A measurement method for the complete set of angular coefficients for the W boson at the LHC

Physics motivation

Few remarks on the choice of the frame

Measurements so far: Tevatron, LHC

A path to measure complete set of coefficients.

Bonus slides: Mustraal frame

What are A_i's

In the rest frame of the W or Z we can decompose differential cross-section in the base of orthonormal polynomials of the at most second order.

$$
\begin{aligned}
\frac{\mathrm{d}^4\sigma}{\mathrm{d}(p_T^W)^2\ \mathrm{d}y\ \mathrm{dcos}\theta_{CS}\ \mathrm{d}\phi_{CS}} \quad = \quad & \frac{3}{16\pi}\frac{\mathrm{d}^2\sigma^{TOT}}{\mathrm{d}(p_T^W)^2\mathrm{d}y}[(1+\cos^2\theta_{CS})+ \\
& +\frac{1}{2}A_0(1-3\cos^2\theta_{CS}) - QA_1\sin 2\theta_{CS}\cos\phi_{CS} + \\
& +\frac{1}{2}A_2\sin^2\theta_{CS}\cos 2\phi_{CS} + A_3\sin\theta_{CS}\cos\phi_{CS} + \\
& -QA_4\cos\theta_{CS} + A_5\sin^2\theta_{CS}\sin 2\phi_{CS} + \\
& -QA_6\sin 2\theta_{CS}\sin\phi_{CS} + A_7\sin\theta_{CS}\sin\phi_{CS}],
\end{aligned}
$$

Where A_i's are functions of (p_T, Y, m_{ll}).
Carry information on the QCD dynamics of the production process.

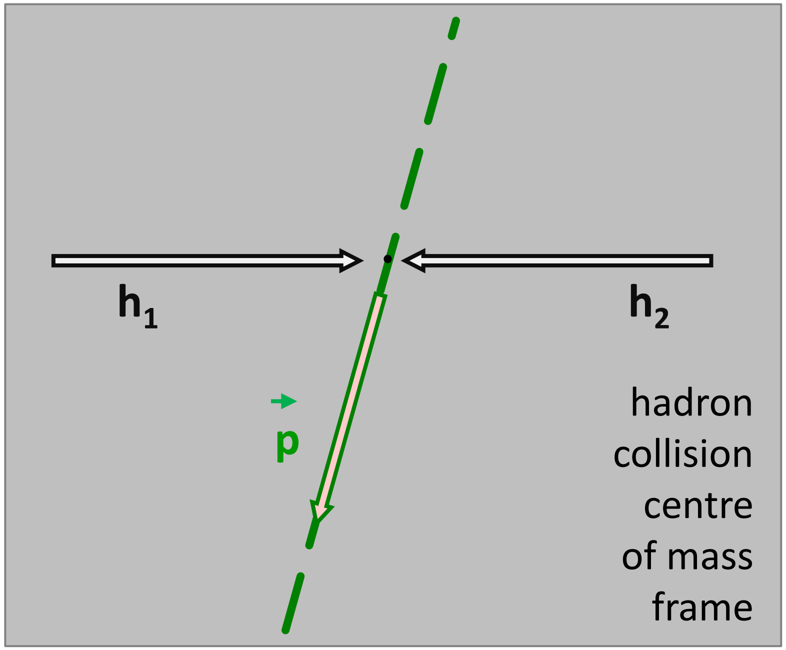
Why interesting to measure A_i's

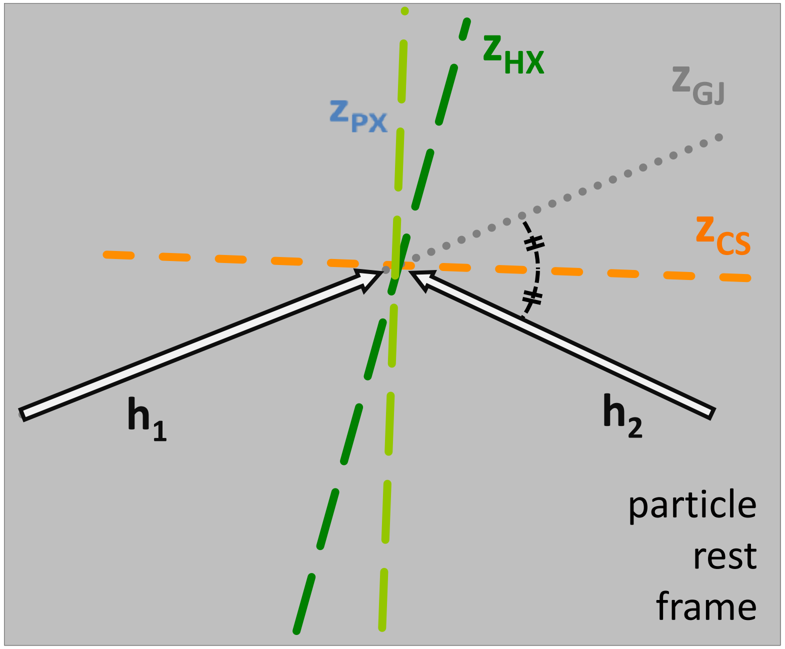
- Probe the effect of QCD corrections on the spin structure of W production. The m_W precise measurements using m_T or p_T^l is sensitive to it!
  - Focus point of this workshop
- Fraction of left-, right- and longitudinaly polarised W's. Expecting dominantly left-handed at high p_T and increasing longitudinal fraction at high p_T.
- Measuring P-odd and T-odd coefficients may play an important role in revealing direct CP violation effects in W production and decay.
- Knowledge on angular distributions can be used to test new models.

Choice of the polarisation frame

The rest frame of W or Z boson: the z-axis is defined as

- **Helicity axis (HX): outgoing Z or W direction in the lab frame (f_L, f_R, f_0)**
- **Collins-Soper axis (CS): average of the two beam directions in the W/Z rest frame (A_i's)**
- **Gottfried-Jackson axis (GJ): direction of one or the other beam in the W/Z rest frame**
- **Perpendicular helicity axis (PX): perpendicular to CS**





BONUS slides: Mustraal frame as a much more elaborated version of GJ frame.

A bit of history: Tevatron (2000-2003)

CDF Collaboration, hep-ex/0311050
DO Collaboration, hep-ex/0009034

Decomposition in Collins-Soper frame

$$\frac{d^4\sigma}{d(p_T^W)^2\, dy\, d\cos\theta_{CS}\, d\phi_{CS}} =$$

$$\frac{3}{16\pi}\frac{d^2\sigma^{TOT}}{d(p_T^W)^2 dy}[(1+\cos^2\theta_{CS}) +$$
$$+\frac{1}{2}A_0(1-3\cos^2\theta_{CS}) - QA_1\sin 2\theta_{CS}\cos\phi_{CS} +$$
$$+\frac{1}{2}A_2\sin^2\theta_{CS}\cos 2\phi_{CS} + A_3\sin\theta_{CS}\cos\phi_{CS} +$$
$$-QA_4\cos\theta_{CS} + A_5\sin^2\theta_{CS}\sin 2\phi_{CS} +$$
$$-QA_6\sin 2\theta_{CS}\sin\phi_{CS} + A_7\sin\theta_{CS}\sin\phi_{CS}],$$

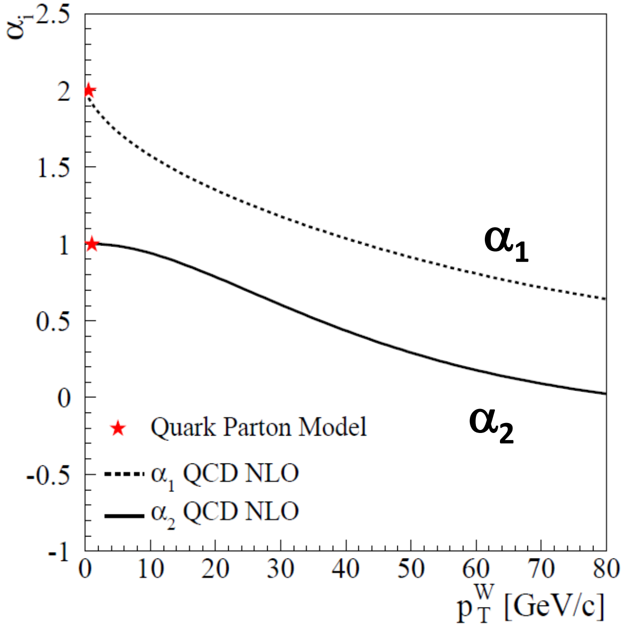


FIG. 2: Theoretical NLO-QCD calculation of α_2 and α_1 vs. p_T^W. The limit $p_T^W \to 0$ GeV/c is the Quark Parton Model, for which $\alpha_2 = 1$ and $\alpha_1 = 2$.

Polar angle measurement: sensitive to A_0, A_4

$$\frac{d\sigma}{d\cos\theta_{CS}} \propto (1 - Q\alpha_1\cos\theta_{CS} + \alpha_2\cos^2\theta_{CS}),$$

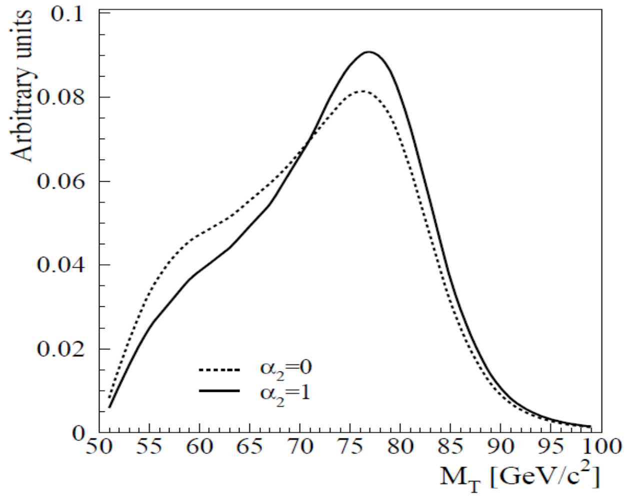
$$\alpha_1 = \frac{2A_4}{2+A_0}, \quad \boxed{\alpha_2 = \frac{2-3A_0}{2+A_0},}$$

Due to complexity of solving equation for neutrino momenta and two-fold solution for sign of cos θ, the sensitivity of m_T to α_2 is used insteadfor measurement.
The sensitivity to α_1 is residual only, predicted from MC and accounted for in the systematics.

