Two issues with spin in version 1 lattice

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Problem Description

What happens to the vertical spin component if quadrupoles are misplaced?

Simulation	Vertical spin precession rate
Yann D. (Bmad with PTC)	$\approx 300\mu$rad/s
Yann D. (Bmad with RK4)	$\approx 0.6\mu$rad/s
Selcuk H. (RK4)	$\approx 0.7\mu$rad/s
Emerey V. (COSY Infinity)	$\approx 100\mu$rad/s

- Various people simulated supposedly same lattice and got rather different results
- Question: Is it a numerical issue?

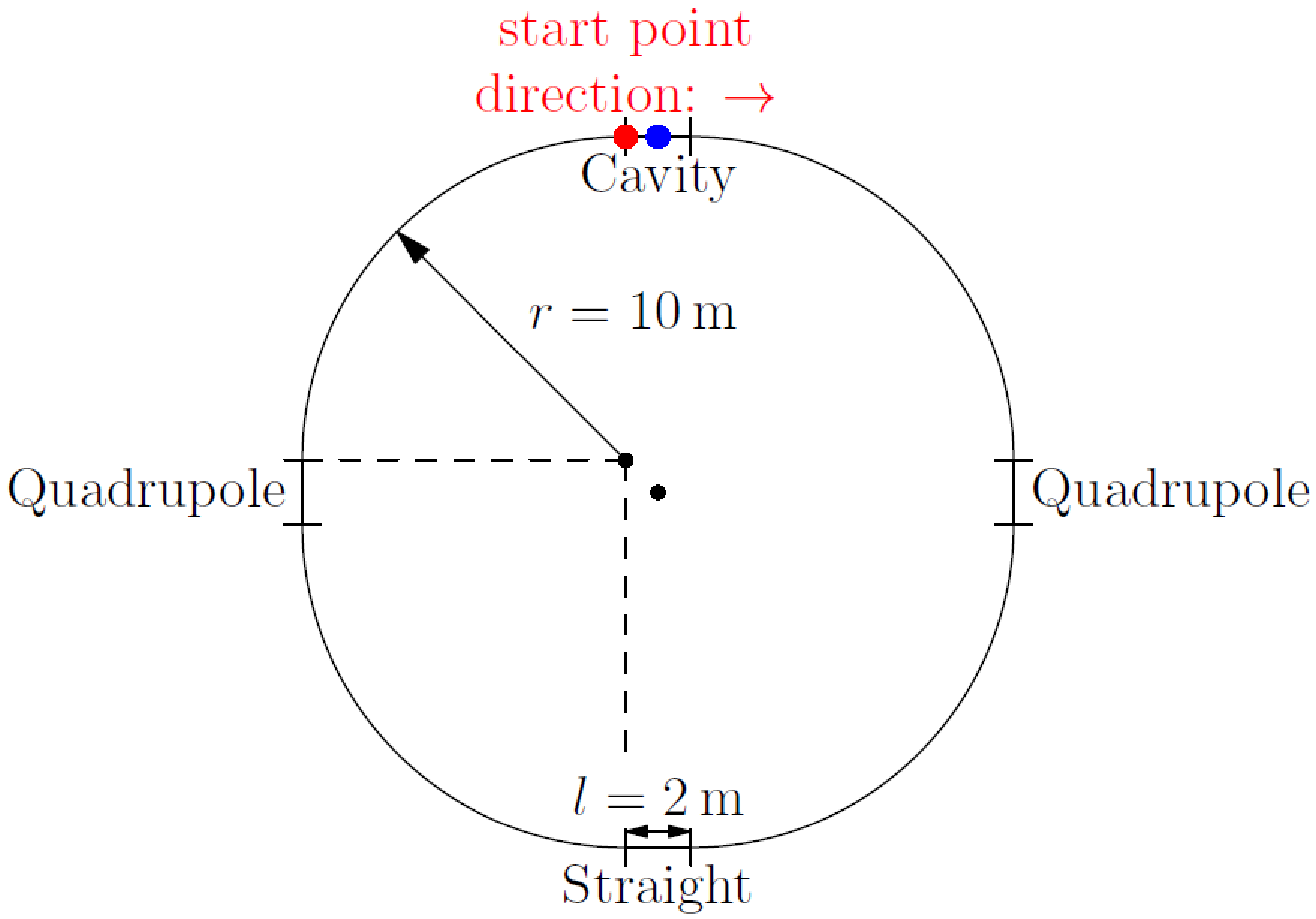
Overview: My Program

- Simple but very modular program
- Can use different integration algorithms
- Can use different data types (double, long double, arbitrary precision)

→ easy to change all kinds of things
→ ideal to study numerical issues
→ ideal to understand single particle motion
→ not so good to calculate beam properties

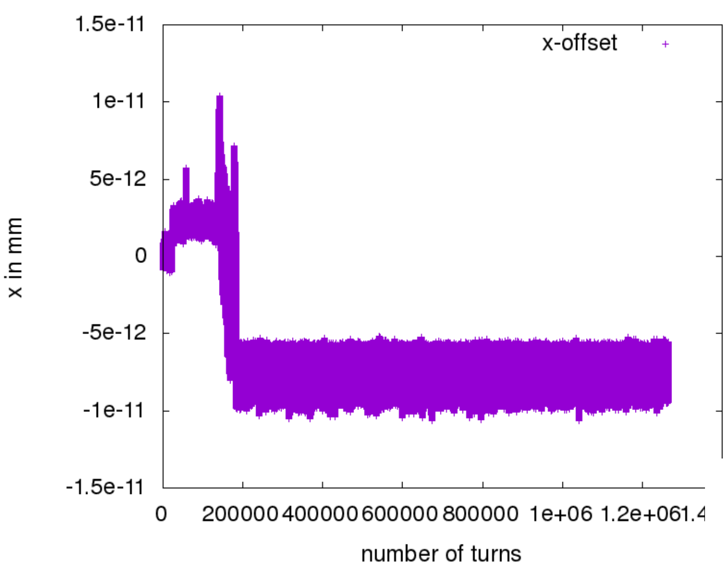
Test Lattice

Use simple test lattice:

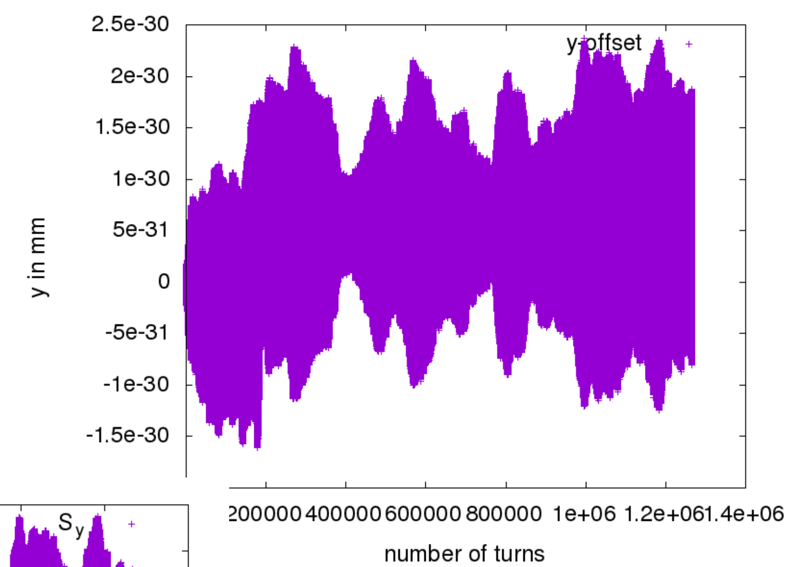


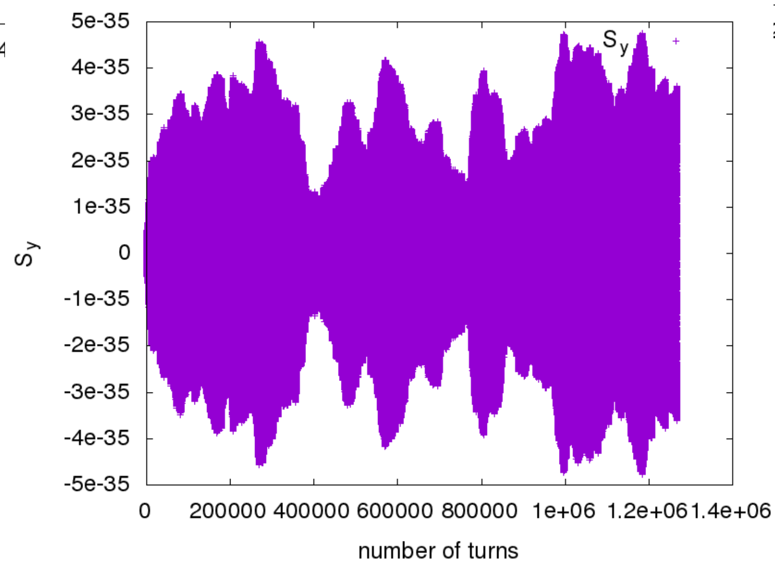
Test Steps

- Run tests with RK8 integrator with different step sizes and different data types → choose dt = 3e-9, long double (accuracy vs. speed)
- Confirm that ideal particle does what it should do



