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QCD theta term and holography

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Plan

- The θ-angle in Yang-Mills and QCD
- Holographic Yang-Mills at finite θ-angle
- Holographic QCD at finite θ-angle

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Effects of the θ parameter interesting but challenging.

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2}\theta\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2}\theta\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

- θ term breaks P,T and hence CP.
- θ multiplies topological charge density, whose 4d integral is integer.
- Physics periodic under $\theta \longrightarrow \theta + 2\pi$
- Hard problem: need non-perturbative tools to study θ dependence

The θ-angle in Yang-Mills

- Euclidean Lagrangian

$$\mathcal{L}_\theta = \frac{1}{2g_{YM}^2} \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2} \theta \, \epsilon^{\mu\nu\rho\sigma} \, \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

- Effects: vacuum structure, CP violating effects in QGP [Kharzeev et al], mass and interactions of η' meson in QCD [Witten-Veneziano], cosmology, axions

- Challenging on the Lattice (sign problem: imaginary term)
- Can get results only for few terms in expansion around θ ≈ 0.

Lattice results

[Pisa Lattice QCD group: Bonati, D'Elia, Vicari et al; years 2006-present]

- Ground state energy density

$$f(\theta) - f(0) = \frac{1}{2}\chi_g \theta^2 \left[1 + \bar{b}_2 \frac{\theta^2}{N_c^2} + \bar{b}_4 \frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right] \quad \chi_g = f''(\theta)|_{\theta=0} \neq 0 \text{ topological susceptibility}$$

$$\boxed{\bar{b}_2 \approx -0.2} \qquad \boxed{b_4 \equiv \frac{\bar{b}_4}{16} \approx 6(2) \cdot 10^{-4}} \text{ (N}_c\text{=2, new 2018)} \qquad |b_4| < 0.001 \text{ (Nc=3)}$$

- String tension

$$T_s(\theta) = T_s(0)\left[1 + \bar{s}_2 \frac{\theta^2}{N_c^2} + \mathcal{O}(\theta^4)\right], \quad \bar{s}_2 \approx -0.9 \text{ (from N}_c\text{=3,...,6 data)}$$

- Lowest 0^{++} glueball mass

$$M_{0^{++}}(\theta) = M_{0^{++}}(0)\left[1 + g_2\theta^2 + \mathcal{O}(\theta^4)\right], \quad g_2 \approx -0.06(2), \quad (N_c = 3)$$

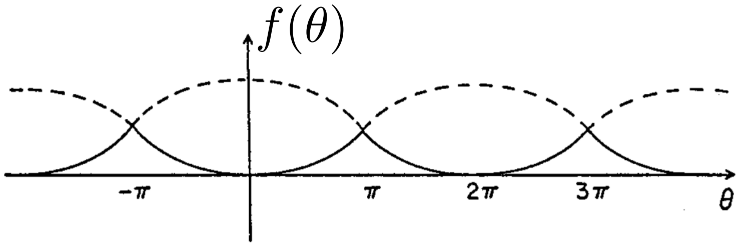
- Deconfinement temperature

$$T_c(\theta) = T_c(0)\left[1 + R_\theta\theta^2 + \mathcal{O}(\theta^4)\right], \quad R_\theta \approx -0.0175(7), \quad (N_c = 3)$$

The θ-angle in large N Yang-Mills

- $\mathcal{L} = \frac{N_c}{2\lambda}\left[\mathrm{Tr}F^2 - i\frac{\lambda}{8\pi^2}\frac{\theta}{N_c}\mathrm{Tr}F\tilde{F}\right]$ 't Hooft limit: must take θ/N fixed.

