

Parton distributions, nonperturbative functions, quark-mass effects in W mass measurements

Pavel Nadolsky

Southern Methodist University

Based on the work with

C. Brock, A. Belyaev, S. Berge, S. Doyle, M. Guzzi, N. Kidonakis, A. Konychev, J. Gao, T. J. Hobbs, T.-J. Hou, G. Ladinsky, F. Olness, B. T. Wang, B. W. Wang, C.-P. Yuan

CTEQ-TEA group / ResBos developers



1. Parton distributions in TMD factorization for W, Z boson production

- ...introduce a systematic uncertainty of order $\delta M_W \gtrsim 10$ MeV
- Two kinds:
 - a. $f_a(x, \mu)$ -- collinear PDFs [e.g., CT14 NNLO]
 - b. $\mathcal{P}_a(x, \vec{k}_T)$ and $\bar{\mathcal{P}}_a(x, \vec{b})$ -- transverse-momentum-dependent (TMD) or transverse-position-dependent PDFs
- $\bar{\mathcal{P}}_a(x, \vec{b})$ is related to $f_a(x, \mu)$ at $b^2 \ll 1/\Lambda^2$, where $\Lambda \sim 1$ GeV

$$\bar{\mathcal{P}}_a(x, \vec{b}) = \int_x^1 \frac{d\xi}{\xi} C_{a/a'} \left(\frac{x}{\xi}, \frac{b\mu_b}{2e^{-\gamma_E}}; \alpha_s(\mu_b) \right) f_{a'} \left(x, \mu_b \sim \frac{1}{b} \right) + O(\Lambda^2 b^2)$$

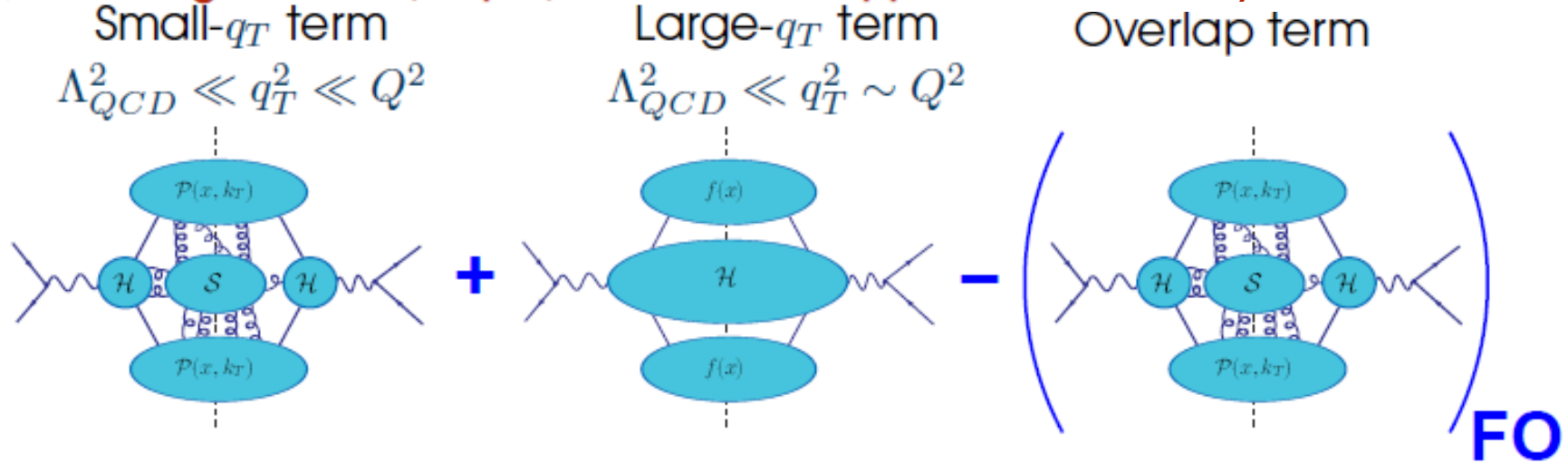
The power-suppressed terms of order $\Lambda^{2p} b^{2p}$ must be constrained by data just as the collinear PDFs $f_a(x, \mu)$

⇒ global analyses of q_T distributions (in the future, PDFs+ q_T distributions)

⇒ **constraining $f_a(x, \mu)$ and $\bar{\mathcal{P}}_a(x, \vec{b})$ at NNLO is a large coupled problem!**

QCD factorization as a function of q_T

(according to Collins, Soper, and Sterman approach in ResBos)



■ k_T -dependent PDFs

$\mathcal{P}_a(x, \vec{k}_T)$ ✨ ✨

■ Sudakov function

$\mathcal{S}(x, \vec{k}_T)$ ✨ ✨

▷ actually, their impact parameter (b) space transforms

$\mu_Q \sim Q, \mu_b \sim 1/b$

■ Collinear PDFs

$f_a(x, \mu)$ ✨

■ hard matrix elements

\mathcal{H} of order N

$\mu \sim Q \sim q_T$

■ Truncated perturbative expansion

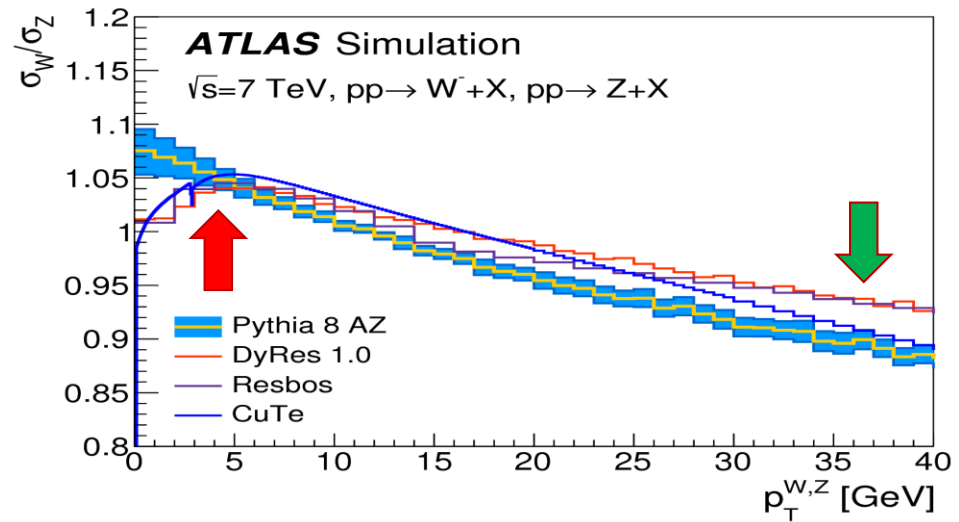
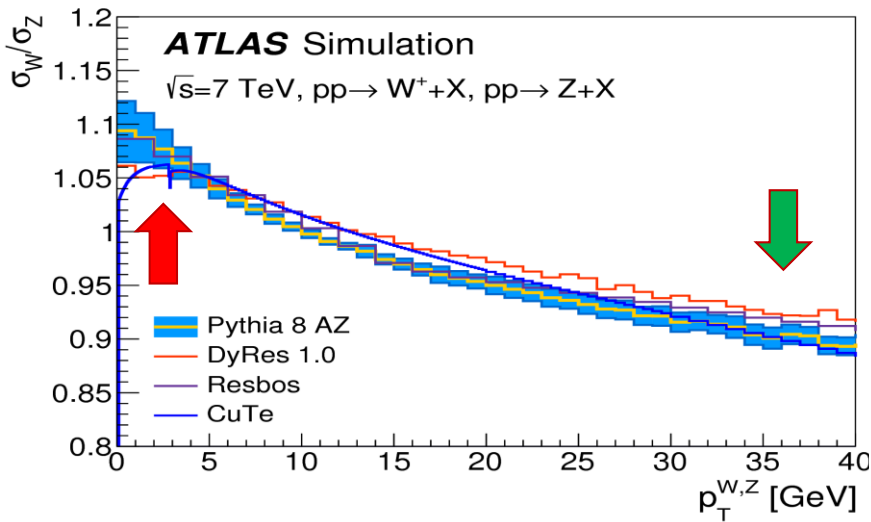
$$\sum_{k=0}^N \alpha_s^k \sum_{m=0}^{2k-1} c_{km} \ln^m \left(\frac{q_T^2}{Q^2} \right)$$

[also depends on $f_a(x, \mu)$ ✨]

✨ long-distance
 ✨ depends on $f_a(x, \mu)$

✨ flavor-independent

Example: $\sigma(W^+/Z^0)$ & $\sigma(W^-/Z^0)$ vs. $p_T^{W,Z}$



Various flavor combinations of $f_a(x, \mu)$ and $\bar{P}_a(x, \vec{b})$ enter in a range of QCD scales from 1 GeV to $\gtrsim M_{W,Z}$; do not cancel in some xsec ratios!

