^{\text{YM-X}}(1,2,\ldots,i{-}1,\underbrace{q}_{\text{gluon}},i,\ldots,j,\underbrace{p}_{\text{gluon}},j{+}1,\ldots,$$

$$-\,\frac{(\epsilon_p\cdot\epsilon_q)}{2}\sum_{l=1}^{n-1}(p\cdot k_l)\sum_{1=i\leq j}^{l}A^{\text{YM-X}}(1,2,\ldots,i{-}1,\underbrace{q}_{\text{gluon}},i,\ldots,j{-}1,\underbrace{p}_{\text{gluon}},j,\ldots,$$

$$+\,(p\leftrightarrow q)\Big]$$

- And higher single trace generalizations. Multi-trace is unclear at this point.

One Loop Einstein-Yang-Mills



with D. Nandan and G. Travaglini

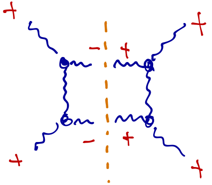
Rational one-loop amplitudes

- Gluon & Graviton amplitudes of same helicity or max. one helicity flip:

$$\boxed{A_n(\pm,+,+,\ldots,+) = M_n(\pm,+,+,\ldots,+) = 0}$$

  Due to hidden supersymmetry. True @ tree-level for any QFT! In susy field theories true to all loops

- In non-susy theories @ one-loop these amplitudes are finite, rational expressions = look like trees. Reason: Unitarity cuts vanish in 4d

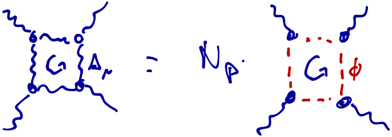


$= A_{MHV} \cdot 0 = 0$ $\Rightarrow$ must be finite rational functions

- Elegant way to construct: Employ cuts in $D = 4 - 2\epsilon$. Rational term $\mathcal{R}$ in 4D will acquire discontinuity in D dimensions: [Bern Morgan]

$$\mathcal{R} \;\to\; \mathcal{R}(-s)^{-\epsilon} \;=\; \mathcal{R}\left[1 - \epsilon\log(-s)\right] + \cdots$$

Computational tricks

- SUSY Ward identity implies: t 

$$\boxed{A_n^{\text{any state in loop}}(1,2,\ldots,n) = N_p\, A_n^{\text{scalar in loop}}(1,2,\ldots,n)}$$

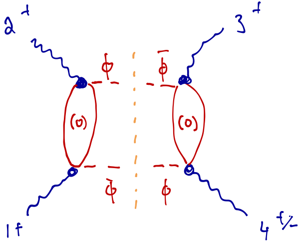
with $N_p = \sum_{\text{d.o.f.}}(-)^F$

- Take scalars to be massive in order to simulate D-dimensions:

$$L^2 = 0 = l_{(4)}^2 - \underbrace{l_{(-2\epsilon)}^2}_{\mu^2} = l_{(4)}^2 - \mu^2$$

Applied in two particle cuts:

- Fuse trees with internal massive complex scalars



Pure gluon results

- Pure YM: All multiplicity expression [Bern, Chalmers, Dixon, Kosower] [Mahlon]

$$A_n^{\text{1-loop}}(1^+,\ldots,n^+) = \frac{iN_p}{96\pi^2} \sum_{1\le k_1<k_2<k_3<k_4\le n} \frac{\langle k_1k_2\rangle[k_2k_3]\langle k_3k_4\rangle[k_4k_1]}{\langle 12\rangle\langle 23\rangle\cdots\langle n1\rangle}$$

- Spinor helicity: $\boxed{p^\mu \to p^{\alpha\dot\alpha} = \bar\sigma_\mu^{\alpha\dot\alpha}\, p^\mu = \lambda^\alpha\tilde\lambda^{\dot\alpha}}$ (makes $p^\mu p_\mu = 0$ manifest)

$$\lambda^\alpha = \frac{1}{\sqrt{p^0+p^3}}\begin{pmatrix} p^0+p^3 \\ p^1+ip^2 \end{pmatrix}, \quad \tilde\lambda^{\dot\alpha} = (\lambda^\alpha)^\dagger, \quad \langle ij\rangle = \epsilon_{\alpha\beta}\lambda_i^\alpha\lambda_j^\beta, \quad [ij] = \epsilon_{\beta\alpha}\tilde\lambda_i^\alpha\tilde\lambda_j^\beta$$

- State of the art: $A_n^{\text{2-loop}}$ for n=4 [Bern, Dixon, Kosower] and n=5 [Henn, Gehrmann, Presti, 2016]
- Similar one-loop results for all multiplicities for single minus helicity:

$$A_n^{\text{1-loop}}(1^-,2^+,\ldots,n^+)$$ [Bern, Dixon, Kosower] [Mahlon]

Pure graviton results - our goal

- All plus helicity @ all multiplicities conjectured (checked up to $n \leq 7$) [Bern, Dixon, Perelstein, Rozowsky]

$$M_n^{\text{1-loop}}(1^+, 2^+, \dots, n^+)$$

- Single minus helicity via recursive formula [Dunbar, Ettle, Perkins]

$$M_n^{\text{1-loop}}(1^-, 2^+, \dots, n^+)$$

- Nothing known for pure EYM amplitudes!

