Systematic Errors on the Emittance Measurement

C Hunt

MICE Analysis Workshop

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Imperial College
London

Introduction

- Investigate and measure the systematic bias and uncertainty on the emittance reconstruction,
- Concentrating on the effects of Field Miaslignment, Scale and Uniformity,
- All three are modelled for the reconstruction and any deviation will affect the emittance calculation,
- Other sources of systematic error have been covered by Victoria to a high precision.

The Basics

1. Tracker-Field Alignment Algorithm

   Works to high precision, but with difficult to quantify systematics.

   Luckily that doesn't matter for this study!

2. Official CDB Geometries with Comsol Field Map

   Can vary the alignments, and move things around in MC.

3. Official CDB Geometry with MAUS Field Model

   An alternative field map - no PRY effects. A significant overestimate for variations in uniformity

4. Scale Factors!

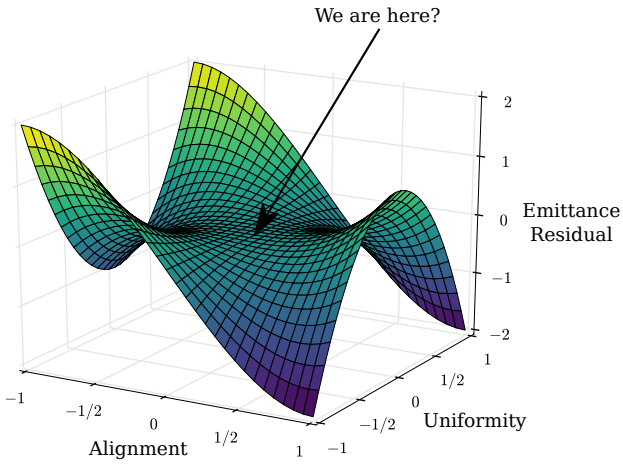
   Can arbitrariry scale fields in MC and see how the reconstruction changes.

Concept

Estimate the sytematic bias and uncertainty for a geometry that we trust.

Then demonstrate that the residuals don't change across variations in the geometries.

Assert that we trust the systematic errors we estimate as they don't change within the space of reasonable geometry models.



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Job List

Type	Job	Testing	Official
Analysis	Estimate the Tracker-Field Alignment	Done	Done
Analysis	Systematic Error Estimate	Done	Done
MC	CDB + 1-Sigma Misalignment	Done	Ready
MC	CDB + MAUS Fields MC	Done	Ready
MC	CDB + 1-Sigma Scale Factors	Done	Ready

Can run with higher stats if needed. But local MC agrees with Official Batch MC.

The Analysis

Aim to distinguish two measurements from the proceedure:

1. Estimate of a Systematic Bias — *Residual from MC study*

   A fixed offset from the expected value

2. Estimate of a Systematic Error — *Chi-Square Minimisation Algorithm*

   A broadening of the measurement variance

The Analysis

With MC, we make many independent measurements of an emittance.

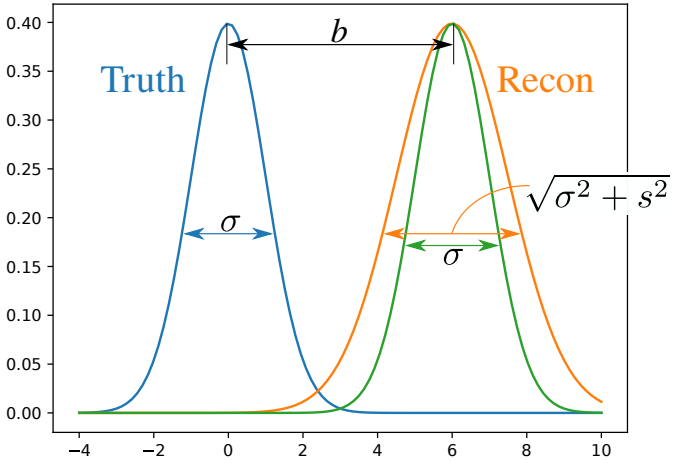
A true emittance sample has mean, x, and variance, σ^2.

Assume measurement introduces a bias, b, and a systematic error contribution, s^2.

So each emittance measurement is transformed by, $(x, \sigma^2) \rightarrow (x + b,\ \sigma^2 + s^2)$

Assuming normally distributed systematics.

The Analysis



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The Analysis

1. Bias, b,

   Difference between true mean and measured mean.

