Soft Heisenberg Hair

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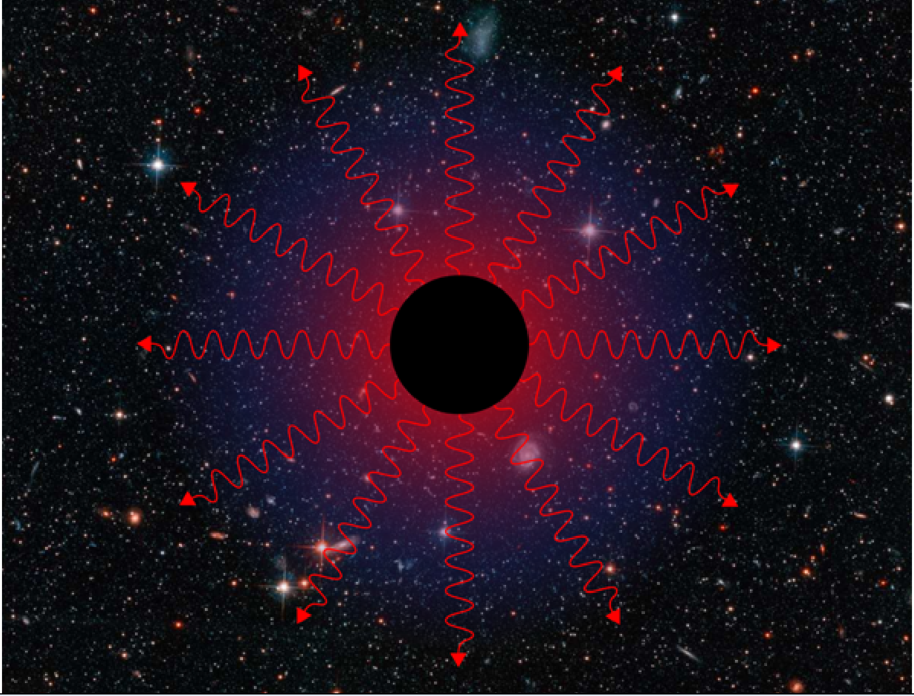

Two simple punchlines

1. Heisenberg algebra

$$[X_n,\, P_m] = i\,\delta_{n,\,m}$$

fundamental not only in quantum mechanics
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at least in three spacetime dimensions

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2. Black hole microstates identified as specific "soft hair" descendants
at least in three spacetime dimensions

based on work (2016-2018) with

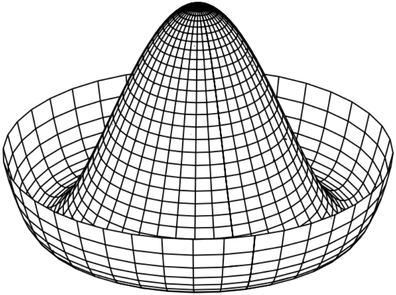
- Hamid Afshar, Shahin Sheikh-Jabbari, Zahra Mirzaiyan [IPM Teheran]
- Martin Ammon [U. Jena]
- Stephane Detournay, Wout Merbis, Stefan Prohazka, Max Riegler [ULB]
- Hernan Gonzalez, Philip Hacker, Raphaela Wutte, Céline Zwikel [TU Wien]
- Alfredo Perez, David Tempo, Ricardo Troncoso [CECS Valdivia]
- Hossein Yavartanoo [ITP Beijing]

Outline

Taxonomy of symmetries

- Discrete

- Continuous



Taxonomy of symmetries

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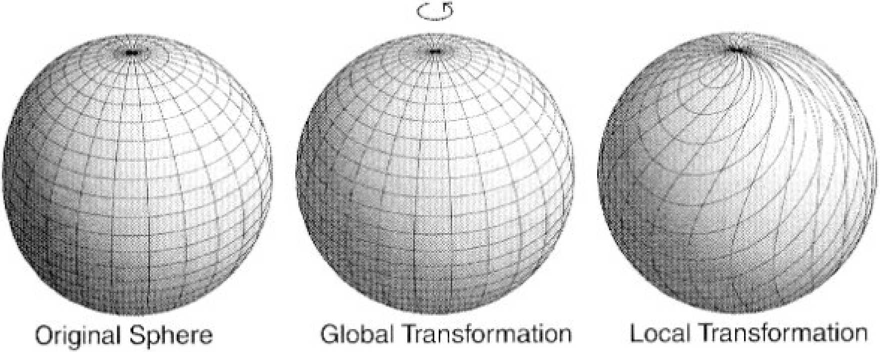
- Global

