BENCHMARKS

for EW lineshape, loops + boxes corrections

in different codes

EW settings

Table 45: Different options for EW parameters setting in $\mathscr{A}^{Born+EW}$ matrix element as defined in formula (64). Indicated is the value of the form-factors. Only those fulfilling Standard Model tree-level relation $\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1$ can be used with default version of DYTURBO. $\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}$.

EW LO reference

α_0 (LO) (EWopt=1)	*LEP* (EWopt=3)	*LEP with improved norm.* (EWopt=4)
$G_\mu = 1.166389 \cdot 10^{-5}$	$G_\mu = 1.166389 \cdot 10^{-5}$	$G_\mu = 1.166389 \cdot 10^{-5}$
$M_Z = 91.1876$ GeV	$M_Z = 91.1876$ GeV	$M_Z = 91.1876$ GeV
$\alpha = 1/137.03604$	$\alpha = 1/128.8667$	$\alpha = 1/128.8667$
$s_W^2 = 0.21211538$	$s_W^2 = 0.23152$	$s_W^2 = 0.23152$
$\rho_{\ell f} = 1.0$	$\rho_{\ell f} = 1.0$	$\rho_{\ell f} = 1.005$
$K_i = 1.0$	$K_i = 1.0$	$K_i = 1.0$
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1$	$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1$	$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} = 1$
α_0 (Dizet) (EWopt=2)	α_0 (Dizet) + *EW (loops)* (EWopt=5)	α_0 (Dizet) + *EW (loops + boxes)* (EWopt=6)
$G_\mu = 1.166389 \cdot 10^{-5}$	$G_\mu = 1.166389 \cdot 10^{-5}$	$G_\mu = 1.166389 \cdot 10^{-5}$
$M_Z = 91.1876$ GeV	$M_Z = 91.1876$ GeV	$M_Z = 91.1876$ GeV
$\alpha = 1/137.03604$	$\alpha = 1/137.03604$	$\alpha = 1/137.03604$
$s_W^2 = 0.22351946$	$s_W^2 = 0.22351946$	$s_W^2 = 0.22351946$
$\rho_{\ell f} = 1.0$	$\rho_{\ell f}(s)$	$\rho_{\ell f}(s,t)$
$\rho_{\ell f} = 1.0$	$K_i(s)$	$K_i(s,t)$
$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} != 1$	$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} != 1$	$\frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} != 1$

EW HO „best”

Benchmark table: EW 1loop

Table 47: Results of calculations: masses, couplings, etc. *Tuned* comparison: predictions with EW 1-loop corrections, configuration of `Dizet 6.21` shown in Table 39.

Parameter	`Powheg_ew` (EW 1-loop)	`Dizet 6.21` (EW 1-loop)	Description
m_W s_W^2 c_W^2	?	80.4912759 0.22084586 0.77915414	mass of W boson $s_W^2 = 1 - m_W^2/m_Z^2$
$\alpha_{QED}(M_Z)$ $1/\alpha_{QED}(M_Z)$	?	0.007534 132.7316	
$\alpha_s(M_Z)$ $\alpha_s(m_t)$	?	0.1250 0.1134	
$ZPAR(1) = \Delta r$ $ZPAR(2) = \Delta r_{rem}$	?	0.0286451 0.0	the loop corrections to G_μ the remainder contribution $O(\alpha)$
$ZPAR(17)$ $ZPAR(18)$ $ZPAR(19)$ $ZPAR(20)$ $ZPAR(21)$ $ZPAR(22)$ $ZPAR(23)$ $ZPAR(24)$ $ZPAR(25)$ $ZPAR(26)$ $ZPAR(27)$ $ZPAR(28)$ $ZPAR(29)$ $ZPAR(30)$	?	1.04204102 1.05001972 1.04144942 1.03347072 1.04205621 1.04986350 1.04144942 1.03347072 1.04204134 1.05002004 1.04201352 1.02762415 -0.00002603 0.21220659	

Benchmark table: EW 1loop

Table 48: Common for Dizet 6.21 and MCSANC convention, real part of form-factors for $\sqrt{s} = m_Z$ and $cos\theta = 0$. Shown are *tuned* (EW 1-loop) and best default predictions of both programs.