- θ/N scaling + periodicity in θ: multi-branched energy density f(θ)[Witten, 1980]



$$f(\theta) = b\,N^2 \min_k \left(\frac{\theta + 2k\pi}{N}\right)^2 + O\,(\theta^4/N^4)$$

- First order transitions at θ=(2k+1)π.
- CP spontaneously broken at θ=(2k+1)π
- Hence there will be domain walls

Theta term in QCD

a) Massless quarks:

$$\psi \to e^{i\alpha\gamma_5}\,\psi \qquad [d\psi][d\bar{\psi}] \to \exp\left(\frac{-i\,\alpha\,g^2\,N_f}{32\pi^2}\int d^4x\,\epsilon_{\mu\nu\rho\sigma}F^a_{\mu\nu}F^a_{\rho\sigma}\right)[d\psi][d\bar{\psi}]$$

$$\theta \to \theta - 2\alpha N_f$$

- Theta rotated away by chiral rotation.
- No theta dependence: topological susceptibility is zero

b) Massive quarks: $\bar{\theta} = \theta + \arg\det \mathrm{M}$

- Neutron Electric Dipole Moment: $d_n \sim \theta\frac{em_f}{m_n^2} \sim \theta\frac{em_\pi^2}{m_n^3} \approx 10^{-16}\theta\, e\,\mathrm{cm}$
- Theory: very hard to compute from first principles.
- Experiments: $|d_n| < 2.9\times 10^{-26} e\,\mathrm{cm}$ so $|\theta| < 10^{-10}$. Strong CP problem.

Plan

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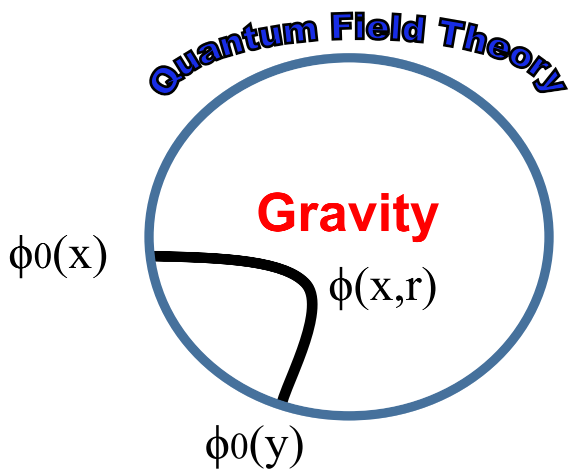
- Holographic QCD at finite θ-angle

$\boxed{\text{Exact } \theta\text{-dependence in a large N Yang-Mills model from holography}}$

Holography

[Maldacena, 97; Witten; Gubser, Klebanov, Polyakov, 98]

Quantum Field Theory in d-dim ⟷ Gravity (string) in d+1 dim



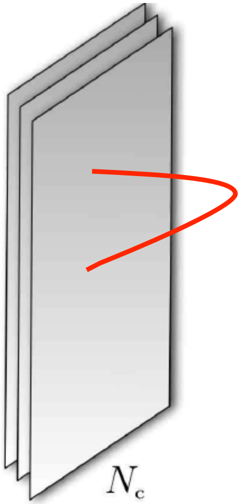
$$\boxed{Z_{QFT}[\phi_0] = Z_{QG/String} \approx e^{iS_{gravity}[\phi_0]}|_{\text{“}\phi(x,r)\to\phi_0(x)\text{”}}}$$