Parton luminosities as percentages of the total cross section:

W^+ : $u\bar{d}$ (~70%), $c\bar{s}$ (20%), gq , ...

Z^0 : $u\bar{u}$ (30%), $d\bar{d}$ (30%), $s\bar{s}$ (15%),

W^- : $d\bar{u}$ (60%), $s\bar{c}$ (25%), gq , ...

$c\bar{c}$ (8%), $b\bar{b}$ (5%)....

q_{sea} and \bar{q}_{sea} dominate the PDF uncertainties; impossible to **guess** which ones

Quark mass effects in c , b channels; included at NLO in ResBos 1

Statistical methods to identify the PDF dependence

1. PDF-driven correlations

[CTEQ6.6, arXiv:0802.0007]

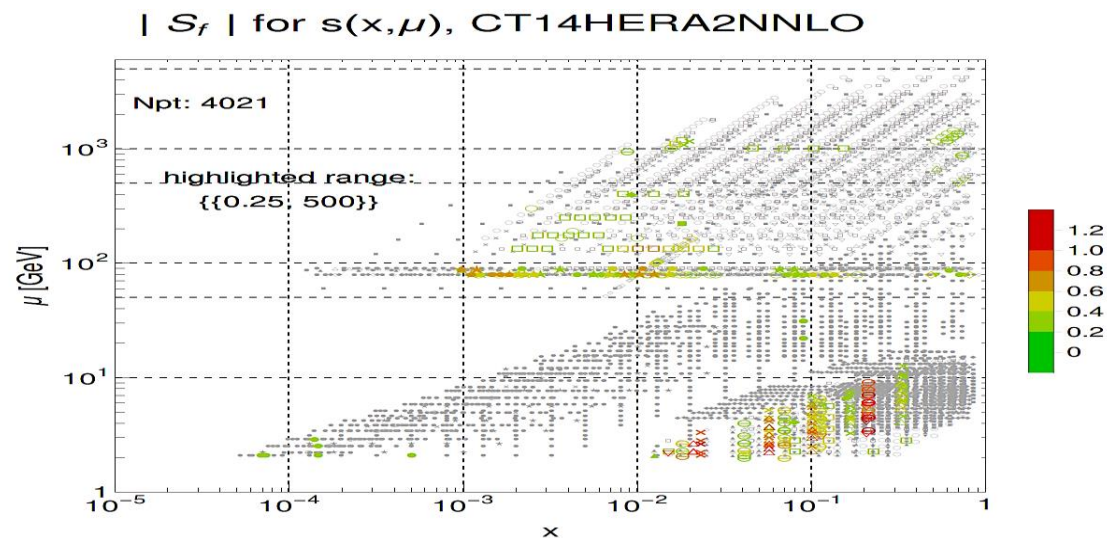
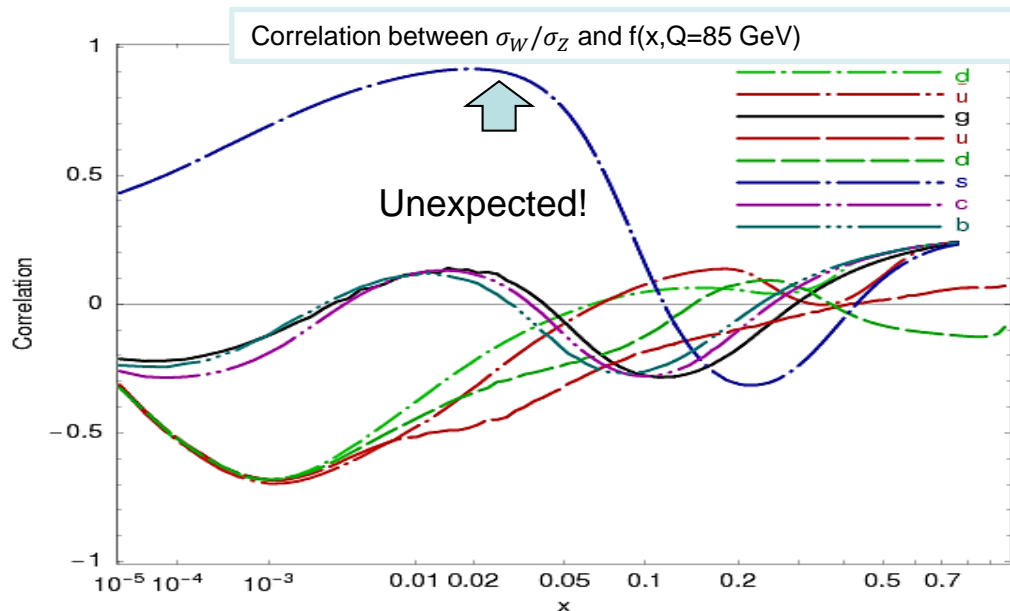
$\cos \varphi > 0.7$ shows that the ratio σ_W/σ_Z at the LHC must be sensitive to the strange PDF $s(x, Q)$

Useful, but incomplete information!

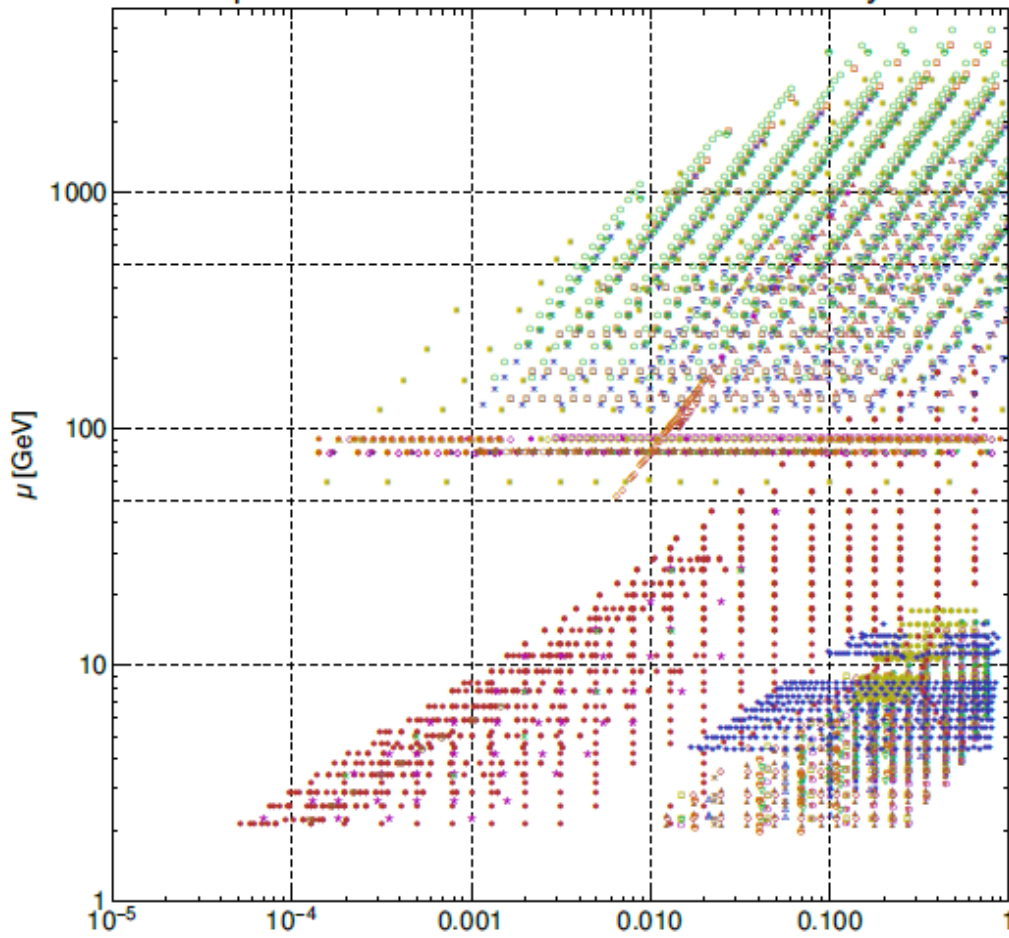
2. Program PDFSense,

B.W. Wang, T.J. Hobbs, et al., :
arXiv:1803.02777

Quickly identifies **sensitivity** S_f of experimental data to any PDF-dependent quantity:
 $\sigma(W)/\sigma(Z)$, $\sigma_{ResBos}/\sigma_{DYRES}$, etc.



