Systematic errors related to radial B-field in the Storage-Ring Proton EDM Experiment

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Christian Carli has issued, in a PowerPoint (pp) document dated November 2017, a number of important issues related to the proton EDM plan regarding the detection of the average radial B-field around the ring at the required level of about 10aT. That level of radial B-field would cause a 5pm beam separation when a vertical tune of 0.1 is used (these numbers come from CC's pp document). The 10aT average B-field limit requirement holds for the integral around the ring and for the average over the total run time of the experiment. We either need to hold the average radial B-field below that level or know the average radial B-field over the ring circumference at that level. In general, the issues raised by CC are all manageable by manipulating the relative beam intensities of the counter-rotating beams as well as their transverse size. This ability is the main reason we require the simultaneous clock-wise and counter-clock-wise beam storage. Statistically, the counter-rotating beam adds, at a great expense, just a factor of two in accumulated statistics; however, its presence is critical in addressing the systematic errors many of which have been pointed out by CC. In addition, the issues raised by CC refer to the appearance of a dc radial B-field at the location of the SQUID pick-up loop, whereas our signal is at the tune modulation frequency. This last point essentially, all but eliminates the systematic errors that are addressed over here. However, there might still be a small remnant signal due to the tune modulation, so we are addressing those issues here.

We are going to address each issue raised separately in enough detail to demonstrate that the errors are under control. The majority of those issues have been raised previously and the answers are scattered in presentations and/or notes, but it is good putting them together in one note here. Some of the raised issues do not really pose a problem since our SQUIDs are only capable of detecting modulated signals at the vertical tune modulation frequency (1-10kHz). Nonetheless, we have expanded the issues raised in the pp document by CC to include the modulation of the beam size due to the tune modulation, the point being that when we modulate the tune, the betatron function is also modulated and therefore the maximum size of the beam as well. This beam size modulation may cause a signal at the SQUID location, even though there may be no radial B-field present around the ring. The beam size modulation signal, however, is a sideband at the betatron plus and minus the tune modulation frequency (study under way).

1st page of CC's pp presentation, the issues concerned are

1. 1 ppm difference in beam intensities and 0.25 mrad orientation error pick-up coil
2. 1 ppm difference in beam intensities and 5 μm common (both beams) orbit offset
3. 1% difference of horizontal emittance and 1 μrad orientation error of pick-up coil
4. Possible other perturbations: horizontal offset between the two beams, different emittances and common vertical offset…

Actually, all the above points regard very similar, equivalent to some extend, effects. The bottom line is that due to a non-perfect cancellation of the counter-rotating currents in the ring there's going to be a certain remnant azimuthal B-field and if our SQUIDs are slightly misaligned there's going to be a false signal. Let's first estimate the effects: 1mA times 10^{-6} and a SQUID at 20 mm away misaligned by 0.25 mrad will pickup

$$B = \frac{\mu_0 I}{2\pi r} = \frac{4\pi \times 10^{-7} \times 10^{-9}}{2\pi \times 0.02} = 10^{-14}\,\mathrm{T} \tag{1}$$

which for 0.25 mrad pick up loop orientation misalignment becomes 2.5 aT. This effect of course is not modulated, it's a dc radial B-field and the SQUID cannot "see" it. The only way for it to appear is due to the beam size modulation via the tune modulation. This is a small effect, plus we can make the two counter-rotating currents unbalanced by a large factor, e.g., ten times as much compared to the normal operating conditions and measure the effect with the SQUIDs to correct for it. The counter-rotating beams will be scraped by applying magnetic or electric fields, the time and spatial dependence of which will be described elsewhere. The bottom line will be to change the current and or size of the counter-rotating beams at will. The beam-size effect is the largest, because each SQUID pick-up loop is on one side of the beams radially, breaking the left/right symmetry, indicating that we would need to make the transverse beam sizes of the counter-rotating beams the same at the 10^{-4} level. However, since we are only sensitive to a signal at the tune modulation frequency, this is going to be much lower and not an issue at all (study under way).

It turns out that all the effects included in the note appear as a systematic effect only when the beam size modulation is included and then they are at most very small, at a ppm level, and in addition they can be further canceled when we take into account that we can measure them by injecting beams with 10 times more effective (net) current into the ring, and so on, up to including the case where only one of the counter-rotating beams is present.