A bit of history: Tevatron (2000-2003)

CDF Collaboration, hep-ex/0311050
DO Collaboration, hep-ex/0009034

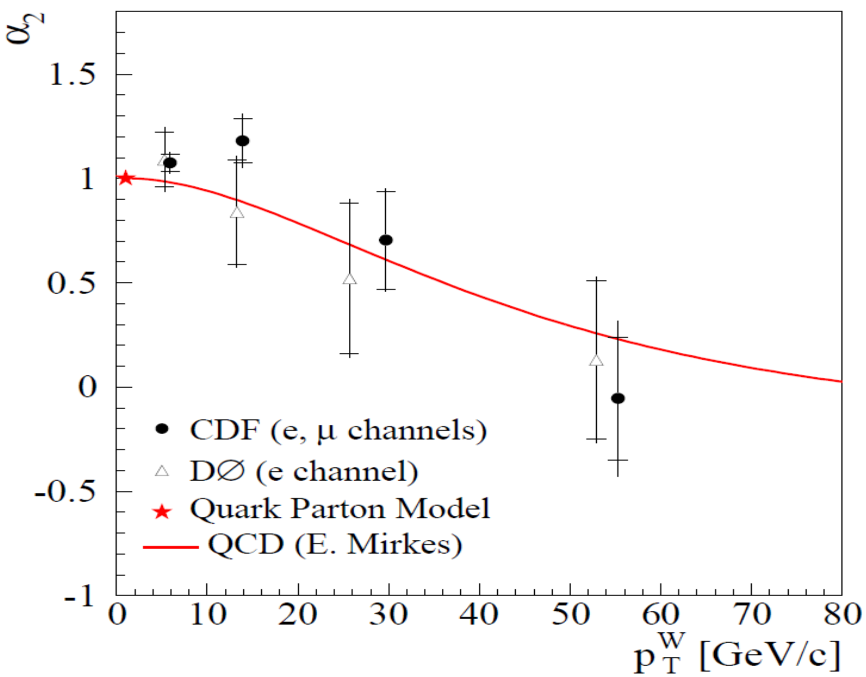
Example of sensitivity



p_T^W [GeV/c]	α_2 (CDF combined)	α_2 (theory)
5.9	1.07±0.04(stat)±0.03(syst)	0.98
13.9	1.18±0.10(stat)±0.06(syst)	0.89
29.7	0.70±0.23(stat)±0.07(syst)	0.61
55.3	-0.05±0.29(stat)±0.21(syst)	0.23

Experimental information used: m_T^W and p_T^W

Measurement



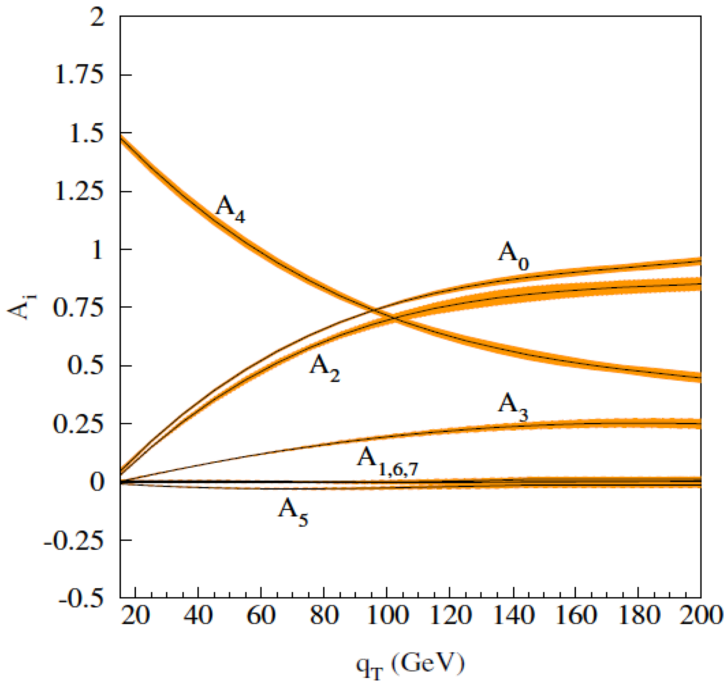
1% uncertainty on α_2 corresponds to 10 MeV shift in m_W
L. Cerrito, PhD thesis 2002

Raises already question of the TH uncertainty on either α_i or A_i and ther impact on m_W

A bit of history: Tevatron (2006)

J. Strologas and S. Errede, Phys. Rev D73 (2006) 052001
CDF Collaboration, Phys. Rev D73 (2006) 052002
J. Strogolas, PhD Thesis, 2002

Theory predictions



Calculated using moments method and DYRAD MC (NLO QCD)
A_4 decreases with p_T increasing
A_0-A_2 not equal due to gluon loops contrib. (Lam-Tung relation), consistently $A_0 > A_2$

- If the W is produced with no p_T, then all coefficients but A_4 are zero.
- If only valence quark contributed to the W^{-+} production, the A_4 would be = 2 (pure V-A coupling) and angular distribution of lepton from W^{-+} would be $\sim(1 \pm \cos\theta)^2$,
- The A_5, A_6, A_7 are non-zero only if gluon loops are present, so represent α_s^2 corrections.

Calculated analytically for W+j at NLO QCD.
E. Mirkes, Nucl. Phys. B387 (1992) 3.
E. Mirkes et al. Phys. Rev. D50(1994) 5692.

A bit of history: Tevatron (2006)

J. Strologas and S. Errede, Phys. Rev D73 (2006) 052001
CDF Collaboration, Phys. Rev D73 (2006) 052002
J. Strogolas, PhD Thesis, 2002

q_T = W tranverse momentum in LAB frame

$$p_z^\nu = \frac{1}{(2p_T^l)^2}\left(Ap_z^l \pm E^l\sqrt{A^2 - 4(p_T^l)^2(p_T^\nu)^2}\right)$$

$$A = M_W^2 + q_T^2 - (p_T^l)^2 - (p_T^\nu)^2$$

Two solutions corespond to the same value of ϕ but opposite sign of $\cos\theta$

Use M_W PDG for solving equation, if no solution possible event rejected.

Azimuthal angle measurement: sensitive to A_2, A_3

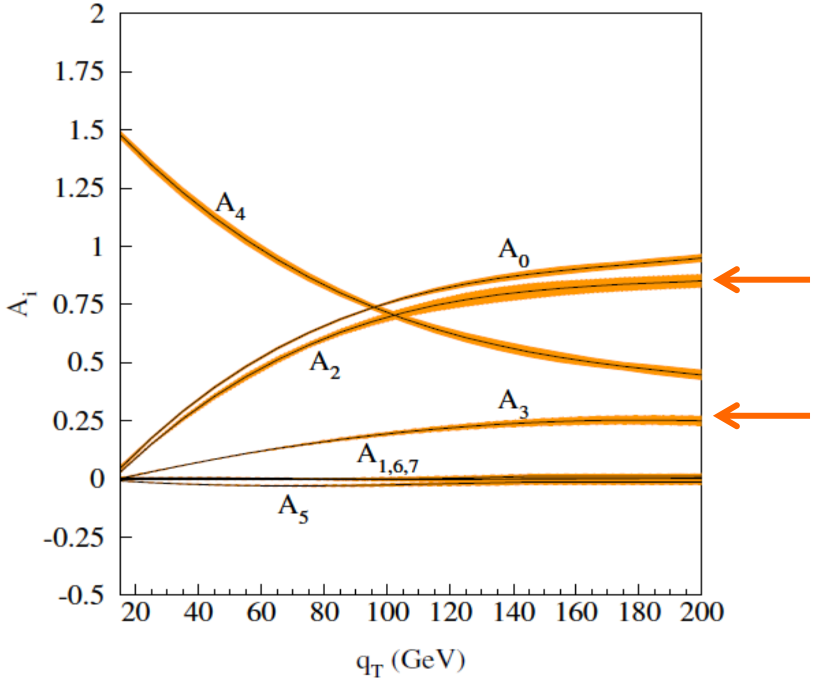
Fold analytical templates with acceptance factors f_i (i=0-7) of the fiducial phase-space derived from MC. Then fit folded templates to the reco data.

$$\frac{d\sigma}{dq_T^2 d\phi} = C'(1 + \beta_1 \cos\phi + \beta_2 \cos 2\phi + \beta_3 \sin\phi + \beta_4 \sin 2\phi)$$

$$C' = \frac{1}{2\pi}\frac{d\sigma}{dq_T^2}, \qquad \boxed{\beta_1 = \frac{3\pi}{16}A_3,} \qquad \boxed{\beta_2 = \frac{A_2}{4},} \qquad \beta_3 = \frac{3\pi}{16}A_7, \qquad \beta_4 = \frac{A_5}{2}.$$

A bit of history: Tevatron (2006)

Theory predictions



J. Strologas and S. Errede, Phys. Rev D73 (2006) 052001
CDF Collaboration, Phys. Rev D73 (2006) 052002
J. Strogolas, PhD Thesis, 2002

For the first time measurement of A_2, A_3 sensitive to azimuthal angle ϕ

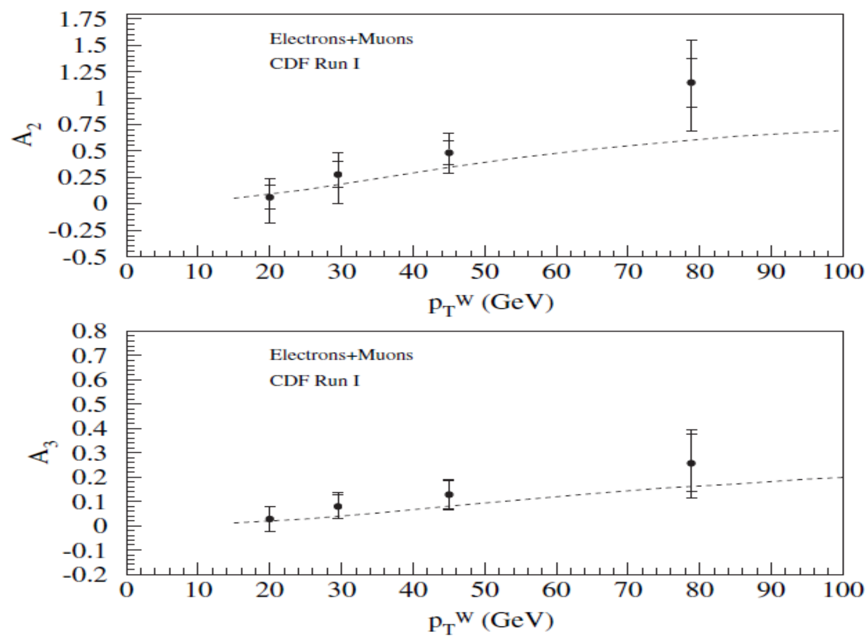


FIG. 33. Measured A_2 and A_3 using the combination of electron and muon measurements. The total (outer) and statistical (inner) uncertainties are shown along with the standard model 1-loop prediction up to order α_s^2 (dashed line).

Calculated using moments method and DYRAD MC (NLO QCD)

A_4 decreases with p_T increasing

A_0-A_2 not equal due to gluon loops contrib. (Lam-Tung relation broken), consistently $A_0 > A_2$

pp collisions at LHC

Only quarks are valence, both quarks and anti-quarks populate sea.
Angular coefficients for W^+ and W^- are not related by CP symmetry.

Low p_T^W picture

At LHC, quarks carry generally larger fraction of the momentum.
This causes W to be boosted in the direction of the initial quark.
In the massles quark approximation, quark must be left-handed,
as results W bosons at large y^W are purely left-handed.
At central rapidity, increasing probability that antiquark carries larger momentum, the helicity state of W becomes mixture of left- and right-handed.

High p_T^W picture

More complicated production mechanism. Polarisation in longitudinal state is possible. Left-handed W dominating at high p_T.