Experimental data in CTEQ-TEA PDF analysis



Experiments

| In CT14HERA2 | | | | New | |
|--------------|-------|-------|-------|-------|-------|
| ● 160 | △ 124 | ● 201 | ◇ 261 | ● 250 | ■ 565 |
| ■ 101 | ▽ 125 | ■ 203 | ○ 266 | ◇ 245 | ◆ 566 |
| ◆ 102 | × 126 | ◆ 204 | ★ 267 | △ 246 | ▲ 567 |
| ▲ 104 | ⊖ 127 | ▲ 225 | ● 268 | ▽ 247 | ▼ 568 |
| ▼ 108 | × 145 | ▼ 227 | ▽ 281 | × 542 | ○ 545 |
| ○ 109 | * 147 | ○ 234 | △ 504 | ⊖ 544 | □ 252 |
| □ 110 | ⊖ 169 | ◆ 240 | ▽ 514 | ○ 249 | ◇ 253 |
| ◇ 111 | ▲ 241 | ■ 535 | □ 538 | | |
| | □ 260 | | | | |

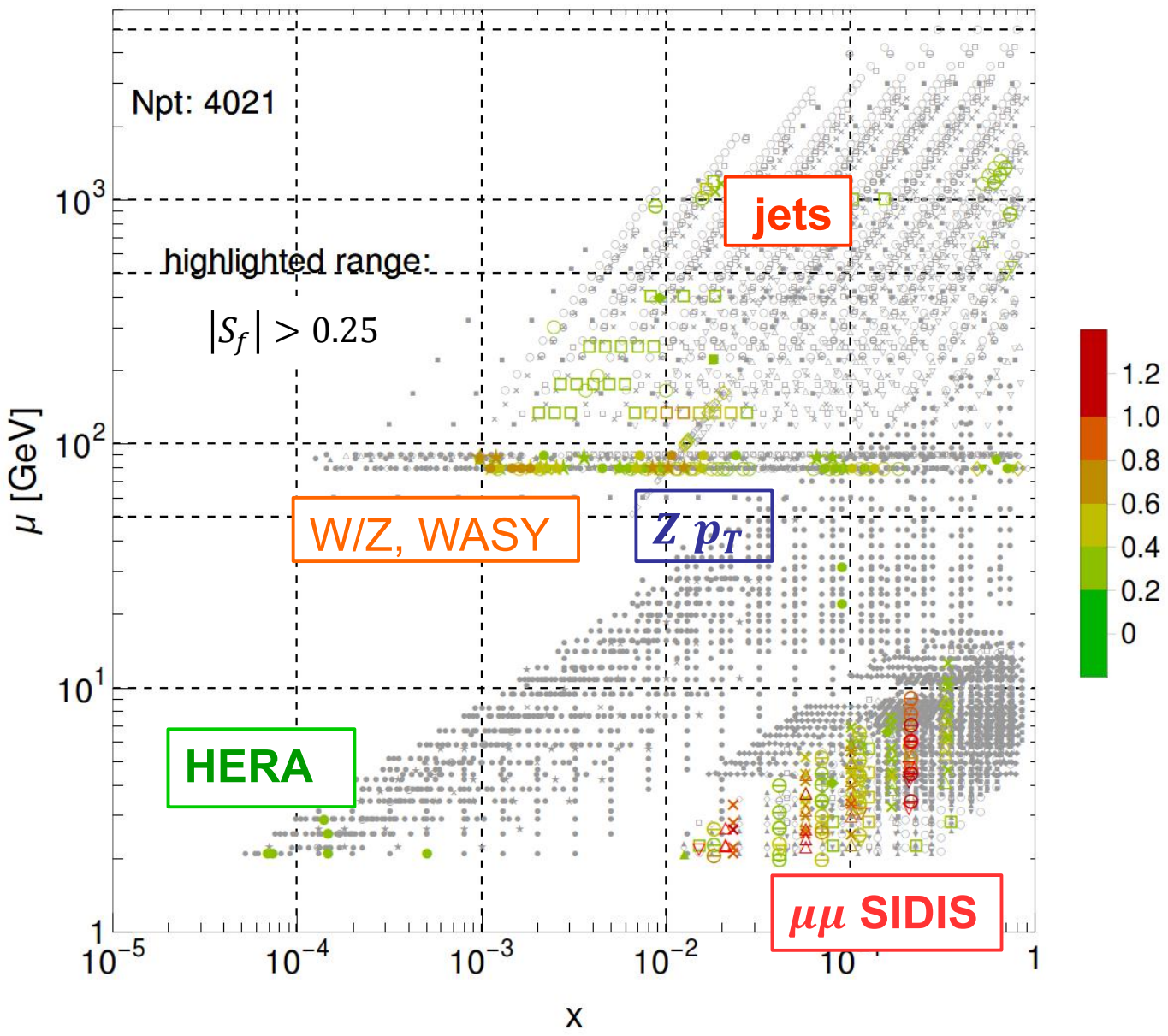
Experiments in the CT14 HERA2 NNLO fit ($N_{pt} = 3258$)

New experiments considered for the fit (ATLAS, CMS, LHCb; $N_{pt} = 734$)

| ID# | Experimental dataset | N_{pt} |
|-----|--|----------|
| 245 | LHCb 7 TeV Z/W muon forward- η Xsec (1.0 fb^{-1}) | [71] 33 |
| 246 | LHCb 8 TeV Z electron forward- η $d\sigma/dy_z$ (2.0 fb^{-1}) | [72] 17 |
| 247 | ATLAS 7 TeV $d\sigma/dp_T^2$ (4.7 fb^{-1}) | [73] 8 |
| 249 | CMS 8 TeV W muon, Xsec, $A_\mu(\eta^\mu)$ (18.8 fb^{-1}) | [74] 33 |
| 250 | LHCb 8 TeV W/Z muon, Xsec, $A_\mu(\eta^\mu)$ (2.0 fb^{-1}) | [75] 42 |
| 252 | ATLAS 8 TeV Z ($d^2\sigma/d y dm_{ll}$) (20.3 fb^{-1}) | [76] 48 |
| 253 | ATLAS 8 TeV ($d^2\sigma/dp_T^2 dm_{ll}$) (20.3 fb^{-1}) | [77] 45 |

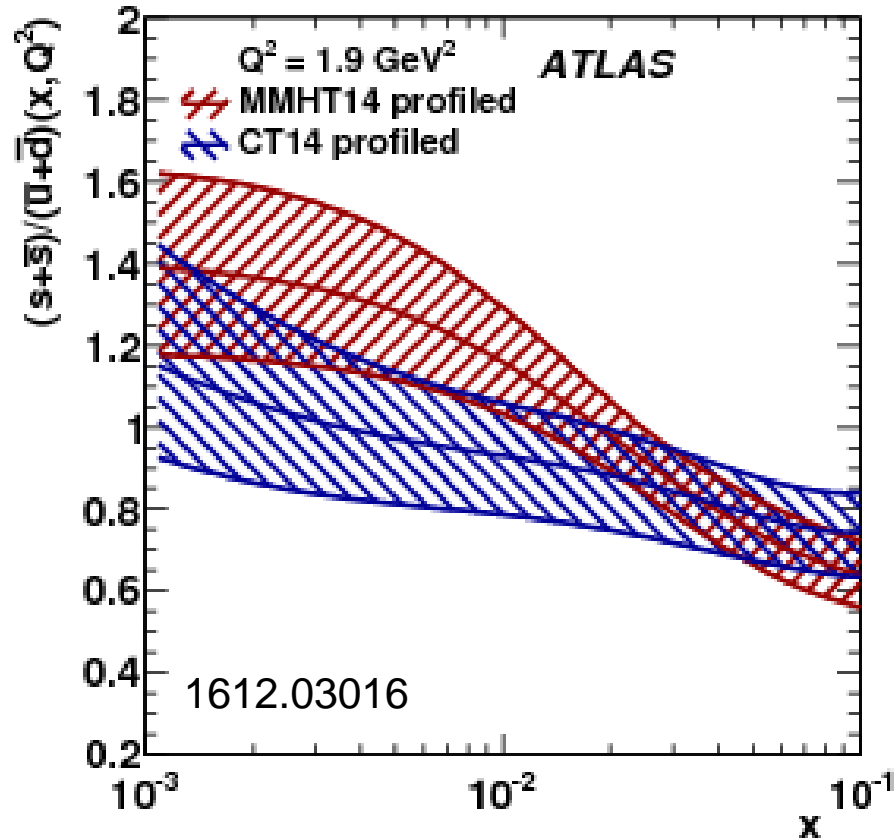
| | | | |
|-----|--|------|----|
| 260 | DØ Run-2 Z $d\sigma/dy_z$ (0.4 fb^{-1}) | [63] | 28 |
| 261 | CDF Run-2 Z $d\sigma/dy_z$ (2.1 fb^{-1}) | [64] | 29 |
| 266 | CMS 7 TeV $A_\mu(\eta)$ (4.7 fb^{-1}) | [65] | 11 |
| 267 | CMS 7 TeV $A_e(\eta)$ (0.840 fb^{-1}) | [66] | 11 |
| 268 | ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb^{-1}) | [67] | 41 |

