Phenomenology of family-nonuniversal 3HDMs

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Charged Higgs 2018, Uppsala

Motivation for extended Higgs sectors

Despite of remarkable consistency of the SM with to-date observations,
it remains remarkably unsatisfactory.

We are looking for

explanations why/where the SM features as they are/originate from

such as

neutrino sector features | **mass/mixing hierarchies** | **origin of CPV**

Dark Matter | **baryon asymmetry etc**

Non-minimal Higgs sectors can provide natural
explanation to CPV and fermion puzzles

Active model-building activity with non-minimal scalar sectors
(see e.g. Ivanov, 1702.03776)

“Bottom-up” approach

- **Assume a little extra: take the plain SM and add a bit more on top to explain a particular peculiarity of the SM (very rarely most them)**

- **Postulate: such an extension to be anomaly-free (just like the SM itself)**

- **Fit: parameters to the observables to see if it complies with the reality**

- **Extrapolate: such an extension to much higher energies hoping the vacuum remains stable, Higgs mass does not receive GUT-scale corrections and the gauge couplings unify (e.g. MSSM)**

Consider adding a family symmetry and extra Higgs doublets

Example I. CP4 3HDM

- **CP is not uniquely defined in QFT**
  Grimus, Rebelo, 1997; Branco, Lavoura, Silva, 1999

$$J: \quad \phi_i \xrightarrow{CP} X_{ij}\phi_j^* , \quad X \in U(N) , \quad J^2 = XX^* \qquad J^k = \mathbb{I}$$

- **k=2 is usual CP, the first non-trivial case is CP4 with 3HDs**
  Ivanov, Keus, Vdovin, 2012; Ivanov, Silva, 2016

$$V_0 = -m_{11}^2(1^\dagger 1) - m_{22}^2(2^\dagger 2 + 3^\dagger 3) + \lambda_1(1^\dagger 1)^2 + \lambda_2\left[(2^\dagger 2)^2 + (3^\dagger 3)^2\right]$$
$$+\lambda_3(1^\dagger 1)(2^\dagger 2 + 3^\dagger 3) + \lambda_3'(2^\dagger 2)(3^\dagger 3) + \lambda_4\left[(1^\dagger 2)(2^\dagger 1) + (1^\dagger 3)(3^\dagger 1)\right] + \lambda_4'(2^\dagger 3)(3^\dagger 2)$$

$$V_1 = \lambda_5(3^\dagger 1)(2^\dagger 1) + \frac{\lambda_6}{2}\left[(2^\dagger 1)^2 - (3^\dagger 1)^2\right] + \lambda_8(2^\dagger 3)^2 + \lambda_9(2^\dagger 3)\left[(2^\dagger 2) - (3^\dagger 3)\right] + h.c.$$

$$V = V_0 + V_1 \qquad X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & i \\ 0 & -i & 0 \end{pmatrix} , \quad J^2 = \mathrm{diag}(1, -1, -1) , \quad J^4 = \mathbb{I}$$

Flavour CP4 3HDM

- **CP4 can be extended to the Yukawa sector and must be spontaneously broken leading to particular patterns in the flavour sector**

  Ferreira, Ivanov, Jimenez, Pasechnik, Serodio, 2017

$$\psi_i \to Y_{ij}\psi_j^{CP}, \quad \psi^{CP} = \gamma^0 C\bar{\psi}^T \qquad -\mathcal{L}_Y = \bar{q}_L \Gamma_a d_R \phi_a + \bar{q}_L \Delta_a u_R \phi_a^* + h.c.$$

CP4 invariance: $(Y^L)^\dagger \Gamma_a Y^d X_{ab} = \Gamma_b^*, \quad (Y^L)^\dagger \Delta_a Y^u X_{ab}^* = \Delta_b^*$ $\qquad Y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & e^{i\alpha} \\ 0 & e^{-i\alpha} & 0 \end{pmatrix}$

Example: case B_1 $\quad \Gamma_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ g_{31} & g_{31}^* & g_{33} \end{pmatrix}, \quad \Gamma_2 = \begin{pmatrix} g_{11} & g_{12} & g_{13} \\ g_{21} & g_{22} & g_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad \Gamma_3 = \begin{pmatrix} -g_{22}^* & -g_{21}^* & -g_{23}^* \\ g_{12}^* & g_{11}^* & g_{13}^* \\ 0 & 0 & 0 \end{pmatrix}$