Our goal: Compute the mostly plus EYM amplitudes @ 4 points

Pure graviton results - our goal

- All plus helicity @ all multiplicities conjectured (checked up to $n \leq 7$) [Bern, Dixon, Perelstein, Rozowsky]

$$M_n^{\text{1-loop}}(1^+, 2^+, \ldots, n^+)$$

- Single minus helicity via recursive formula [Dunbar, Ettle, Perkins]

$$M_n^{\text{1-loop}}(1^-, 2^+, \ldots, n^+)$$

- Nothing known for pure EYM amplitudes!

Our goal: Compute the mostly plus EYM amplitudes @ 4 points

All rational one-loop EYM amplitudes at 4 pts.

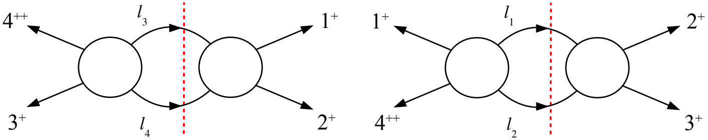
- Compute the leading in κ results for all rational four-point EYM amplitudes:

$$\langle +,+,+;++\rangle \quad \langle +,+,+,;--\rangle \quad \langle -,+,+;++\rangle \quad \langle +,+;++,++\rangle$$

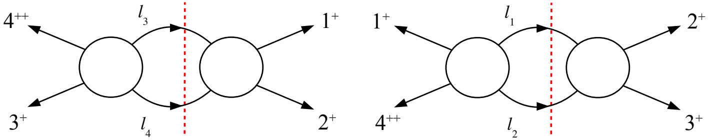
$$\langle -,+;++,++\rangle \quad \langle +,+;++,--\rangle \quad \langle \pm;++,++,++\rangle \quad \langle +;++,++,--\rangle$$

  In total 9 amplitudes to compute!

- Method: Two-particle cuts
- Discuss simplest case here: All plus single graviton: $\langle 1^+, 2^+, 3^+; 4^{++}\rangle$



Needs the gluon-graviton-massive scalar amplitudes at four points $\Rightarrow$ previous results handy! Follow directly from gluon-scalar amplitudes.



- s-channel cut

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{(1234),s} = \mathcal{A}(3^+,4^{++},l_{3,\phi},l_{4,\bar\phi})A(1^+,2^+,-l_{4,\phi},-l_{3,\bar\phi})$$

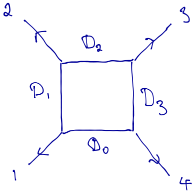
$$= 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\,\frac{\langle 3|l|4]}{\langle 34\rangle}\,\frac{\big[(2\pi)\delta(D_0)\big]\,\big[(2\pi)\delta(D_2)\big]}{D_1\,D_3}$$

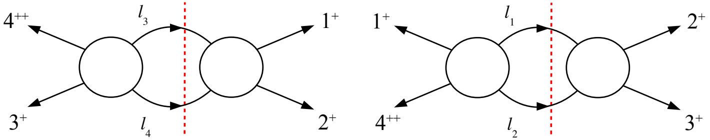
- t-channel cut

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{(1234),t} = \mathcal{A}(4^{++},1^+,l_{1,\phi},l_{2,\bar\phi})A(2^+,3^+,-l_{2,\phi},-l_{1,\bar\phi})$$

$$= 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\,\frac{\langle 1|l|4]}{\langle 14\rangle}\,\frac{\big[(2\pi)\delta(D_1)\big]\,\big[(2\pi)\delta(D_3)\big]}{D_0\,D_2}$$

- With $D_i = (l-q_i)^2 - \mu^2$





- s-channel cut

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{(1234),s} = \mathcal{A}(3^+,4^{++},l_{3,\phi},l_{4,\bar{\phi}})A(1^+,2^+,-l_{4,\phi},-l_{3,\bar{\phi}})$$

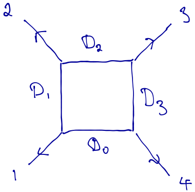
$$= 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \frac{\langle 3|l|4]}{\langle 34\rangle} \frac{\left[(2\pi)\delta(D_0)\right]\left[(2\pi)\delta(D_2)\right]}{D_1\, D_3}$$