Measurement done by ATLAS and CMS @ 7 TeV, used helicity frame

ATLAS @ 7 TeV (2012)

ATLAS Collaboration, Eur. Phys. J. C72 (2012) 2001

Used helicity frame; measured helicity fractions: f_0, f_L, f_R

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{3D}} = \frac{3}{8}[(1+\cos^2\theta_{3D}) + A_0\frac{1}{2}(1-3\cos^2\theta_{3D}) + A_4\cos\theta_{3D}].$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{3D}} = \frac{3}{8}f_L(1\mp\cos\theta_{3D})^2 + \frac{3}{8}f_R(1\pm\cos\theta_{3D})^2 + \frac{3}{4}f_0\sin^2\theta_{3D}$$

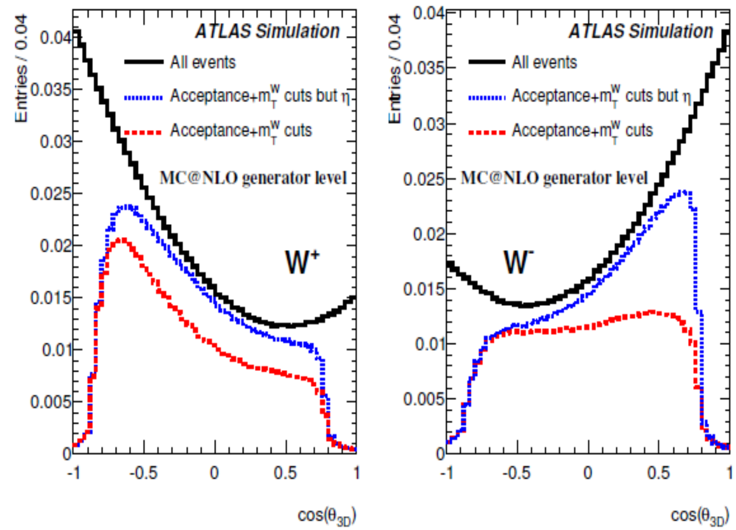
$$f_L(y_W, p_T^W) = \frac{1}{4}(2 - A_0(y_W, p_T^W) \mp A_4(y_W, p_T^W))$$

$$f_R(y_W, p_T^W) = \frac{1}{4}(2 - A_0(y_W, p_T^W) \pm A_4(y_W, p_T^W))$$

$$f_0(y_W, p_T^W) = \frac{1}{2}A_0(y_W, p_T^W)$$

$$f_L - f_R = \mp\frac{A_4}{2}.$$

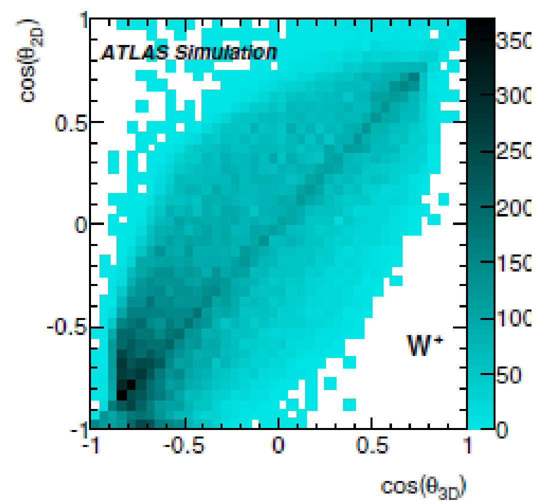
f_0 expected to vanish at p_T^W=0 and p_T^W = infinity



Experimental technique:

to avoid complexity of solving neutrino equation used „transverse helicity" angle θ_{2D}

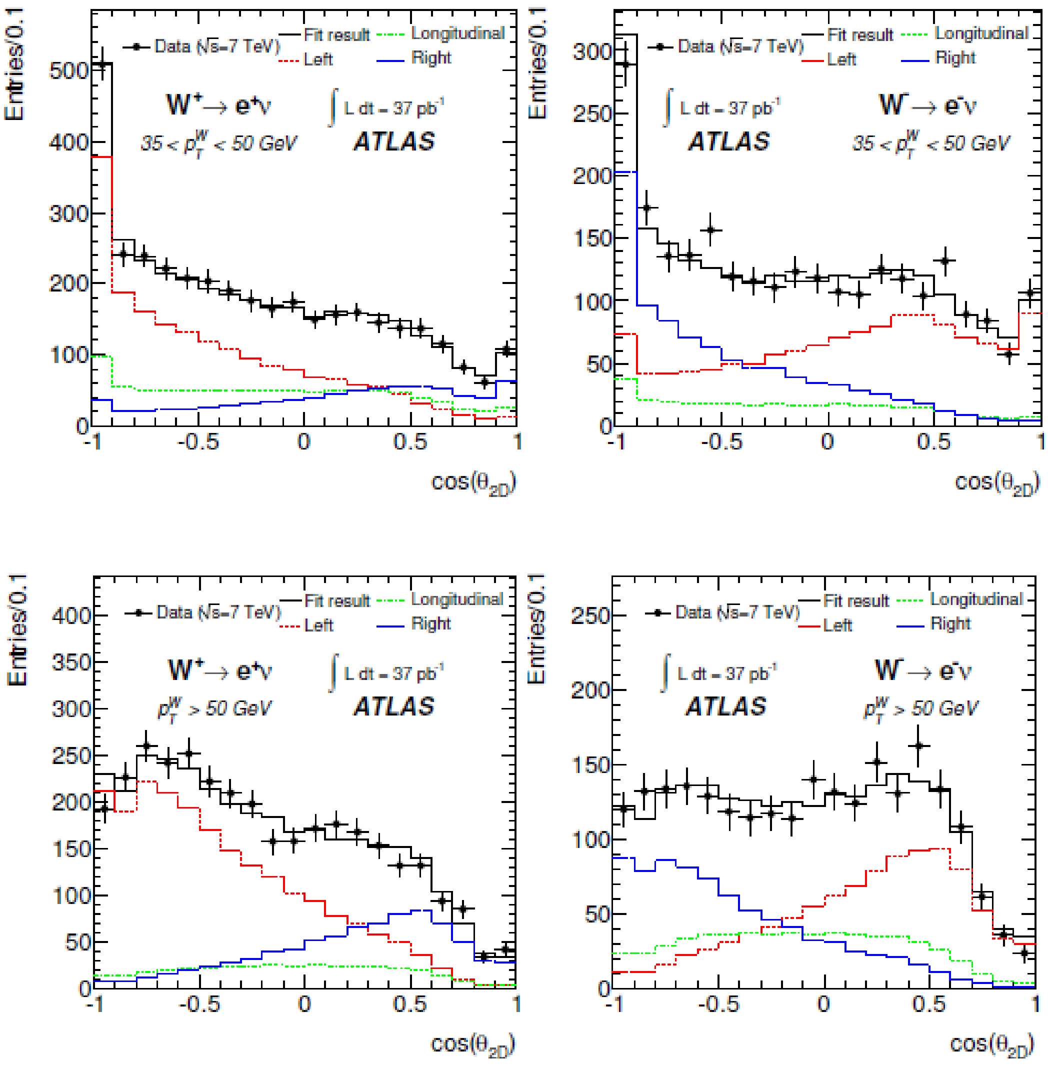
$$\cos\theta_{2D} = \frac{\vec{p}_T^{\,\ell *}\cdot\vec{p}_T^{\,W}}{|\vec{p}_T^{\,\ell *}|\,|\vec{p}_T^{\,W}|},$$



transverse W momenta, transverse W rest frame

ATLAS @ 7 TeV (2012)

ATLAS Collaboration, Eur. Phys. J. C72 (2012) 2001



Measurement in e and μ channels.
Created f_L, f_R, f_0 templates from MC.
Fold templates into fiducial region.
Fit of templates to the data.
Extracted polarization fractions.

Measurement performed in two p_T bins: 35-50 GeV, > 50 GeV

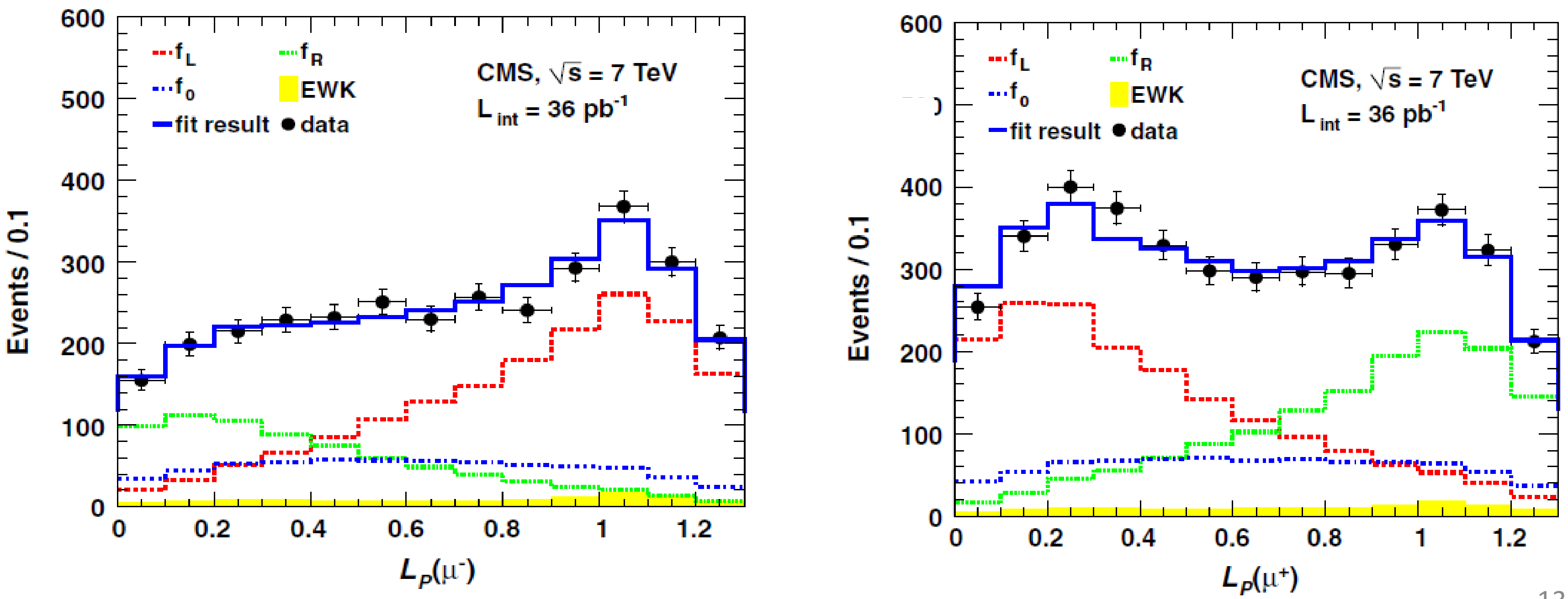
CMS @ 7 TeV (2011)

CMS Collaboration, Phys. Rev. Lett. 107 (2011) 021802

Focus on $p_T^W > 50$ GeV
Define lepton projection variable L_p and corresponding polar angle θ^*

$$L_P = \frac{\vec{p}_T(\ell) \cdot \vec{p}_T(W)}{|\vec{p}_T(W)|^2} \qquad \cos\theta^* = 2(L_P - \tfrac{1}{2})$$

Measurement in e and μ channels. Fit templates to the data. Extract f_L, f_R, f_0 fractions.