- **Tree-level FCNCs**

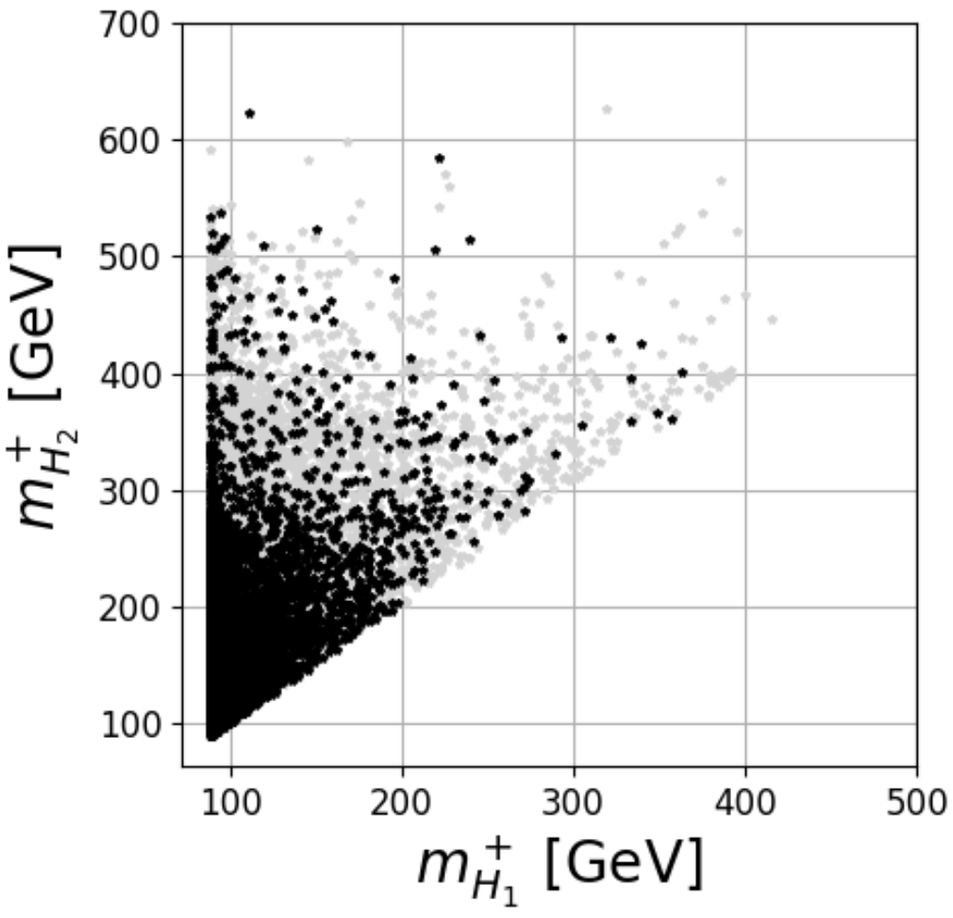
One avoids FCNCs from h125 by imposing the scalar alignment condition:

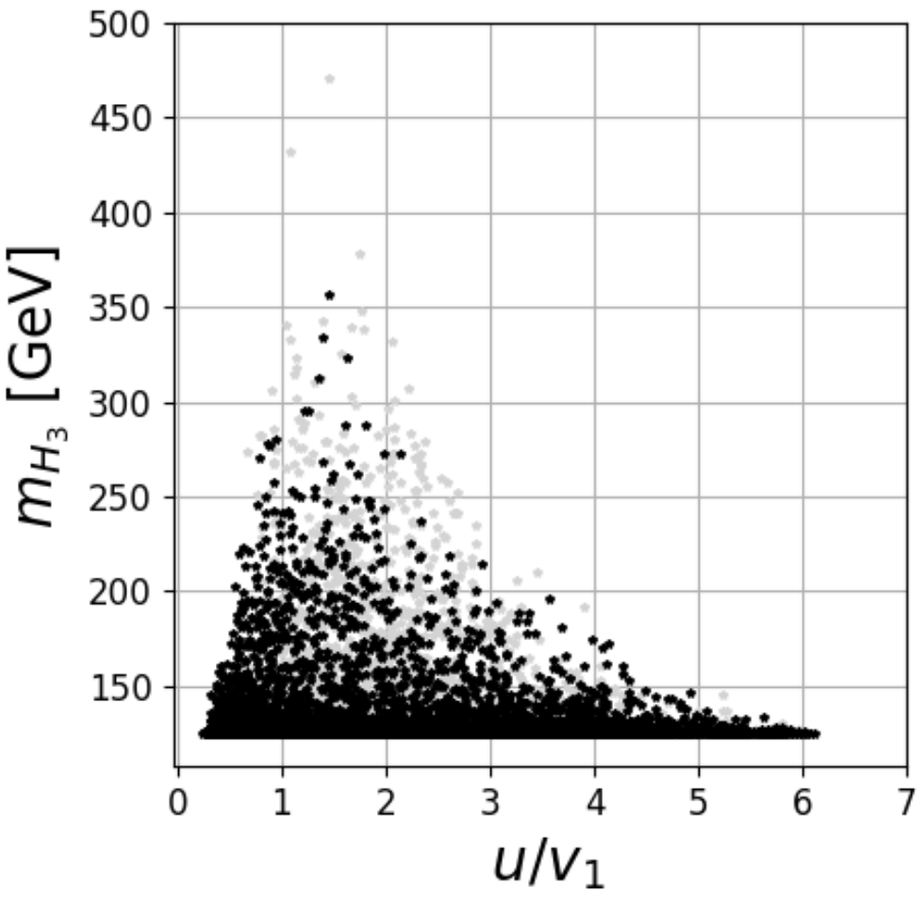
$$m_{11}^2 = m_{22}^2$$

Flavour CP4 3HDM: extra Higgses

Scalar sector scan

- Stick to the scalar alignment, take h125 to be the lightest scalar, vary nine free parameters
- Checks for the boundedness from below and perturbativity
- Check that S,T,U parameters are within 3σ of experimental values