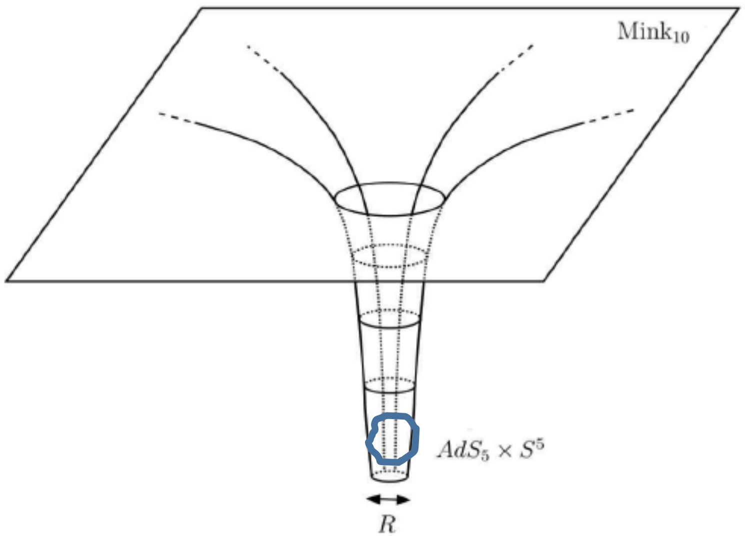
$$Z_{QFT}[\phi_0] = \int D\Psi \exp\left(i[S_{QFT} + \int d^Dx\, \phi_0(x)\mathcal{O}[\Psi](x)]\right) \qquad \langle\mathcal{O}(x)\mathcal{O}(y)\rangle \sim \frac{\delta^2 S_{grav}[\phi_0]}{\delta\phi_0(x)\delta\phi_0(y)}|_{\phi_0=0}$$

Holography

[Maldacena, 97; Witten; Gubser, Klebanov, Polyakov, 98]

Concrete realization in string theory: open/closed string duality





Master example: AdS/CFT

- Open strings attached to N_c D3-branes (left): 4d SU(N_c) susy CFT
- Closed strings (right): gravity on Anti-de-Sitter 5d background (times S^5)

- N_c>>1, $\lambda = g^2_{YM}N_c$>>1 in QFT ⟺ Classical theory of gravity

Holographic Yang-Mills [Witten 1998]

$\boxed{SU(N_c) \text{ Yang-Mills in 3+1 dimensions + massive adjoint KK fields}}$

- Need to go beyond AdS/CFT

- Start with N_c D4-branes : 5d $SU(N_c)$ QFT
- Compactify on a circle $S^1{}_{x4}$ of radius $R_4 = 1/M_{KK}$ with antiperiodic fermions.
- Low energy: 4d non-susy $SU(N_c)$ Yang-Mills + massive adjoint KK modes

- Compute gravity solution sourced by wrapped D4-branes
- Can add θ term to the model, no sign problem.
- To leading order in θ/N_c done in [Witten 1998].

Holographic Yang-Mills [Witten 1998]

- Gravity action (closed string description)

$$S = \frac{1}{2k_0^2} \int d^{10}x \sqrt{-g} \left[e^{-2\phi} \left(\mathcal{R} + 4(\partial\phi)^2 \right) - \frac{1}{2}|F_4|^2 - \frac{1}{2}|F_2|^2 \right]$$

- Gauge theory action (open string description, IR)

$$S = \frac{1}{8\pi g_s l_s M_{KK}} \int d^4x \, \mathrm{Tr} F_{\mu\nu} F^{\mu\nu} - \frac{i}{32\pi^2 l_s} \int_{S_{x^4}} C_1 \int d^4x \, \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} F_{\mu\nu} F_{\rho\sigma}$$

$$\boxed{F_2 = d\, C_1} \qquad \boxed{\theta + 2\pi k = \frac{1}{l_s} \int_{S_{x_4}} C_1}$$

- Classical gravity picture dual to gauge theory at $\lambda_4 >> 1$, $N_c >> 1$

$$\lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c$$

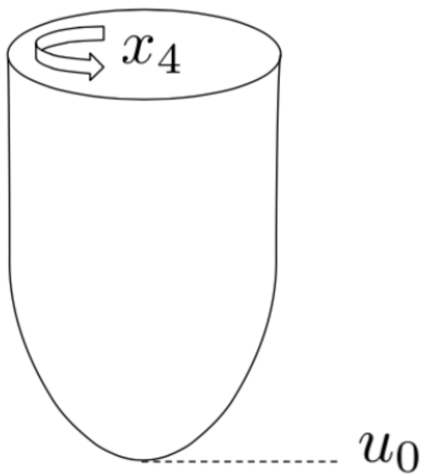
The gravity solution

[Barbon, Pasquinucci 99; Dubovsky, Lawrence, Roberts 2011] ($x_4 \sim x_4 + 2\pi/M_{KK}$)