Form-factors	Powheg_ew (EW 1-loop)	Dizet 6.21 (EW 1-loop)	Dizet 6.21 Default (KKMC)	Comments
ρ_{eu}	?	1.0064962	1.005408	
K_e		1.0415753	1.036649	
K_u		1.0411421	1.036172	
K_{eu}		1.0827174	1.074146	
ρ_{ed}	?	1.0069491	1.005894	
K_e		1.0415753	1.036649	
K_d		1.0406299	1.035603	
K_{ed}		1.0822052	1.073556	

Benchmark table: best predictions

Table 46: Results of calculations: masses, couplings, etc. Default (best) predictions of both programs.

Parameter	Powheg_ew (EW best)	Dizet 6.21 Default (KKMC)	Description
m_W	?	80.353 GeV	mass of W boson
s_W^2		0.22351946	$s_W^2 = 1 - m_W^2/m_Z^2$
c_W^2			
$\alpha_{QED}(M_Z)$	0.007751	0.007759	
$1/\alpha_{QED}(M_Z)$	129.02086	128.88259	
$ZPAR(1) = \Delta r$	?	0.03694272	the loop corrections to G_μ
$ZPAR(2) = \Delta r_{rem}$		0.01169749	the remainder contribution $O(\alpha)$
$sin^2\theta_W^{eff}$ neutrino	?	0.23137443	$ZPAR(5)$
$sin^2\theta_W^{eff}$ electron		0.23175590	$ZPAR(6)$
$sin^2\theta_W^{eff}$ muon		0.23175590	$ZPAR(7)$
$sin^2\theta_W^{eff}$ tau		0.23175590	$ZPAR(8)$
$sin^2\theta_W^{eff}$ u-quark		0.23164930	$ZPAR(9)$
$sin^2\theta_W^{eff}$ d-quark		0.23152214	$ZPAR(10)$
$sin^2\theta_W^{eff}$ c-quark		0.23164930	$ZPAR(11)$
$sin^2\theta_W^{eff}$ s-quark		0.23152214	$ZPAR(12)$
$sin^2\theta_W^{eff}$ b-quark		0.23307588	$ZPAR(14)$

Lineshape: ratio to EW LO

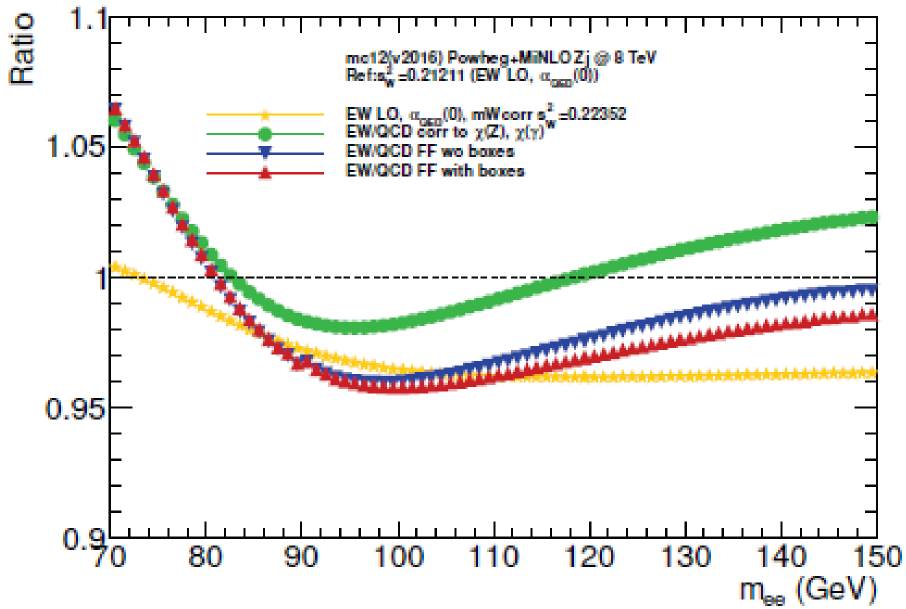


Table 49: Normalisation corrections to the total cross-section. Shown are individual normalisation factors vs predicted with EW LO $\alpha(0)$ scheme `EWopt = 1`.