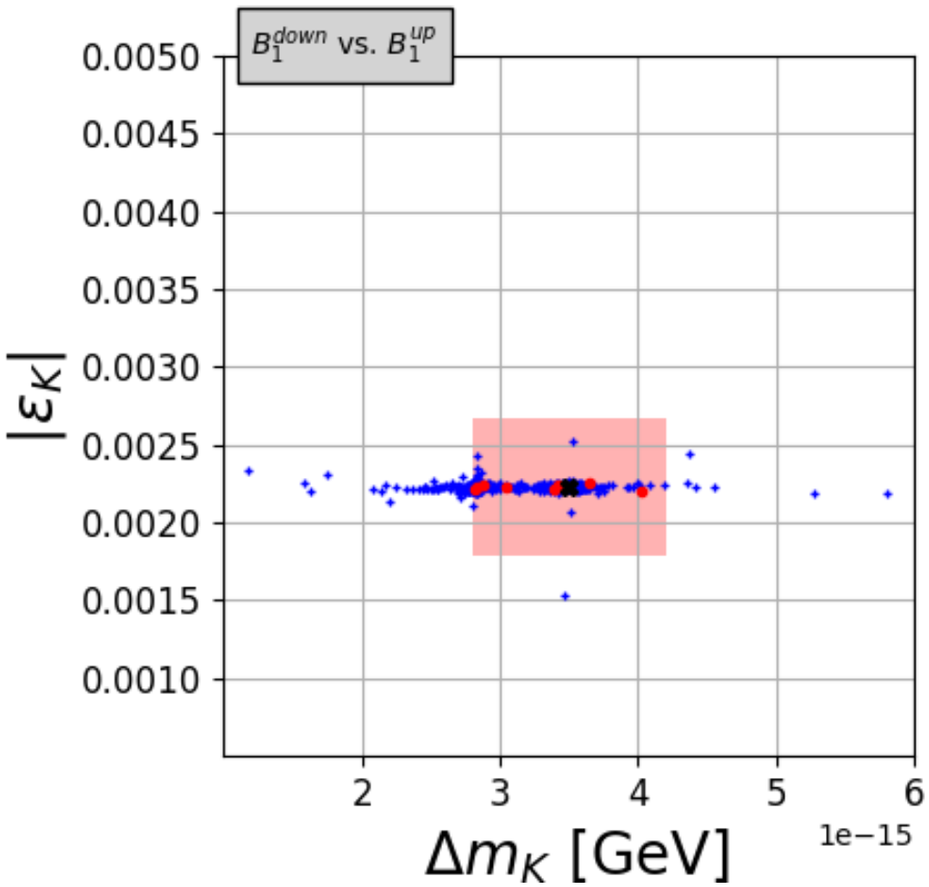


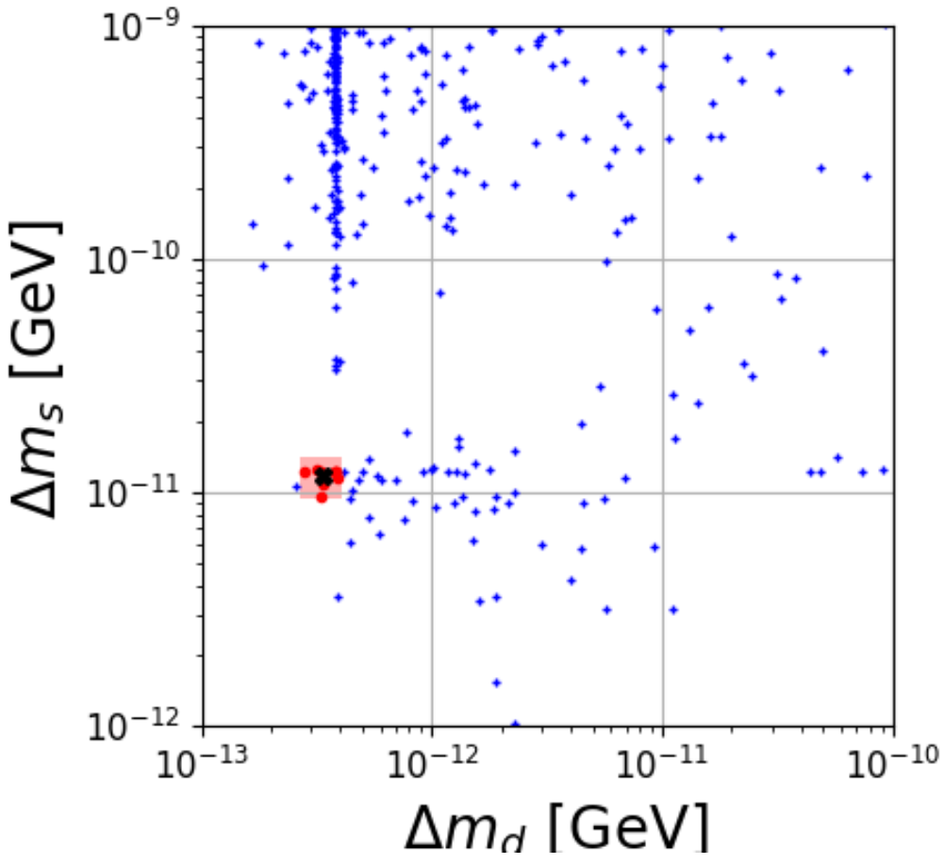


Flavour CP4 3HDM: flavour observables

Yukawa sector scan

- Fit all quark masses, mixing, and CPV phase
- Add K and B oscillation parameters $|\epsilon_K|$, Δm_K, Δm_{B_d}, Δm_{B_s}
  from Buras et al, 2013 (tree-level contributions from neutral Higges only)





Typical points have light Higgses (< 150 GeV); a few points have moderately heavy Higgses.

