Neutrino Physics

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1. *neutrino masses in the Lagrangian*
   - Majorana, Dirac, phases, 2s and all that
2. *neutrinos oscillate* $\Leftrightarrow$ *have mass*
   - atmospheric oscillations
   - usual 2 flavour oscillations
   - (why can one use a Schrodinger Eqn ?)
3. oscillations in matter
   - solar (2-flav) oscillations
   - no supernovae :(
   - 3 generations, or leptogenesis

(hypothetical /known) history of neutrinos (shy in the lab, relevant in cosmo)

- ...
- inflation (gives large scale CMB fluctuations) (?driven by sneutrino ?)
- baryogenesis (excess of matter over anti-matter)via leptogenesis ?
- relic density of (cold) Dark Matter (?heavy neutrinos ?)Shaposhnikov
- Big Bang Nucleosynthesis ($H, D,^3 He,^4 He,^7 Li$ at $T \sim$ MeV))
  $\Leftrightarrow$ 3 species of relativistic ν in the thermal soup
- decoupling of photons — $e + p \to H$ (CMB spectrum today)
  cares about radiation density $\leftrightarrow N_\nu, m_\nu$
- for 10^{10} yrs —stars are born, radiate (γ, ν), and die
- supernovae explode (?thanks to ν ?) spreading heavy elements

- 1930 : Pauli hypothesises the "neutrino", to conserve E in $n \to p + e(+\nu)$
- 1953 Reines and Cowan : neutrino CC interactions in detector near a reactor
- invention of the Standard Model (SM) : massless ν
-
- **neutrinos have mass ! There is more in the Lagrangian than the SM...**

References...

Giunti website "neutrino unbound" : http ://www.nu.to.infn.it/

fits : http ://www.nu-fit.org/

Raffelt talks (astropart) :http ://wwwth.mpp.mpg.de/members/raffelt/

Plots thanks to Strumia + Vissani : hep-ph/0606054

simple 3-gen probabilities for LBL :Cervera etal 0002108 (+ later versions)

current state of oscillation measurements : Gonzalez-Garcia @ CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

neutrino cosmology : Lesgourgues at CERN ν plafform kickoff : https ://indico.cern.ch/event/572831/

Definitions and such...

I use Dirac spinors, with 4 degrees of freedom(dof) labelled by $\{\pm E, \pm s\}$, in *chiral* decomposition

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \ \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\psi_L = \ P_L\,\psi \quad \text{avec} \quad P_L = \frac{(1-\gamma_5)}{2} \quad , \ \psi_R = \ P_R\,\psi$$

chirality is *not* an observable ($\rightarrow$ helicity $= \pm\hat{s}\cdot\hat{k} = \pm 1/2$ in relativistic limit), but $P_{L,R}$ simple to calculate with :)

notation : $\overline{(\psi_R)} = (P_R\psi)^\dagger\gamma_0 = \psi^\dagger P_R\gamma_0 = \psi^\dagger\gamma_0 P_L = (\overline{\psi})_L$

$$(\psi^c)_L = P_L(-i\gamma_0\gamma_2\gamma_0\psi^*) = -i\gamma_0\gamma_2\gamma_0\psi^*_R$$

To write a neutrino mass

To write a mass for ν_L...Lorentz Invariance

Before discussing oscillations and the kinematics of m_ν, think about how to write a mass term for neutrinos in $\mathcal{L}$...
Cosmology says : $\sum m_i \lesssim$ eV. Oscillations say :

(global fits of www.nu-fit.org)

$$
\begin{aligned}
|\Delta^2_{31j}| &= |m_3^2 - m_j^2| = 2.52 \pm 0.04 \times 10^{-3} \ \mathrm{eV}^2 \\
&\gg \Delta m^2_{21} = 7.50 \pm 0.2 \times 10^{-5} \ \mathrm{eV}^2 \\
&\sqrt{\Delta m^2_{31}} \simeq 0.05 \ \mathrm{eV} \qquad \sqrt{\Delta m^2_{21}} \simeq 0.008 \ \mathrm{eV}
\end{aligned}
$$

Low scale so work in effective theory of SM below m_W (neglect SU(2) invariance). Mass must be Lorentz invariant. Only possibility for a four-component fermion ψ :

$$
m\overline{\psi}\,\psi = m\overline{\psi_L}\,\psi_R + m\overline{\psi_R}\,\psi_L
$$

To write a Dirac mass for ν_L

Work in effective theory below m_W. Neutrino mass must be Lorentz invariant. For four-component fermion ψ :

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

1. Dirac masss term : SM has only ν_L, 2-component chiral fermion $\Rightarrow$ introduce chiral gauge singlet fermion ν_R
Construct fermion number conserving mass term like for other SM fermions :

$$m\overline{\nu_L}\,\nu_R \;+ m\overline{\nu_R}\,\nu_L$$

In full SM : $\lambda(\overline{\nu_L}, \overline{e_L}) \begin{pmatrix} H_0 \\ H_- \end{pmatrix} \nu_R \equiv \lambda(\overline{\ell} H) e_R \rightarrow \; m = \lambda \langle H_0 \rangle$

Diagonalising Dirac mass matrix in flavour space

Like for charged leptons and quarks : $[\lambda]_{\sigma I}$ arbitrary 3×3 matrix in flavour space

Diagonalise with different unitary transformations on left and right of Yukawa :

$$U^T [\boldsymbol{\lambda}] U^\dagger_{R\nu} = D_\nu$$

• $U = 3 \times 3$ leptonic version of CKM called PMNS matrix (Pontecorvo, Maki, Nakagawa and Sakata) : U_{PMNS}.