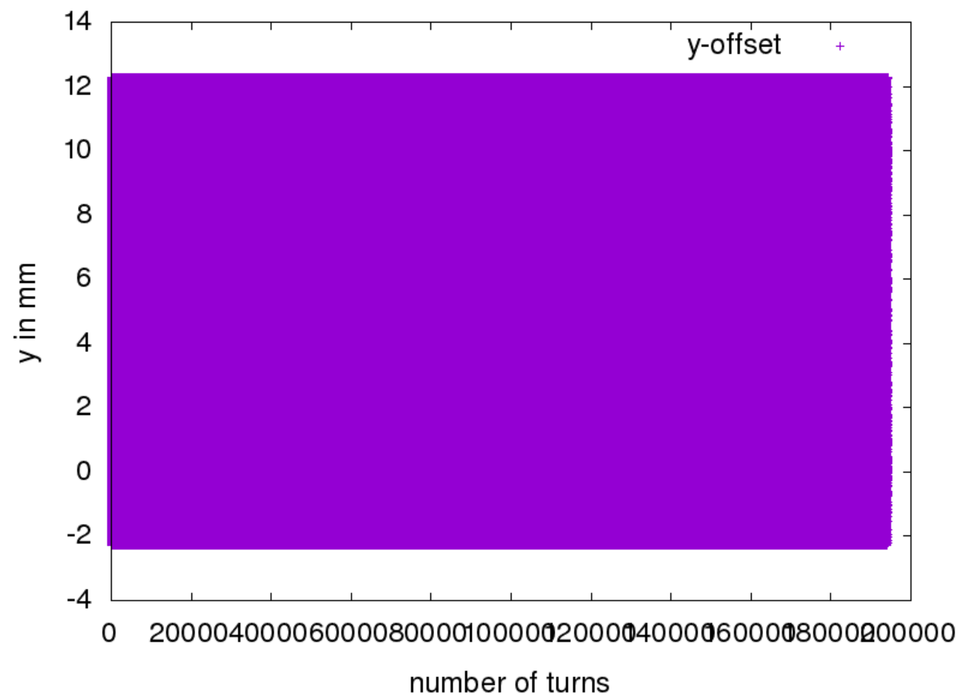
Design particle, no cavity, T = 0.5s

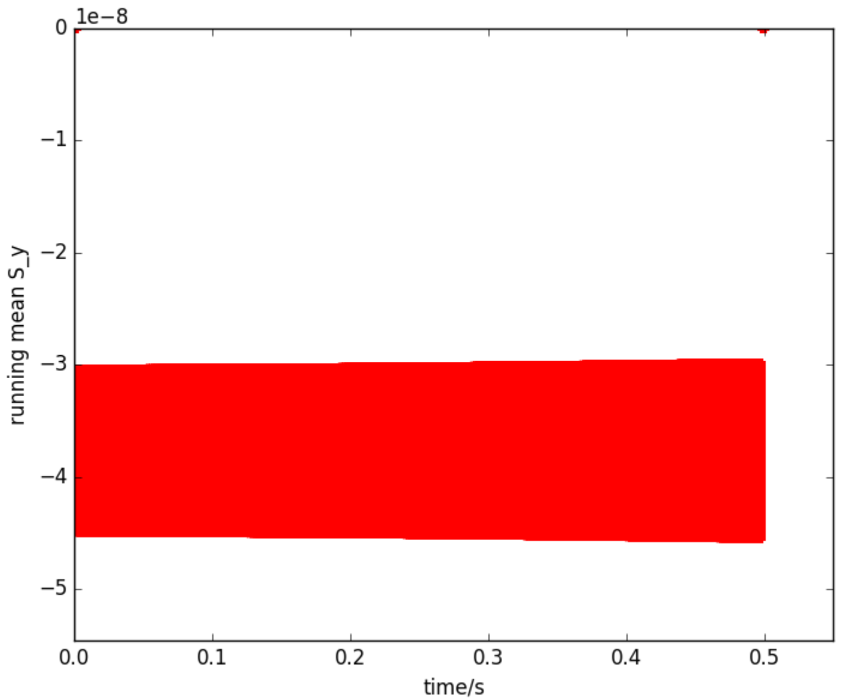




Test Steps

- Misplace one quad vertically by 10mm
- Start particle with x- and y-offset (10mm)
- Still no cavity

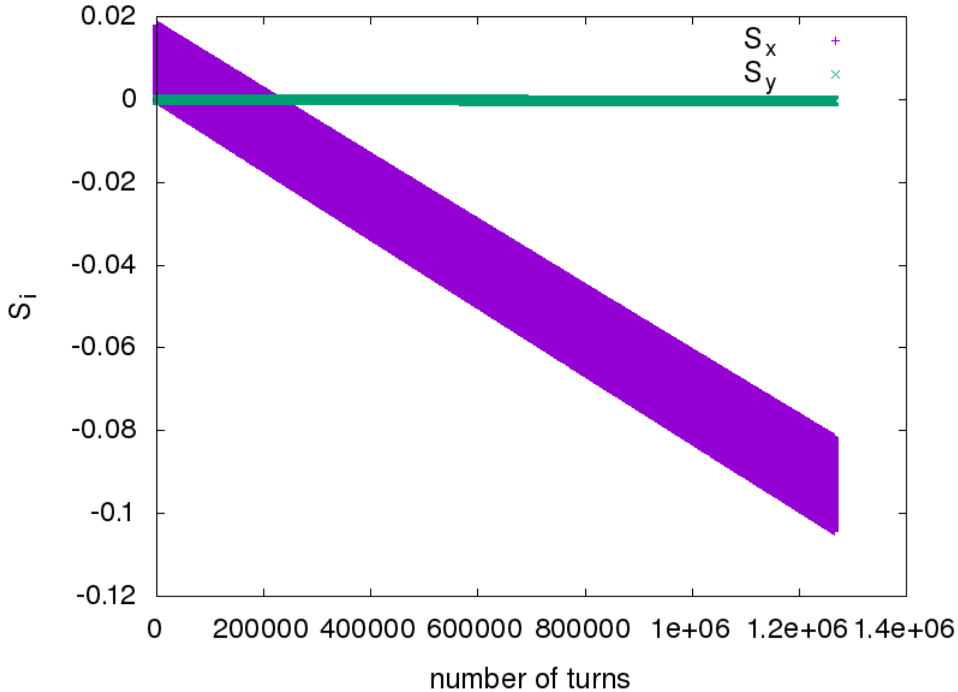


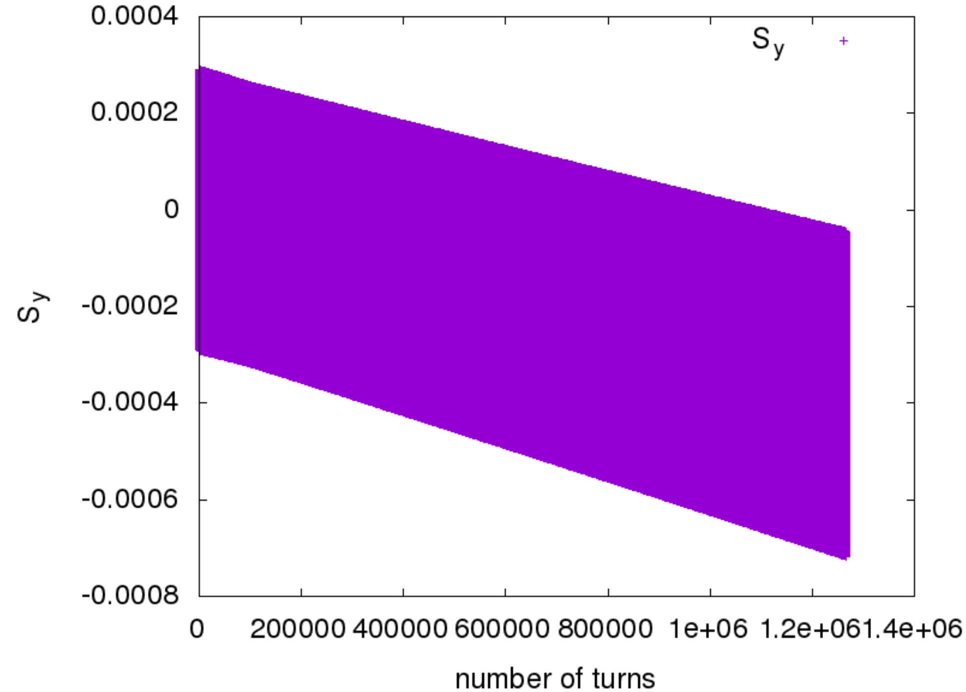


→ no vertical spin build-up

Test Steps

- Misplace one quad vertically by **1mm**
- Start particle with x- and y-offset (10mm)
- **Now with cavity**