- Continuous
- Local



Taxonomy of symmetries

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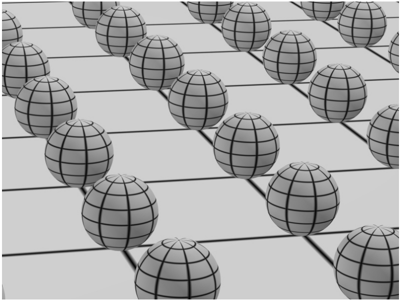
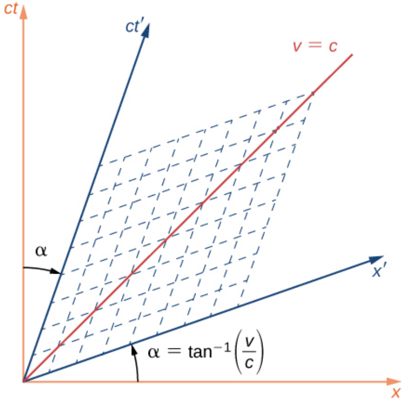


- Continuous
- Local
- Spacetime



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Main interest of this talk: asymptotic symmetries!

Definition of asymptotic symmetries

All boundary condition preserving gauge transformations (bcpgt's) modulo trivial gauge transformations

Let us break this down for electrodynamics (abelian connection A):

Asymptotic symmetries

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- Gauge transformations generated by ϵ

$$A \to A + \mathrm{d}\epsilon$$

Asymptotic symmetries

Definition of asymptotic symmetries

All boundary condition preserving gauge transformations modulo trivial gauge transformations

Let us break this down for electrodynamics (abelian connection A):

- Gauge transformations generated by ϵ
- Boundary condition

  For instance

$$A|_{r\to\infty} = \mathcal{O}(1)$$

  where r = radial coordinate; asymptotic boundary: $r \to \infty$

Asymptotic symmetries

Definition of asymptotic symmetries

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Let us break this down for electrodynamics (abelian connection A):

- Gauge transformations generated by ϵ
- Boundary condition
- Boundary condition preserving gauge transformations

  All gauge transformations generated by some ϵ that preserve the boundary conditions, e.g.

$$\epsilon = \epsilon(t, \theta, \varphi) + \mathcal{O}(1/r) \text{ as } r \to \infty$$

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All boundary condition preserving gauge transformations modulo trivial gauge transformations

Let us break this down for electrodynamics (abelian connection A):

- Gauge transformations generated by ϵ
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- Boundary condition preserving gauge transformations
- Trivial gauge transformations
  All gauge transformations generated by some ϵ that fall off sufficiently fast near the boundary

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What is the precise meaning of sufficiently fast?

Canonical boundary charges

- Field theories with gauge symmetries lead to constraints $G[\epsilon]$

  Example: Gauss constraint in electrodynamics

$$G[\epsilon] = -\int_\Sigma \mathrm{d}^3x\, \epsilon(x)\, \partial_i E^i(x)$$

Canonical boundary charges

- Field theories with gauge symmetries lead to constraints $G[\epsilon]$
- Constraints in general not functionally differentiable
  (i.e., their variation leads to total derivative terms)

$$\delta G[\epsilon] = \text{volume term} - \delta Q[\epsilon]$$

Example:

$$\delta G[\epsilon] = \int_\Sigma \mathrm{d}^3x\, \big(\partial_i \epsilon(x)\big) \delta E^i(x) - \oint_{\partial\Sigma} \mathrm{d}^2x_i\, \epsilon(x)\, \delta E^i(x)$$
$$= \text{volume term} - \oint_{\partial\Sigma} \mathrm{d}^2x_i\, \epsilon(x)\, \delta E^i(x)$$

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- Improved generator $\Gamma[\epsilon]$ is functionally differentiable

$$\delta \Gamma[\epsilon] = \delta G[\epsilon] + \delta Q[\epsilon] = \text{volume term}$$