To write a Majorana mass for ν_L

Lorentz-invar mass term for a four-component fermion ψ :

$$m\overline{\psi}\,\psi \;=\; m\overline{\psi_L}\,\psi_R \;+ m\overline{\psi_R}\,\psi_L$$

2. Majorana mass term : the charge conjugate of ν_L is right-handed ! check !

$\Rightarrow$ write a mass term with ν_L ; *no new fields*, but lepton number violating mass :

$$\begin{aligned} \frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] &= \frac{m}{2}[(\nu_L)^\dagger\gamma_0(\nu_L)^c + ((\nu_L)^c)^\dagger\gamma_0\nu_L] \\ &= -i\frac{m}{2}[\nu_L^\dagger\sigma_2\nu_L^* + \nu_L^T\sigma_2\nu_L] \equiv \frac{m}{2}\nu_L\nu_L + h.c. \end{aligned}$$

(2nd line = 2 comp notn) *Non*-renormalisable in full SM :

$$\mathcal{L} = ... + \frac{K}{2M}(\ell H)(\ell H) + h.c. \rightarrow \frac{m}{2}\nu_L\nu_L + h.c. \quad , \quad m = \frac{K}{M}\langle H_0\rangle^2$$

$\Rightarrow$ *requires New Heavy Particles*

Majorana mass term : $(\nu_L)^c$ is right-handed

Recall that :

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \quad , \; \{\gamma^\alpha\} = \left\{ \begin{bmatrix} 0 & I \\ I & 0 \end{bmatrix}, \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix} \right\}$$

$$\{\sigma_i\} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Majorana mass term : the charge conjugate of ν_L is right-handed

$$\psi^c = -i\gamma_0\gamma_2\bar{\psi}^T = -i\gamma_0\gamma_2\gamma_0\psi^* = i\gamma_2^*\psi^*$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \begin{pmatrix} \psi_L^* \\ \\ \end{pmatrix} \\ \begin{pmatrix} \psi_R^* \\ \\ \end{pmatrix} \end{pmatrix}$$

$$(\nu_L)^c = \begin{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{pmatrix} \nu_L^* \\ \\ \end{pmatrix} \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 0 \\ \\ \end{pmatrix} \\ \begin{pmatrix} -i\sigma_2\nu_L^* \\ \\ \end{pmatrix} \end{pmatrix}$$

Diagonalising Majorana mass matrix

With multiple generations, Majorana

$$\frac{1}{2}\overline{\nu_L}^{\alpha}[m]_{\alpha\beta}(\nu_L)^c_{\beta}$$

is *symmetric*, diagonalise as :

$$U^T m U = D_m$$

• $m^\dagger m$ hermitian, for non-degen eigenvalues can obtain U from $U^\dagger m^\dagger m U = D_m^2$.

(in 2-comp notn :

$$\frac{1}{2}\nu_{L\alpha}[m]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{L\alpha}[U^* U^T m U U^\dagger]_{\alpha\beta}\nu_{L\beta} + h.c. = \frac{1}{2}\nu_{Li} m_i \nu_{Li} + h.c.$$

fermion fields anti-commute. But for ρ, σ 2-comp spinor indices,

$\nu^\rho_{Li}\varepsilon_{\rho\sigma}\nu^\sigma_{Lj} = -\nu^\sigma_{Lj}\varepsilon_{\rho\sigma}\nu^\rho_{Li} = \nu^\sigma_{Lj}\varepsilon_{\sigma\rho}\nu^\rho_{Li}$)

Muddle for theorists : Majorana 2s in the Lagrangian

Recall : usual to distribute $\frac{1}{2}$s for identical fields in $\mathcal{L}$, in order that F-rules and physical parameters not contain 2s : $\frac{m}{2}\nu_L\nu_L + h.c.$, $\frac{\kappa}{2M}(\ell H)\ell H) + h.c.$ (like for real scalar masses) because get F-rules as $\delta^n\mathcal{L}/\delta\nu^n$...