ATLAS @ 7 TeV (2012)

ATLAS Collaboration, Eur. Phys. J. C72 (2012) 2001

	$f_L - f_R$ (%)		f_0 (%)	
	$35 < p_T^W < 50$ GeV	$p_T^W > 50$ GeV	$35 < p_T^W < 50$ GeV	$p_T^W > 50$ GeV
Data	$23.8 \pm 2.0 \pm 3.4$	$25.2 \pm 1.7 \pm 3.0$	$21.9 \pm 3.3 \pm 13.4$	$12.7 \pm 3.0 \pm 10.8$
MC@NLO	27.1 ± 0.7	26.2 ± 0.5	17.9 ± 1.2	21.0 ± 1.0
POWHEG	19.9 ± 1.0	21.2 ± 0.8	22.9 ± 1.0	19.4 ± 0.8

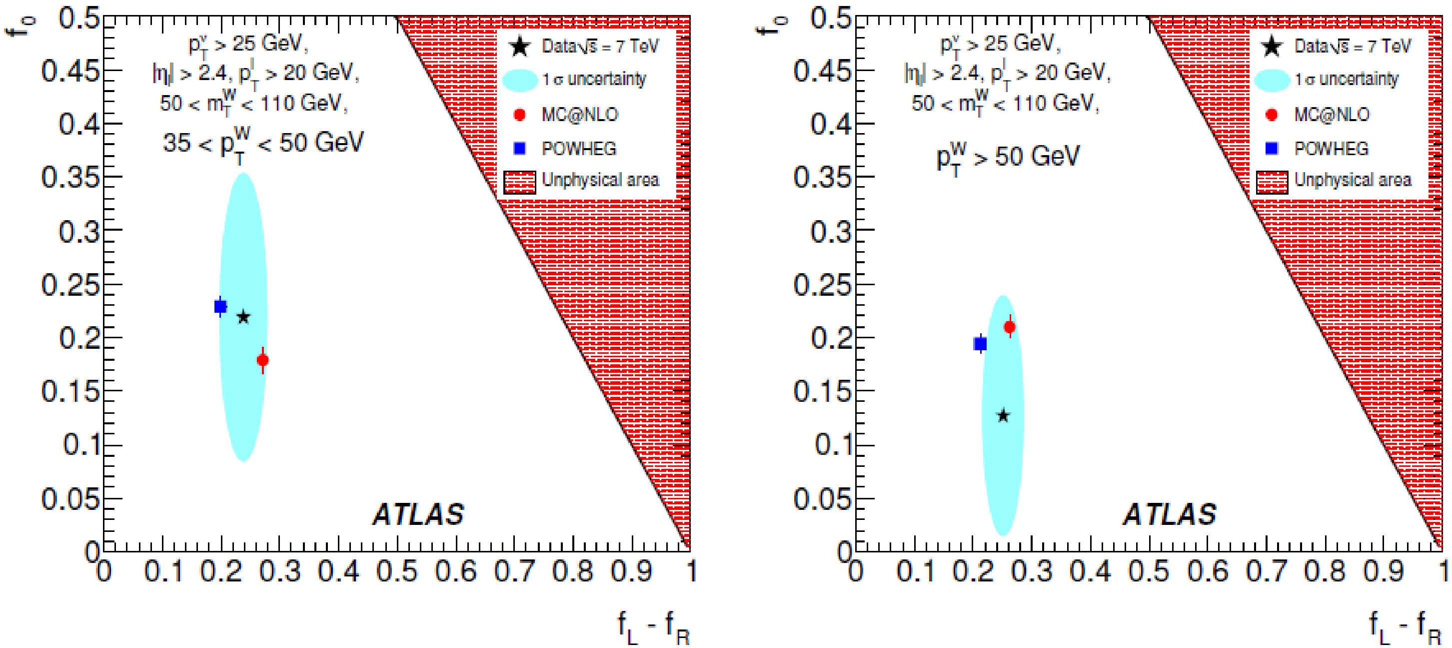
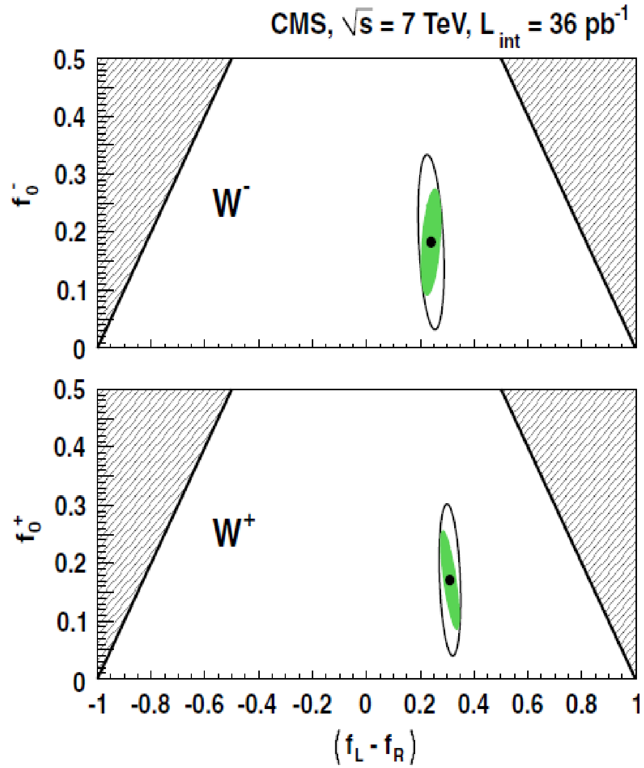


Fig. 9 Measured values of f_0 and $f_L - f_R$after corrections (Table 11), within acceptance cuts, for $35 < p_T^W < 50$ GeV (left) and $p_T^W > 50$ GeV (right), compared with the predictions of MC@NLO and POWHEG. The ellipses around the data points correspond to one standard deviation.

CMS @ 7 TeV (2011)

CMS Collaboration, Phys. Rev. Lett. 107 (2011) 021802



CMS measurement: The muon fit result yields the most precise measurement: $(f_L - f_R)^- = 0.240 \pm 0.036(\mathrm{stat}) \pm 0.031(\mathrm{syst})$ and $f_0^- = 0.183 \pm 0.087(\mathrm{stat}) \pm 0.123(\mathrm{syst})$ for negatively charged W bosons and $(f_L - f_R)^+ = 0.310 \pm 0.036(\mathrm{stat}) \pm 0.017(\mathrm{syst})$ and $f_0^+ = 0.171 \pm 0.085(\mathrm{stat}) \pm 0.099(\mathrm{syst})$ for positively charged W bosons.

green - stat error

ATLAS measurement: $f_0 = 0.127 \pm 0.030 \pm 0.108$

(p_T> 50 GeV) $f_L - f_R = 0.252 \pm 0.017 \pm 0.030$

Both measurements consistent.
Confirm expected non-zero fraction of longitudinal polarisation (helicity frame).

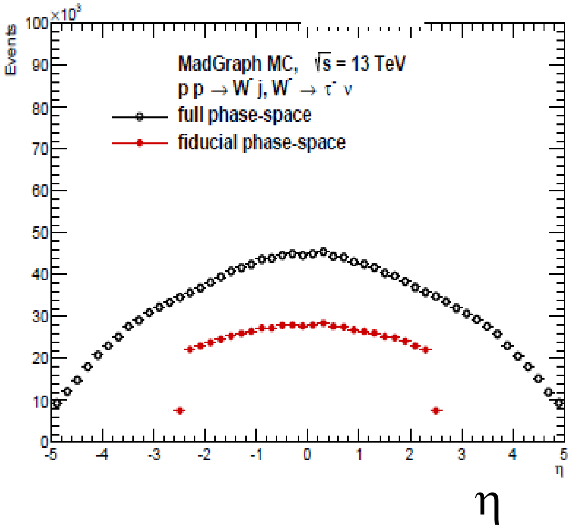
A path to measure complete set of A_i's

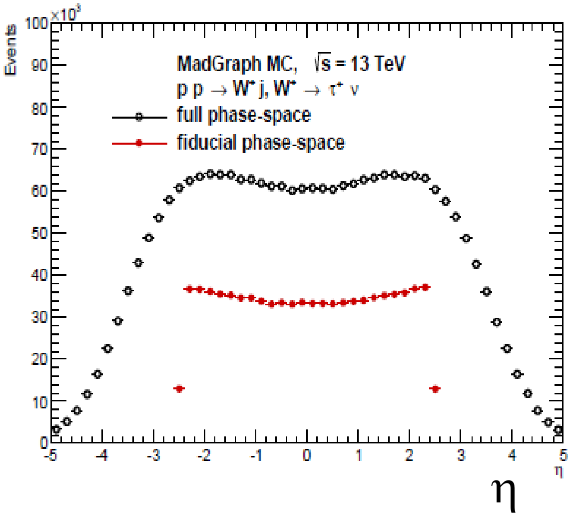
ERW, Z. Was, arXiv: 1609.02536

1) Solve system for neutrino p_z^ν momenta. If two solutions possible use randomly chosen one. Then construct rest-frame of the W boson.
2) Build P_i polynomial templates using unweighted MC events. For unweighting use moments method to estimate A_i's in MC. Fold into fiducial phase-space.
3) Fit to the data and extract complete set of A_i's coefficients.

Measurement will give A_i's in the full phase-space.

Much more elaborated version of (2)-(3) was used for measurement of A_i's in Z->ll events, published by ATLAS, JHEP 08 (2016) 159.





Fiducial phase-space:

$p_T^l > 25$ GeV, $|\eta| < 2.5$

$p_T^\nu > 25$ GeV

$m_T > 40$ GeV

Solving equation for neutrino momenta

ERW, Z. Was, arXiv: 1609.02536

Simple formulas:

$$p_z^\nu = \frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2a},$$

$$\begin{aligned}
a &= 4 \cdot p_z^\ell - 4 \cdot E^\ell, \\
b &= 4 \cdot (m_W^2 + (p_x^\ell + p_x^\nu)^2 + (p_y^\ell + p_y^\nu)^2 - (E^\ell)^2 + (p_z^\ell)^2 - (p_T^\ell)^2) \cdot p_z^\ell, \\
c &= (m_W^2 + (p_x^\ell + p_x^\nu)^2 + (p_y^\ell + p_y^\nu)^2 - (E^\ell)^2 + (p_z^\ell)^2 - (p_T^\ell)^2)^2 - 4 \cdot (E^\ell)^2 \cdot (p_T^\ell)^2 \\
p_T^\ell &= \sqrt{(p_x^\ell)^2 + (p_y^\ell)^2}.
\end{aligned}$$

For solving equation use $m_W = m_W^{PDG}$, use $m_\nu = 0$ constrain to calculate E_ν

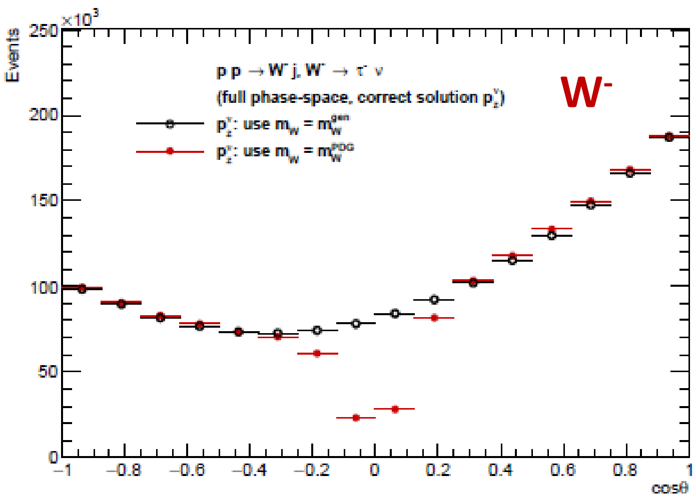
$$E^\nu = \sqrt{(p_x^\nu)^2 + (p_y^\nu)^2 + (p_z^\nu)^2}.$$