$|S_f|$ for $s(x, \mu)$, CT14HERA2NNLO

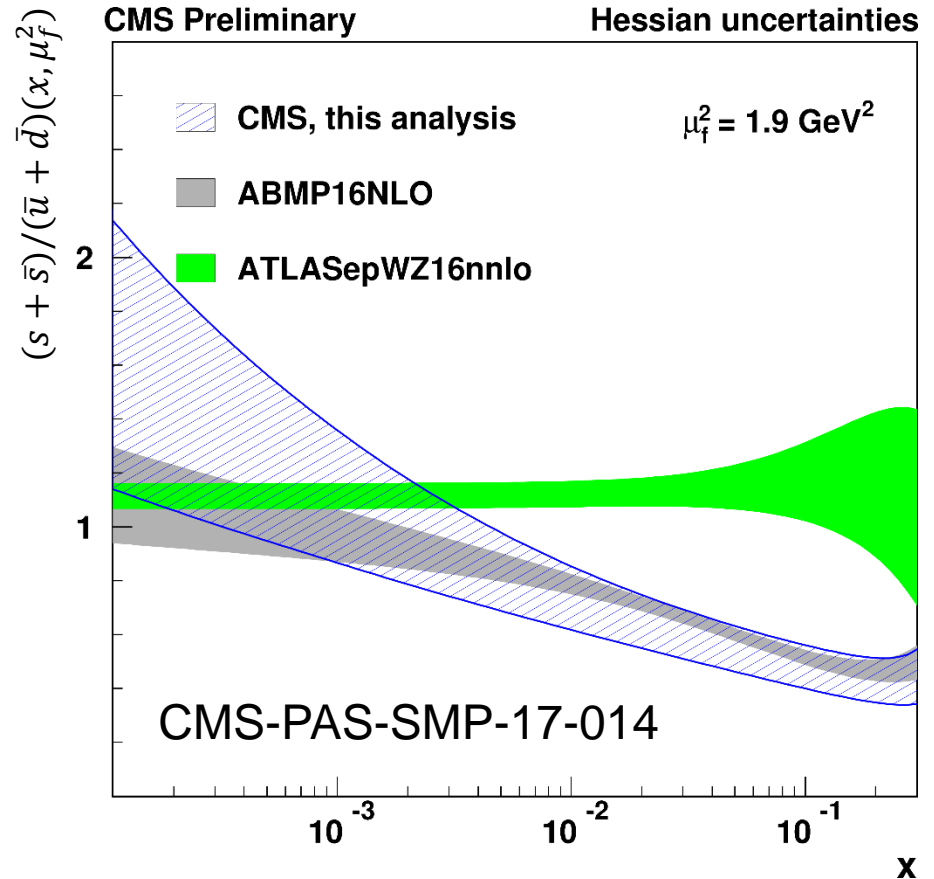


- Points with a large S_f have a stronger sensitivity to $s(x_i, \mu_i)$
- Constraints on $s(x, \mu)$ are weaker than on the other flavors
- NuTeV, CCFR dimuon SIDIS, HERA DIS most sensitive
- Combined $|S_f|$ of CMS7+8 jet data comparable to $|S_f|$ of one of NuTeV, CCFR data sets
- W ASYmmetry, σ_W , σ_Z are weakly sensitive

But, wait, LHC experiments do not agree on strangeness



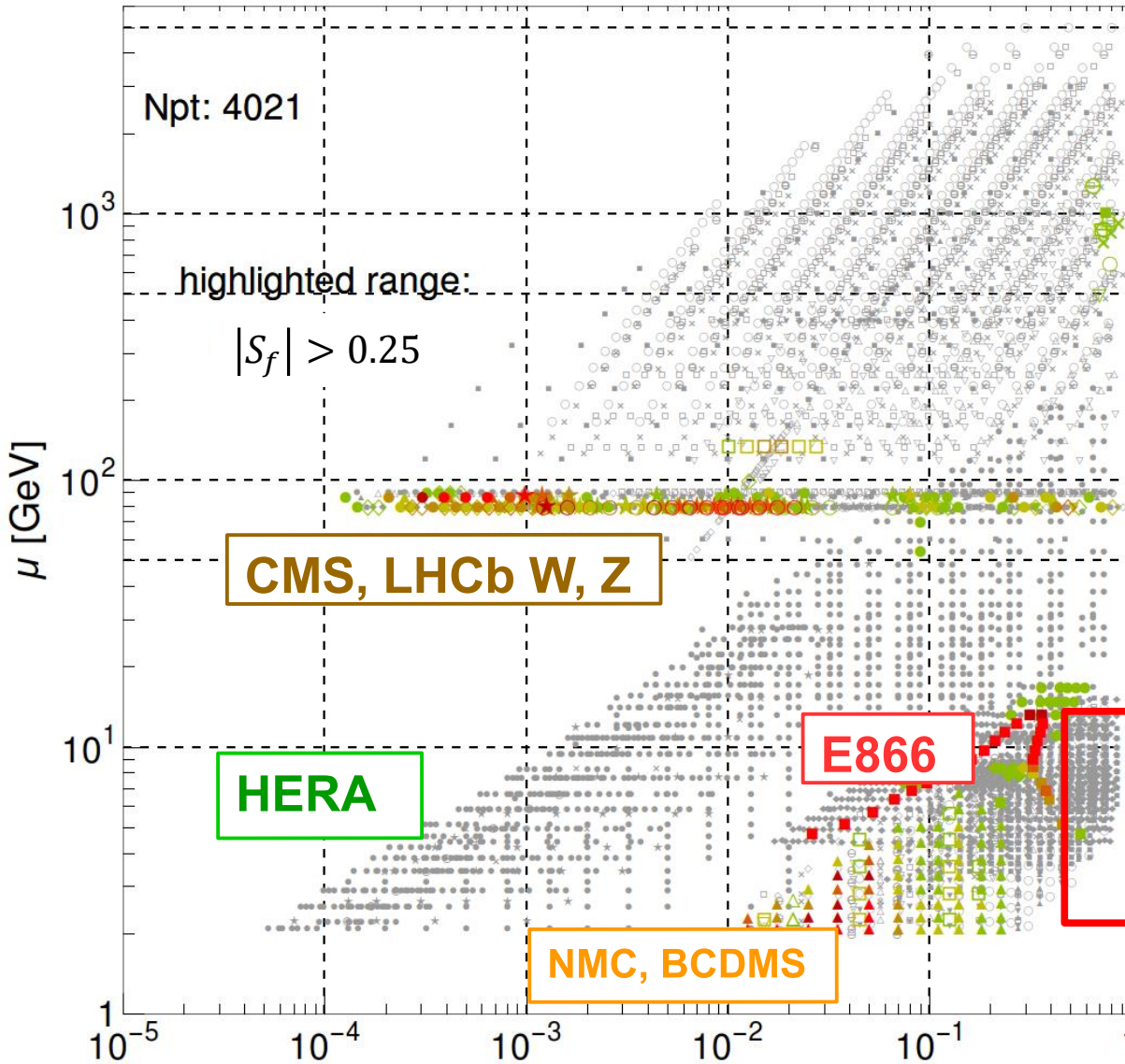
CT14 and MMHT14 NNLO PDFs profiled using ATLAS 7 TeV (4.6 fb^{-1}) W^\pm, Z xsecs prefer $s(x, 1.9 \text{ GeV}) \sim 1$ @ $x = 0.023$