Version	Corrections to cross-section	m_{ee} = 89 - 93 GeV	m_{ee} = 80 - 100 GeV	Comments	Comments
$EWopt = 1$	EW $\alpha(0)$	1.0	1.0	reference	$s_W^2 = 0.21211$
$EWopt = 2$	EW $\alpha(0) + m_W corr$	0.97102	0.98245		$s_W^2 = 0.22352$
	EW $\alpha(0) + m_W corr$ + FF (prop.)	0.98245	0.98344		$s_W^2 = 0.22352$
$EWopt = 5$	EW $\alpha(0) + m_W corr$ + FF (loops)	0.96526	0.96645		$s_W^2 = 0.22352$
$EWopt = 6$	EW $\alpha(0) + m_W corr$ + FF (loops + boxes)	0.96526	0.96648		$s_W^2 = 0.22352$

SPARES

From Zfitter/Dizet documentation

D. Bardin et al., Comp. Phys. Comun. 133 (2001) 229-395

Zfitter is a **semi-analytical program** for calculating total cross-sections and pseudo-observables (eg. A_{fb}, $\sin^2\theta_W^{eff}$), used by LEP1, and to a lesser degree by LEP2.
DIZET is a library for calculating form-factors and some other corrections. Provides complete EW $O(\alpha)$ weak-loop corrections supplemented with selected higher order terms (eg. vacum polarisation, $\alpha_{QED}(Q^2)$).
For analyses at LEP1, LEP2 used aways in parallel with **MC generators (KoralZ, KoralW)** eg. to evaluate systematics of simplified cuts used in analysis integration.

one loop amplitude

$$\begin{aligned} A_Z^{OLA}(s,t) = & \; i\sqrt{2}G_\mu I_e^{(3)} I_f^{(3)} M_Z^2 \chi_Z(s) \rho_{ef}(s,t) \Big\{ \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu(1+\gamma_5) \\ & -4|Q_e|s_W^2 \kappa_e(s,t) \gamma_\mu \otimes \gamma_\mu(1+\gamma_5) - 4|Q_f|s_W^2 \kappa_f(s,t) \gamma_\mu(1+\gamma_5) \otimes \gamma_\mu \\ & +16|Q_e Q_f| s_W^4 \kappa_{e,f}(s,t) \gamma_\mu \otimes \gamma_\mu \Big\}. \end{aligned} \quad (A.4.75)$$

$$A_\gamma^{OLA} = i\chi_\gamma(s)\alpha(s)\gamma_\mu \otimes \gamma_\mu \,. \quad (2.2.36)$$

Dyson summation leads to the change of α into $\alpha(s)$:

$$\alpha(s) = \frac{\alpha(0)}{1-\Delta\alpha^{\rm fer}(s)} = \frac{\alpha(0)}{1-\Delta\alpha^{(5)}(s) - \Delta\alpha^{t}(s) - \Delta\alpha^{\alpha\alpha_s}(s)}, \quad (2.2.37)$$

Vacuum polarisation corrections

From Zfitter/Dizet documentation

D. Bardin et al., Comp. Phys. Comun. 133 (2001) 229-395

After some trivial algebra one derives the final expressions:

$$\rho_{ef} = 1 + \frac{g^2}{16\pi^2}\Big\{ -\Delta\rho_Z^F + \mathcal{D}_Z^F(s) + \frac{5}{3}B_0^F(-s;M_W,M_W) - \frac{9}{4}\frac{c_W^2}{s_W^2}\ln c_W^2 - 6 + \frac{5}{8}c_W^2\left(1+c_W^2\right) + \frac{1}{4c_W^2}\left(3v_e^2+a_e^2+3v_f^2+a_f^2\right)\mathcal{F}_Z(s) + \hat{\mathcal{F}}_W^0(s) + \hat{\mathcal{F}}_W(s) - \frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right] - c_W^2(R_Z-1)\,s\mathcal{B}^d_{WW}(s,t)\Big\}, \qquad (A.4.80)$$