- **Drop an event if no solution exists:** $\Delta = (b^2 - 4a \cdot c)$
  - **-- About 10% lost at truth level (recoverable but needed?)**
- **Choose randomly one if two solutions. Each class of solutions, correct or wrong, separately preserves information about polarisation.**

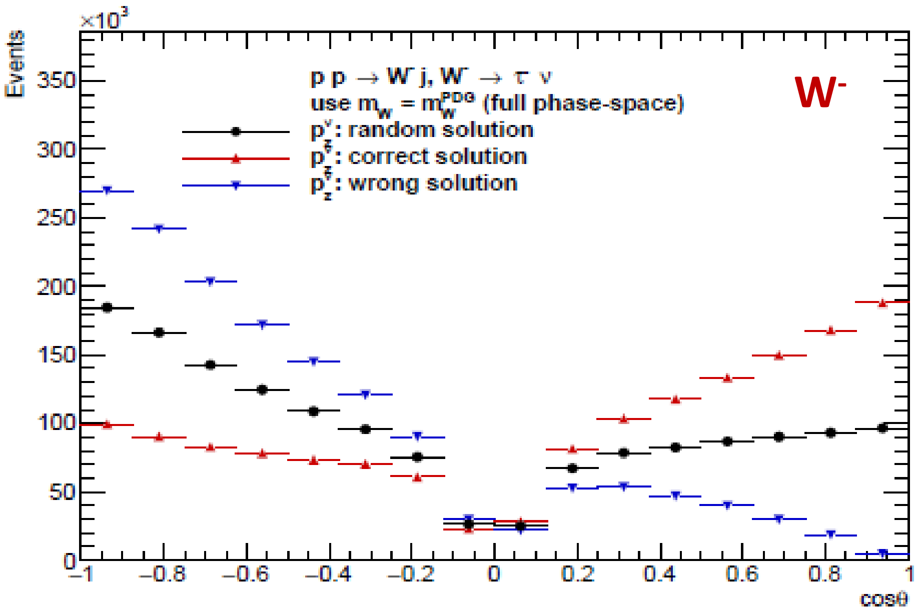
Collins-Soper frame

ERW, Z. Was, arXiv: 1609.02536

Use Collins-Soper fame, define z-axis orientation as one the charged lepton. Same formulas for cosθ, ϕ definitions as used for Z->ll events, as we have solved already equation for neutrino momenta.



Random/Correct/Wrong solution used

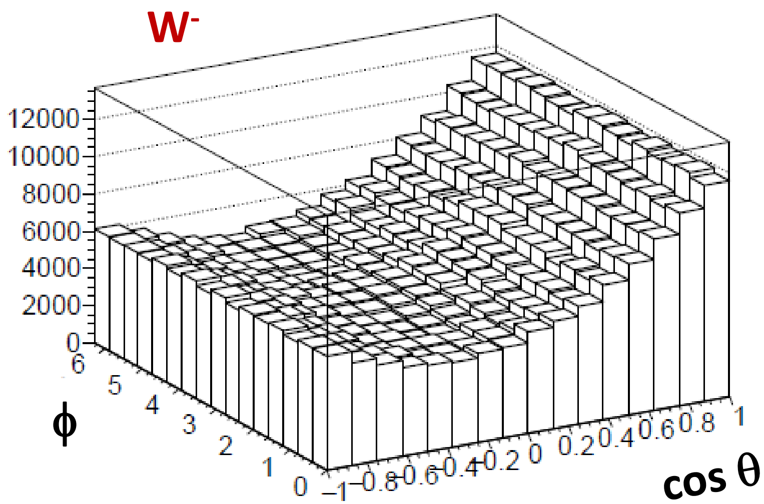


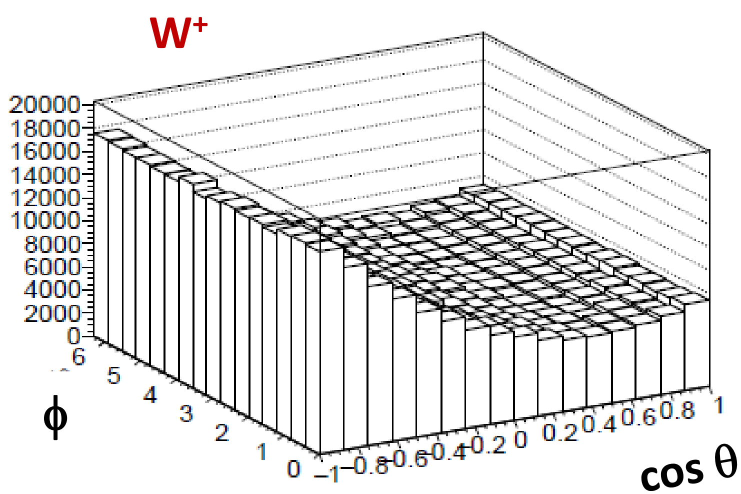
Red – 10% of events lost due to no solution for neutrino momenta. Mostly around cos θ=0 region

Collins-Soper frame

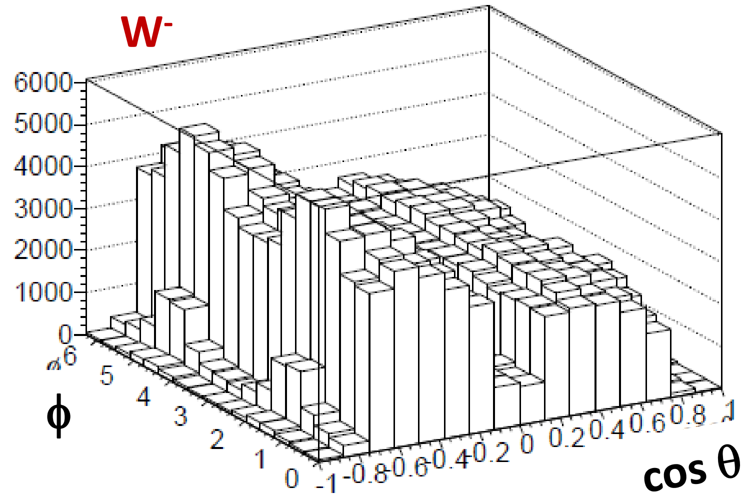
ERW, Z. Was, arXiv: 1609.02536

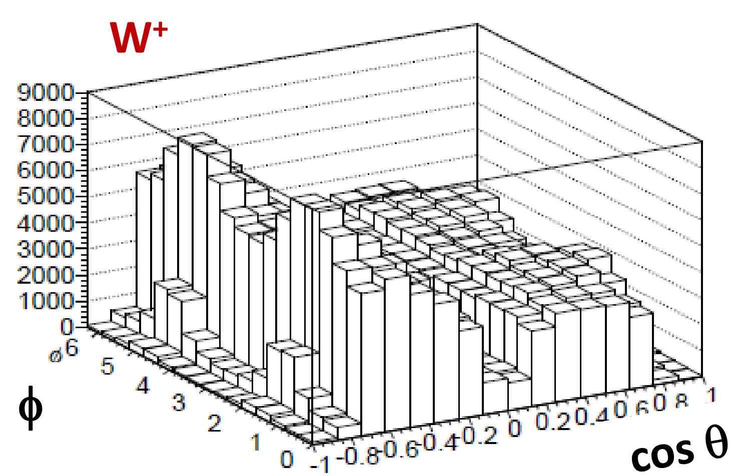
Generated events, full phase-space





Random p_z^ν solution, fiducial phase-space





Polynomial templates

ERW, Z. Was, arXiv: 1609.02536

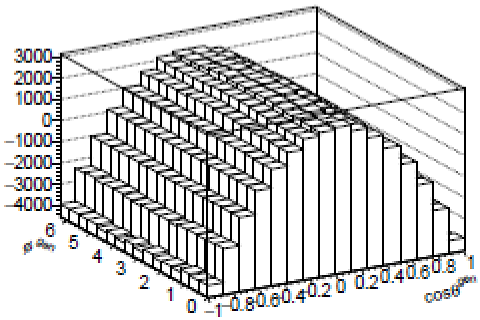
Use unweighted MC events, weighted to different P_i shapes.
For unweighting use A_i^s extracted with moments method.

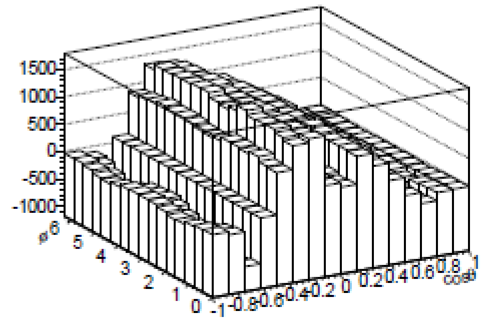
$$wt_{\Sigma AiPi} = \frac{1}{\Sigma_{i=0}^{i=8} A_i P_i(\cos\theta, \phi)}$$

$$\langle P_i(\cos\theta,\phi)\rangle = \frac{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, P_i(\cos\theta,\phi) d\sigma(\cos\theta,\phi)}{\int_{-1}^{1} d\cos\theta \int_0^{2\pi} d\phi\, d\sigma(\cos\theta,\phi)}.$$