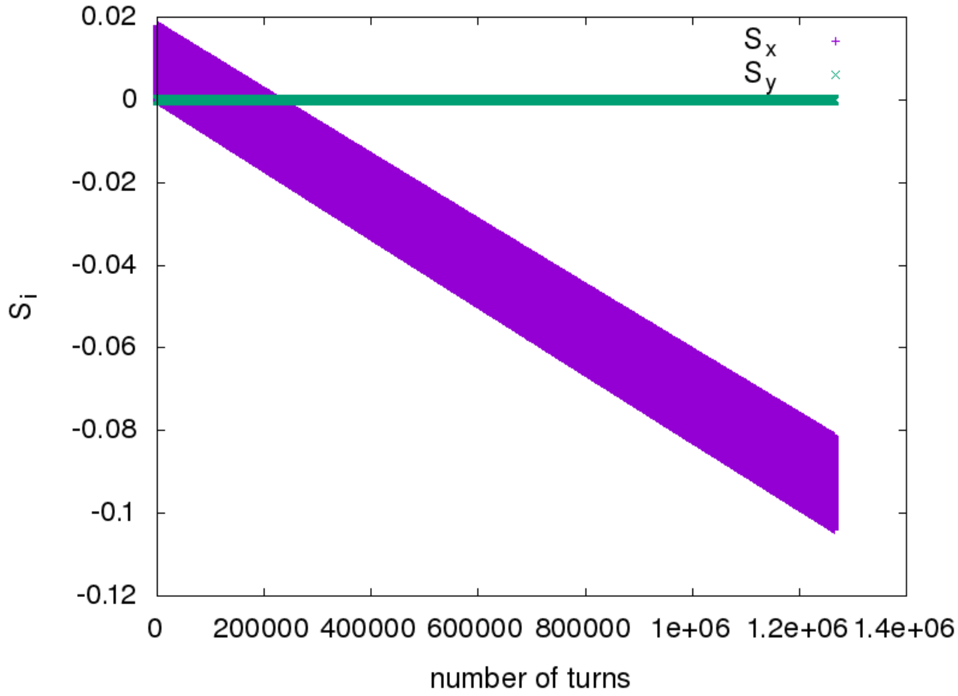


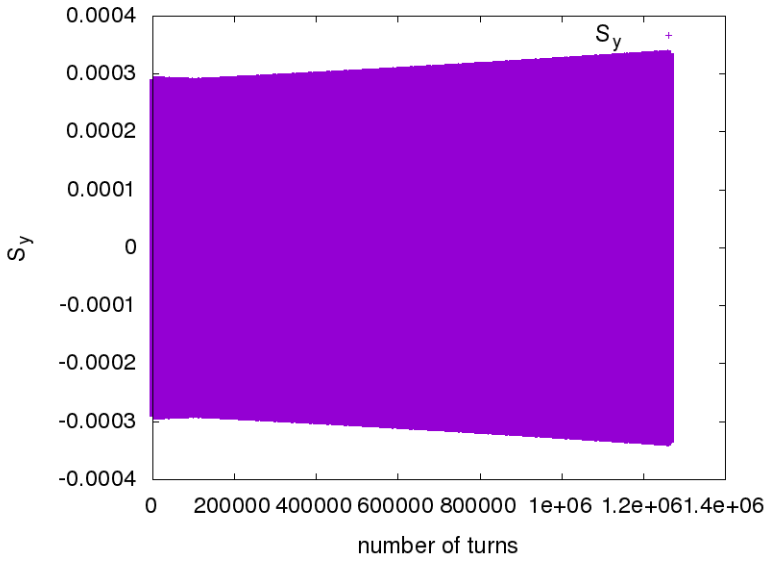


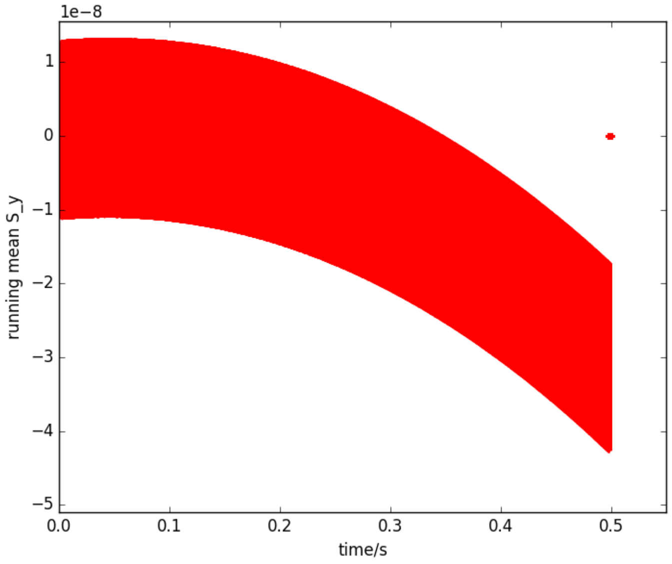
→ Get significant vertical spin build-up, comparable to high end estimates of other people (from different lattice!)