2. Systematic Uncertainty, s,

   Minimize the expected $\chi^2 - N_{dof}$ from the measured mean.

Without Systematics

$$\chi^2 = \sum_i \frac{(x_i - \hat{x})^2}{\sigma^2}$$

Modified For Systematics

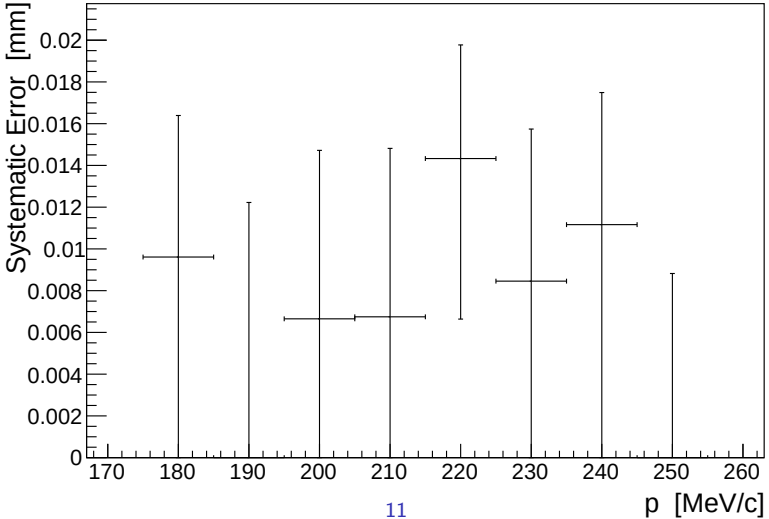
$$\chi^2 = \sum_i \frac{(x_i - \hat{x} - b)^2}{\sigma^2 + s^2}$$

Systematic Uncertainty

- Proven to be statistically unstable,
- Requires a huge amount of data to constrain,
- Remains consistent with zero for all MC simulations,

The conclusion is that the uncertainty on the emittance measurement has a negligible contribution from the field model. It is zero!

Systematic Uncertainty



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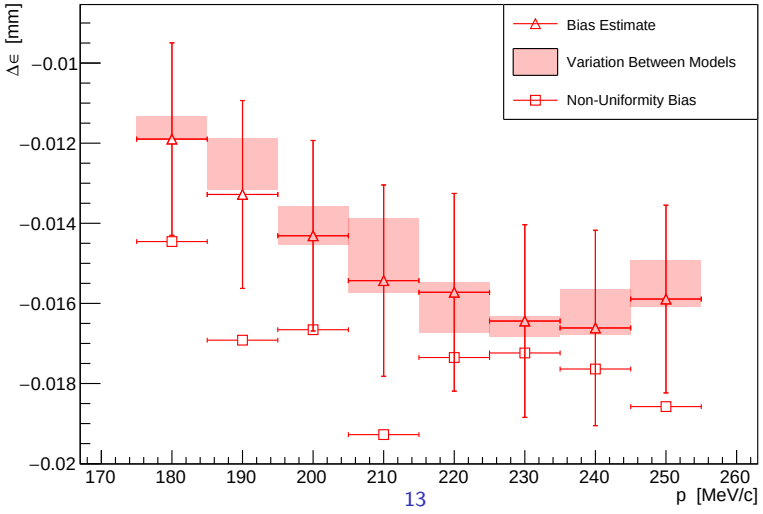
Systematic Bias

- Chose to use the variation across the different field models as an estimate for the accuracy for the bias measurement,
- The MAUS Fields non-uniformity model was not included, and presented separately as it is a non-physical departure from the known field map,
- Ideally we would have a better probe of non-uniformity, but we don't have one at present.

In any case the variation between the different models is smaller than the statistical error, so we can predict the systematic bias precisely, and apply it to the reconstruction used in the Emittance Paper.

Systematic Bias



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Conclusions

The Good:

- Algorithm is well tested and performs to the required precision,
- Able to predict the systematic effects on the uncertainty for Scale, Alignment and Uniformity,

The Bad:

- Requires a huge amount of data - more than was reasonably achieveable from the offical MC,
- Lack of knowledge of Comsol makes it difficult to probe the effects of non-uniformity,

Conclusions

- Could use the recent field map data to improve the understanding of the effects of non-uniformity.

  Not for the Emittance Paper however!

- There is still some small uncertainty in the beam model that was used. Could use the actual muons.