Considerations on EDM lattices

Content and Conclusion

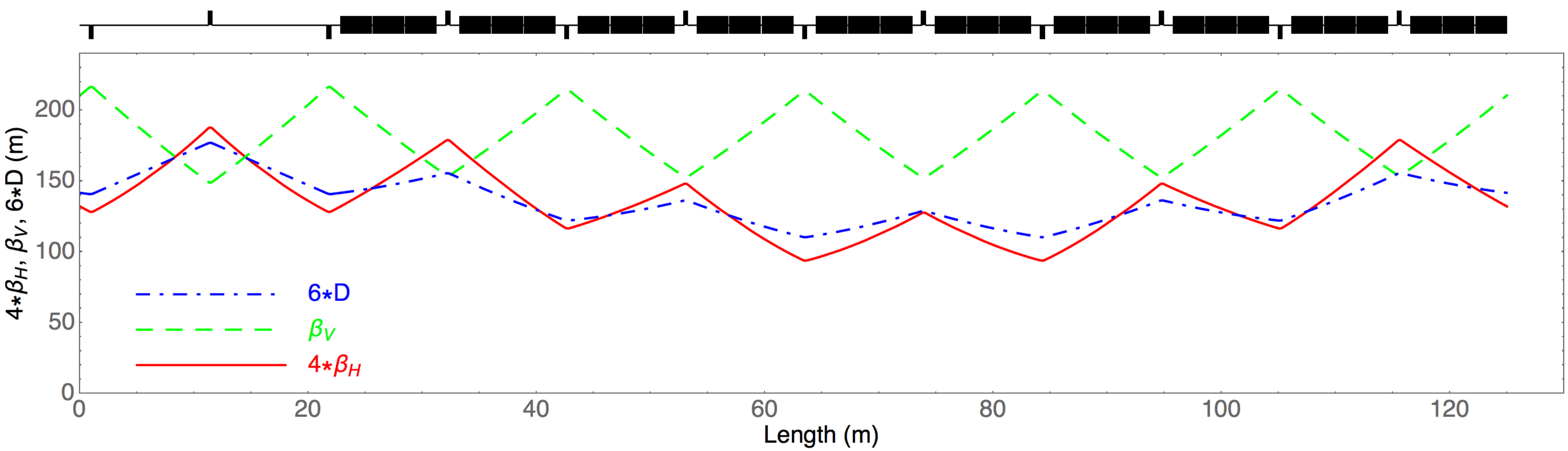
- Strong focusing lattice as proposed by V. Lebedev
  - Most properties (Twiss functions, transition energy, momentum compaction) confirmed for nominal gradients – small discrepancy on IBS
  - Gradients adjusted (same geometry) for lower vertical tune (probably not the intention of this proposal)

  => one can find a lattice, which works on paper and in simulations

  => large vertical beam sizes to become manageable for IBS and sensitivity
- Perturbations for low vertical tunes – using latest version of prototype proposal by R. Talman
  - Orbit very sensitive to tilts, less to vertical alignments
  - Coupling due to electrode misalignement possibly strong
- Appendix: derivation of equations of motion for fully electric ring
  - Found exactly the same than used e.g. by Lebedev

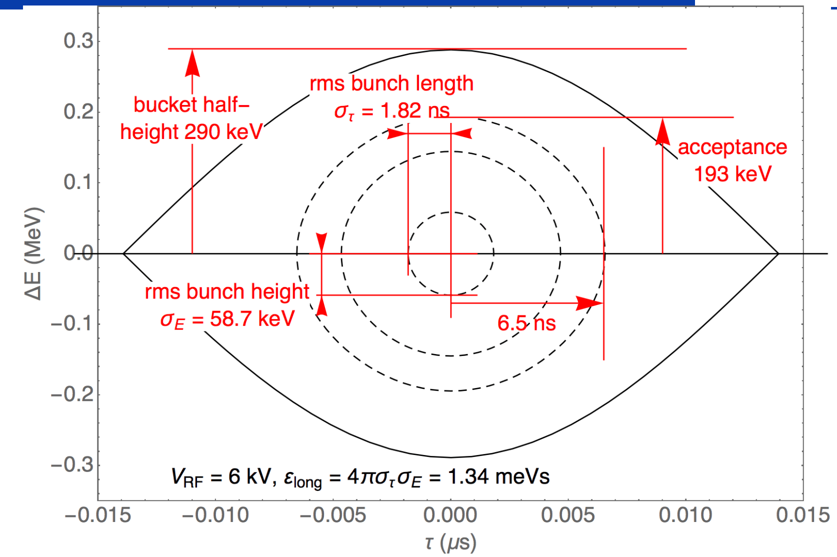
Strong focusing all electric pEDM lattice as in presentation by Valeri Lebedev