Example:

$$\delta Q[\epsilon] = \oint_{\partial\Sigma} \mathrm{d}^2 x_i \, \epsilon(x) \, \delta E^i(x)$$

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- Integrate $\delta Q[\epsilon]$ in field space to obtain canonical boundary charges

$$\delta Q[\epsilon] \qquad \Rightarrow \qquad Q[\epsilon]$$

Example:

$$Q[\epsilon] = \oint_{\partial\Sigma} \mathrm{d}^2 x_i \, \epsilon(x) \, E^i(x)$$

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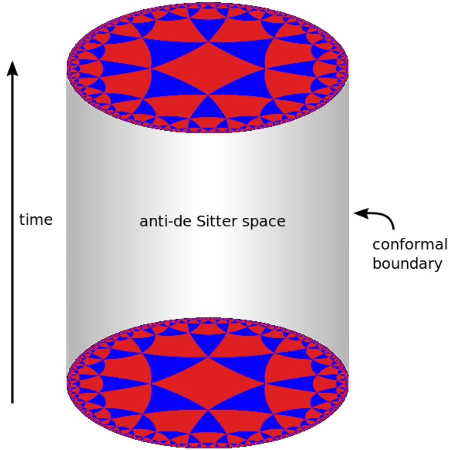
Definition of asymptotic symmetry algebra

Quotient algebra of all bcpgt's modulo trivial gauge transformations

AdS_3/CFT_2 example

Brown & Henneaux '86



AdS_3 Penrose diagram (Wikipedia)

Note: set AdS-radius to unity

- Impose AdS_3 fall-off on metric

$$\mathrm{d}\rho^2 + \big(e^{2\rho}\,\eta_{\mu\nu} + t_{\mu\nu} + \mathcal{O}(e^{-2\rho})\big)\;\mathrm{d}x^\mu\,\mathrm{d}x^\nu$$

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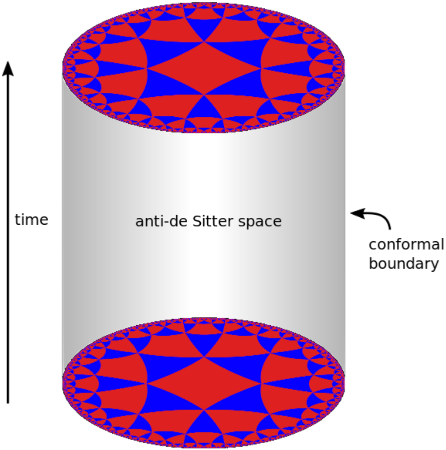

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- Determine bcpgt's

$$\mathcal{L}_\xi g_{\mu\nu} = \delta t_{\mu\nu} + \mathcal{O}(e^{-2\rho})$$

In lightcone gauge:

$$\xi^\pm = \epsilon^\pm(x^\pm) + \mathcal{O}(e^{-2\rho})$$

$\epsilon^\pm$: asymptotic symmetries
$\mathcal{O}(e^{-2\rho})$: trivial gauge symm.

AdS$_3$/CFT$_2$ example

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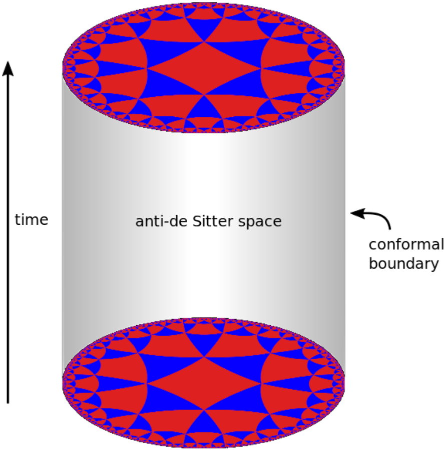

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- Determine boundary charges

$$Q^\pm[\epsilon^\pm] \propto \oint \mathrm{d}\varphi\,\epsilon^\pm\,t_{\pm\pm}$$