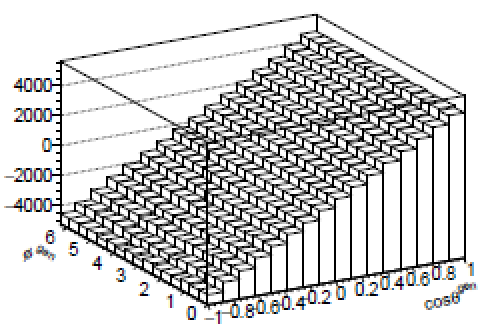
Full phase-space

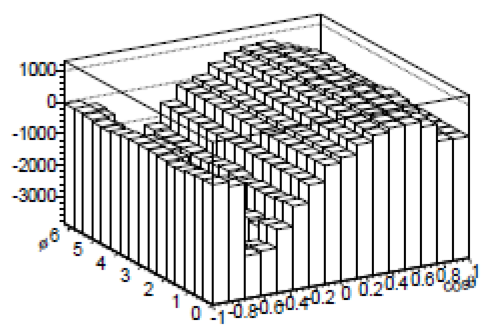
Fiducial phase-space





P_o shape



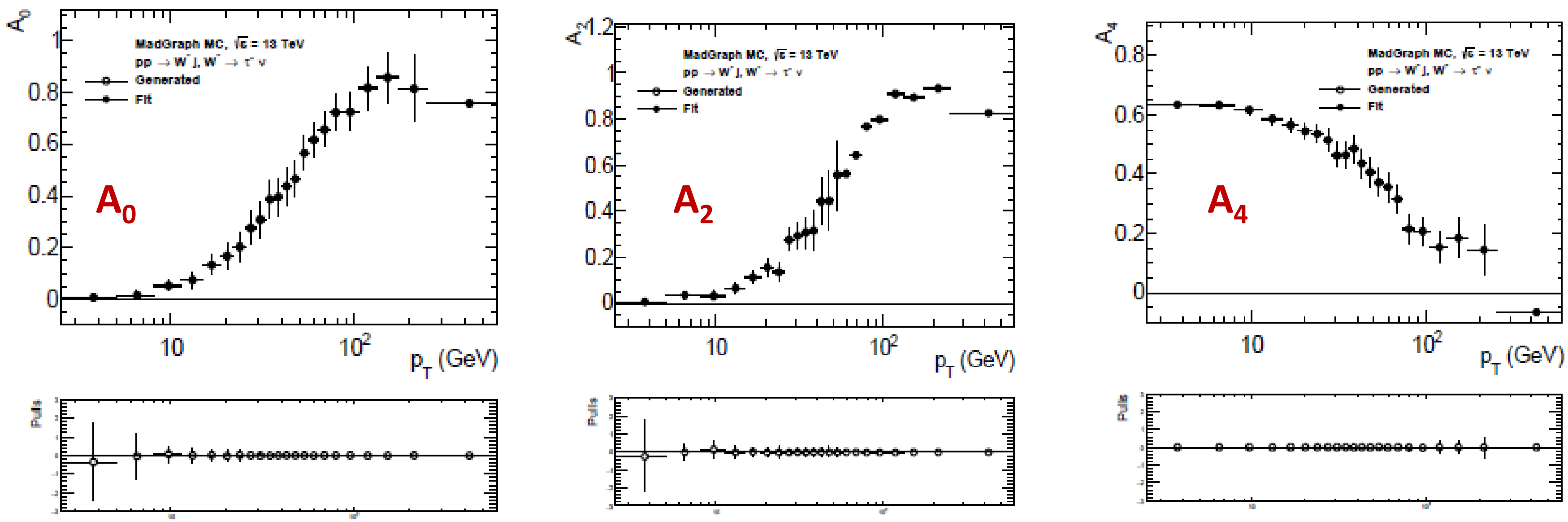


P_4 shape

Proof-of-concept: fit result

ERW, Z. Was, arXiv: 1609.02536

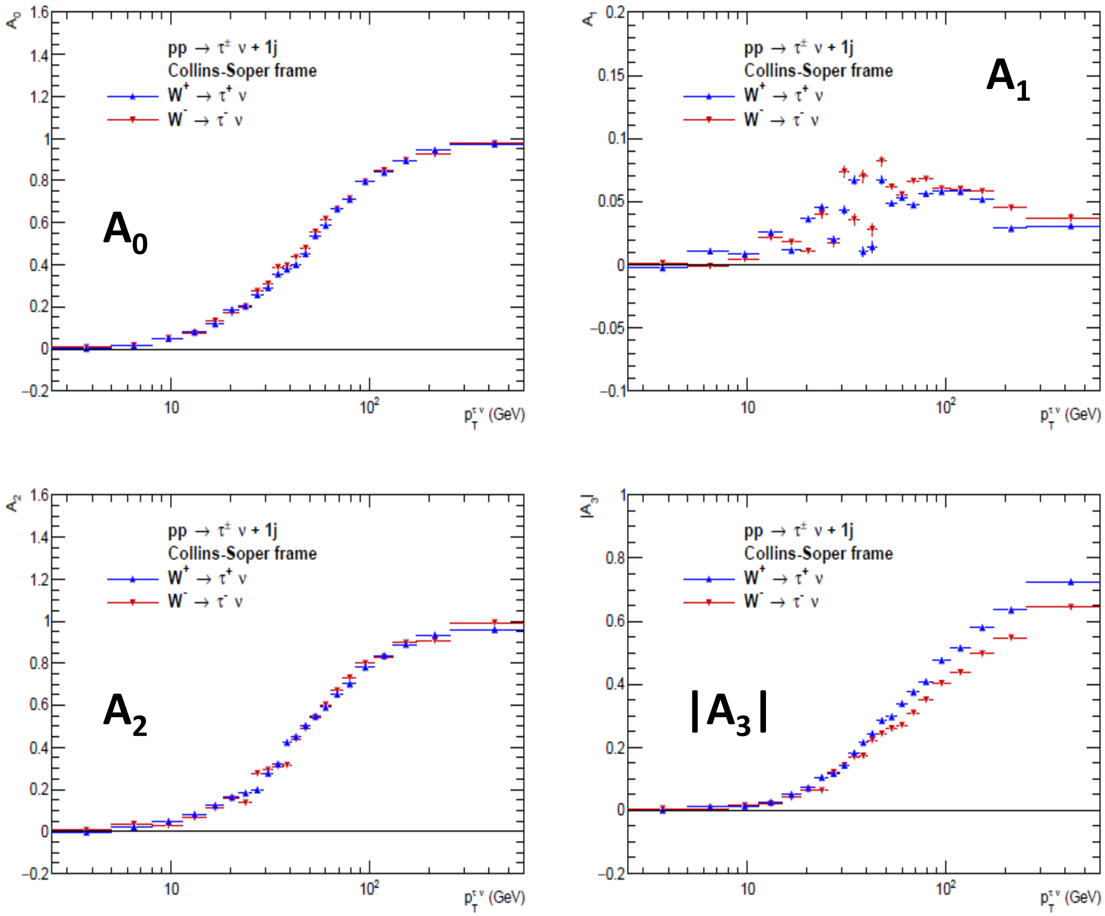
Closure test: generated and fit results for A_i^s

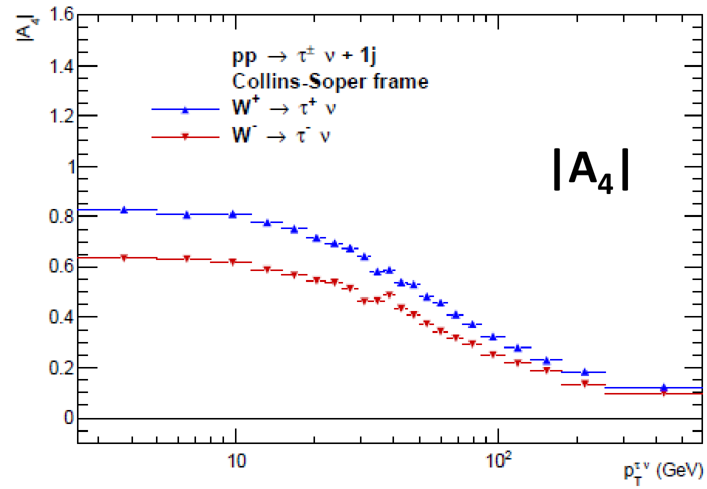


Same tests repeated using „only wrong" and „only correct" solutions for p_z^ν
Confirmed that either case has the same sensitivity to the polarisation.

Predictions: MadGraph W+1j

ERW, Z. Was, arXiv: 1609.02536





Compared $|A_i|$ for W^+, W^-.

With convention chosen, A_3, A_4 are negative for W^+.

Summary

- **Presented proof-of-concept that measurement of complete set of Ai's coefficients can be performed also in case of W boson, despite two-fold solution for neutrino momenta.**
  - **Use randomly choosen solution for p_z^ν**
  - **Follow strategy as for Z->ll case**
- **Limitation for precision and granularity in p_T binning of experimental measurement will come from resolution of E_T^{miss} reconstruction.**

BONUS SLIDES: Mustraal frame

Mustraal frame

[18] F. A. Berends, R. Kleiss, and S. Jadach, *Comput. Phys. Commun.* **29** (1983) 185–200.

$s = 2p_+ \cdot p_-, \quad t = 2p_+ \cdot q_+, \quad u = 2p_+ \cdot q_-$

$s' = 2q_+ \cdot q_-, \quad t' = 2p_- \cdot q_-, \quad u' = 2p_- \cdot q_+$

Mustraal: Monte Carlo for $e^+ e^- \to \mu^+ \mu^-$ (γ)

$$\sigma_{\mathrm{hard}} = \int \mathrm{d}\tau \left(X_{\mathrm{i}} + X_{\mathrm{f}} + X_{\mathrm{int}} \right),$$

The explicit forms of the three terms in σ_{hard} read:

$$X_{\mathrm{i}} = \frac{Q^2\alpha}{4\pi^2 s} \frac{1-\Delta}{k_+ k_-} s'^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t, u) + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s', t', u') \right], \tag{3.4}$$

$$X_{\mathrm{f}} = \frac{Q'^2\alpha}{4\pi^2 s} \frac{1-\Delta'}{k'_+ k'_-} s^2 \left[\frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t, u') + \frac{\mathrm{d}\sigma^B}{\mathrm{d}\Omega}(s, t', u) \right], \tag{3.5}$$

$$X_{\mathrm{int}} = \frac{QQ'\alpha}{4\pi^2 s} W \frac{\alpha^2}{2ss'} \left[(u^2 + u'^2 + t^2 + t'^2) \tilde{f}(s, s') + \tfrac{1}{2}(u^2 + u'^2 - t^2 - t'^2) \tilde{g}(s, s') \right]$$
$$+ \frac{QQ'\alpha^3}{4\pi^2 s} \frac{(s - s')M\Gamma}{k_+ k_- k'_+ k'_-} \epsilon_{\mu\nu\rho\sigma} p_+^\mu p_-^\nu q_+^\rho q_-^\sigma \left[\tilde{E}(s, s')(t^2 - t'^2) + \tilde{F}(s, s')(u^2 - u'^2) \right], \tag{3.6}$$

Resulting optimal frame used to minimise higher order corrections from initial state radiation in $e^+e^- \to Z/\gamma^* \to \mu\ \mu$ for algorithms of genuine EW corrections implementation in LEP time Monte Carlo's like Koral Z.

Mustraal frame for pp case

Rest-frame of outgoing leptons from V decay.

Proven in 80's (F.A.Berends at al. Comp. Phys. Com 29(1983) 185) that for qqbar->Z->ll and single spin-1 emission in initial state (gluon or photon) matrix element can be presented as weighted sum of Borns.

Addapted (ERW&ZW, Eur. Phys. J C76 (2016) 473) to pp case + added definition of azimuthal angle.

