# Transverse momentum spectra of gauge bosons

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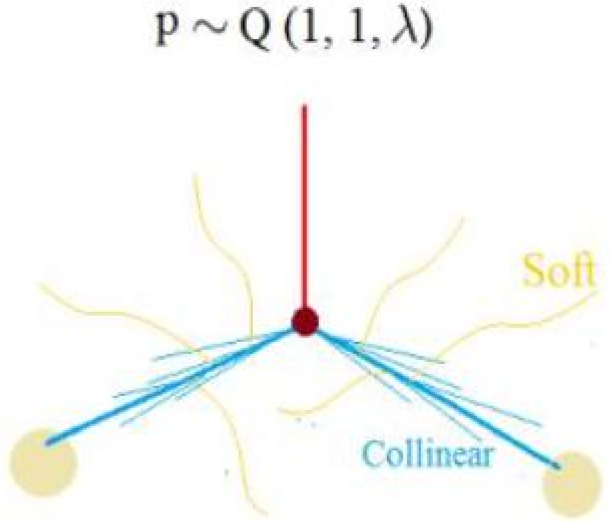
# Transverse spectrum of gauge bosons

P+P $\to$ H+X, P+P $\to$ $l^+ + l^-$ +X.

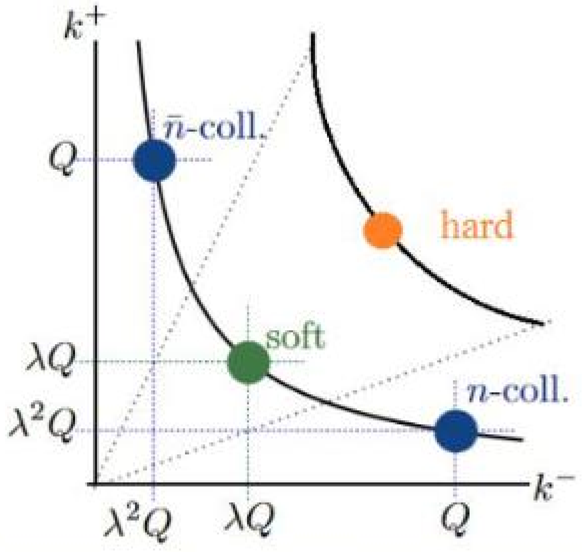


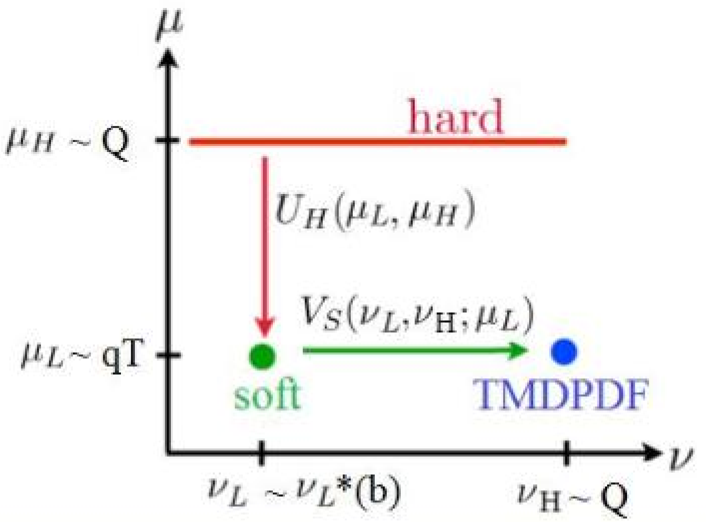

$$p_c \sim Q\,(1, \lambda^2, \lambda)\,,$$
$$p_{\bar{c}} \sim Q\,(\lambda^2, 1, \lambda)\,,$$
$$p_s \sim Q\,(\lambda, \lambda, \lambda)\,,$$
$$\lambda = qT / Q$$

**EFT for resummation of large logs of qT/Q**

$$\frac{d\sigma}{d^2 q_T dy} = \sigma_0 C_t^2(M_t^2, \mu) H(Q^2;\mu) \int d^2\vec{q}_{Ts} d^2\vec{q}_{T1} d^2\vec{q}_{T2} \delta^2(\vec{q_T} - (\vec{q_T}_s + \vec{q}_{T1} + \vec{q}_{T2}))$$
$$\times S(\vec{q}_{Ts};\mu,\nu) f_1^{\perp}\left(\vec{q}_{T1}, x_1, p^-;\mu,\nu\right) f_2^{\perp}\left(\vec{q}_{T2}, x_2, p^+;\mu,\nu\right),$$