Example II. U(1) x U(1) 3HDM

- **The most constraining realisable Abelian symmetry of 3HDM**
  Keus, King, Moretti 2014; Ivanov, Keus, Vdovin, 2012

- **Promote this symmetry to be a family symmetry of the fermion sector**
  Camargo-Molina, Mandal, Pasechnik, Wessen, 2018

softly broken

$\mathrm{U}(1)_\mathrm{X} \times \mathrm{U}(1)_\mathrm{Z}$

- No tree-level FCNCs
- Cabbibo-like mixing at tree-level
- Fermion mass hierarchies are related to hierarchy of VEVs
- New scalar states couple dominantly to the second quark family (exotic collider signatures)

$$V_0 = -\sum_{i=1}^{3} \mu_i^2 |H_i|^2 + \sum_{i,j=1}^{3} \left(\frac{\lambda_{ij}}{2} |H_i|^2 |H_j|^2 + \frac{\lambda'_{ij}}{2} |H_i^\dagger H_j|^2 \right) , \quad V_{\text{soft}} = \sum_{i=1}^{3} \frac{1}{2} (m_{ij}^2 H_i^\dagger H_j + \text{c.c})$$

All the parameters can be taken real!

$$\lambda_{ij} = \lambda_{ji} , \quad \lambda'_{ij} = \lambda'_{ji} , \quad m_{ij}^2 = m_{ji}^2 ,$$

$$\lambda'_{11} = \lambda'_{22} = \lambda'_{33} = 0 , \quad m_{11}^2 = m_{22}^2 = m_{33}^2 = 0 .$$

Flavour U(1) x U(1) 3HDM: Yukawa sector

$$\mathcal{L}^{\mathrm{q}}_{\mathrm{Yukawa}} = \sum_{i,j=1}^{2} \left\{ y^{\mathrm{d}}_{ij} \bar{d}^{i}_{\mathrm{R}} H_1^{\dagger} Q^{j}_{\mathrm{L}} - y^{\mathrm{u}}_{ij} \bar{u}^{i}_{\mathrm{R}} \tilde{H}_2^{\dagger} Q^{j}_{\mathrm{L}} \right\} + y_{\mathrm{b}} \bar{b}_{\mathrm{R}} H_3^{\dagger} Q^{3}_{\mathrm{L}} - y_{\mathrm{t}} \bar{t}_{\mathrm{R}} \tilde{H}_3^{\dagger} Q^{3}_{\mathrm{L}} + \mathrm{c.c.}$$

	$\mathrm{U(1)_Y}$	$\mathrm{U(1)_X}$	$\mathrm{U(1)_Z}$
H_1	$\frac{1}{2}$	-1	$-\frac{2}{3}$
H_2	$\frac{1}{2}$	1	$\frac{1}{3}$
H_3	$\frac{1}{2}$	0	$\frac{1}{3}$
$Q^{1,2}_{\mathrm{L}}$	$\frac{1}{6}$	γ	δ
Q^{3}_{L}	$\frac{1}{6}$	β	α
$u^{1,2}_{\mathrm{R}}$	$\frac{2}{3}$	$1+\gamma$	$\frac{1}{3}+\delta$
t_{R}	$\frac{2}{3}$	β	$\frac{1}{3}+\alpha$
$d^{1,2}_{\mathrm{R}}$	$-\frac{1}{3}$	$1+\gamma$	$\frac{2}{3}+\delta$
b_{R}	$-\frac{1}{3}$	β	$-\frac{1}{3}+\alpha$

$v_3 \gg v_{1,2}$ ⇒ **Heavy third generation!**

The model is treatable fully analytically in this limit!

Dim-6 operators:

$$\bar{d}^{1,2}_{\mathrm{R}} \left(H_i^{\dagger} Q^{3}_{\mathrm{L}}\right) \left(H_j^{\dagger} H_k\right), \quad \bar{u}^{1,2}_{\mathrm{R}} \left(\tilde{H}_i^{\dagger} Q^{3}_{\mathrm{L}}\right) \left(H_j^{\dagger} H_k\right)$$

$$\bar{b}_{\mathrm{R}} \left(H_i^{\dagger} Q^{1,2}_{\mathrm{L}}\right) \left(H_j^{\dagger} H_k\right), \quad \bar{t}_{\mathrm{R}} \left(\tilde{H}_i^{\dagger} Q^{1,2}_{\mathrm{L}}\right) \left(H_j^{\dagger} H_k\right)$$

⇒ **Full CKM?**

$$(\beta-\gamma, \alpha-\delta) \notin \{(-1,-1), (-1,0), (0,0), (1,0), (1,1), (2,1)\}$$

Flavour U(1) x U(1) 3HDM: exotic channels

$$\xi \equiv \frac{\sqrt{v_1^2 + v_2^2}}{v_3}$$

$$\xi \ll 1$$

SM-like Higgs: $\mathcal{L} \supset \sum_q \frac{m_q}{v_3}\, \bar{q} q\, h_{125} + \mathcal{O}(\xi)$

Additional scalars' Yukawa couplings:

$$\mathcal{L} \supset \cos\theta_{\rm C} \frac{\sqrt{2} m_{\rm s}}{v_1} \bar{s}_{\rm R} c_{\rm L} H_{\rm a}^- - \cos\theta_{\rm C} \frac{\sqrt{2} m_{\rm c}}{v_2} \bar{c}_{\rm R} s_{\rm L} H_{\rm b}^+ + {\rm c.c.} + \mathcal{O}(\xi)$$
$$+ \frac{m_{\rm s}}{v_1} \bar{s} s h_{\rm a} - \frac{m_{\rm c}}{v_2} \bar{c} c h_{\rm b} + {\rm i} \frac{m_{\rm s}}{v_1} \bar{s} \gamma^5 s A_{\rm a} - {\rm i} \frac{m_{\rm c}}{v_2} \bar{c} \gamma^5 c A_{\rm b} + \mathcal{O}(\xi)\,,$$

Charged Higgses are mostly produced via $c\bar{s}$ fusion!