$$ds^2_{10} = \left(\frac{u}{R}\right)^{3/2}\left[\sqrt{H_0}\, dx_\mu dx^\mu + \frac{f}{\sqrt{H_0}} dx_4^2\right] + \left(\frac{R}{u}\right)^{3/2}\sqrt{H_0}\left[\frac{du^2}{f} + u^2 d\Omega_4^2\right]$$

$$e^\Phi = g_s \frac{u^{3/4}}{R^{3/4}} \qquad \int_{S^4} F_4 = 8\pi^3 \alpha'^{3/2} g_s N$$

$$f = 1 - \frac{u_0^3}{u^3}, \qquad H_0 = 1 - \frac{u_0^3}{u^3}\frac{\Theta^2}{1+\Theta^2} \qquad \boxed{\Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta + 2k\pi}{N_c}\right)} \qquad \boxed{\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1}$$



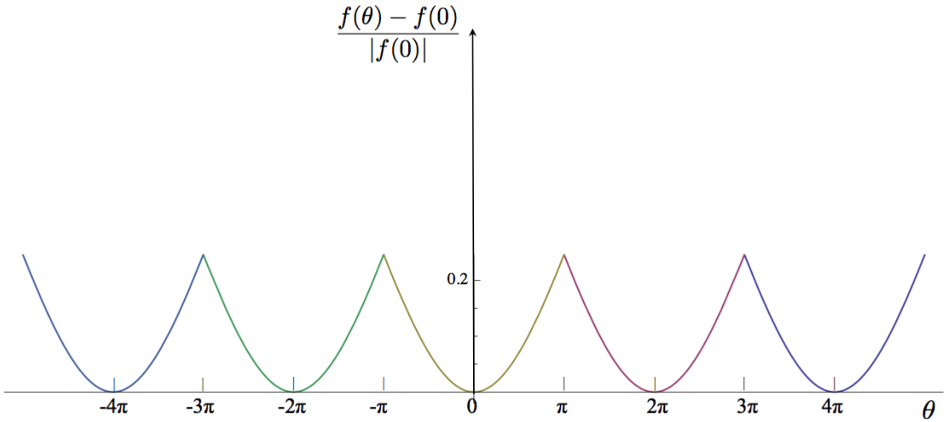
- (u,x_4) subspace is a cigar
- $g_{00}(u_0) \neq 0$ (regular) : confinement
- KK modes NOT decoupled

$$\boxed{u_0 = \frac{4R^3}{9} M_{KK}^2 \frac{1}{1+\Theta^2}}$$

The ground-state energy density

From holographic relation $Z = e^{-V_4 f(\theta)} \approx e^{-S^{\rm ren}_{E\,\rm on-shell}}$ $\qquad S(\theta) - S(0) \sim \int d^{10}x |F_2|^2$

$$\boxed{f(\theta) = \min_k f(\Theta)} \qquad f(\Theta) = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3} \qquad \Theta \equiv \frac{\lambda_4}{4\pi^2}\left(\frac{\theta+2k\pi}{N_c}\right)$$



Expected structure explicitly realized

$$f(\theta) - f(0) = \frac{1}{2}\chi_g\theta^2\left[1 + \bar{b}_2\frac{\theta^2}{N_c^2} + \bar{b}_4\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right] \qquad \chi_g = \frac{\lambda_4^3 M_{KK}^4}{4(3\pi)^6} \qquad \boxed{\bar{b}_2 = -\frac{\lambda_4^2}{8\pi^4}\,, \quad \bar{b}_4 = \frac{5\lambda_4^4}{384\pi^8}}$$