A majorana mass m appears in $\mathcal{L}$ as (4-comp notn on left, 2-comp notn on right)

$$\frac{m}{2}[\overline{\nu_L}(\nu_L)^c + \overline{(\nu_L)^c}\nu_L] \equiv \frac{m}{2}\nu_L\nu_L + h.c.$$

A dirac mass m appears in $\mathcal{L}$ as

$$m\overline{\psi}\psi + h.c.$$

2.8 eigenvectors of a Majorana mass matrix

eigenvectors $\vec{v}_i$ of a hermitian matrix A, with eigenvalues $\{a_i\}$ from

$$A\vec{v}_i = a_i\vec{v}_i$$

because hermitian : $V^\dagger AV = D_A = diag\{a_1, ...a_n\}$ (V unitary)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} \\ \vec{v}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{v}_3 \\ \\ \end{pmatrix} \end{bmatrix} \begin{bmatrix} a_1 & & \\ & ... & \\ & & a_n \end{bmatrix}$$

Whereas for majorana : $U^T AU = D_A \Rightarrow AU = U^* D_A$ (U unitary $UU^\dagger = 1$)

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \\ \vec{u}_1 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2 \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3 \\ \\ \end{pmatrix} \end{bmatrix} = \begin{bmatrix} \begin{pmatrix} \\ \vec{u}_1^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_2^* \\ \\ \end{pmatrix} \begin{pmatrix} \\ \vec{u}_3^* \\ \\ \end{pmatrix} \end{bmatrix} \begin{bmatrix} a_1 & & \\ & ... & \\ & & a_n \end{bmatrix}$$

For Majorana matrix :

$$A\vec{u}_i = a_i\vec{u}_i^*$$

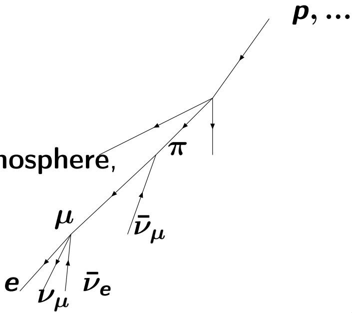
Historical problems...

BUT...historical "problems" : neutrinos disappear...

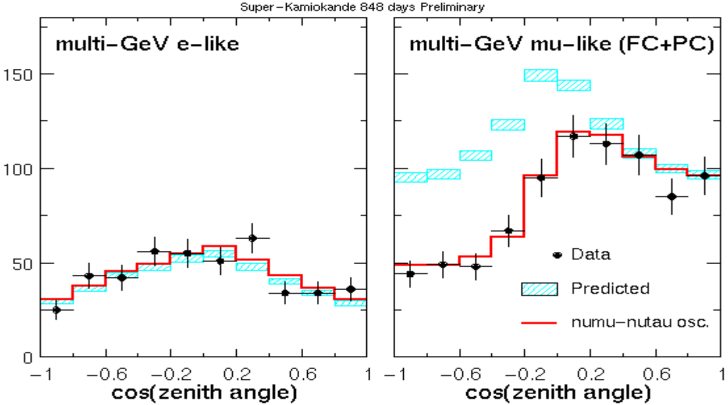
2. the sun produces energy by a network of nuclear reactions, should produce ν_e (lines and continuum) which escape. The photons diffuse to the surface. Observed ν_e flux $\sim .3 \to .5$ expected from solar energy output. Flux in $\sum$ flavours $\sim$ expected (SNO).
$\Rightarrow$ new ν physics (BSM !), that changes ν flavour on way out of sun :

- magnetic moments ?
- wierd new interactions ?
- masses (and mixing angles) in matter
- ...

1. deficit of ν_μ arriving from **earth's atmosphere**, produced in cosmic ray interactions :
expect $N(\nu_\mu + \bar{\nu}_\mu) \simeq 2N(\nu_e + \bar{\nu}_e)$
see deficit of $\nu_\mu, \bar{\nu}_\mu$ from below.



SK-98 : $\nu_\mu + H_2 0 \rightarrow \mu + ..$, deficit in ν_μ from below (PRL 81 (1998) 1562-1567)



upwards $\leftrightarrow$ cos= -1 ; down $\leftrightarrow$ cos= + 1.
L : 20 km $\leftrightarrow$ 10 000 km.

SNO : solar ν_e deficit, but expected $\sum \nu_\alpha$ flux(PRL 89 (2002) 011301)

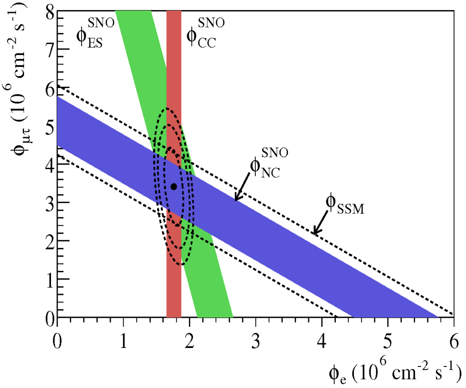


FIG. 3: Flux of ^{8}B solar neutrinos which are μ or τ flavor vs flux of electron neutrinos deduced from the three neutrino reactions in SNO. The diagonal bands show the total ^{8}B flux as predicted by the SSM [11] (dashed lines) and that measured