- Geometry and quadrupole gradients from presentation with electrical beam rigidity $E\rho$ = 419.308 MV in straight section k_{QFS} = (-3.3918 kV/cm^2)/$E\rho$ and k_{QFS} = (4.1756 kV/cm^2)/$E\rho$ and in arc k_{QFA} = (-3.2068 kV/cm^2)/$E\rho$ and k_{QFA} = (3.7306 kV/cm^2)/$E\rho$
  - Extra factors $(1+1/\gamma^2)$ for horizontal focusing (of bend with straight electrodes), generation of dispersion (in addition to enhanced focusing) and momentum compaction => see appendix for derivation (not from first principles) and details
- Gives: working point Q_h = 2.42 and Q_V = 0.44, momentum compaction α_c = 0.450 and momentum slip factor η = -0.192

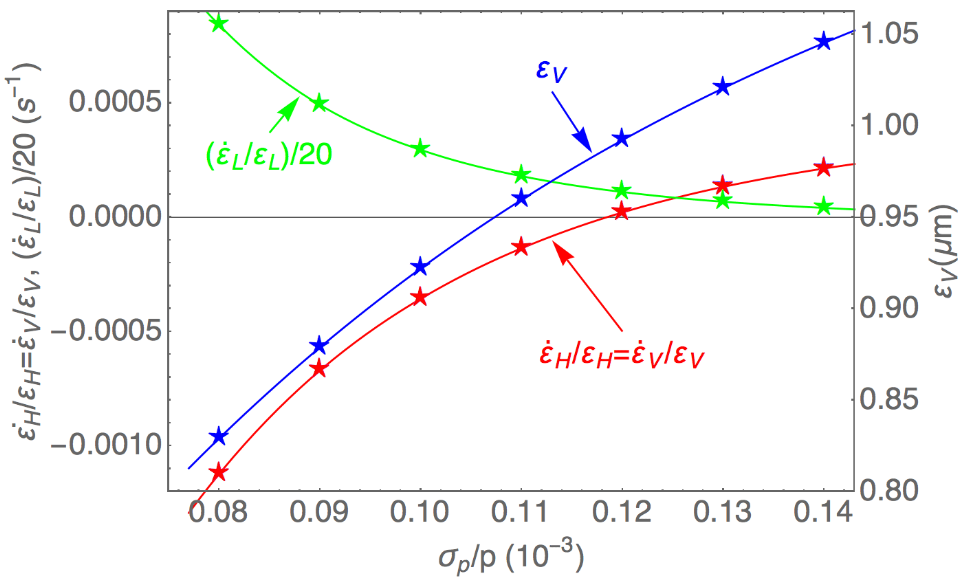


Strong focusing all electric pEDM lattice as in presentation by Valeri Lebedev

- Longitudinal dynamics
  - Buckets and bunches for $V_{RF} = 6$ kV confirmed

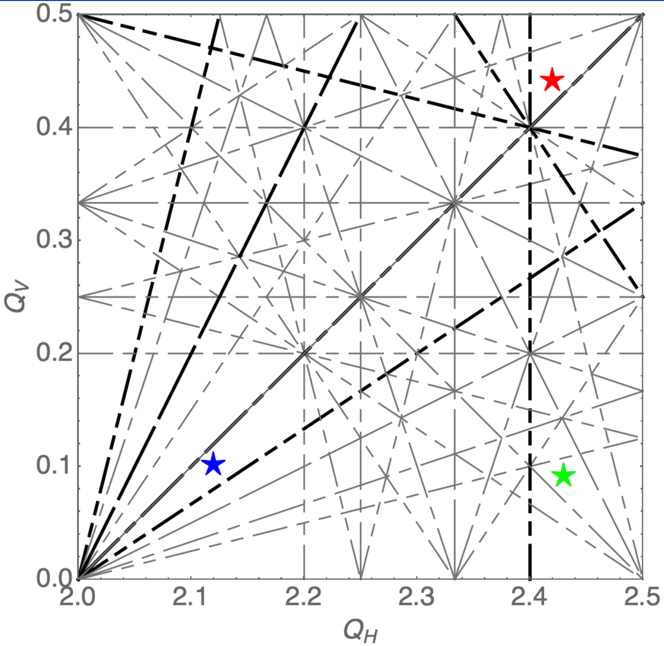


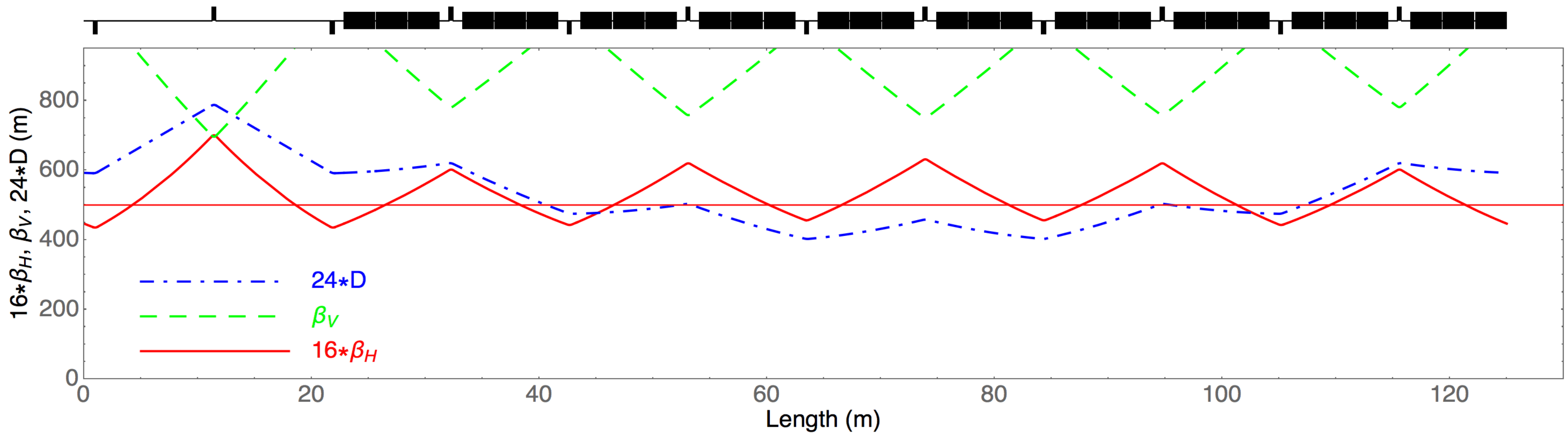
- Intra Beam Scattering
  - Search for zero growth in both transverse planes by adjusting momentum spread and vert. emittance (rms bunch length $\sigma_z = 0.326$ m kept – V_{RF} adjusted)