CMS W,Z, CMS W+c (13 TeV) prefer smaller $s(x, \mu)$ than ATLAS for $x \gtrsim 10^{-3}$

$|S_f|$ for $\bar{d}/\bar{u}(x, \mu)$, CT14HERA2NNLO

HERA DIS:
large $N_{pt} \Rightarrow$
large total $|S_f|$



- the large E866 pd/pp sensitivity *degrades* at larger x
- this is a prime motivation for higher x DY measurements at **E906** (SeaQuest)

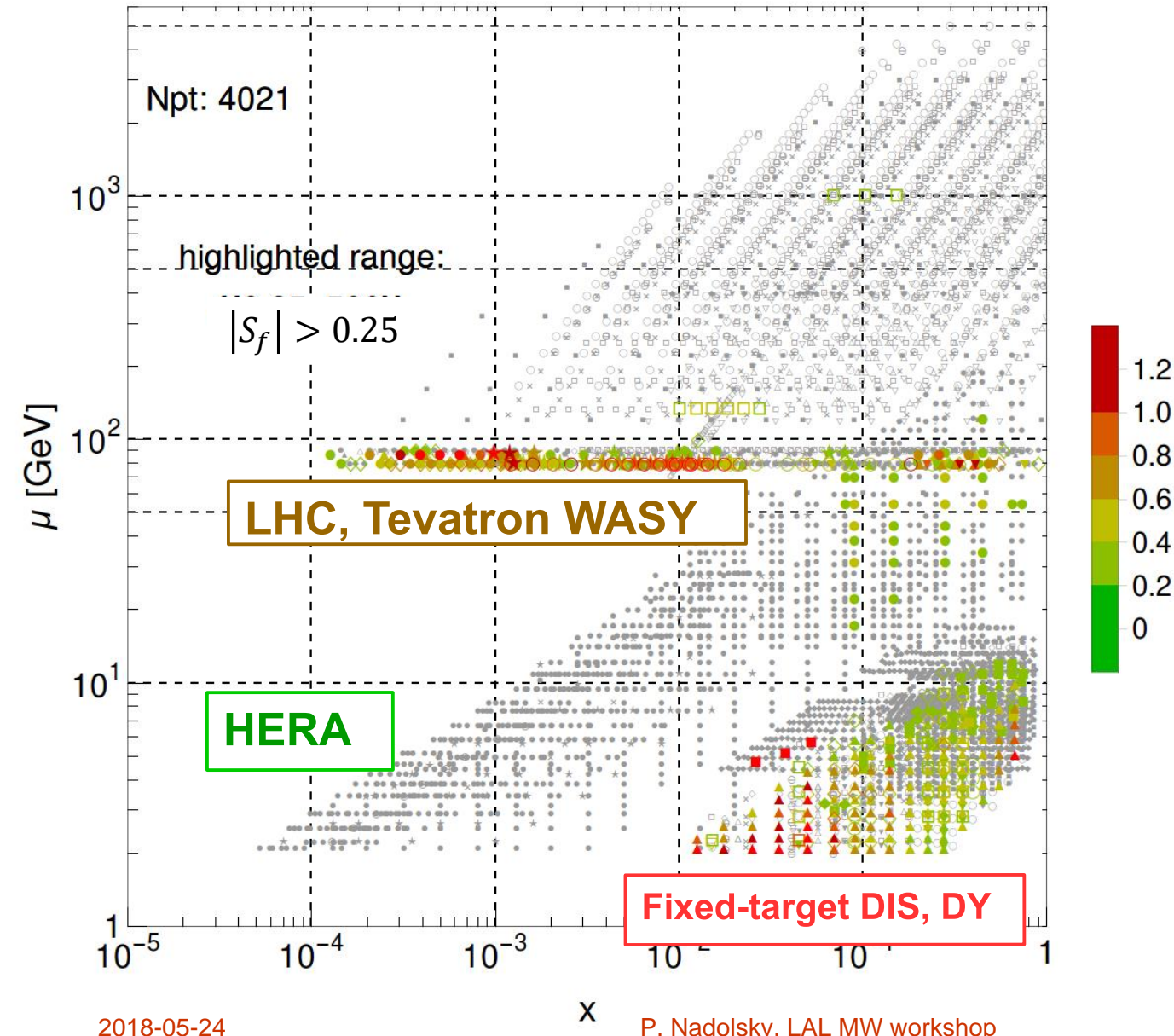
- Inclusive jet production has potential to constrain \bar{d}/\bar{u} in the near future

$|S_f|$ fo $d/u(x, \mu)$, CT14HERA2NNLO

HERA, fixed-target DIS still most sensitive!

Individually, LHC W , Z experiments provide the essential reach to d/u and \bar{d}/\bar{u} at $x \sim 10^{-2}$

In the fit, they do not tangibly reduce the PDF error because they don't quite agree



Questions to address

1. Which experiments constrain the PDFs in the M_W , $\sin \theta_w$ measurements?
2. What needs to be done to reduce the PDF uncertainty on M_W ? To phase out the fixed-target DIS/DY experiments?

[These questions can be studied as a part of the M_W analysis using the PDFSense tool, without relying on PDF reweighting or profiling]

3. Benchmarking exercise for NNLO QCD + NLO EW W, Z rapidity distributions and asymmetries

Similar in spirit to the PDF4LHC benchmarking exercise that reduced $\delta_{PDF}\sigma(H_0)$ from 7% to 3% [arXiv:1510.03865]

All NNLO/resummation/PDF fitting codes must agree on benchmark **inclusive** W, Z cross sections. [Often, they don't.] Check for numerical issues. PDF fits are often done with fast NNLO QCD interfaces for “bare Born” lepton production.

Fitted W, Z experiments must agree with one another, or we cannot reduce the CT14 PDF uncertainty. [They don't.]

2. Nonperturbative parameters in the TMD factorization

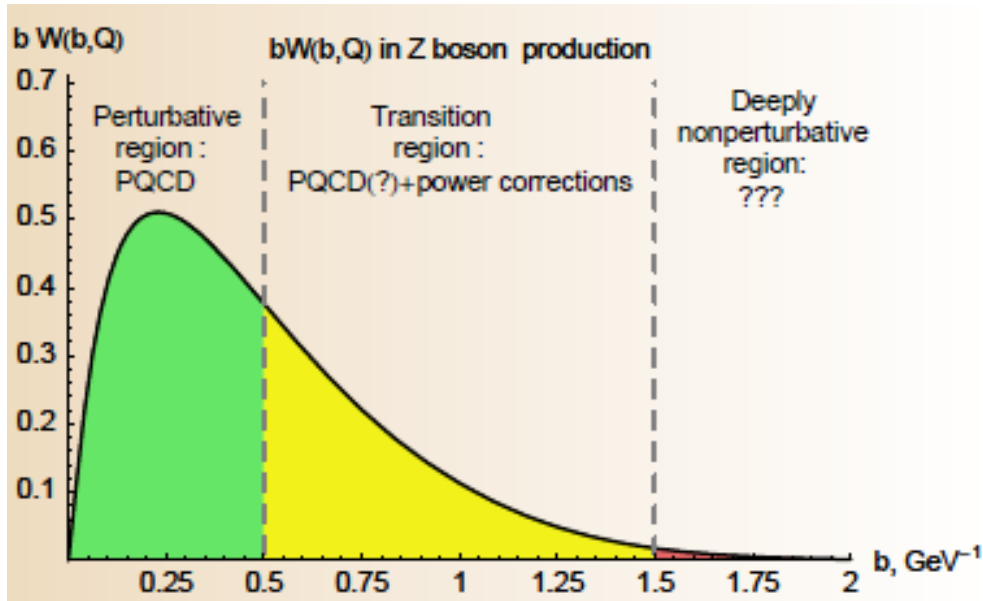
Arise at $b > 0.5 \text{ GeV}^{-1}$ from...