The beam separation due to the 10aT radial B-field is going to be:

$$y(\vartheta) = \sum_{N=0}^{\infty} \frac{\beta c R_0 B_{R,N}}{E_0 \left(Q_y^2 - N^2\right)} \cos\left(N\vartheta + \varphi_N\right) = \frac{0.6 \times 3 \times 10^8 \left(\text{m/s}\right) \times 80\text{m} \times 10\text{aT}}{5.3\text{MV/m} \times \left(0.1\right)^2} = 2.7\text{pm} \quad (2)$$

and the total separation between the counter-rotating beams is twice that, i.e., 5.4pm.

The radial B-field generated due to the vertical offset in the *ideal* case in which both beams have the exact same current and shape is

$$\Delta B = 2 \times \mu_0 \frac{I}{2\pi r} \frac{y}{r} = 2 \times 2 \times 10^{-7} \frac{1\text{mA}}{0.02\text{m}} \frac{2.7\text{pm}}{0.02\text{m}} = 2.7\text{aT} \quad (3)$$

This is the *total* radial magnetic field generated due to the vertical offset of the two beams. The modulated B-field, at a frequency of our choice, e.g., 1-10kHz due to the 10% of tune modulation will be modulated at the same frequency with 20% amplitude (due to the square dependence of the vertical separation). The modulated radial B-field will have amplitude of 0.54aT at the location of the SQUIDs, 2cm away from the center of the beam.

In principle, in the absence of a net radial B-field around the ring, there's no modulation of the radial B-field generated due to the vertical beam separation at the location of the SQUIDs. However, the beams have a transverse size, which is modulated at the tune modulation frequency, since the tune determines the beam beta-function at the SQUID location. The beam size modulation appears at frequency, which is at the betatron plus & minus the tune modulation frequency and in reality, it poses no problem at all. Any effect at the tune modulation frequency itself would be negligible and it can be further eliminated by additional measures. The proposed actions to further eliminate the above-mentioned systematic errors are:

1. The estimated effect due to the beam size modulation at the SQUID location is at most at the ppm level (estimation note is under way) and it depends linearly on the intensity difference, while its dependence on the SQUID orientation misalignment is weak. Whereas the counter-rotating beam contributes only a factor of two to the genuine signal from eq. (3), it can be reduced below our sensitivity by special, systematic error runs. During the run of the experiment we will be taking those special runs, for about 1 hour long every few days, very much like the NMR trolley runs in the muon g-2 experiments at FNAL. These studies will give us information about the relative SQUID orientation with respect to the ideal position.
2. This effect is equivalent to the one above and therefore the same procedure will be enough to eliminate it.
3. The horizontal emittance may be different between the counter-rotating beams. Again, we have the option to make this difference larger by a factor of 3-30 and observe its effect on the modulated radial B-field and therefore its dependence on the SQUID pickup loop orientation.
4. The horizontal split between the two beams will indicate the presence of a vertical B-field. We will be able to monitor it, although with less sensitivity, with a SQUID whose pickup loop is vertically offset by several centimeters.

2nd page, the issues concerned are

1. Estimation of the ring parameters, points one and two. They all look reasonable.
2. Third point: estimation of the force equation in the vertical direction. The radial B-field force is balanced by a vertical electric field due to the electric quadrupoles. The signs of the fields are estimated explicitly in Appendix

below. The same explanation was shown to Valery Lebedev at the time of his report and he accepted that our sign is correct.

3. Points 4, 5, and 6 seem reasonable, with the correct value for the 6th one of 9.3aT, as per the Appendix below.

3rd page, the issues concerned are

The equation used here seems different than eq. (2) above but must be equivalent since the results are close enough.

4th page, the issues concerned are

Again, the results are consistently close so we are in agreement.

5th page, the issues concerned are

The estimation of the dc effect seems to be correct and the result is proportional to the total (net) current circulating in the ring and the misalignment angle of the SQUID pickup loop. There's an effect due to the modulation of the beam sizes (under estimation), which will show up at the SQUID location, but reduced at least at the ppm level (study in progress). Above we mentioned that the way we are going to further address this effect is by running the experiment at different total (net) currents in the ring. The background can be changed by orders of magnitude, whereas the genuine signal is only a factor of two larger when both CW and CCW beams are present.

6th page, the issues concerned are

The estimation (of the dc effect) is again correct and the plan to address the systematic error is the same as above since this error is equivalent to the one from above.