  - Found for $\varepsilon_H = 0.21$ μm, $\varepsilon_V = 0.987$ μm, $\sigma_p/p = 0.118\ 10^{-3}$ (outside bends, smaller than value $0.14\ 10^{-3}$ quoted by Lebedev)
  - Longitudinal growth with time constant 420 s (about a factor two smaller than V. Lebedev, which may caused by different definition)



Strong focusing all electric pEDM lattice tuned to lower vertical tune

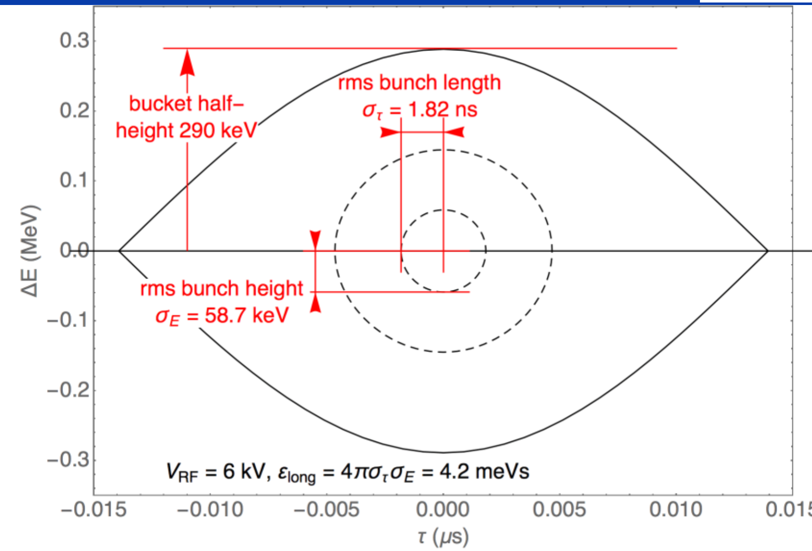
- Geometry and quadrupole gradients from presentation with electrical beam rigidity $E\rho = 419.308$ MV in straight section $k_{QFS} = (-3.5911\ \text{kV/cm}^2)/E\rho$ and $k_{QFS} = (5.0759\ \text{kV/cm}^2)/E\rho$ and in arc $k_{QFA} = (-2.8949\ \text{kV/cm}^2)/E\rho$ and $k_{QFA} = (3.4220\ \text{kV/cm}^2)/E\rho$
- Gives working point $Q_h = 2.43$ and $Q_V = 0.09$, momentum compaction $\alpha_c = 0.431$ and momentum slip factor $\eta = -0.1211$
- Low vertical tune will make machine sensitive to errors! (motivation to propose larger Q_V ?!?)