- Qualitative agreement with Lattice
- Prediction: $b_4 > 0$ (confirmed on the lattice for $N_c=2$) [Bonanno, Bonati, D'Elia 2018]

Other observables

- String tension (from rectangular Wilson loop) $\boxed{T_s = \frac{2\lambda_4}{27\pi} M_{KK}^2 \frac{1}{(1+\Theta^2)^2}}$

$$T_s = \frac{2\lambda_4}{27\pi} M_{KK}^2 \left(1 - \frac{\lambda_4^2}{8\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{256\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

- Light scalar glueball mass $\boxed{M(\Theta) = \frac{M(\Theta=0)}{\sqrt{1+\Theta^2}}}$

$$M(\theta) = M(\theta=0)\left(1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right)$$

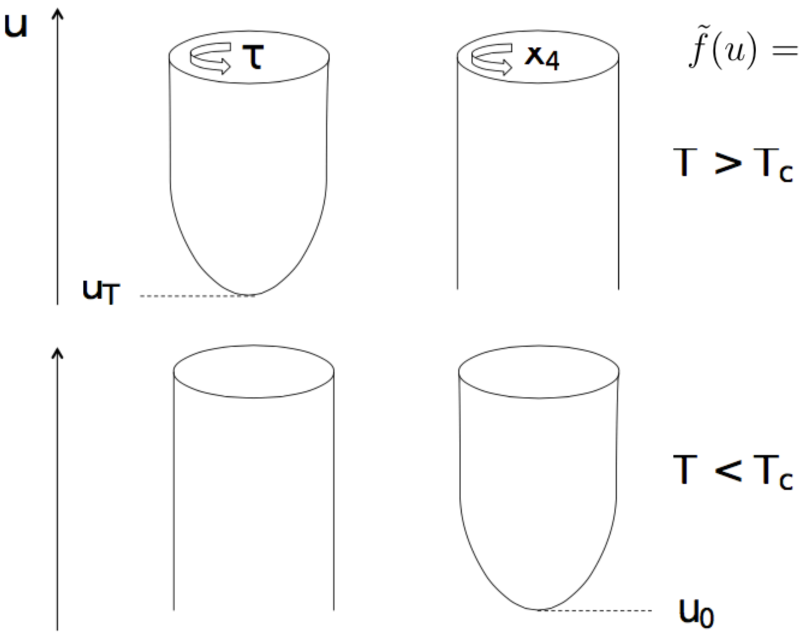
- Again: qualitative agreement with Lattice. Prediction on subleading coefficients

Finite temperature

Two possible gravity solutions, with Euclidean time circle of length 1/T

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[\tilde{f}(u)dx_0^2 + dx_a dx^a + dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{\tilde{f}(u)} + u^2 d\Omega_4^2\right],$$