Introduce Fourier modes:

$$L_n^\pm = Q^\pm[e^{inx^\pm}]$$

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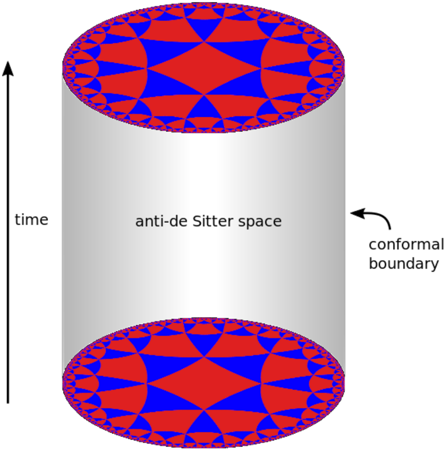

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$$[L_n^\pm,\,L_m^\pm] = (n-m)\,L_{n+m}^\pm + \frac{1}{4G}\,n^3\,\delta_{n+m,\,0}$$

AdS$_3$/CFT$_2$ example

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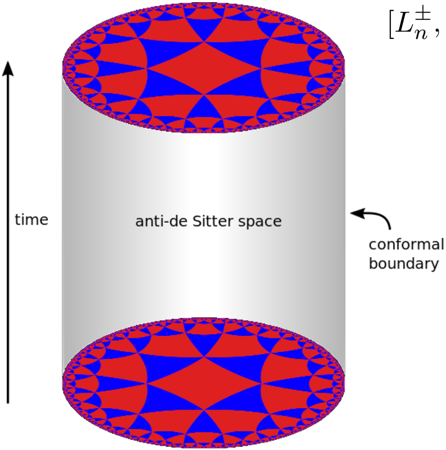

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consists of two Virasoro algebras with central charge

$$c = \frac{3}{2G}$$

(for Einstein gravity)

$\mathrm{AdS}_3/\mathrm{CFT}_2$ example

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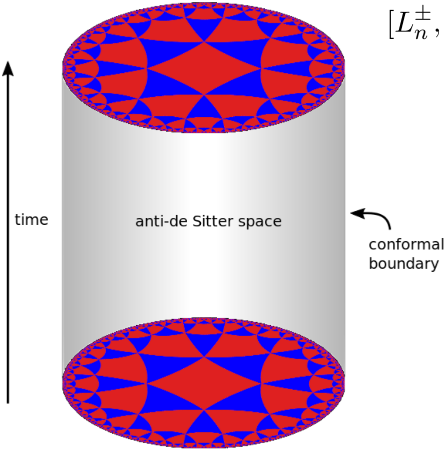

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- $=$ Conformal algebra in 2d

Physical Hilbert space falls into representations of two copies of Virasoro $=$ physical Hilbert space of some CFT_2!

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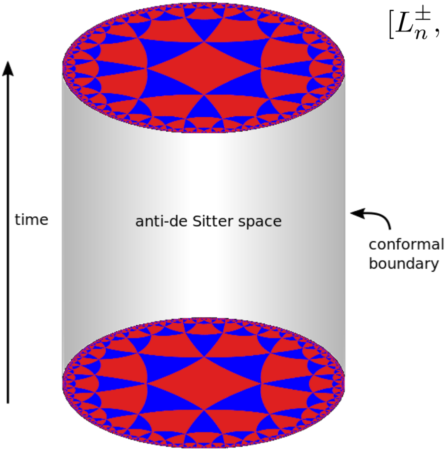

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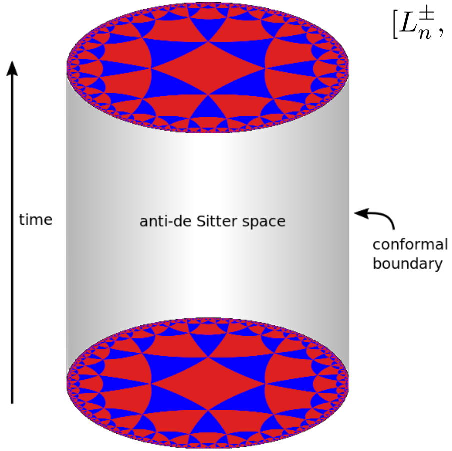