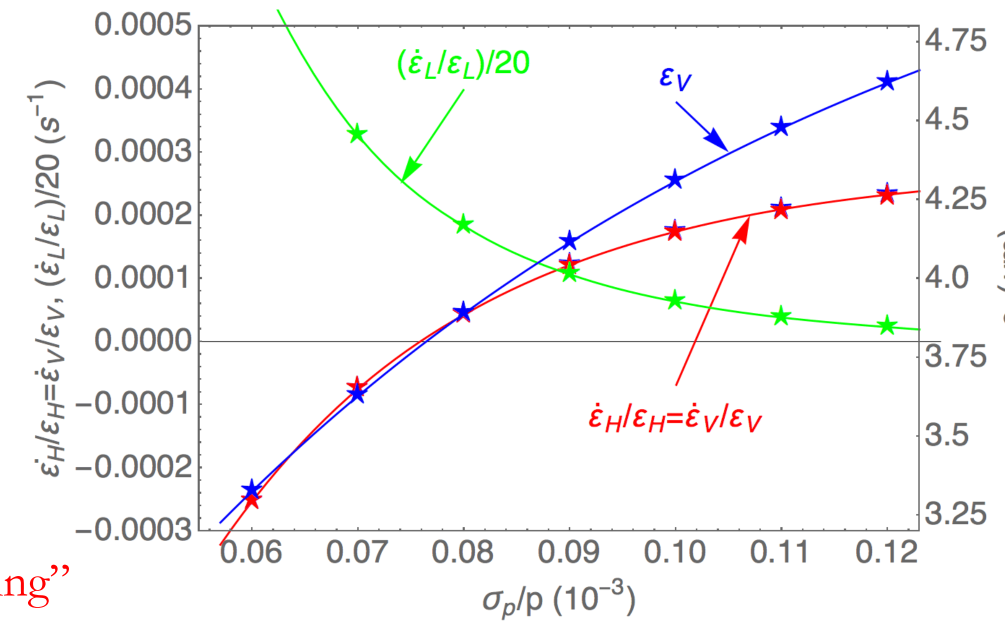


Strong focusing all electric pEDM lattice tuned to lower vertical tune

- Longitudinal dynamics
  - Buckets and bunches for $V_{RF} = 6.6$ kV (larger momentum slip factor compensated by larger voltage)