$$\kappa_e = 1 + \frac{g^2}{16\pi^2}\Big\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi^F_{Z\gamma}(s) - \frac{1}{6}B_0^F(-s;M_W,M_W) - \frac{1}{9} - \frac{v_e\sigma_e}{2c_W^2}\mathcal{F}_Z(s) - \hat{\mathcal{F}}_W^0(s) + (R_Z-1)\Big[\frac{|Q_f|}{2}\left(1-4|Q_f|s_W^2\right)\mathcal{F}_Z(s) + c_W^2\Big[\hat{\mathcal{F}}_{W_n}(s) - |Q_{f'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\Big]\Big\}, \qquad (A.4.81)$$

$$\kappa_f = 1 + \frac{g^2}{16\pi^2}\Big\{ -\frac{c_W^2}{s_W^2}\Delta\rho^F - \Pi^F_{Z\gamma}(s) - \frac{1}{6}B_0^F(-s;M_W,M_W) - \frac{1}{9} - \frac{v_f\sigma_f}{2c_W^2}\mathcal{F}_Z(s) - \hat{\mathcal{F}}_W(s) + (R_Z-1)\Big[\frac{|Q_e|}{2}\left(1-4|Q_e|s_W^2\right)\mathcal{F}_Z(s) + c_W^2\Big[\hat{\mathcal{F}}^0_{W_n}(s) - |Q_{e'}|\mathcal{F}_{W_a}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\Big] - \frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right]\Big\}, \qquad (A.4.82)$$

$$\kappa_{ef} = 1 + \frac{g^2}{16\pi^2}\Big\{ -2\frac{c_W^2}{s_W^2}\Delta\rho^F - 2\Pi^F_{Z\gamma}(s) - \frac{1}{3}B_0^F(-s;M_W,M_W) - \frac{2}{9} - \frac{1}{4c_W^2}\Big[\frac{\delta_e^2+\delta_f^2}{s_W^2}(R_W-1) + 3v_e^2+a_e^2+3v_f^2+a_f^2\Big]\mathcal{F}_Z(s) - \hat{\mathcal{F}}_W^0(s) - \hat{\mathcal{F}}_W(s) - \frac{r_t}{4}\left[B_0^F(-s;M_W,M_W)+1\right] + c_W^2(R_Z-1)\Big[\frac{2}{3} - \hat{\Pi}^{bos,F}_{\gamma\gamma}(s) + s\hat{\mathcal{B}}^d_{WW}(s,t)\Big]\Big\}. \qquad (A.4.83)$$

Interference

Fermionic loops in γ* propagator

Box

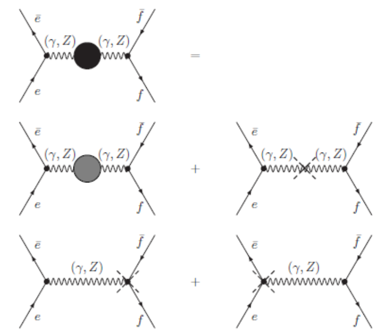


Figure A.11. *Bosonic self-energies and bosonic counter-terms for* $e\bar{e} \to (Z,\gamma) \to f\bar{f}$

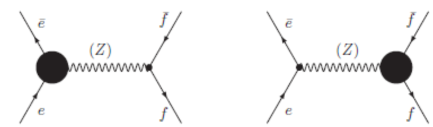


Figure A.10. *Electron (a) and final fermion (b) vertices in* $e\bar{e} \to (Z) \to f\bar{f}$

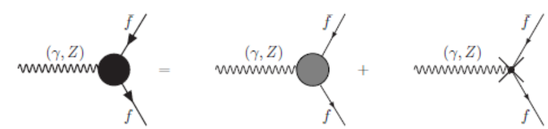


Figure A.6. *Off-shell* $Zf\bar{f}$ *and* $\gamma f\bar{f}$ *vertices*

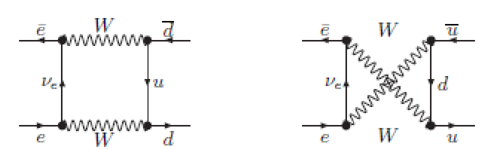


Figure A.7. *The WW boxes*

etc. etc.