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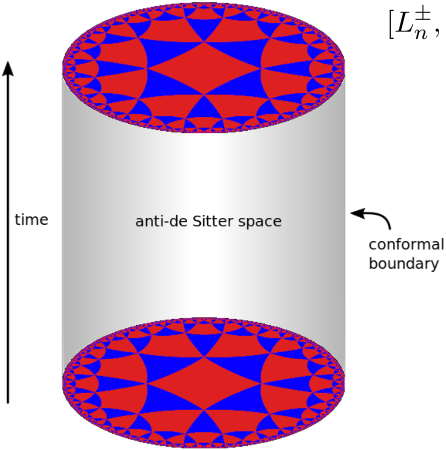

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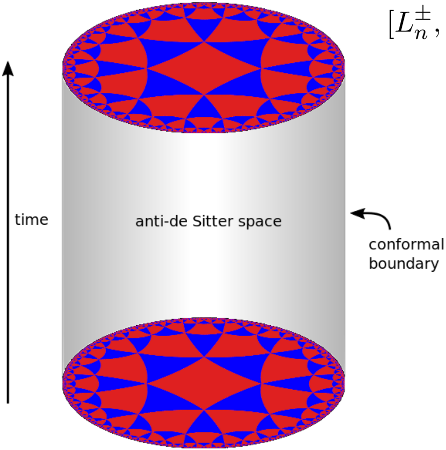

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- $=$ Conformal algebra in 2d
- Precursor of AdS/CFT!
- Generalizable to other theories
- or to other asymptotic backgrounds
- or when relevant boundary is not asymptotic

Motivation for near horizon boundary conditions

Old idea by Carlip

Main idea

Impose existence of non-extremal horizon as boundary condition on state space

Motivations:

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- Want to ask conditional questions "given a black hole, what are the probabilities for some scattering process"

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$$S_{\mathrm{BH}} = \frac{A}{4G} + \mathcal{O}(\ln(A/G))$$

1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Explicit form of near horizon boundary conditions

See Donnay, Giribet, Gonzalez, Pino '15 and Afshar et al '16

Postulates of near horizon boundary conditions:

Explicit form of near horizon boundary conditions

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Postulates of near horizon boundary conditions:

1. Rindler approximation

$$\mathrm{d}s^2 = -\kappa^2\rho^2\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \Omega_{ab}(t,\,x^c)\,\mathrm{d}x^a\,\mathrm{d}x^b + \dots$$

$\rho \to 0$: Rindler horizon
κ: surface gravity
Ω_{ab}: metric transversal to horizon
$\dots$: terms of higher order in ρ or rotation terms

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$$\delta\kappa = 0$$

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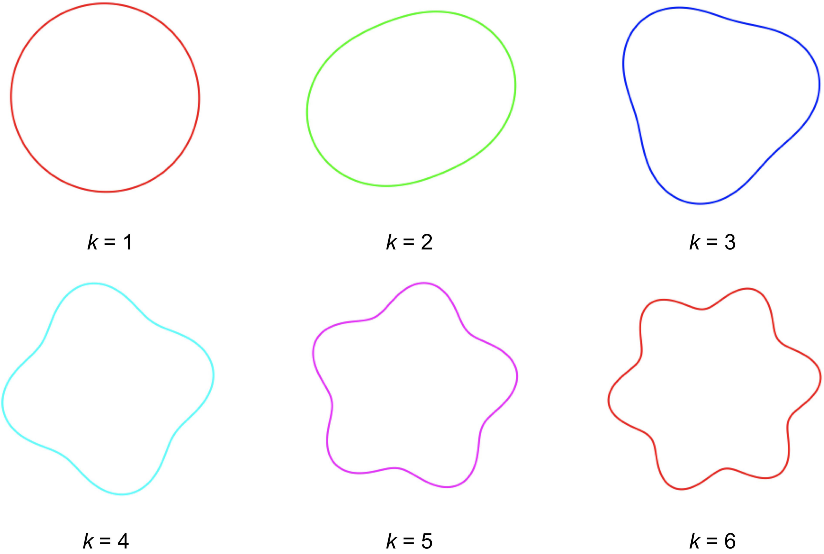
4. Remaining terms fixed by consistency of canonical boundary charges

Black holes can be deformed into black flowers Afshar et al. 16

Horizon can get excited by area preserving shear-deformations



Near horizon symmetries = “asymptotic symmetries” for near horizon bc’s

Restrict for the time being to AdS_3 black holes (BTZ)

Simplification in 3d:

$$\mathrm{d}s^2 = \Big[-\kappa^2 \rho^2 \, \mathrm{d}t^2 + \mathrm{d}\rho^2 + \gamma^2(\varphi) \, \mathrm{d}\varphi^2 + 2\kappa \, \omega(\varphi) \, \rho^2 \, \mathrm{d}t \, \mathrm{d}\varphi \Big] \big(1 + \mathcal{O}(\rho^2)\big)$$

- Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi) \, \mathrm{d}\varphi = \mathrm{d}\tilde{\varphi}$

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- Trivial or non-trivial?
  Answer provided by boundary charges!