Main focus: $c\bar{s} \to H^+ \to W^+ h_{125}$ $\qquad m_{H^\pm} > 200$ GeV

Flavour U(1) x U(1) 3HDM: results

$$pp \to H^\pm \to W^\pm h_{125} \to \ell^\pm + \not{E}_T + b\bar{b}$$

Typical backgrounds:

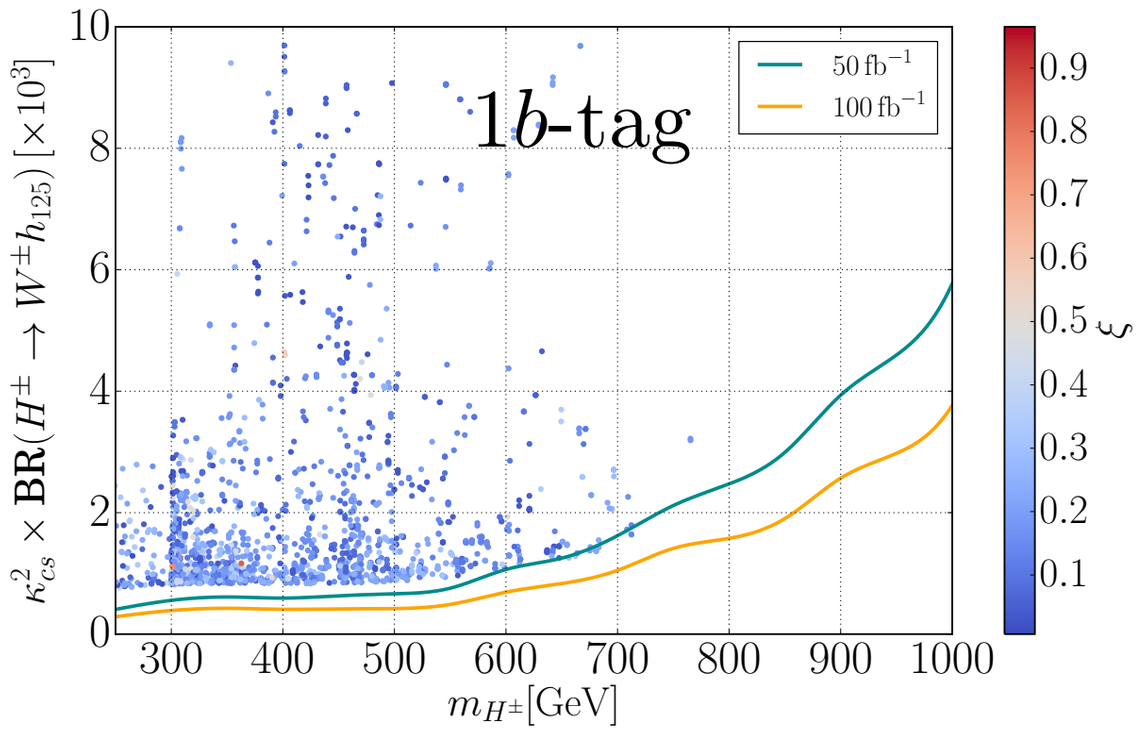
LHC ($\sqrt{s} = 13$ TeV)

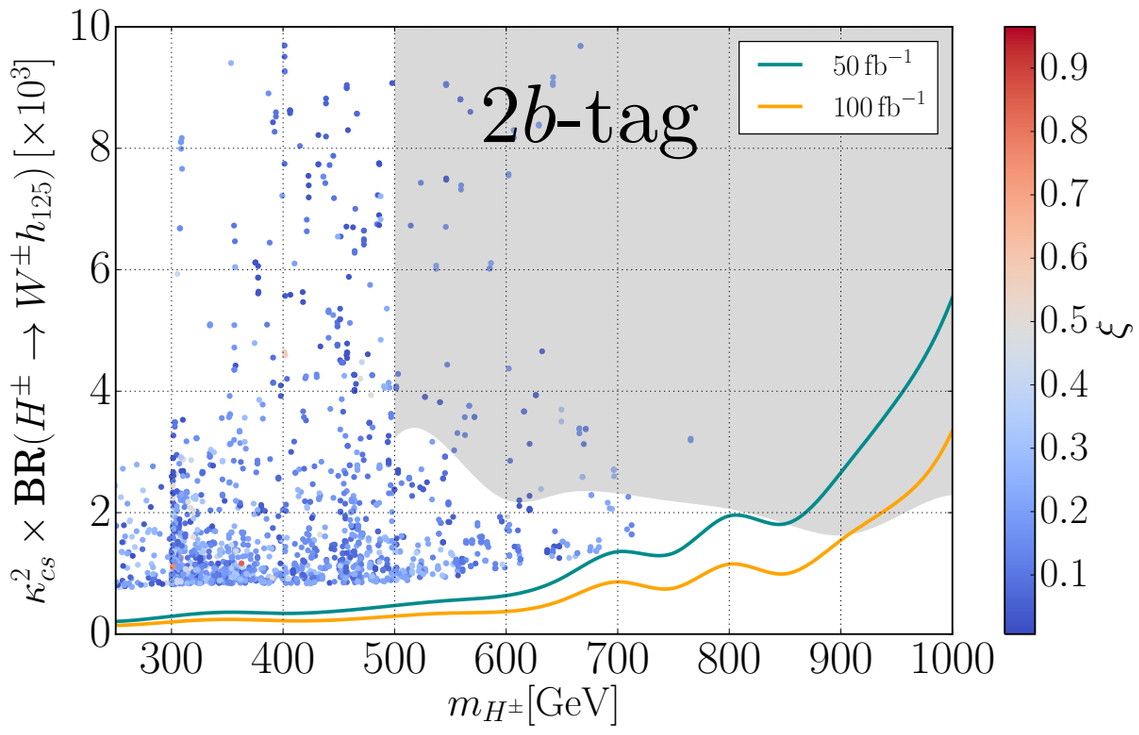
Process	$W + n\ j$	Wbj	$Wb\bar{b}$	$t\bar{t} + n\ j$	tj	tb	tW	WW	WZ	Wh_{125}
x-sec (pb)	1.53×10^5	308.9	41.7	431.3	174.6	2.6	54.0	67.8	25.4	1.1