...the soft (Sudakov) function $S(b, Q)$

flavor-independent, x -independent; linear in $\ln(Q/Q_{ref})$; dominate at $Q = M_{W,Z}$; shift M_W by $\times 100 \text{ MeV}$

...TMD PDFs $\bar{\mathcal{P}}_a(x, \vec{b})$

depend on the flavor & x , not on Q ; marginal, poorly known; shift M_W by $\times 1 \text{ MeV}$



Universal in e^+e^- , SIDIS, DY & compatible with \overline{MS} PDFs in the CSS framework [Collins, Metz, 2004];

not automatically universal in all resummation/event generator frameworks

A Gaussian nonperturbative function, example

$$F_{NP}(b, Q) = b^2 a(Q, x_A, x_B)$$

Adequate in DY processes for $Q > 5$ GeV

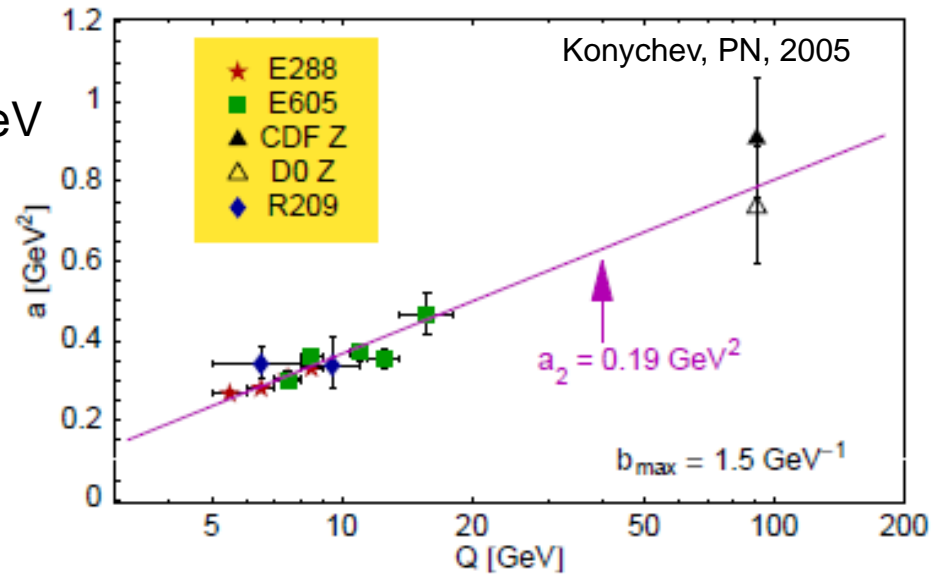
Guzzi, PN, Wang [arXiv: 1309.1393]:

$$Q_{ref} = M_Z, b_{max} = 1.5 \text{ GeV}^{-1};$$

NNLL+ $O(\alpha_s^2)$ (ResBos 1)

Fitted to Tevatron Run-2 $\frac{d\sigma(Z)}{dyd\varphi_\eta^*}$ using

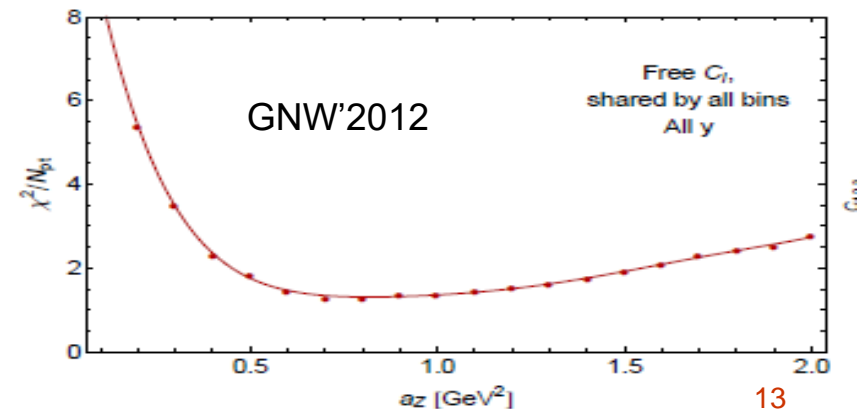
fixed or free scales $C_{1,2,3}$



$$a(Q, \sqrt{s}) = a_Z(1.96 \text{ TeV}) + a_2 \ln\left(\frac{Q}{M_Z}\right) + a_3 \ln\left(\frac{Q^2}{M_Z^2} \frac{s}{(1.96 \text{ TeV})^2}\right).$$

$$a_Z = 0.79_{-0.03}^{+0.2}, a_2 = 0.17 \pm 0.03, \\ a_3 = -0.03 \pm 0.02 \text{ (in GeV}^2\text{)}$$

For $Q \approx M_{W,Z}$: $a(Q, \sqrt{s}) \approx a_Z$ --
a 1-parameter fit with the
scales $C_{1,2,3}$ as nuisance
parameters



Some alternative nonperturbative functions

- BLNY form with small- x broadening in ResBos

[Berge, PN, Olness, Yuan, hep-ph/0410375]

$$F_{NP}(b, Q, x_A, x_B) = b^2 \left[0.21 + 0.68 \ln \left(\frac{Q}{3.2 \text{ GeV}} \right) - 0.126 \ln \frac{x_A x_B}{0.1^2} + g_3 \left(\frac{1}{x_A} + \frac{1}{x_B} \right) \right]$$

- Joint form for DY+SIDIS

[Sun, Isaacson, Yuan, Yuan, 1406.0373]

$$F_{NP}(b, Q, x_A, x_B) = b^2 \left[0.212 + 0.84 \ln \left(\frac{Q}{1.55 \text{ GeV}} \right) + g_3 \left(\left(\frac{0.01}{x_A} \right)^{0.2} + \left(\frac{0.01}{x_B} \right)^{0.2} \right) \right]$$

Current Drell-Yan data are compatible with $g_3 = 0$

- The Gaussian approximation fails at $Q < 5 \text{ GeV}$. A more complete parametrization is discussed, e.g., in J. Collins, T. Rogers, 1412.3820
- A variety of other forms were proposed, hard to discriminate by data

3. TMD PDFs with quark mass dependence

(PN, Kidonakis, Olness, Yuan, : hep-ph/0210082; Berge, PN, Olness, hep-ph/0509023;
Recent work in SCET at NNLL-NNLO by Pietrulewicz et al., 1703.09702)

In ResBos 1, finite-mass effects are included in $\bar{\mathcal{P}}_a(x, \vec{b})$ for $a = c, b$ at NLO in the S-ACOT- χ mass scheme – the scheme used to determined CT14 PDFs