EW $\alpha(0)$ scheme (Dizet)

1) Start with tree parameters at their best measured values :

G_μ, $\alpha(0)$, M_Z

2) Calculate m_W using iterative formula including higher-order corrections

Parameter	Value
M_Z	91.1876 GeV
m_h	125.0 GeV
m_t	173.0 GeV
m_b	4.7 GeV
$1/\alpha(0)$	137.0359895(61)
G_μ	$1.166389(22) \cdot 10^{-5}$ GeV^{-2}

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

M_W	80.353 GeV
s_W^2	0.22351946

$$s_W^2 = 1 - M_W^2/M_Z^2$$

On-mass-shell relation valid to all orders

$\alpha(0)$ important to consistently separate QED corrections from genuine EW ones. No need to introduce two α scales.

Spin amplitude: EW Improved Born (IBA)

$$
\begin{aligned}
\mathscr{A}^{Born+EW} = \frac{\alpha}{s}\{ \quad & [\bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{v}\gamma^{\nu} u] \cdot (q_\ell \cdot q_f) \cdot \boxed{\Gamma_{V_\Pi}} \cdot \chi_\gamma(s) \\
& +[\bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{v}\gamma^{\nu} u \cdot (v_\ell \cdot v_f \cdot vv_{\ell f}) + \bar{u}\gamma^{\mu} v g_{\mu\nu} \bar{v}\gamma^{\nu}\gamma^5 u \cdot (v_\ell \cdot a_f) \\
& +\bar{u}\gamma^{\mu}\gamma^5 v g_{\mu\nu} \bar{v}\gamma^{\nu} u \cdot (a_\ell \cdot v_f) + \bar{u}\gamma^{\mu}\gamma^5 v g_{\mu\nu} \bar{v}\gamma^{\nu}\gamma^5 u \cdot (a_\ell \cdot a_f)] \cdot \boxed{Z_{V_\Pi}} \cdot \chi_Z(s) \quad \}
\end{aligned}
$$

$$\chi_\gamma(s) = 1$$

$$\chi_Z(s) = \frac{G_\mu \cdot M_z^2 \cdot \Delta^2}{\sqrt{2} \cdot 8\pi \cdot \alpha} \cdot \frac{s}{s - M_Z^2 + i \cdot \Gamma_Z \cdot M_Z}$$

$$
\begin{aligned}
v_\ell &= (2 \cdot T_3^\ell - 4 \cdot q_\ell \cdot s_W^2 \cdot \boxed{K_\ell(s,t)})/\Delta \\
v_f &= (2 \cdot T_3^f - 4 \cdot q_f \cdot s_W^2 \cdot \boxed{K_f(s,t)})/\Delta \\
a_\ell &= (2 \cdot T_3^\ell)/\Delta \\
a_f &= (2 \cdot T_3^f)/\Delta
\end{aligned}
$$

$$\Delta = \sqrt{16 \cdot s_W^2 \cdot (1 - s_W^2)}$$

$$\boxed{Z_{V_\Pi}} = \rho_{e,f}(s,t)$$

EW form-factors , functions of $(s,t)=(m_{ll}, \cos\theta)$

Calculated with Dizet 6.21 library

$$\boxed{\Gamma_{V_\Pi}} = \frac{1}{2 - (1 + \boxed{\Pi_{\gamma\gamma}(s)})}$$

Vacuum polarisation corrections, used low-energy experiment input.

Warning: problem for analytic continuation

$$
\begin{aligned}
vv_{\ell f} = \quad & \frac{1}{v_\ell \cdot v_f}[(2 \cdot T_3^\ell)(2 \cdot T_3^f) - 4 \cdot q_\ell \cdot s_W^2 \cdot \boxed{K_f(s,t)}(2 \cdot T_3^\ell) - 4 \cdot q_f \cdot s_W^2 \cdot \boxed{K_\ell(s,t)}(2 \cdot T_3^f) \\
& +(4 \cdot q_\ell \cdot s_W^2)(4 \cdot q_f \cdot s_W^2)\boxed{K_{\ell f}(s,t)}]\frac{1}{\Delta^2}
\end{aligned}
$$