Selection cuts:

- Lepton: $p_T(\ell) > 25$ GeV, $|\eta(\ell)| < 2.5$
- Jet: $p_T(J) > 25$ GeV, $|\eta(J)| < 4.5$
- Missing transverse energy: $\not{E}_T > 25$ GeV
- ΔR separation: $\Delta R(J_1, J_2) > 0.4$, $\Delta R(\ell, J) > 0.4$

5σ discovery contours





"Top-down" approach

- **Postulate**: a consistent SUSY GUT valid at high energy scales

- **Explore**: the larger the symmetry, the fewer free parameters, the higher degree of unification

- **Study**: to what extent the SM emerges as an EFT at low energies. Anomalies of the visible sector with additional gauge groups in the EFT need not necessarily cancel

- **Hope**: to explain seemingly arbitrary features of the SM (number of families, mass/mixing hierarchies, neutrino status, cancellation of gauge anomalies in the observed sectors etc)

Any promising candidates?

Example III. GUT-inspired Higgs sectors

Our proposal: Extend the SUSY trinification model (Georgi, Glashow and De Rujula 1984) with a local family $\mathrm{SU}(3)_\mathrm{F}$ symmetry

A. Morais, A. Ordell, J.-E. Camargo-Molina, RP, J. Wessen, PRD95 (2017) 075031

$$[\mathrm{SU}(3)_\mathrm{C} \times \mathrm{SU}(3)_\mathrm{L} \times \mathrm{SU}(3)_\mathrm{R}] \rtimes \mathbb{Z}_3 \times \mathrm{SU}(3)_\mathrm{F}$$

We refer to the model as **Supersymmetric Higgs-Unified Trinification**

- Use the minimal field content:

$$(1,\mathbf{3},\overline{\mathbf{3}},\mathbf{3}) = \left(L^i\right)^l{}_r = \begin{pmatrix} H_\mathrm{u}^0 & H_\mathrm{d}^- & e_\mathrm{L} \\ H_\mathrm{u}^+ & H_\mathrm{d}^0 & \nu_\mathrm{L} \\ e_\mathrm{R} & \nu_\mathrm{R} & \phi \end{pmatrix}^i, \quad (\mathbf{3},\overline{\mathbf{3}},1,\mathbf{3}) = \left(Q_\mathrm{L}^i\right)^x{}_l = \begin{pmatrix} u_\mathrm{L}^x & d_\mathrm{L}^x & D_\mathrm{L}^x \end{pmatrix}^i,$$

$$(\overline{\mathbf{3}},1,\mathbf{3},\mathbf{3}) = \left(Q_\mathrm{R}^i\right)^r{}_x = \begin{pmatrix} u_{\mathrm{R}x}^c & d_{\mathrm{R}x}^c & D_{\mathrm{R}x}^c \end{pmatrix}^{\top\, i}.$$

- Higgs and leptons unified in L due to SUSY

full unification of Yukawa couplings

- $W_1 = \lambda_{27}\,\varepsilon_{ijk}\left(Q_\mathrm{L}^i\right)^x{}_l \left(Q_\mathrm{R}^j\right)^r{}_x \left(L^k\right)^l{}_r$
  - > **One** family of quarks and **all** leptons massless at tree-level $\rightarrow$ **radiatively generated**,
  - > Exact Yukawa unification for all three families.

- $\mathrm{SU}(3)_\mathrm{F}$ also fits neatly into an $\mathrm{E}_8 \subset \mathrm{E}_6 \times \mathrm{SU}(3)_\mathrm{F}$ embedding

$\mathbf{SU(3)_F \times E_6}$ $\qquad \mathbf{248} = (\mathbf{8},\mathbf{1}) \oplus (\mathbf{1},\mathbf{78}) \oplus (\mathbf{3},\mathbf{27}) \oplus (\overline{\mathbf{3}},\overline{\mathbf{27}})$ $\quad \Rightarrow$ **No gauge anomalies**.