- t-channel cut

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{(1234),t} = \mathcal{A}(4^{++},1^+,l_{1,\phi},l_{2,\bar{\phi}})A(2^+,3^+,-l_{2,\phi},-l_{1,\bar{\phi}})$$

$$= 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \frac{\langle 1|l|4]}{\langle 14\rangle} \frac{\left[(2\pi)\delta(D_1)\right]\left[(2\pi)\delta(D_3)\right]}{D_0\, D_2}$$

- With $D_i = (l - q_i)^2 - \mu^2$



Constructing the integrand

- Comparing

$$A^{(1)}\Big|_{(1234),s} = 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\, \frac{\langle 3|l|4]}{\langle 34\rangle}\, \frac{\big[(2\pi)\delta(D_0)\big]\,\big[(2\pi)\delta(D_2)\big]}{D_1\,D_3}$$

$$A^{(1)}\Big|_{(1234),t} = 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\, \frac{\langle 1|l|4]}{\langle 14\rangle}\, \frac{\big[(2\pi)\delta(D_1)\big]\,\big[(2\pi)\delta(D_3)\big]}{D_0\,D_2}$$

- Note the identity

$$\frac{\langle 3|l|4]}{\langle 34\rangle} = \frac{\langle 1|l|4]}{\langle 14\rangle} + \frac{[24]}{[32]\langle 34\rangle}\,(D_3 - D_0)$$

- Inserting this into the s-cut expression & droping the D_0 term yields the integrand lifted off the cuts using $(2\pi)\delta(D) \to i/D$

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{1234} =$$

$$-2i\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\int \frac{d^4l}{(2\pi)^4}\,\frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}}\,\frac{\mu^4}{D_0D_1D_2D_3}\left[\frac{\langle 1|l|4]}{\langle 14\rangle} + \frac{[24]}{[32]\langle 34\rangle}\,D_3\right]$$

Linear box + triangle

- Passarino-Veltman reducing this yields $\boxed{A^{(1)}(1^+,2^+,3^+;4^{++}) = 0}$ in 4D!!

Constructing the integrand

- Comparing

$$A^{(1)}\Big|_{(1234),s} = 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \frac{\langle 3|l|4]}{\langle 34\rangle} \frac{\big[(2\pi)\delta(D_0)\big]\,\big[(2\pi)\delta(D_2)\big]}{D_1\,D_3}$$

$$A^{(1)}\Big|_{(1234),t} = 2i\mu^4 \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \frac{\langle 1|l|4]}{\langle 14\rangle} \frac{\big[(2\pi)\delta(D_1)\big]\,\big[(2\pi)\delta(D_3)\big]}{D_0\,D_2}$$

- Note the identity

$$\frac{\langle 3|l|4]}{\langle 34\rangle} = \frac{\langle 1|l|4]}{\langle 14\rangle} + \frac{[24]}{[32]\langle 34\rangle}(D_3 - D_0)$$

- Inserting this into the s-cut expression & droping the D_0 term yields the integrand lifted off the cuts using $(2\pi)\delta(D) \to i/D$

$$A^{(1)}(4^{++};1^+,2^+,3^+)\Big|_{1234} =$$

$$-2i\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\int \frac{d^4l}{(2\pi)^4}\frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}}\frac{\mu^4}{D_0D_1D_2D_3}\left[\frac{\langle 1|l|4]}{\langle 14\rangle} + \frac{[24]}{[32]\langle 34\rangle}D_3\right]$$

Linear box + triangle

- Passarino-Veltman reducing this yields $\boxed{A^{(1)}(1^+,2^+,3^+;4^{++}) = 0}$ in 4D!!

Same method applied to the other 8 cases yields the compact results:

$$A^{(1)}(1^+,2^+,3^+;4^{++})\Big|_{\kappa g^3} = 0\,,$$

$$A^{(1)}(1^+,2^+,3^+;4^{--})\Big|_{\kappa g^3} = -\frac{i}{(4\pi)^2}\,\frac{[12][34]}{\langle 12\rangle\langle 34\rangle}\,(\langle 42\rangle[23]\langle 34\rangle)^3\,\frac{s^2+t^2+u^2}{6\,s^2\,t^2\,u^2}\,,$$

$$A^{(1)}(1^-,2^+,3^+;4^{++})\Big|_{\kappa g^3} = \frac{i}{(4\pi)^2}\,\frac{[24][34]}{\langle 24\rangle\langle 34\rangle}\,\frac{1}{\langle 23\rangle[21][31]}\,\frac{1}{6}\,(s^2+u^2)\,,$$

$$A^{(1)}(1^+,2^+;3^{++},4^{++})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[12]}{\langle 12\rangle}\,\frac{[34]^2}{\langle 34\rangle^2}\,\frac{s}{6}\,,$$