# Transverse spectrum of gauge bosons




$$\frac{d\sigma}{dq_T^2 dy} \propto H(\frac{\mu}{Q}) \int bdbJ_0(bq_T)S(b,\mu,\nu)f_1^{\perp}(x_1,b,\mu,\nu,Q)f_2^{\perp}(x_2,b,\mu,\nu,Q)$$

- **CSS resumation scheme: Choose μ ~ 1/b, v ~1/b , Landau pole cut off , numerical implementation of cross section**

- **Implement a resummation scheme with v chosen in b space, μ in momentum space.**
- **Obtain an analytical expression for the cross section**

# Resummed spectrum

- **Resummed cross section**

$$\frac{d\sigma}{dq_T^2 dy} = \frac{\sigma_0}{2} C_t^2(M_t^2, \mu_T) H(Q^2, \mu_H) U(\mu_L, \mu_H, \mu_T) I_b(q_T, Q; \mu_L, \nu_L^*, \nu_H)$$

$$I_b(q_T, Q; \mu_L, \nu_L^*, \nu_H) \equiv \int_0^\infty db\, b J_0(bq_T) \widetilde{F}(b, x_1, x_2, Q; \mu_L, \nu_L^*, \nu_H) V_\Gamma(\nu_L^*, \nu_H; \mu_L)$$

Fixed order terms — resummed exponent

$$V_\Gamma = C e^{-A \ln^2(\mu_L b_0 \chi)}$$

Quadratic in log b

$$I_b^0 = \int_0^\infty db\, b J_0(bq_T)\, e^{-A \ln^2(\Omega b)}$$

$$\Omega \equiv \mu_L e^{\gamma_E} \chi / 2. \qquad b_0 = \frac{b e^{\gamma_E}}{2}$$

- **Fixed order terms can be obtained from this master integral by taking derivatives**

# Computing the master integral

- **Mellin Barnes representation of the Bessel function**

$$J_0(z) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt \frac{\Gamma[-t]}{\Gamma[1+t]}\left(\frac{1}{2}z\right)^{2t}$$

- **Integral in b space can be done exactly**

$$I_b^0 = \int_0^\infty db\, b J_0(bq_T)\, e^{-A\ln^2(\Omega b)} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} dt\, \frac{\Gamma[-t]}{\Gamma[1+t]}\int_0^\infty db\, b\left(\frac{bq_T}{2}\right)^{2t} e^{-A\ln^2(\Omega b)}$$

$$I_b^0 = \frac{2}{iq_T^2}\frac{e^{-AL^2}}{\sqrt{\pi A}}\int_{c-i\infty}^{c+i\infty} dt\, \frac{\Gamma[-t]}{\Gamma[1+t]} e^{\frac{1}{A}(t-t_0)^2}$$

$t_0 = -1 + AL$ $\qquad$ $L = \ln(2\Omega/q_T)$

# Expansion in Hermite basis

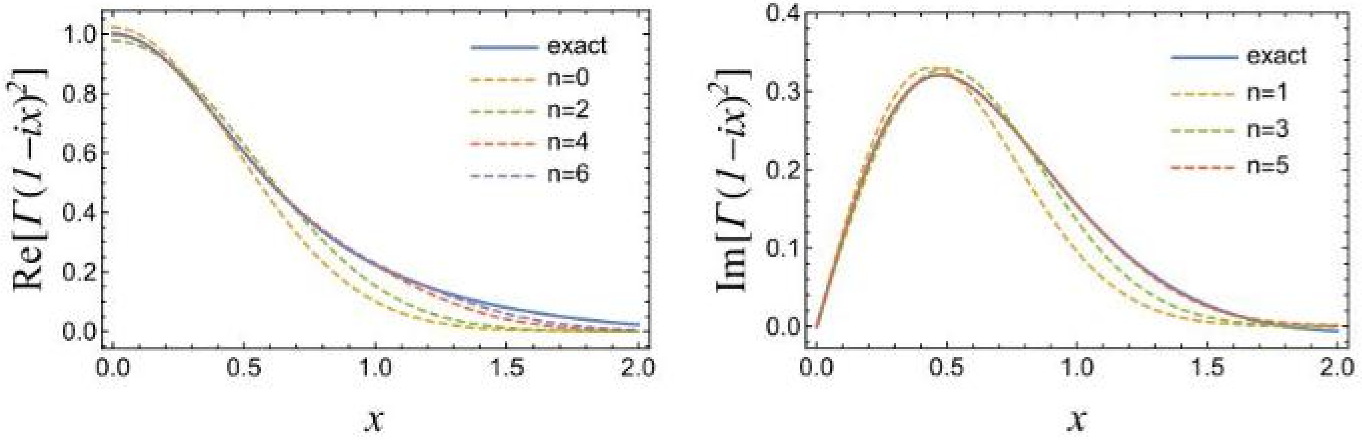
$$I_b^0 = \frac{2}{\pi q_T^2 \sqrt{\pi A}} \,\mathrm{Im}\left\{ e^{-A(L-i\pi/2)^2} \int_{-\infty}^{\infty} dx\, \Gamma(-c-ix)^2 e^{-\frac{1}{A}\left[x+\frac{A\pi}{2}-i(c-t_0)\right]^2} \right\}$$