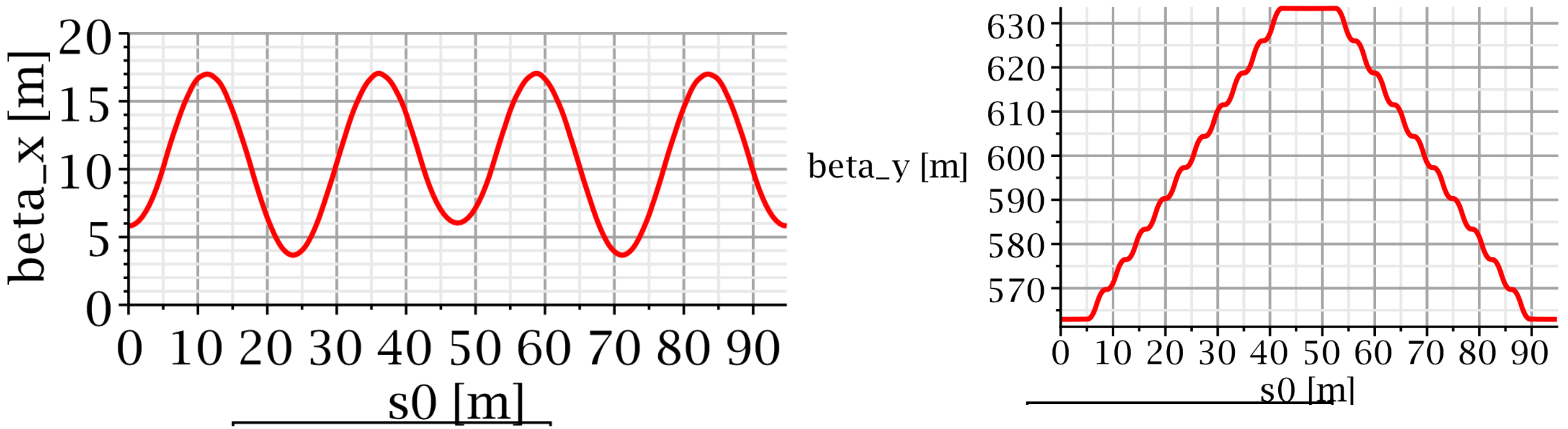
- Intra Beam Scattering
  - Search for zero growth in both transverse planes by adjusting momentum spread and vert. emittance (rms bunch length $\sigma_z = 0.326$ m kept – V_{RF} adjusted)
  - Found for $\varepsilon_H = 0.21$ μm, $\varepsilon_V = 3.79$ μm, $\sigma_p/p = 0.0759\ 10^{-3}$ (outside bends)
  - Longitudinal growth with time constant 216 s



- Strong focusing lattice with low Q_v "on paper" but is it realistic?
  - Sensitivity to imperfections/perturbations
  - Vertical aperture requirements
- Expect similar issues with "smooth focusing"

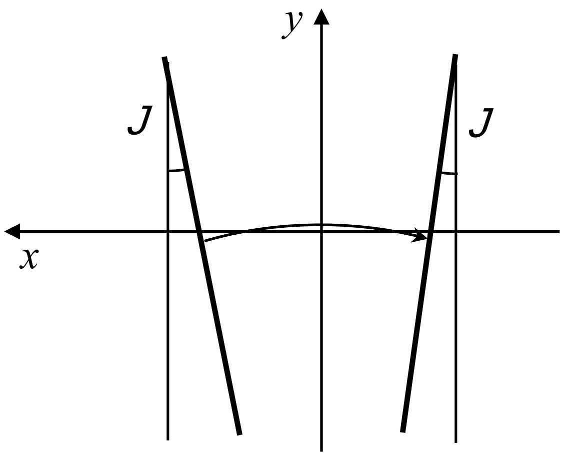
On sensitivity of low vertical tune lattices to perturbations

- For numerical evaluations latest proton prototype version sent by Richard Talman - assuming that it used with only electric fields (no "frozen spin")
  - Working point Q_H=1.73 and Q_V=0.0254, circumference C = 94.8 m and bending radius ρ = 10 m
  - Typical average betatron functions $\beta_H = C/(2\pi\, Q_H)$=594 m and $\beta_V = C/(2\pi\, Q_V)$ = 8.7 m
  - 40 bends with deflection per bend a = 2π/40 = 0.157 rad = 9° and field index m = +/- 0.2



- Tilt of one bending element by $\vartheta = 0.1$ mrad gives
  - Vertical deflection $\Delta y' = a\vartheta = 0.0157$ mrad amplitude of orbit deformation $\Delta y = \dfrac{b_V \Delta y'}{2\sin(\pi Q_V)} \approx \dfrac{a\vartheta}{2\pi Q_V} = 5.8$ cm
- Vertical offset of one bending element by $\Delta y = 0.1$ mm gives
  - Vertical deflection $\Delta y' = (a\rho)(m/\rho^2)\Delta y = 0.00031$ mrad and orbit deformation $\Delta y \approx \dfrac{a\vartheta}{2\pi Q_V} = 1.2$ mm
- In a realistic machine all bends are misaligned and contribute to orbit distortions