7th page, the issues concerned are

1. In this one, there's an effect on a single SQUID even though the counter-rotating beams have the same intensity and thus the total current is zero. The difference comes from the fact that the beam emittance may be slightly different and therefore the magnetic field amplitude may also be different. As we have explained earlier, we will "shave" the beams individually[1] and will produce runs with different beam horizontal sizes circulating the ring. The effect again originates due to the beam size modulation (at most at the ppm level, currently under evaluation), which we will pin down by taking data with larger differences between the two beam sizes. Taking the average between SQUIDS in opposite (symmetric) positions will also provide additional information.
2. It turns out this effect is the largest from all effects addressed here. Nonetheless, manipulating their relative beam sizes will give us the required information. Attention will also be paid to make the CW vs. CCW beam injections as similar as possible and thus end up with as close emittances as possible. Again, our signal is at the tune modulation frequency whereas this effect appears at the vertical betatron frequency plus and minus the tune modulation signals, which the SQUID will be blind to (low pass filter). Overall, again this systematic error should also be negligible.

Conclusions

Christian Carli pointed out important issues regarding the method suggested to probe the average radial B-field around the ring. Those issues have an effect on the specifications of the equality of the intensities and their transverse sizes of the counter-rotating beams. Ideally, at the start of the run, we would just inject one of the two beams (CW or CCW) and adjust the SQUID pick up loop orientations to show

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[1] By applying a vertical B-field we have split the beams horizontally and scrape them individually at will. We can do the same vertically, even though the scraping at the target region and the polarimeter counting rates CW vs. CCW will always give us information about their vertical phase-space distribution.

zero signal at the tune modulation signal. Then, we would inject the counter-rotating beam to detect the effect of the net (average) radial B-field integrated around the ring.

In addition, every couple of days we would cross check the SQUID pick up loop orientation by having special runs with beam intensities or beam sizes of the counter-rotating beams individually manipulated by significant factors to pin down the indicated systematic errors. The largest effect comes from the horizontal beam size differences when only one side SQUIDS are used in the evaluation. Overall, we expect the systematic error effects to be several orders of magnitude below our sensitivity level.

Studies with specific ring parameters, counter-rotating beam intensities and transverse sizes need to be done before we finalize the ring design. More general studies are currently under way and will be send around soon.

Finally, the counter-rotating beams, in the presence of only E-fields and due to T-invariance symmetry should trace, on average, the same positions around the ring, except due to the effect of the earth's rotation, which is negligibly small, of order 1nm. Therefore, we plan to reduce all the beam position differences by probing the effect of B-fields using SQUIDS as beam position monitors. Overall, the indicated systematic errors should be at a negligible level.

Appendix

The radial B-field effect on the vertical spin precession of the proton is, naively, given by the equation

$$\omega_V = \frac{eg\langle B_r \rangle}{2m} \tag{1}$$

The point is that since the particle does not actually move vertically, the spin precession rate is not proportional to (*g*-2) but directly proportional to *g*. However, the particle doesn't move vertically since there is a vertical electric field acting on it. This electric field originates from the electric focusing system, which exactly compensates the magnetic force due to the radial B-field:

$$\vec{F}_V = q\left(\vec{E}_V + \vec{\beta} \times \vec{B}_r\right) = 0 \Rightarrow \vec{E}_V = -\vec{\beta} \times \vec{B}_r \tag{2}$$

That vertical electric field, together with the lab frame radial B-field it is partially transformed [1] to a radial B-field in the proton rest frame:

$$\begin{aligned} &\vec{B}' = \gamma\left(\vec{B}_r - \vec{\beta} \times \vec{E}_V\right) = \gamma\left(\vec{B}_r + \vec{\beta} \times \left(\vec{\beta} \times \vec{B}_r\right)\right) \\ &\Rightarrow B' = \gamma\left(1 - \beta^2\right) B_r = \gamma \frac{B_r}{\gamma^2} \end{aligned} \tag{3}$$

Since in the rest frame time is slowed down by one factor of gamma, we lose it from the numerator, and eq. (1) finally gets to be

$$\omega_V = \frac{eg\langle B_r \rangle}{2m\gamma^2} \tag{4}$$

With γ=1.25, γ^2=1.56, reducing the vertical spin precession due to a net radial B-field.

References

1. J.D. Jackson, "Classical Electrodynamics", p. 552, 2nd ed., 1975.