**Writing t= c + i x**

$$A \sim \Gamma_{cusp}(\alpha_s(\mu))$$

**We make a choice c= -1.** $\Gamma(-c-ix)^2$ **contributes only in the region -2 > x >-2**

$$\Gamma(1-ix)^2 = e^{-a_0 x^2} \sum_{n=0}^{\infty} c_{2n} H_{2n}(\alpha x) + \frac{i\gamma_E}{\beta} e^{-b_0 x^2} \sum_{n=0}^{\infty} c_{2n+1} H_{2n+1}(\beta x)$$



# Computing the master integral

$$I_b^0 = \frac{2}{\pi q_T^2} \sum_{n=0}^{\infty} \mathrm{Im}\left\{ c_{2n}\mathcal{H}_{2n}(\alpha, a_0) + \frac{i\gamma_E}{\beta} c_{2n+1}\mathcal{H}_{2n+1}(\beta, b_0) \right\}$$

- **An explicit all orders expression for the master integral**

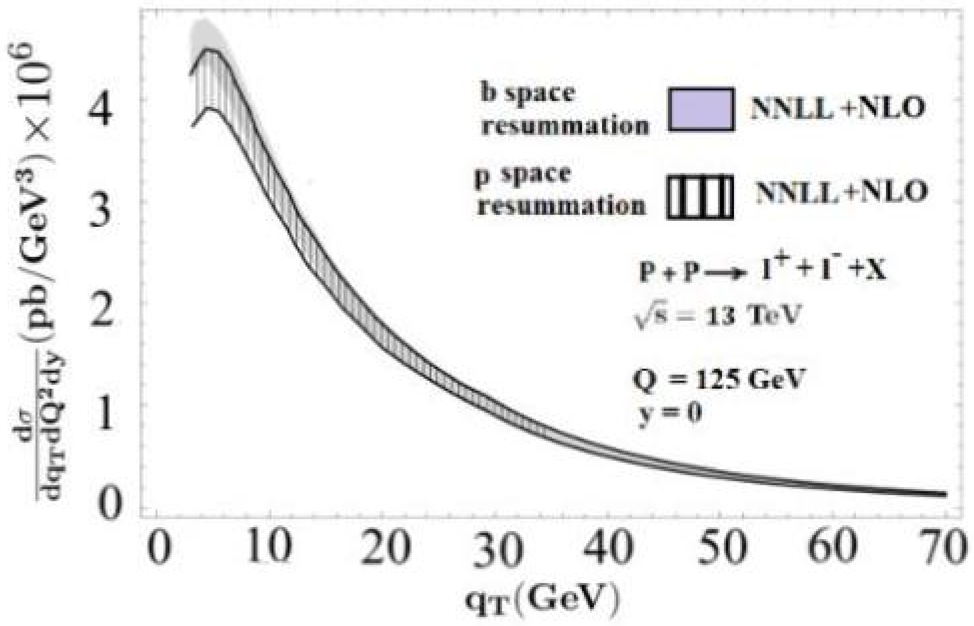
$$\mathcal{H}_n(\alpha, a_0) = \mathcal{H}_0(\alpha, a_0)\frac{(-1)^n n!}{(1+a_0 A)^n} \sum_{m=0}^{\lfloor n/2 \rfloor} \frac{1}{m!}\frac{1}{(n-2m)!}\Big\{[A(\alpha^2 - a_0) - 1](1 + a_0 A)\Big\}^m (2\alpha z_0)^{n-2m}$$

$$z_0 = A(\pi/2 + iL)$$

$$\frac{d\sigma}{dq_T^2 dy} = \frac{1}{2}\sigma_0 C_t^2(M_t^2, \mu_T) H(Q^2, \mu_H) U(\mu_H, \mu_T)\, C \times \sum_{n=0}^{\infty}\sum_{k=0}^{2n} \left(\frac{\alpha_s(\mu_{\text{run}})}{4\pi}\right)^n \widetilde{F}_k^{(n)}\, I_b^k(q_T)$$

$$I_b^k = \left[\hat{\partial}_\chi\right]^k I_b^0 \qquad I_b^0 = \int_0^\infty db\, b J_0(bq_T)\, e^{-A\ln^2(\Omega b)} \qquad \Omega \equiv \mu_L e^{\gamma_E}\chi/2.$$