$$A^{(1)}(1^-,2^+;3^{++},4^{++})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[24]^2[34]^2\langle 14\rangle^2}{\langle 34\rangle^2}\,\frac{s}{6\,t\,u}\,,$$

$$A^{(1)}(1^+,2^+;3^{++},4^{--})\Big|_{\kappa^2 g^2} = \frac{i}{(4\pi)^2}\,\frac{[12][1,3]^4\langle 14\rangle^4}{\langle 12\rangle}\,\frac{t^2+u^2}{6\,s\,t^2\,u^2}\,,$$

$$A^{(1)}(1^\pm;2^{++},3^{++},4^{++})\Big|_{\kappa^3 g} = 0\,,$$

$$A^{(1)}(1^+;2^{++},3^{++},4^{--})\Big|_{\kappa^3 g} = 0\,.$$

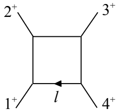
Double copy construction

- $A_{\mathsf{EYM}} = A_{\mathsf{YM}} \otimes A_{\mathsf{YM}+\phi^3}$ [Chiodaroli,Gunaydin Johansson,Roiban]
- $\mathsf{YM} + \phi^3$ theory has biadjoint scalars $\phi^{A\hat{a}}$ next to the gluons $A^{\hat{a}}_{\mu}$

$$\mathcal{L}_{\mathsf{YM}+\phi^3} = -\frac{1}{4} F^{\hat{a}}_{\mu\nu} F^{\mu\nu\,\hat{a}} + \frac{1}{2} (D_\mu \phi^A)^{\hat{a}} (D^\mu \phi^A)^{\hat{a}} + \frac{1}{3!}\,\lambda\, g\, F^{ABC}\, f^{\hat{a}\hat{b}\hat{c}}\,\phi^{A\hat{a}}\,\phi^{B\hat{b}}\,\phi^{C\hat{c}} \\ -\frac{g^2}{4}\, f^{\hat{a}\hat{b}\hat{d}}\, f^{\hat{d}\hat{c}\hat{d}}\,\phi^{A\hat{a}}\,\phi^{B\hat{b}}\,\phi^{A\hat{c}}\,\phi^{B\hat{d}}\,.$$

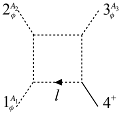
- Redo the $\langle 1^+, 2^+, 3^+, 4^{++}\rangle$ using double copy:

$$A^{(1)}(1^+,2^+,3^+,4^+) =$$



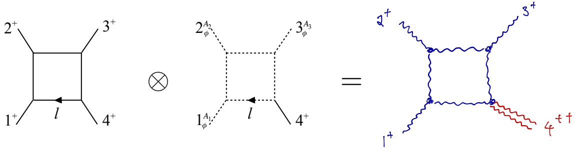
$$= \frac{[12][34]}{\langle 12\rangle\langle 34\rangle} \int \frac{d^4 l}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}} \frac{\mu^4}{D_0 D_1 D_2 D_3}$$

$$A^{(1)}(1^{A_1}_{\phi}, 2^{A_2}_{\phi}, 3^{A_3}_{\phi}, 4^+)\Big|_{\mathsf{boxes}} =$$



$$= i \int \frac{d^4 l}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}} \frac{f^{A_1 A_2 A_3}}{D_0 D_1 D_2 D_3} \frac{\langle q|l|4]}{\langle q4\rangle} + \ \mathsf{cycl(1,2,3)}\,.$$

Double copy construction of $\langle 1^+, 2^+, 3^+; 4^{++} \rangle$



- Double copy:

$$A^{(1)}(1^+_{A_1}, 2^+_{A_2}, 3^+_{A_3}, 4^{++}) =$$
$$i f^{A_1 A_2 A_3} \frac{[12][34]}{\langle 12 \rangle \langle 34 \rangle} \int \frac{d^4 l}{(2\pi)^4} \frac{d^{-2\epsilon}\mu}{(2\pi)^{-2\epsilon}} \frac{\mu^4}{D_0 D_1 D_2 D_3} \frac{\langle q|l|4]}{\langle q4 \rangle} + \text{cycl(1,2,3)}$$

- Indeed integrates to zero in 4D as well!

Summary & Outlook:

Tree level Einstein-YM-Matter amplitudes reduced to YM-Matter ones

- Yukawa and ϕ^n-couplings also possible from diagramatics. No known color basis.
- Multi-trace tree-amplitudes?
- Loop level implications?

All Rational one-loop Einstein-Yang-Mills @ 4 points computed

- MHV amplitudes?
- Gravitons in the loop? $\Rightarrow$ higher κ-orders. Do zeros persist?
- General multiplicity results?

Thank you!