Restoring Born-like structure of the event :

– **All A_i's except A_4 close to zero at low and high p_T.**

Formulas of Mustraal frame

ERW, Z. Was, arXiv: 1605.05450

- The 4-vectors of incoming partons and outgoing leptons are boosted into lepton-pair rest frame.
- To fix orientation of the event we use versor $\hat{x}_{lab}$ of the laboratory reference frame. It is boosted into lepton-pair rest frame as well. It will be used in definition of azimuthal angle ϕ, which has to extend over the range $(0, 2\pi)$.
- We first calculate $\cos\theta_1$ (and $\cos\theta_2$) of the angle between the outgoing lepton and incoming quark (outgoing anti-lepton and incoming anti-quark) directions.

$$\cos\,\theta_1 = \frac{\vec{\tau_1}\cdot\vec{p_1}}{|\vec{\tau_1}||\vec{p_1}|}, \qquad \cos\,\theta_2 = \frac{\vec{\tau_2}\cdot\vec{p_2}}{|\vec{\tau_1}||\vec{p_2}|} \tag{6}$$

- The azimuthal angles ϕ_1 and ϕ_2 corresponding to θ_1 and θ_2 are defined as follows. We first define $e_{\vec{y}1,2}$ versors and with their help later $\phi_{1,2}$ as:

$$\vec{e_y} = \frac{\vec{x_{lab}}\times\vec{p_2}}{|\vec{e_y}|}, \qquad \vec{e_x} = \frac{\vec{e_y}\times\vec{p_2}}{|\vec{e_x}|}$$

$$\cos\,\phi_1 = \frac{\vec{e_x}\cdot\vec{\tau_1}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_1})^2+(\vec{e_y}\cdot\vec{\tau_1})^2}} \qquad \sin\,\phi_1 = \frac{\vec{e_y}\cdot\vec{\tau_1}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_1})^2+(\vec{e_y}\cdot\vec{\tau_1})^2}} \tag{7}$$

and similarly for ϕ_2:

$$\vec{e_y} = \frac{\vec{x_{lab}}\times\vec{p_1}}{|\vec{e_y}|}, \qquad \vec{e_x} = \frac{\vec{e_y}\times\vec{p_1}}{|\vec{e_x}|}$$

$$\cos\,\phi_2 = \frac{\vec{e_x}\cdot\vec{\tau_2}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_2})^2+(\vec{e_y}\cdot\vec{\tau_2})^2}} \qquad \sin\,\phi_2 = \frac{\vec{e_y}\cdot\vec{\tau_2}}{\sqrt{(\vec{e_x}\cdot\vec{\tau_2})^2+(\vec{e_y}\cdot\vec{\tau_2})^2}}. \tag{8}$$

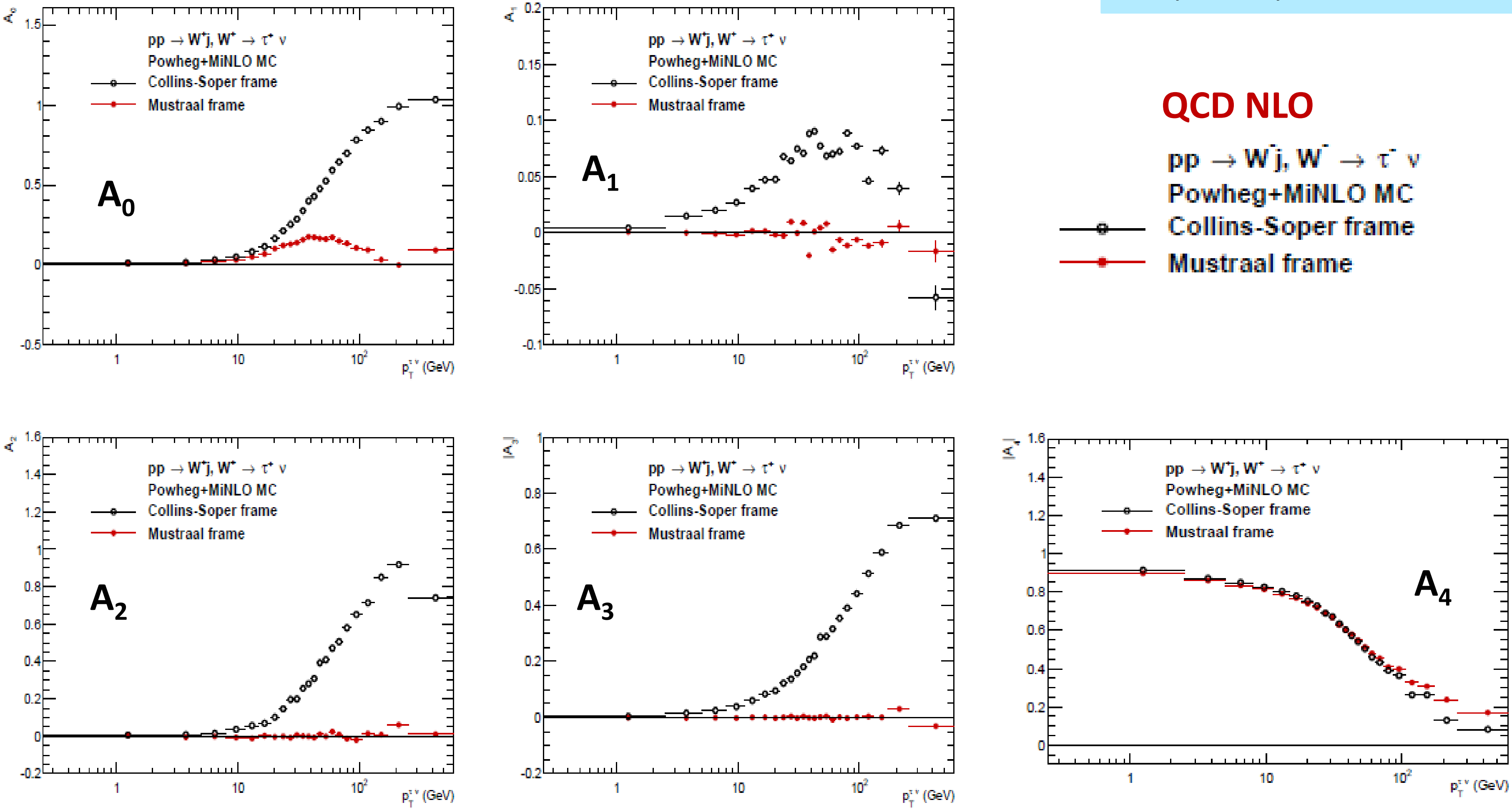
- Each event contributes with two Born-like kinematics configurations $\theta_1\phi_1$, $(\theta_2\phi_2)$, respectively with wt_1 (and wt_2) weights; $wt_1 + wt_2 = 1$ where

$$wt_1 = \frac{E_{p1}^2(1+\cos^2\theta_1)}{E_{p1}^2(1+\cos^2\theta_1)+E_{p2}^2(1+\cos^2\theta_2)}, \qquad wt_2 = \frac{E_{p2}^2(1+\cos^2\theta_2)}{E_{p1}^2(1+\cos^2\theta_1)+E_{p2}^2(1+\cos^2\theta_2)}. \tag{9}$$

In the calculation of the weight, incoming partons energies E_{p1}, E_{p2} in the rest frame of lepton pair are used, but not their couplings or flavours. That is also why, instead of $\sigma_B(s, \cos\theta)$ the simplification $(1+\cos^2\theta)$ is used in Eq. (9). Dependence on the sign of $\cos\theta$ drops out[3].

A_i's in Mustraal frame

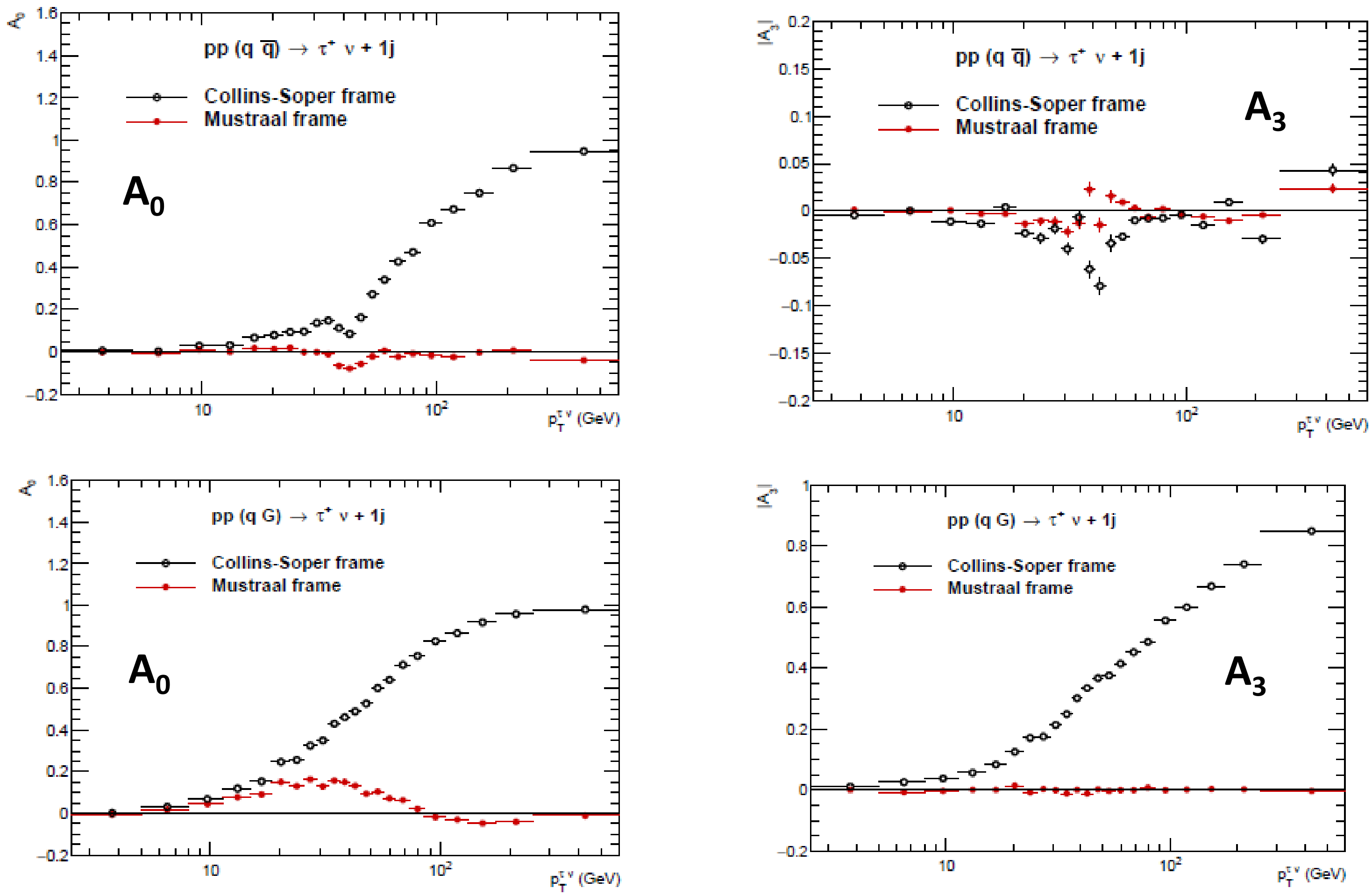
ERW, Z. Was, arXiv: 1609.02536



Only A_4 coefficient significantly different from zero at low p_T.
Frame suitable for factorising EW and QCD corrections.