Effective weak mixing angle

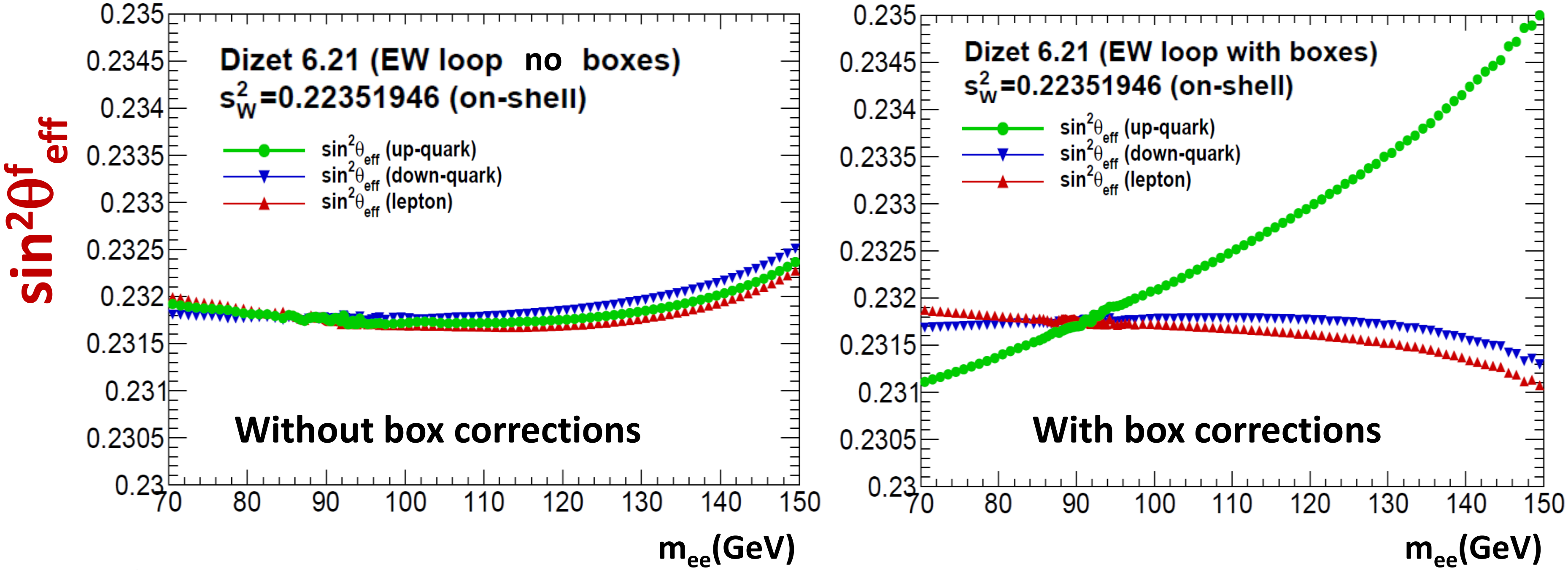
Here convoluted with line-shape and cosθ* distribution of MC events.

$$\sin^2\theta_W^{eff} = Re(K_Z^f)s_W^2 + I_f^2$$

$$I_f^2 = \alpha^2(s)\frac{35}{18}[1 - \frac{8}{3}Re(K_Z^f)s_W^2]$$ **= ~ 10^{-4}**

$$s_W^2 = 1 - M_W^2/M_Z^2$$

$\sin^2\theta_W^{eff}$ **defines ratio of couplings** $g_Z^f = \frac{v_f}{a_f} = 1 - 4|q_f|(K_Z^f s_W^2 + I_f^2)$



De-mystifying EW schemes

EW $\alpha(0)$ scheme:

Input: $\alpha(0)$, M_Z, G_μ

$$M_Z = 91.1876\ \text{GeV}$$
$$\alpha = 1/137.03599$$
$$G_\mu = 1.1663787 \cdot 10^{-5} GeV^{-2}$$

$$M_W = 80.353\ \text{GeV}$$
$$s_W^2 = 0.22351946$$

$$M_W = \frac{M_Z}{\sqrt{2}} \sqrt{1 + \sqrt{1 - \frac{4A_0^2}{M_Z^2(1-\Delta r)}}}$$

$$A_0 = \sqrt{\frac{\pi\alpha(0)}{\sqrt{2}G_\mu}}$$

$$s_W^2 = 1 - M_W^2/M_Z^2$$

EW G_μ scheme:

Input: M_Z, M_W, G_μ

$$M_Z = 91.1876\ \text{GeV}$$
$$M_W = 80.385\ \text{GeV}$$
$$G_\mu = 1.1663787 \cdot 10^{-5}\ \text{GeV}^{-2}$$

$$\alpha = 1/132.2332$$
$$s_W^2 = 0.222897$$

$$\alpha = G_\mu \sqrt{2} M_W^2 (1 - M_W^2/M_Z^2)/\pi$$

$$s_W^2 = 1 - M_W^2/M_Z^2$$

Most commonly used scheme in MC generators todays.