Test Steps

- Misplace one quad vertically by 1mm
- Start particle with x- and y-offset (10mm)
- Now with cavity, **set cavity B-field to zero**





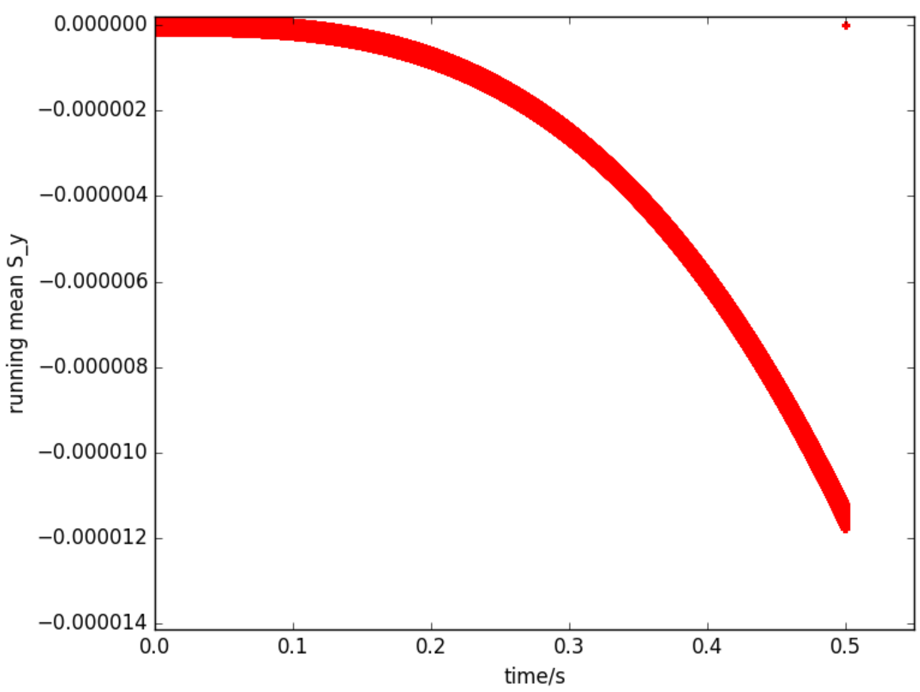


→ Vertical spin build-up is (almost) gone!

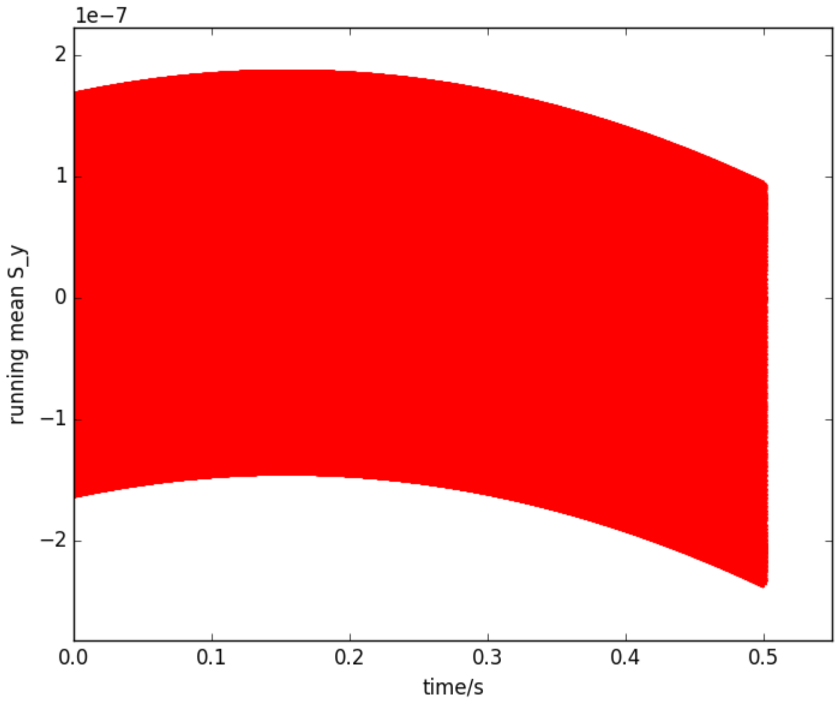
Test Steps

- Misplace one quad vertically by 1mm
- Start particle **on design orbit**
- **Put additional sextupole in empty straight section**

No cavity:



Cavity without B-field:



Explanation

- Longitudinal velocity component in quads leads to vertical spin motion that does average out as long as spin stays essentially in forward direction
- Vertical quad offset shifts equilibrium orbit vertically
- Quad offset doesn't change averaging out of spin build-up because quadrupole fields are linear
- Doesn't work anymore if there are components with nonlinear (vertical) field dependence (sextupoles, fringe fields, etc.)!
- Beam doesn't go through center of cavity → sees average horizontal B-field → vertical spin build-up
- Explains why setting B=0 in cavity removes spin build-up
- Horizontal spin precession rate depends on presence of cavity → fast horizontal rotation can lead to (temporary) build-up

→ Vertical build-up in different programs depends a lot on small details (cavity yes/no, magnitude of betatron/synchrotron oscillations, implementation with or without fringe fields, cavity implementation with/without B-field & nonlinear radial field dependence, etc.)