On sensitivity of low vertical tune lattices to perturbations



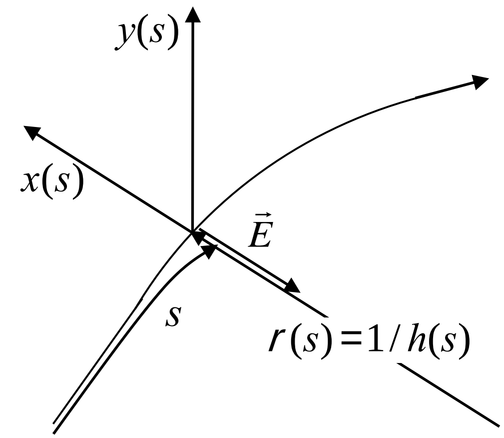
- Divergence of electrodes of one bend (adding a small finite field index will give very similar result) with half of electrode spacing $a = 6$ cm
- Gives additional vertical deflection $\Delta y' = \alpha\vartheta\, x/a$ and horizontal deflection $\Delta x' \approx \alpha\left(\frac{2a}{2a+2y\vartheta} - 1\right) \approx -\alpha\vartheta\, y/a$
- Integrated skew quadrupole with strength $-\alpha\vartheta/a$
- Used to compute 4x4 one-turn transfer matrix

$$M = \begin{pmatrix} \cos(2\pi Q_H) & \beta_H \sin(2\pi Q_H) & 0 & 0 \\ \sin(2\pi Q_H)/\beta_H & \cos(2\pi Q_H) & 0 & 0 \\ 0 & 0 & \cos(2\pi Q_V) & \beta_V \sin(2\pi Q_V) \\ 0 & 0 & \sin(2\pi Q_V)/\beta_V & \cos(2\pi Q_V) \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -\alpha\vartheta/a & 0 \\ 0 & 0 & 1 & 0 \\ \alpha\vartheta/a & 0 & 0 & 1 \end{pmatrix}$$

  - Which becomes unstable for $\vartheta \approx 1.6$ mrad
  - Situation with all 40 bends misaligned?
  - Impact of coupling, such that motion is still stable?

Appendix A:
Equations of Motion in an electrostatic Synchrotron

- Linearized equations of motion in an electrostatic structure
  - To be taken into account (in addition to terms in magnetic structures):
    - Energy and momentum change due to electric potential of bends
    - Velocity change due to momentum offset and electric potential
  - Electric field in bend $E_x = -\beta_m^2 \gamma_m E_r / (e\,\rho)$ leads to
    - Energy change $\Delta E_{kin,d} = -e\,U(x) \approx e\,E_x x = -\left(\beta_m^2 \gamma_m E_r / \rho\right) x$
    - Relative momentum offset $\Delta p_d / p = \left(\Delta E_{kin} / \beta_m c\right) / p = e\,E_x x = -x/\rho$
    - Relative velocity change $\Delta \beta_d / \beta = \Delta E_{kin} / \left(\beta_m^2 \gamma_m^3 E_r\right) = -\left(1/\gamma_m^2\right) x/\rho$
  - In addition gradient $k = -\left(dE_y / dy\right) / \left(\beta_m^2 \gamma_m E_r / e\right)$
    and velocity change due to rel. momentum offset $\Delta\beta/\beta = \left(1/\gamma_m^2\right) \Delta p / p$



- Finally the linearized equations of motion
  - Horizontal equations of motion $x'' = -k\,x + \frac{1}{\rho}\frac{\Delta p_d + \Delta p}{p} + \frac{1}{\rho}\frac{\Delta\beta_d + \Delta\beta}{\beta} = -k\,x - \left(1 + \frac{1}{\gamma_m^2}\right)\frac{x}{\rho^2} + \left(1 + \frac{1}{\gamma_m^2}\right)\frac{1}{\rho}\frac{\Delta p}{p}$

    $$x'' + \left[k + \left(1 + \frac{1}{\gamma_m^2}\right)\frac{1}{\rho^2}\right] x = \left(1 + \frac{1}{\gamma_m^2}\right)\frac{1}{\rho}\frac{\Delta p}{p}$$
  - Vertical equation of motion $y'' - k\,y = 0$
  - Contribution of velocity change $\Delta\beta_d / \beta$ on revolution time of off-momentum particles and, in consequence, momentum compaction, which becomes

    $$\alpha_C = \frac{1}{\gamma_{tr}^2} = \frac{1}{C}\oint_o^C ds \left(\frac{D}{\rho} + \frac{1}{\gamma_m^2}\frac{D}{\rho}\right) = \left(1 + \frac{1}{\gamma_m^2}\right)\frac{1}{C}\oint_o^C ds \frac{D}{\rho}$$