EW $\alpha(M_Z)$ scheme:

Input: $\alpha(M_Z)$, M_Z, G_μ

$$M_Z = 91.1876\ \text{GeV}$$
$$\alpha = 1/128.8793$$
$$G_\mu = 1.1663787 \cdot 10^{-5} GeV^{-2}$$

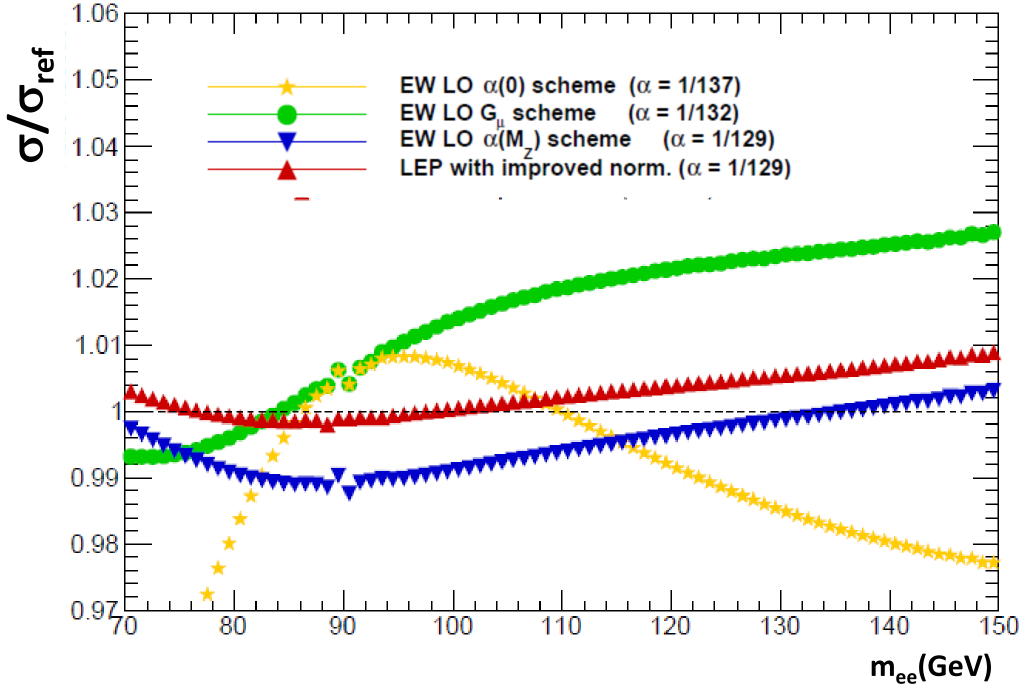
$$M_W = 79.9968\ \text{GeV}$$
$$s_W^2 = 0.2311538$$