# Comparison with other schemes




## Sources of errors/uncertainties

- Perturbative expansion in the strong coupling
- Expansion in $\Lambda_{QCD}$/ qT.
- Expansion in qT/Q
- Variation between pdf sets
- finite quark mass corrections

# W/Z qT spectra

- **Perturbative uncertainties**

$$\frac{\frac{d\sigma_W}{dqTdy}}{\frac{d\sigma_Z}{dqTdy}} = \frac{\sigma_{0,W}\,U_H(M_W,\mu_L)\left[\sum_{n=0}^{n=1}\sum_{k=0}^{k=2}\alpha_s^n(\mu_L)F_k I_{b_k}(\mu_L,q_T,M_W)\right]\sum_u\left(f_u(x_1,\mu_L)f_{\bar{d}}(x_2,\mu_L)+x_1\leftrightarrow x_2\right)}{\sigma_{0,Z}\,U_H(M_Z,\mu_L)\left[\sum_{n=0}^{n=1}\sum_{k=0}^{k=2}\alpha_s^n(\mu_L)F_k I_{b_k}(\mu_L,q_T,M_Z)\right]\sum_i\left(f_i(x_1,\mu_L)f_{\bar{i}}(x_2,\mu_L)+x_1\leftrightarrow x_2\right)}$$

Hard function resummation

rapidity resummation

PDF ratio

Scale variation should produce strongly correlated uncertainties for perturbative QCD corrections.

# W/Z qT spectra

## Finite quark mass corrections

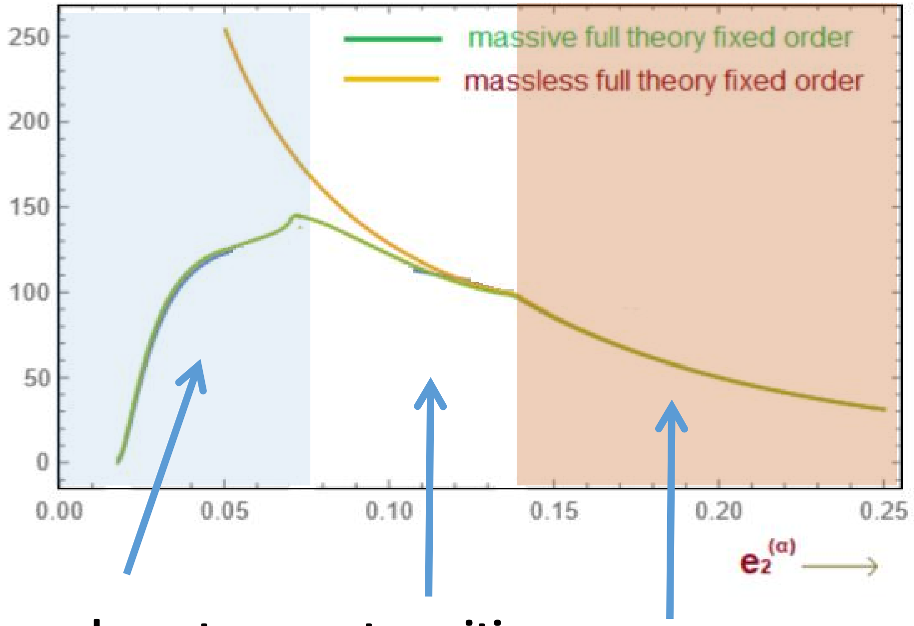
- b quark mass correction for Z boson relevant for qT < m ~ 5 GeV
- Need a multiscale EFT for resumming possible log( qT/m)? matched onto fixed order m/qT corrections.

Possible solution offered by recent jet substructure EFT's for heavy quarks

$\text{SCET}_M$ ( Massive SCET)

↓

HQET (Heavy Quark Effective Theory)




m relevant region

transition

massless region

# W/Z qT spectra

- Non perturbative power correction -> usually handled using a universal non-pertubative model -> relevant for qT ~ $\Lambda_{QCD}$ and hence also strongly correlated.

- Power corrections in qT/Q usually handled by matching to full theory fixed order cross section. However, for percent level accuracy, we need t o resum power suppressed logs ~ qT^2/Q^2 Log(qT^2/Q^2)

SCET at subleading power ->First Subleading Power Resummation for Event Shapes,
I. Stewart et.al. arXiv 1804.08665 -> **Can be adapted for qT resummation**

# Conclusion

- Analytic formula for tranverse momentum spectra of gauge bosons in the perturbative region
- Ratio of W/Z qT spectra sensitive to the differences in the kinematics and production mechanisms.
- EFT can be extended to include finite quark mass effects and Subleading power corrections .
- Borrow EFTs already formulated for jet observables and apply them for qT spectra computation