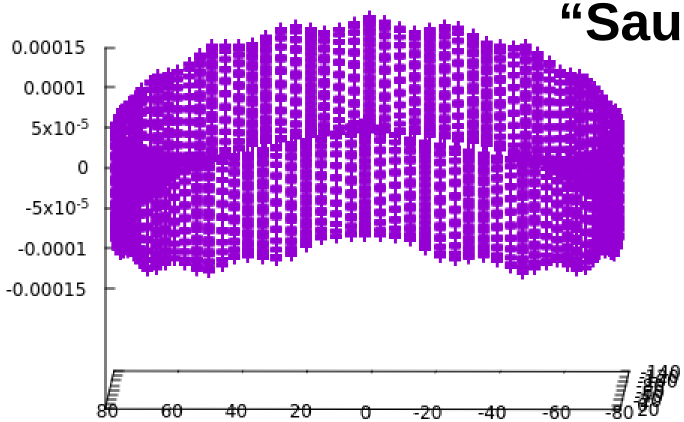
Summary so far

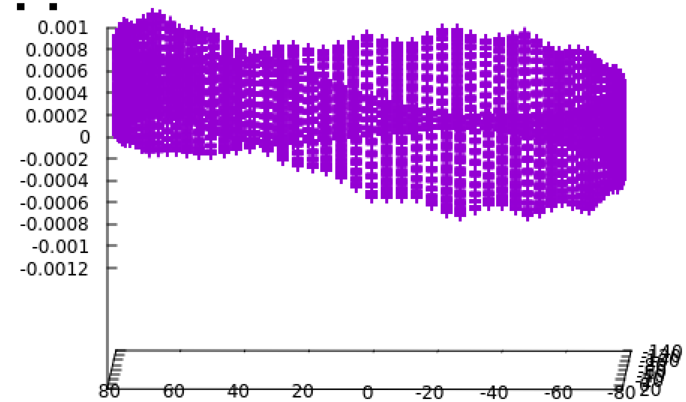
- You will always have a cavity with B-field
- You will always have fields that have nonlinear vertical field dependence
- The horizontal spin component will always drift for some particles

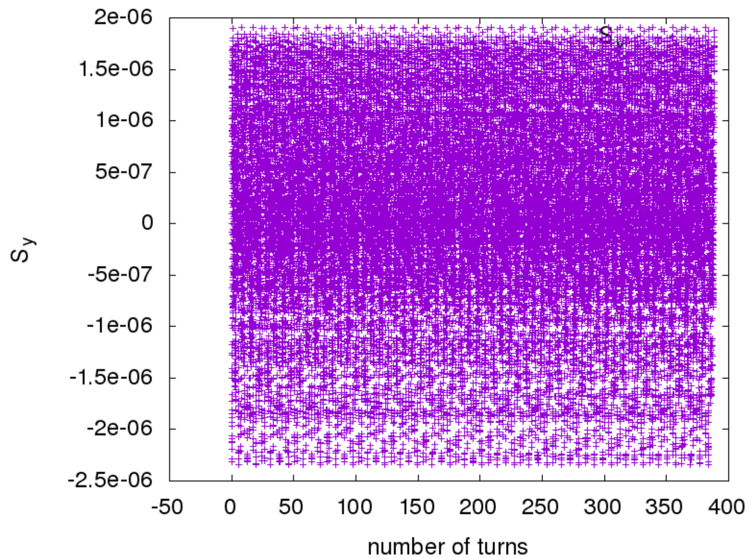
→ In reality there will always be a vertical spin build-up!

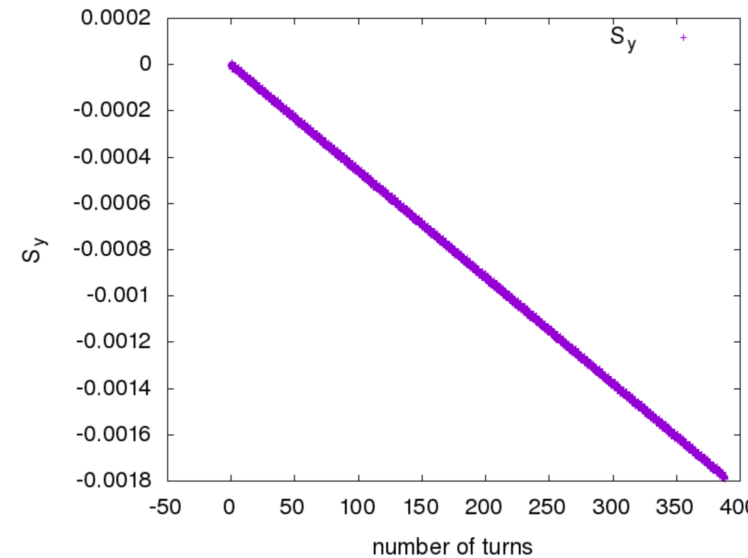
New (at least for me)

“Saucers” & “Twists”???









→ Two seem to be rotated (vs. lattice) versions of each other, important is symmetry with respect to deflectors

CW/CCW operation

CW/CCW operation cancels a lot of systematics:
→ Reverse velocity and spin, fields are equal for both beams

$$\frac{d\vec{s}}{dt} = \vec{s} \times (\vec{\Omega}_{MDM} + \vec{\Omega}_{EDM})$$

$$\vec{\Omega}_{MDM} = \frac{q}{m}\left(G\vec{B} - \frac{\gamma G}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{B}) - \left(G - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right)$$

$$\vec{\Omega}_{EDM} = \frac{\eta q}{2mc}\left(\vec{E} - \frac{\gamma}{\gamma+1}\vec{\beta}(\vec{\beta}\cdot\vec{E}) + c\vec{\beta}\times\vec{B}\right)$$

- No B-field: EDM-term changes sign, MDM-term doesn’t
- B-field of cavity doesn’t change that because it reverses sign for CCW beam
- Static B-fields and rf-E-fields can cause problems

→ If false rotations are small they can be canceled by CW/CCW operation <u>if the beams have the same phase space distribution!</u>

Part 2: Final Ring

1.) Spin Coherence Time

Perfect ring:

- Design particle has frozen spin
- Particle at magic momentum started from position off axis will oscillate
- If it oscillates around design orbit the orbit becomes longer → need higher energy to keep revolution frequency → off magic momentum → spin rotates
- Other option: oscillate about a different equilibrium orbit (negative average x-position) → off magic momentum inside field regions → spin rotates
- Betatron oscillations limit spin coherence time (SCT)