$$s_W^2 = \frac{1 - \sqrt{1 - 4\frac{\alpha\pi}{G_\mu \sqrt{2} M_Z^2}}}{2}$$

$$M_W = M_Z \cdot \sqrt{1 - s_W^2}$$

Line-shape in different EW schemes

Ref.: best predictions with EW O(α) loop+boxes corrections



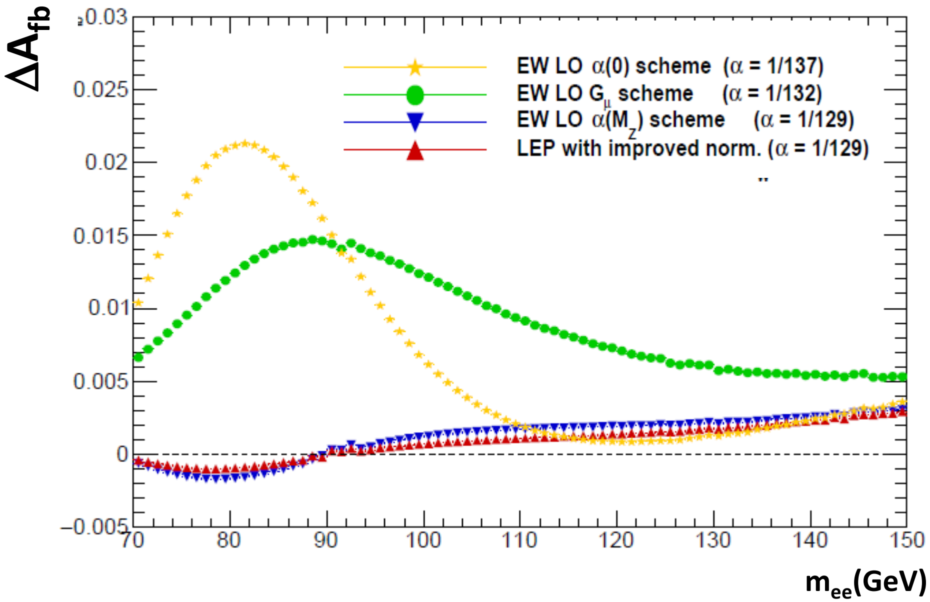
„LEP with improved norm." **(red)** **predicts very well shape and normalisation**

EW LO G_μ (green) requires 0.6% corrections at Z-pole and below , 1.7% above

σ/σ_{ref}	m_{ee} = 70-80 GeV	m_{ee} = 80-100 GeV	m_{ee} = 100-125 GeV
EW LO $\alpha(0)$ scheme	0.9641	1.0052	1.0019
EW LO G_μ scheme	0.9942	1.0061	1.0173
EW LO $\alpha(M_Z)$ scheme	0.9934	0.9891	0.9930
LEP with improv. norm.	1.0007	0.9990	1.0017

A_{fb} in different EW schemes

Reference: best predictions with EW O(α) loop+boxes corrections.



„LEP with improved norm." (red) predicts remarkably well shape and normalisation

EW LO G_μ (green) requires large corrections, above 50% around Z- pole!

$A_{fb} - A_{fb,ref}$	m_{ee} = 70-80 GeV A_{fb} = -0.1990	m_{ee} = 80-100 GeV A_{fb} = 0.0262	m_{ee} = 100-125 GeV A_{fb} = 0.1928
EW LO $\alpha(0)$ scheme	0.0175	0.0146	0.0033
EW LO G_μ scheme	0.0096	0.0143	0.0102
EW LO $\alpha(M_Z)$ scheme	0.0015	0.0002	0.0016
LEP with improv. norm.	-0.0009	0.0001	0.0010

EW α(0) scheme

Tree level inputs and predictions

EW scheme input	EW LO predictions
$M_Z = 91.1876$ GeV $\alpha = 1/137.0359895$ $G_\mu = 1.166389 \cdot 10^{-5}$ GeV^{-2}	$M_W = 80.93875$ GeV $s_W^2 = 0.2121538$
$M_Z = 91.1876$ GeV $\alpha = 1/128.8667$ $s_W^2 = 0.23152$	$M_W = 79.93777$ GeV $G_\mu = 1.165224 \cdot 10^{-5}$ GeV^{-2}
$M_Z = 91.1876$ GeV $\alpha = 1/127.934$ $s_W^2 = 0.23113$	$M_W = 79.958$ GeV $G_\mu = 1.175057 \cdot 10^{-5}$ GeV^{-2}

Often used for configuring MC generators at LHC → (last row)

Input for DIZET (beyond tree level)

1) Starts with tree parameters at best measured value : G_μ, $\alpha(0)$, M_Z
2) Calculate m_W using iterative formula including higher-order corrections

Parameter	Value
M_Z	91.1876 GeV
m_h	125.0 GeV
m_t	173.0 GeV
m_b	4.7 GeV
$1/\alpha(0)$	137.0359895(61)
G_μ	$1.166389(22) \cdot 10^{-5}$ GeV^{-2}
M_W	80.353 GeV
s_W^2	0.22351946