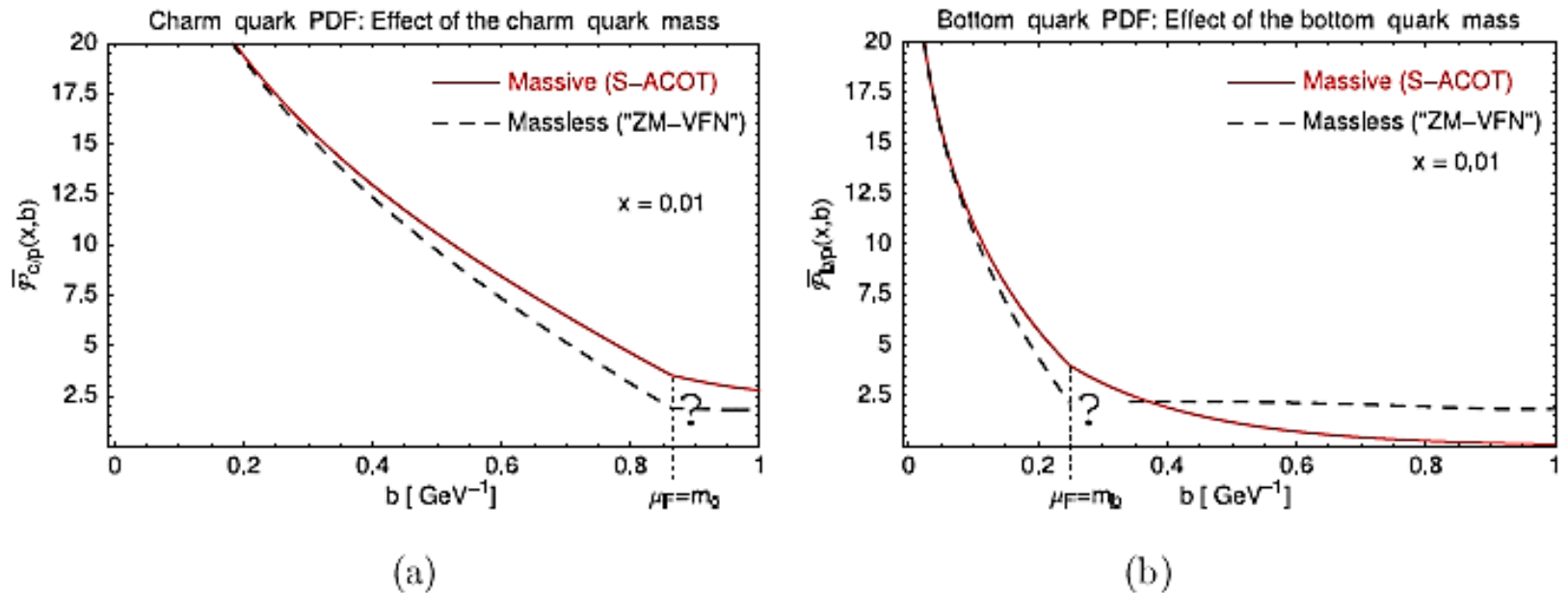
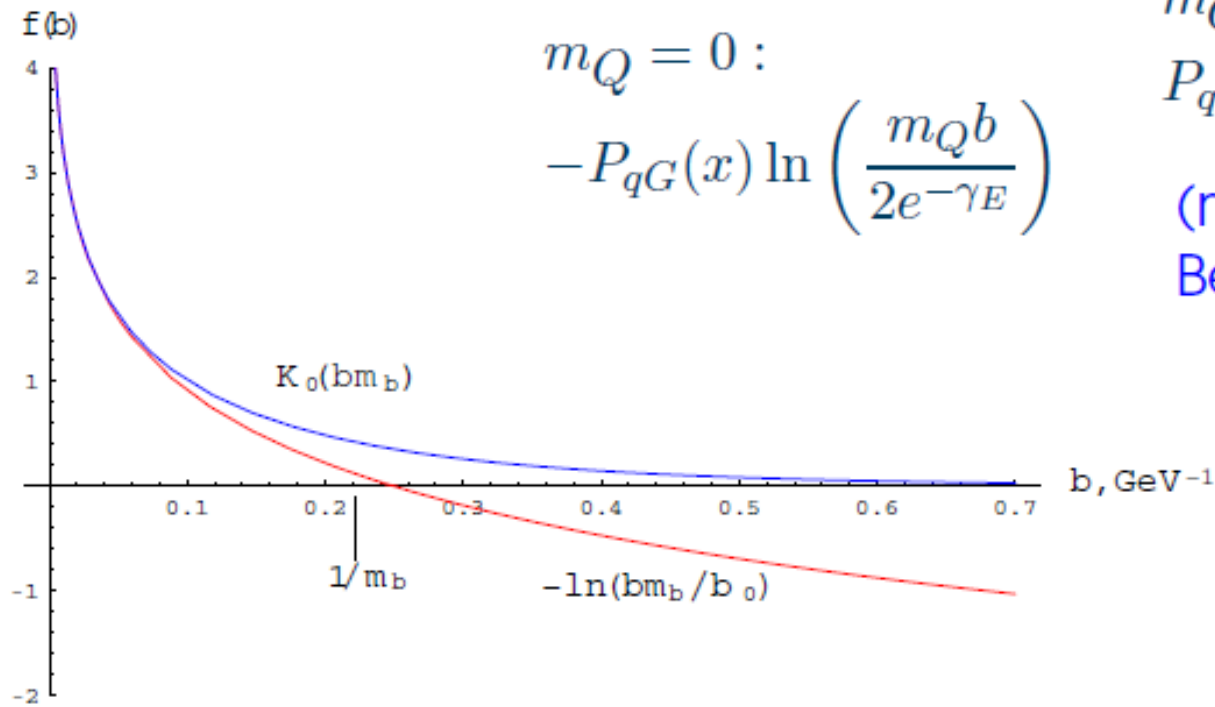


Figure 1: The b -dependent parton densities $\bar{\mathcal{P}}_{Q/A}(x, b, m_Q)$ vs. the impact parameter b for (a) charm quarks and (b) bottom quarks. The solid and dashed curves correspond to the S-ACOT and massless ("ZM-VFN") factorization schemes, respectively.

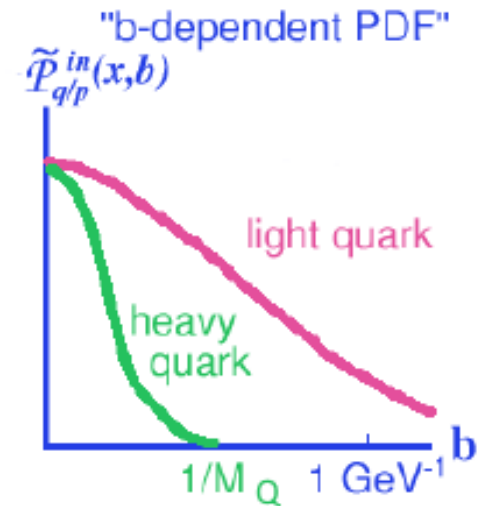
m_Q suppresses contributions from $1/b \lesssim m_Q$



$$m_Q = 0 : \\ -P_{qG}(x) \ln \left(\frac{m_Q b}{2e^{-\gamma_E}} \right)$$

$m_Q \neq 0 :$
 $P_{qG}(x) K_0(m_Q b)$
 (modified Bessel function)

For $m_Q^2 \gg \Lambda_{QCD}^2$, the resummed cross section can be calculated without the nonperturbative input from $b \sim \Lambda_{QCD}$!



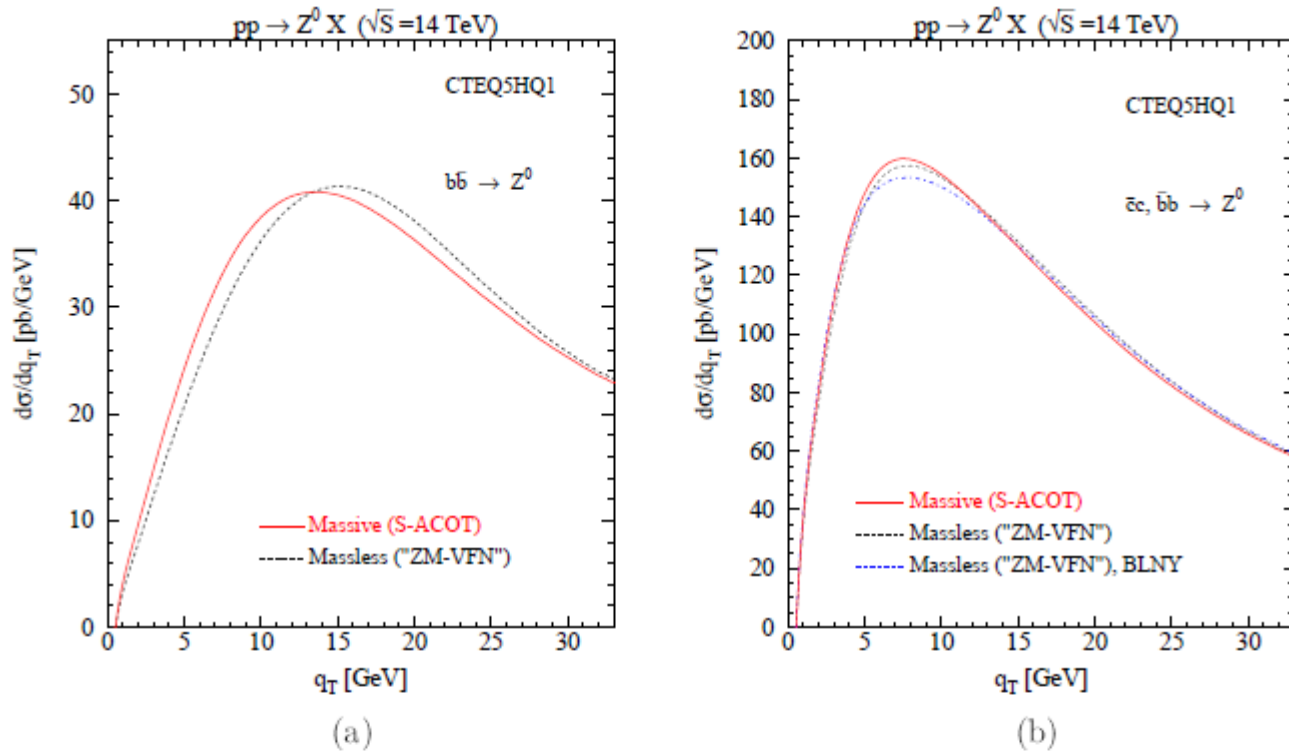


Figure 3: $d\sigma/dq_T$ for $c\bar{c}, b\bar{b} \rightarrow Z^0$ boson production at the LHC: (a) $b\bar{b}$ channel only, (b) combined $c\bar{c}$ and $b\bar{b}$ channels. The solid (red) curve shows the distribution in the massive (S-ACOT) scheme. The dashed (black) curve shows the distribution in the massless ("ZM-VFN") scheme, computed using the parametrization (11) of the nonperturbative function $\mathcal{F}_{NP}(b, Q)$. The dot-dashed (blue) line was calculated in the "ZM-VFN" scheme using an alternative parameterization [48] of the nonperturbative function $\mathcal{F}_{NP}(b, Q)$.