1.) Spin Coherence Time

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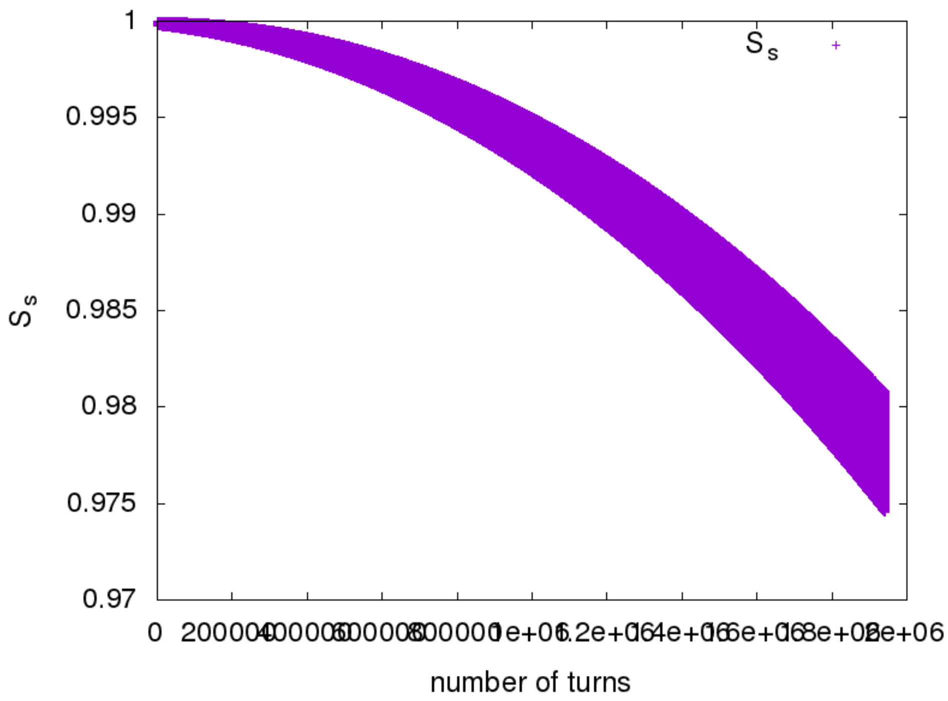
Simulate particle at magic momentum with initial x- and y-offset in straight section of 10mm for T = 5s:



Estimate precession rate:

$$\begin{aligned} \cos(\omega_y T) &\approx 0.978 \\ \Rightarrow \omega_y &= \frac{\arccos(0.978)}{5\,\mathrm{s}} = 0.042\,1/\mathrm{s} \end{aligned}$$

How long does it take for a 90° rotation?

$$\begin{aligned} \cos(\omega_y t) &= 0 \\ \Rightarrow t &= \frac{\arccos(0)}{\omega_y} = 37.4\,\mathrm{s} \end{aligned}$$

→ Very rough estimate for SCT: 10s-100s (depends on phase space distributions)

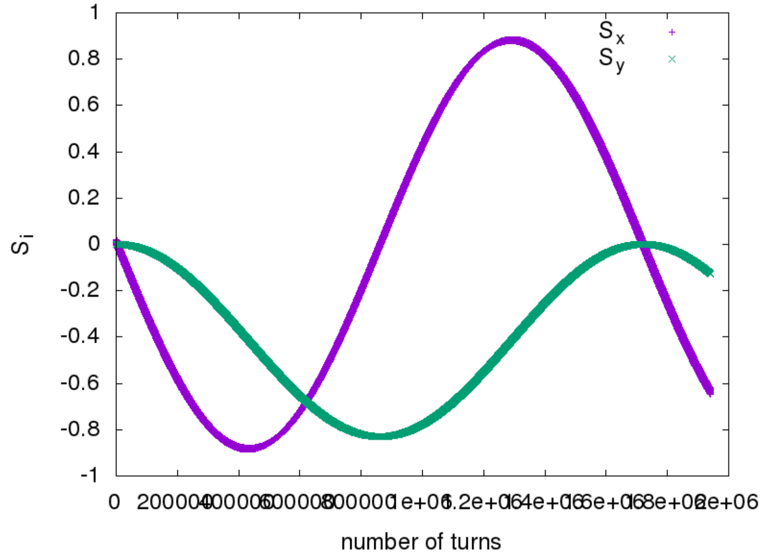
2.) Influence of Imperfections

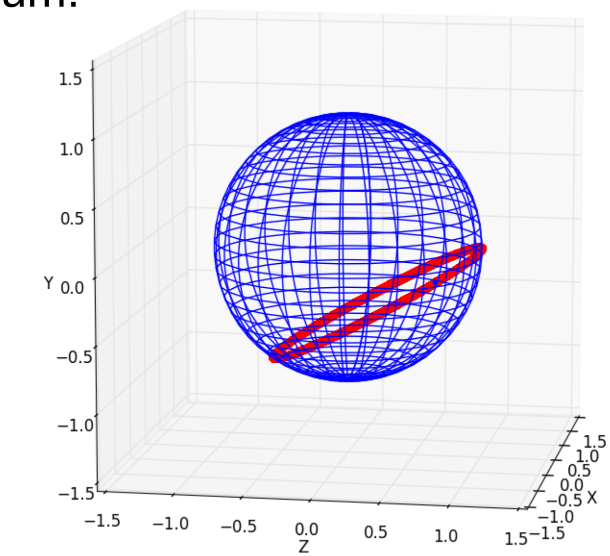
Simulate exactly the same situation as before but with some changes to the ring:

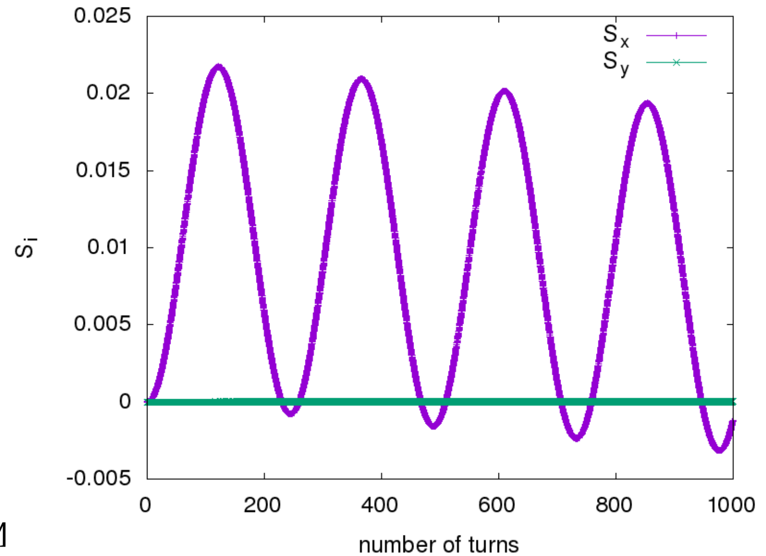
- Randomly misplace all quadrupoles, horizontally and vertically with one sigma offset = 10µm
- Randomly rotate all quads and bends with one sigma rotation of 10µrad
- Randomly change field strength of all elements with one sigma relative field change of 1e-5

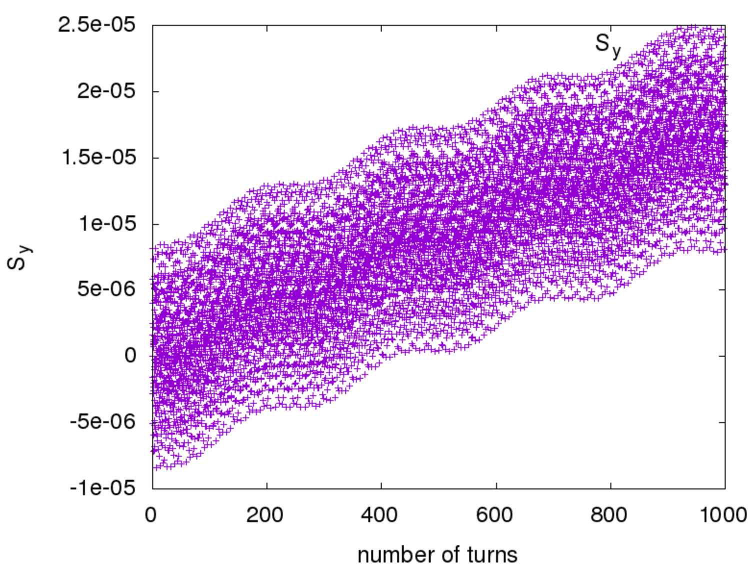
2.) Influence of Imperfections

Particle started on the design orbit with magic momentum:





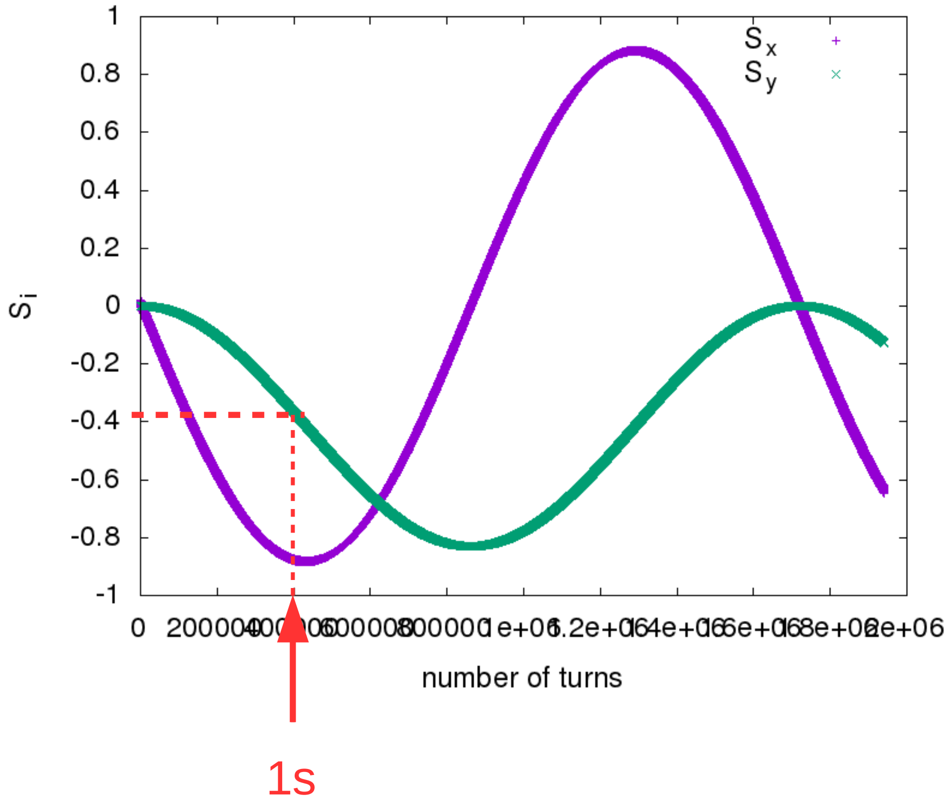




2.) Influence of Imperfections

Particle started on the design orbit with magic momentum:

- **Get extremely large vertical precession rate (compared to expected nrad/s EDM signal)!**
- **May not look that bad in shorter (1ms) simulations!**
- **Cavity needs to be adjusted to freeze horizontal spin → happens only after 1-2 seconds of data taking → need extreme temporal stability of accelerator!**
- **CW/CCW beams will cancel error to a large degree, drifts in parameters will make averaging imperfect!**



2.) Influence of Imperfections

- Ideal case: no spin precession axis → small (random) deviations from ideal case lead to random spin precession axis!
- Leads to vertical (spin) decoherence!
- Some signals won't cancel in CW/CCW operation: fast signals like voltage ripples, signals on harmonic of revolution frequency, e.g. induced by beam itself