A_i's in Mustraal frame

ERW, Z. Was, arXiv: 1609.02536



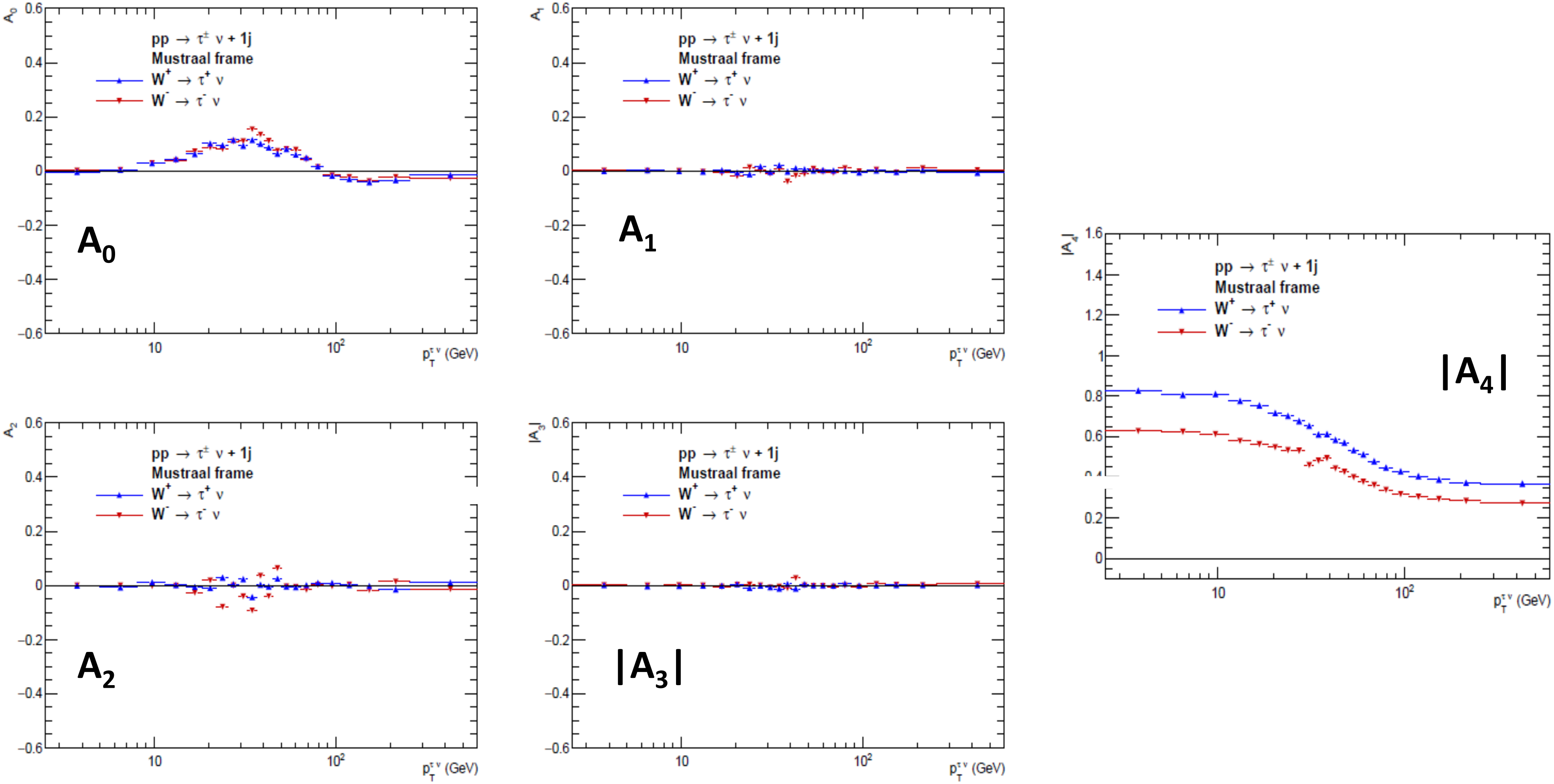
qq -> W j

qG -> W j

Non-zero values of A_0 in Mustraal frame in qG->Wj events only.

A_i's in Mustraal frame

ERW, Z. Was, arXiv: 1609.02536

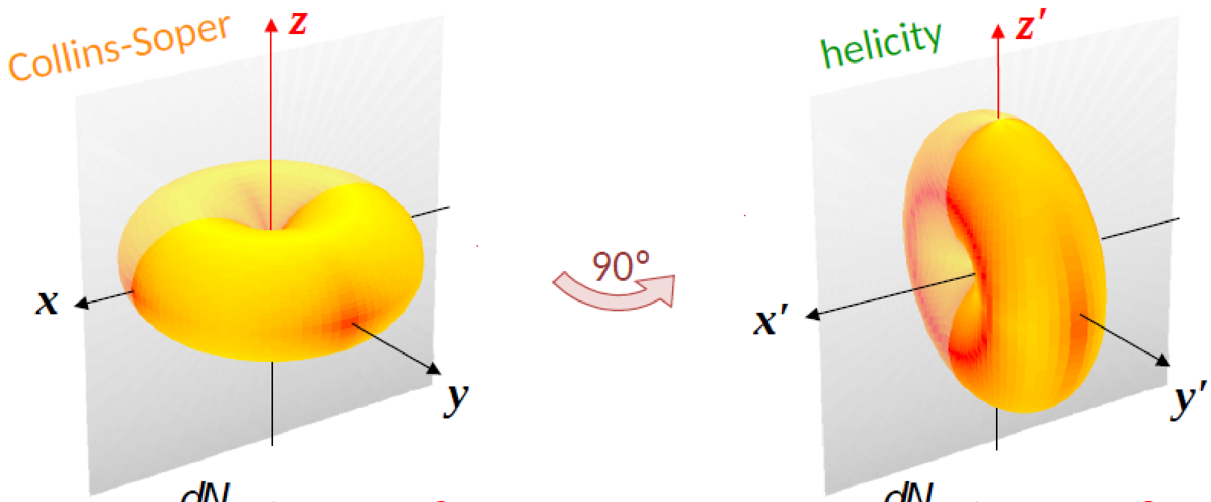


Compared $|A_i|$ for W^+, W^-.
With convention chosen A_3, A_4 are negative for W^+.

SPARES

The observed polarisation depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90°



$$\frac{dN}{d\Omega} \propto 1 - \cos^2\theta$$

longitudinal

$$|\psi\rangle = |0\rangle$$

(pure state)

$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta - \sin^2\theta \cos 2\varphi$$

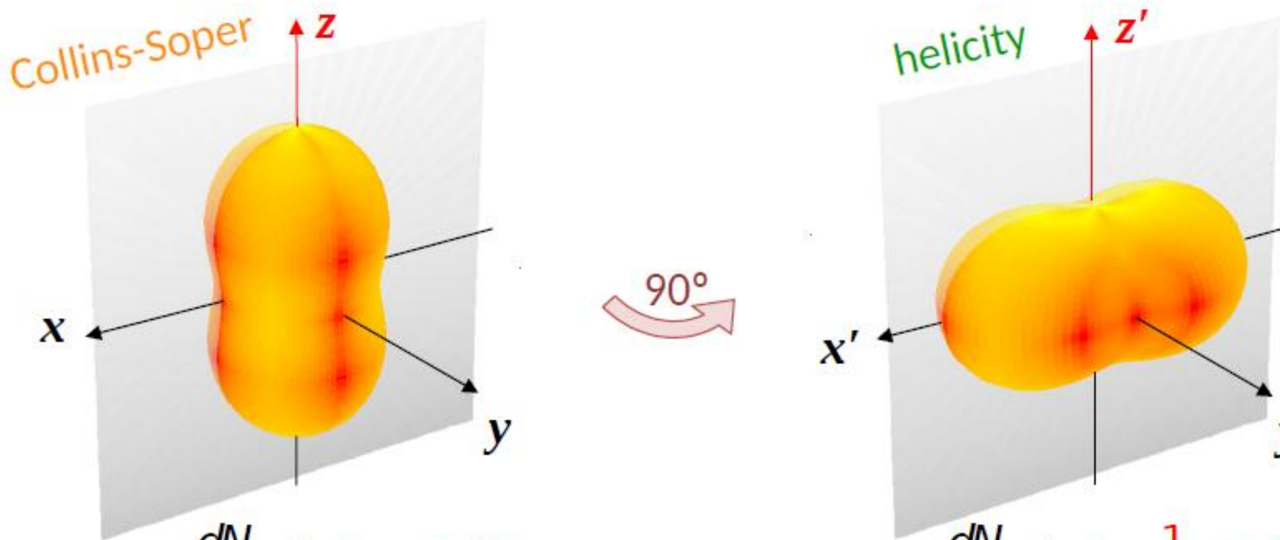
“transverse”

$$|\psi\rangle = \frac{1}{\sqrt{2}}|+1\rangle - \frac{1}{\sqrt{2}}|-1\rangle$$

(mixed state)

The observed polarisation depends on the frame

For $|p_L| << p_T$, the CS and HX frames differ by a rotation of 90°



$$\frac{dN}{d\Omega} \propto 1 + \cos^2\theta$$

transverse

$$|\psi\rangle = |+1\rangle \; or \; |-1\rangle$$

(pure state)

$$\frac{dN}{d\Omega} \propto 1 - \frac{1}{3}\cos^2\theta + \frac{1}{3}\sin^2\theta \cos 2\varphi$$

moderately “longitudinal”

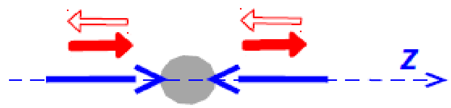
$$|\psi\rangle = \frac{1}{2}|+1\rangle + \frac{1}{2}|-1\rangle \mp \frac{1}{\sqrt{2}}|0\rangle$$

(mixed state)

Drell-Yan, W, Z polarizations

- polarization is *always fully* ***transverse***...

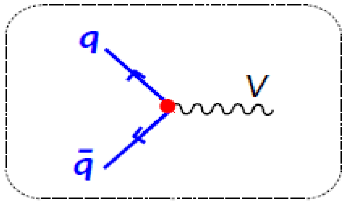
$V = \gamma^*, Z, W$



Due to helicity conservation at the q-$\bar{q}$-V (q-q^*-V) vertex, $J_z = \pm 1$ along the q-$\bar{q}$ (q-q^*) scattering direction z

- ...but with respect to a ***subprocess-dependent quantization axis***

$O(\alpha_s^0)$ →



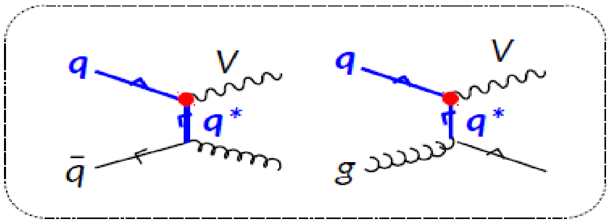
"natural" z = relative dir. of q and $q$bar

→ λ_θ("CS") = +1

wrt *any* axis: $\tilde{\lambda}$ = +1

$O(\alpha_s^1)$

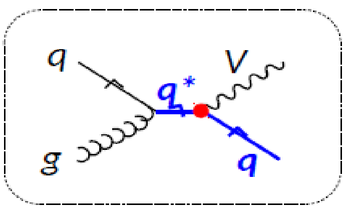
(LO) QCD corrections



z = dir. of *one* incoming quark

→ λ_θ("GJ") = +1

$\tilde{\lambda}$ = +1



z = dir. of outgoing q

→ λ_θ("HX") = +1

$\tilde{\lambda}$ = +1

N.B.: $\tilde{\lambda}$ = +1 in both pp-HX and qg-HX frames!

$\tilde{\lambda}$ = +1 any frame

In all these cases the q-q-V lines are in the production plane (planar processes);
The CS, GJ, pp-HX and qg-HX axes only differ by a rotation in the production plane