On the top-bottom path down to a SM-like theory

non-SUSY multi-scalar and multi-fermion models

Q-GUT:
$[\mathrm{SU(3)_C} \times \mathrm{SU(3)_L} \times \mathrm{SU(3)_R}] \times \mathrm{SU(3)_F}$

Region - I $\left\langle \widetilde{\Delta}^8_{\mathrm{L,R,F}} \right\rangle \sim M_{\mathrm{GUT}}$

SUSY stabilizes the $M_{\mathrm{GUT}} \gg M^{(1)}_{\mathrm{soft}}$ hierarchy.

SUSY-LRF:
$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{SU(2)_F} \times \mathrm{U(1)_L} \times \mathrm{U(1)_R} \times \mathrm{U(1)_F}$

Region - II $\left\langle \widetilde{\phi}^3 \right\rangle \sim M^{(1)}_{\mathrm{soft}} \ll M_{\mathrm{GUT}}$

$T_{\mathrm{L+R}} = T^8_{\mathrm{L}} + T^8_{\mathrm{R}}\,, \quad T_{\mathrm{S}} = T^8_{\mathrm{L}} - T^8_{\mathrm{R}} - 2T_{\mathrm{F}}$

non-SUSY-LRF:
$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{SU(2)_F} \times \mathrm{U(1)_{L+R}} \times \mathrm{U(1)_S}$

Region - III $\left\langle \widetilde{\phi}^2 \right\rangle \sim M^{(2)}_{\mathrm{soft}} \lesssim M^{(1)}_{\mathrm{soft}}$

$T_{\mathrm{V}} = T^3_{\mathrm{F}} - \frac{1}{2\sqrt{2}} T_{\mathrm{S}}$

non-SUSY-LR:
$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{SU(2)_R} \times \mathrm{U(1)_{L+R}} \times \mathrm{U(1)_V}$

Region - IV $\left\langle \widetilde{\nu}^1_{\mathrm{R}} \right\rangle \sim M^{(3)}_{\mathrm{soft}} \lesssim M^{(2)}_{\mathrm{soft}}$

$T_{\mathrm{Y}} = \sqrt{3} T^3_{\mathrm{R}} + T_{\mathrm{L+R}}\,, \quad T_{\mathrm{T}} = T_{\mathrm{V}} - \frac{3}{2} T^3_{\mathrm{R}}$

3HDM EFTs:
$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y} \times \mathrm{U(1)_T}$

Region - V $\langle S \rangle \sim m_{z'} \lesssim M^{(3)}_{\mathrm{soft}}$

3HDM EFTs:
$\mathrm{SU(3)_C} \times \mathrm{SU(2)_L} \times \mathrm{U(1)_Y}$

Natural hierarchy between GUT and EW scales!

All symmetry breaking scales (including the elecro-weak) except v_{GUT} are controlled by SSB parameters $\Rightarrow$ No μ-problem!

Why 3HDMs?

After Q-GUT breaking the SUSY-LRF theory reads

Introduce soft SUSY breaking sector!

$$W = \varepsilon_{ijk}\left\{y_{1-3}\Phi^i \boldsymbol{D}_{\rm L}{}^j \boldsymbol{D}_{\rm R}{}^k + y_{4-6}(\boldsymbol{H}^i)^L{}_R(\boldsymbol{q}_{\rm L}{}^j)_L(\boldsymbol{q}_{\rm R}{}^k)^R \right.$$
$$\left. + y_{7-9}(\boldsymbol{E}_{\rm L}{}^i)^L(\boldsymbol{q}_{\rm L}{}^j)_L \boldsymbol{D}_{\rm R}{}^k + y_{10-12}(\boldsymbol{E}_{\rm R}{}^i)_R \boldsymbol{D}_{\rm L}{}^j(\boldsymbol{q}_{\rm R}{}^k)^R\right\} .$$

The simplest scenario that automatically provides CKM mixing with Cabibbo form.

$$\langle L^1\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \omega & 0 \end{pmatrix} \qquad \langle L^2\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} v_2 & 0 & 0 \\ 0 & v_3 & 0 \\ 0 & 0 & f \end{pmatrix} \qquad \langle L^3\rangle \sim \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & p \end{pmatrix} .$$

$$M_{\rm EW} \sim v_{1,2,3} \ll \omega \lesssim f \lesssim p \lll M_{\rm GUT}, \qquad (p, f, \omega) \sim M_{\rm soft}^{(1,2,3)}$$

Classical approach: (y_{1-12} matched to λ_{27}) Cabibbo form readily at tree-level

$$m^2_{\rm c,t} = \frac{1}{2}\lambda^2_{27}({v_1}^2 + {v_2}^2), \quad m^2_{\rm b} = 3m^2_{\rm s} = \frac{1}{2}\lambda^2_{27}v_3^2, \quad m^2_{\rm u,d} = 0, \quad \tan\theta_{\rm C} = \frac{v_1}{v_2}$$
$$m^2_{\rm B} = \frac{1}{2}\lambda^2_{27}(2p^2 + f^2 + \omega^2), \quad m^2_{\rm S} = \frac{1}{2}\lambda^2_{27}(p^2 + f^2), \quad m^2_{\rm D} = \frac{1}{2}\lambda^2_{27}\omega^2.$$

The impact of quantum corrections: high scales

High-scale loop-level matching + RG evolution (before U(1) family breaking)

$$\mathcal{M}^{\rm u} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -v_2 y_{\rm u}^{(2)} \\ 0 & 0 & v_1 y_{\rm u}^{(1)} \\ v_2 y'^{(2)}_{\rm u} & -v_1 y'^{(1)}_{\rm u} & 0 \end{pmatrix}$$

$$\mathcal{M}^{\rm d} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & -a_{11} v_{\rm d} y_{\rm d}^{(2)} \\ 0 & 0 & 0 \\ v_{\rm d} y'^{(2)}_{\rm d} & 0 & 0 \end{pmatrix}$$