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- Non-trivial diffeo!
- Canonical analysis yields

$$Q^\pm[\epsilon^\pm] \sim \oint \mathrm{d}\varphi\,\epsilon^\pm(\varphi)\,\big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

where $\epsilon^\pm$ are functions appearing in asymptotic Killing vectors

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- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm,\,\mathcal{J}_m^\pm] = \tfrac{1}{2}\,n\,\delta_{n+m,\,0}$$

Two $u(1)$ current algebras! Afshar et al. 16

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- Map from round S^1 to Fourier-excited S^1: diffeo $\gamma(\varphi)\, \mathrm{d}\varphi = \mathrm{d}\tilde{\varphi}$
- Non-trivial diffeo!
- Canonical analysis yields

$$Q^\pm[\epsilon^\pm] \sim \oint \mathrm{d}\varphi\, \epsilon^\pm(\varphi)\, \big(\gamma(\varphi) \pm \omega(\varphi)\big)$$

- Near horizon symmetry algebra Fourier modes $\mathcal{J}_n^\pm = Q^\pm[\epsilon^\pm = e^{in\varphi}]$

$$[\mathcal{J}_n^\pm,\, \mathcal{J}_m^\pm] = \tfrac{1}{2}\, n\, \delta_{n+m,\,0}$$

- Isomorphic to Heisenberg algebras plus center

$$[X_n,\, P_m] = i\, \delta_{n,m} \qquad\qquad [P_0,\, X_n] = 0 = [X_0,\, P_n]$$

$P_0 = \mathcal{J}_0^+ + \mathcal{J}_0^-$, $X_n = \mathcal{J}_n^+ - \mathcal{J}_{-n}^-$, $P_n = 2i/n(\mathcal{J}_{-n}^+ + \mathcal{J}_n^-)$ for $n \neq 0$

Soft Heisenberg hair for BTZ

- Black flower excitations = hair of black holes
  Algebraically, excitations from descendants

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> Call it "soft Heisenberg hair"

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  Note: entropy is observer independent, so can be calculated for asymptotic or near horizon observer and results must agree

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with Hawking–Unruh-temperature

$$T = \frac{\kappa}{2\pi}$$

δ refers to any variation of phase space variables allowed by the boundary conditions

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Can we understand entropy law microscopically?

Semi-classical microstates?

Given our soft Heisenberg hair, attack now entropy questions

1. Why only semi-classical input for entropy?
2. What are microstates?
3. Semi-classical construction of microstates?
4. Does counting of microstates reproduce S_{BH}?

Regarding 1. and 3.: may expect decoupling of scales so that description of microstates does not need info about UV completion, but rather only some semi-classical "Bohr-like" input

Evidence for this: universality of BH entropy for large black holes

$$S_{\text{BH}} = \frac{A}{4G} + \dots$$

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- possible resolution: do not consider asymptotic but near horizon observer (i.e., employ near horizon bc's and symmetry algebra)

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Semi-classical BTZ black hole microstates as near horizon descendants of vacuum

Highest weight vacuum $|0\rangle$

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derived from Bohr-type quantization conditions

- quantization of central charge $c = 3/(2G)$ in integers
- quantization of conical deficit angles in integers over c
- black hole/particle correspondence
  (black hole = gas of coherent states of particles on AdS_3)

Check of fluff proposal

Microstates for BTZ black hole with mass M and angular momentum J:

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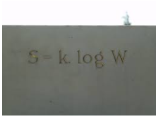

(we set $k = 1$ and $W = p$)

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  may work more generally, but so far only checked BTZ black hole; extremal black holes?

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Thanks for your attention!