Total $\Delta M_W \sim 10$ MeV [~ 0 MeV] for $d\sigma/dp_T^e$ [$d\sigma/dM_T^{e\nu}$] at 14 TeV due to $m_{c,b} \neq 0$

Extra details

Experiments in the CT14 HERA2 fit

| ID# | Experimental dataset | N_d |
|-----|---|-------|
| 101 | BCDMS F_2^p [47] | 337 |
| 102 | BCDMS F_2^d [48] | 250 |
| 104 | NMC F_2^d/F_2^p [49] | 123 |
| 108 | CDHSW F_2^p [50] | 85 |
| 109 | CDHSW F_3^p [50] | 96 |
| 110 | CCFR F_2^p [51] | 69 |
| 111 | CCFR $x F_3^p$ [52] | 86 |
| 124 | NuTeV $\nu\mu\mu$ SIDIS [40] | 38 |
| 125 | NuTeV $\bar{\nu}\mu\mu$ SIDIS [40] | 33 |
| 126 | CCFR $\nu\mu\mu$ SIDIS [41] | 40 |
| 127 | CCFR $\bar{\nu}\mu\mu$ SIDIS [41] | 38 |
| 145 | H1 σ_r^b (57.4 pb ⁻¹) [53][54] | 10 |
| 147 | Combined HERA charm production (1.504 fb ⁻¹) [39] | 47 |
| 160 | HERA1+2 Combined NC and CC DIS (1 fb ⁻¹) [6] | 1120 |
| 169 | H1 F_L (121.6 pb ⁻¹) [55] | 9 |

| ID# | Experimental dataset | N_d |
|-----|---|-------|
| 201 | E605 DY [56] | 119 |
| 203 | E866 DY, $\sigma_{pd}/(2\sigma_{pp})$ [57] | 15 |
| 204 | E866 DY, $Q^3 d^2\sigma_{pp}/(dQdx_F)$ [58] | 184 |
| 225 | CDF Run-1 $A_e(\eta^e)$ (110 pb ⁻¹) [59] | 11 |
| 227 | CDF Run-2 $A_e(\eta^e)$ (170 pb ⁻¹) [60] | 11 |
| 234 | DØ Run-2 $A_\mu(\eta^\mu)$ (0.3 fb ⁻¹) [61] | 9 |
| 240 | LHCb 7 TeV W/Z muon forward- η Xsec (35 pb ⁻¹) [62] | 14 |
| 241 | LHCb 7 TeV W $A_\mu(\eta^\mu)$ (35 pb ⁻¹) [62] | 5 |
| 260 | DØ Run-2 Z $d\sigma/dy_Z$ (0.4 fb ⁻¹) [63] | 28 |
| 266 | CMS 7 TeV $A_\mu(\eta)$ (4.7 fb ⁻¹) [64] | 11 |
| 267 | CMS 7 TeV $A_e(\eta)$ (0.840 fb ⁻¹) [65] | 11 |
| 268 | ATLAS 7 TeV W/Z Xsec, $A_\mu(\eta)$ (35 pb ⁻¹) [66] | 41 |
| 281 | DØ Run-2 $A_e(\eta)$ (9.7 fb ⁻¹) [67] | 13 |
| 504 | CDF Run-2 incl. jet ($d^2\sigma/dp_T^j dy_j$) (1.13 fb ⁻¹) [36] | 72 |
| 514 | DØ Run-2 incl. jet ($d^2\sigma/dp_T^j dy_j$) (0.7 fb ⁻¹) [37] | 110 |
| 535 | ATLAS 7 TeV incl. jet ($d^2\sigma/dp_T^j dy_j$) (35 pb ⁻¹) [68] | 90 |
| 538 | CMS 7 TeV incl. jet ($d^2\sigma/dp_T^j dy_j$) (5 fb ⁻¹) [69] | 133 |

Candidate experiments in the CTEQ-TEA fit

| ID# | Experimental dataset | N_d |
|-----|--|-------|
| 245 | LHCb 7 TeV Z/W muon forward- η Xsec (1.0 fb ⁻¹) [70] | 33 |
| 246 | LHCb 8 TeV Z electron forward- η $d\sigma/dy_Z$ (2.0 fb ⁻¹) [71] | 17 |
| 247 | ATLAS 7 TeV $d\sigma/dp_T^Z$ (4.7 fb ⁻¹) [72] | 8 |
| 249 | CMS 8 TeV W muon, Xsec, $A_\mu(\eta^\mu)$ (18.8 fb ⁻¹) [73] | 33 |
| 250 | LHCb 8 TeV W/Z muon, Xsec, $A_\mu(\eta^\mu)$ (2.0 fb ⁻¹) [74] | 42 |
| 252 | ATLAS 8 TeV Z ($d^2\sigma/d y _Z dm_{ll}$) (20.3 fb ⁻¹) [75] | 48 |
| 253 | ATLAS 8 TeV ($d^2\sigma/dp_T^Z dm_{ll}$) (20.3 fb ⁻¹) [76] | 45 |
| 542 | CMS 7 TeV incl. jet, R=0.7, ($d^2\sigma/dp_T^j dy_j$) (5 fb ⁻¹) [34] | 158 |
| 544 | ATLAS 7 TeV incl. jet, R=0.6, ($d^2\sigma/dp_T^j dy_j$) (4.5 fb ⁻¹) [33] | 140 |
| 545 | CMS 8 TeV incl. jet, R=0.7, ($d^2\sigma/dp_T^j dy_j$) (19.7 fb ⁻¹) [35] | 185 |
| 565 | ATLAS 8 TeV $t\bar{t}$ $d\sigma/dp_T^t$ (20.3 fb ⁻¹) [38] | 8 |
| 566 | ATLAS 8 TeV $t\bar{t}$ $d\sigma/dy_{<t\bar{t}>}$ (20.3 fb ⁻¹) [38] | 5 |
| 567 | ATLAS 8 TeV $t\bar{t}$ $d\sigma/dm_{t\bar{t}}$ (20.3 fb ⁻¹) [38] | 7 |
| 568 | ATLAS 8 TeV $t\bar{t}$ $d\sigma/dy_{t\bar{t}}$ (20.3 fb ⁻¹) [38] | 5 |

N_d is the number of data points

Q_T distribution for $AB \rightarrow VX$

$$\frac{d\sigma_{AB \rightarrow VX}}{dQ^2 dy dq_T^2} = \sum_{a,b=g, \overset{(-)}{u}, \overset{(-)}{d}, \dots} \int \frac{d^2b}{(2\pi)^2} e^{-i\vec{q}_T \cdot \vec{b}} \widetilde{W}_{ab}(b, Q, x_A, x_B) + Y(q_T, Q, x_A, x_B)$$

$$\widetilde{W}_{ab}(b, Q, x_A, x_B) = |\mathcal{H}_{ab}|^2 e^{-\mathcal{S}(b, Q)} \overline{\mathcal{P}}_a(x_A, b) \overline{\mathcal{P}}_b(x_B, b)$$

\mathcal{S} is the soft (Sudakov) function:

$$\mathcal{S}(b, Q) = \int_{b_0^2/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2} \left[\mathcal{A}(\alpha_s(\bar{\mu})) \ln \frac{\bar{\mu}^2}{Q^2} + \mathcal{B}(\alpha_s(\bar{\mu})) \right], \quad b_0 = 2e^{-\gamma_E} \approx 1.12$$

$\overline{\mathcal{P}}_a(x, b)$ are b -dependent PDF's; if $b^2 \ll Q^{-2}$,

$$\overline{\mathcal{P}}_a(x, b) = \sum_c [C_{a/c} \otimes f_c] \left(x, b, \mu_F = \frac{b_0}{b} \right)$$

Y is the difference of the finite-order and overlap (asymptotic) terms