Cabbibo mixing

$$\tan\theta_{\rm C} = \frac{y_{\rm u}^{(1)} v_1}{y_{\rm u}^{(2)} v_2},$$

Mass-degeneracies are gone!

$$\frac{m_{\rm t}^2}{m_{\rm c}^2} = \frac{{y'^{(1)}_{\rm u}}^2 {v_1}^2 + {y'^{(2)}_{\rm u}}^2 {v_2}^2}{{y_{\rm u}^{(1)}}^2 {v_1}^2 + {y_{\rm u}^{(2)}}^2 {v_2}^2}$$

$$\frac{m_{\rm b}^2}{m_{\rm s}^2} = \frac{{y'^{(2)}_{\rm d}}^2}{a_{11}^2 {y_{\rm d}^{(2)}}^2},$$

Massless u,d

$$m^2_{\rm u,d} = 0.$$

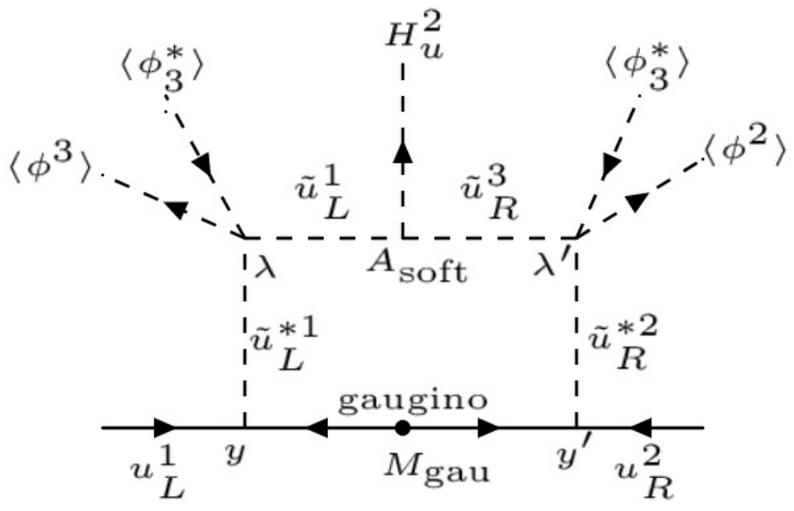


Figure: Example of radiative generation of Yukawa interactions: $y_u^{(2)} H_u^2 u_{\rm L}^1 u_{\rm R}^2$

Fit to the quark masses and Cabbibo angle:

$$\mathcal{M}^{\rm u} = \begin{pmatrix} 0 & 0 & -0.58 \\ 0 & 0 & 0.14 \\ 174 & 0.2 & 0 \end{pmatrix}, \quad \mathcal{M}^{\rm d} = \begin{pmatrix} 0 & 0 & 0.058 \\ 0 & 0 & 0 \\ 4.1 & 0 & 0 \end{pmatrix}$$

$$V_{\rm CKM} \simeq \begin{pmatrix} -0.972 & 0.235 & 0 \\ 0.235 & 0.972 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$m_t = 174\ {\rm GeV},\ m_c = 0.597\ {\rm GeV}\,,\ m_u = 0\,,$$
$$m_b = 4.1\ {\rm GeV},\ m_s = 0.058\ {\rm GeV},\ m_d = 0\,.$$

Loop-matching for Yukawas is very important!

The impact of quantum corrections: high and TeV scales

loop-level matching + RG evolution at both high and low scales

$$M_u = \begin{pmatrix} -6.0 & 0.005 & -0.8 \\ 0.25 & 0.000011 & 0.116 \\ 171 & 0.2 & 0.01 \end{pmatrix}$$

$$M_d = \begin{pmatrix} 0 & 0.03 & 0.056 \\ 0.003 & 0.0078 & 0.0035 \\ 2.6 & 1.2 & 0.39 \end{pmatrix}$$

$$V_{\rm CKM} \simeq \begin{pmatrix} -0.974 & -0.225 & -0.003 \\ 0.225 & -0.973 & 0.042 \\ 0.007 & 0.041 & 0.999 \end{pmatrix}$$

$$m_t = 171.1\ \text{GeV},\ m_c = 0.807\ \text{GeV}\ , m_u = 1.45\ \text{MeV}$$
$$m_b = 2.89\ \text{GeV},\ m_s = 0.061\ \text{GeV},\ m_d = 3.98\ \text{MeV}\ .$$

- It is possible to get one generation of VLQ close to TeV scale $m_{\rm D} \propto y_{\rm D}\omega$
- Charged lepton masses all loop-generated: 3 chiral, $m_{\rm chi} \propto v_{1,2,3}$, and 3 vector-like, $m_{\rm vec} \propto \omega, f, p$.
- Neutrino masses contain both loop and See-Saw effects.

Work in progress..

Summary

- **Several benchmark family-nonuniversal 3HDMs have been proposed**

- **Made a first few steps, much more work is needed towards both phenomenological validation and new yet unexplored signatures**

- **There is a potentially large benefit from synergies between "top-down" and "bottom-up" approaches in the search for the most optimal multi-Higgs extension of the SM**