$$\tilde{f}(u) = 1 - \frac{u_T^3}{u^3}.$$



- black hole solution
- $g_{00}(u_T) = 0$: deconfinement
- no theta dependence

$$\boxed{C_1 \sim \theta dx_4,\;\; F_2 = 0 \quad [(u, x_4) : \text{cylinder}]}$$

- Euclidean version of T=0 one
- Theta-dependence
- Confinement

Phase diagram

Compare the free energy densities of confined and deconfined phase

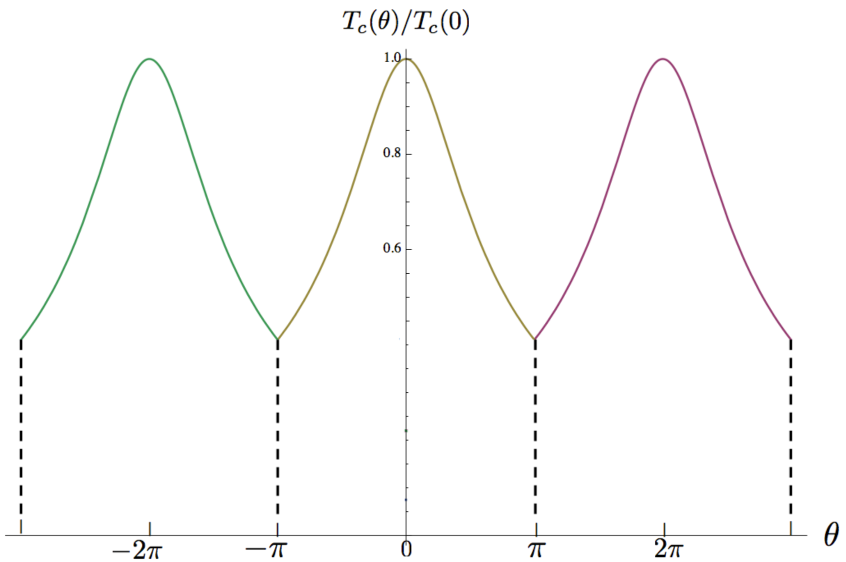
$$f = -p = -\frac{2N_c^2\lambda_4}{3^7\pi^2}\frac{M_{KK}^4}{(1+\Theta^2)^3} \equiv \frac{f(0)}{(1+\Theta^2)^3}$$

$$f_{dec} = -p_{dec} = -\frac{1}{6}\frac{256N_c^2\pi^4\lambda_4}{729M_{KK}^2}T^6$$

$$\boxed{T_c(\Theta) = \frac{M_{KK}}{2\pi}\frac{1}{\sqrt{1+\Theta^2}}}$$

$$T_c(\theta) = \frac{M_{KK}}{2\pi}\left[1 - \frac{\lambda_4^2}{32\pi^4}\frac{\theta^2}{N_c^2} + \frac{3\lambda_4^4}{2048\pi^8}\frac{\theta^4}{N_c^4} + \mathcal{O}(\theta^6)\right]$$

$$T_c(\theta)_{lat} = T_c(0)_{lat}\left[1 - R_\theta\theta^2 + \mathcal{O}(\theta^4)\right], \quad R_\theta = 0.0175(7)$$



Cusps: tri-critical points
Colored: deconf. first order transition
Dashed: CP-breaking first order transition

Phase diagram as expected

Plan

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- Holographic QCD at finite θ-angle

θ-dependence in Witten's model + quarks (Witten-Sakai-Sugimoto model)

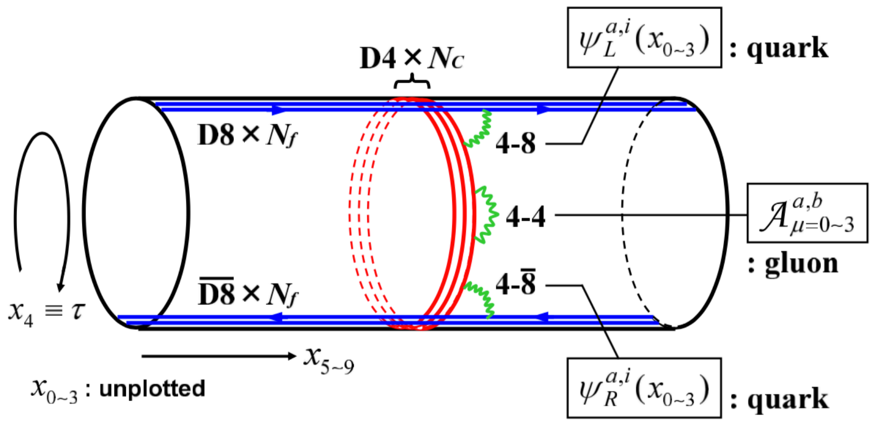
Holographic computation of Neutron Electric Dipole Moment

Domain walls, Chern-Simons theories and 3d dualities

Holographic QCD

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks [Sakai,Sugimoto 2004]

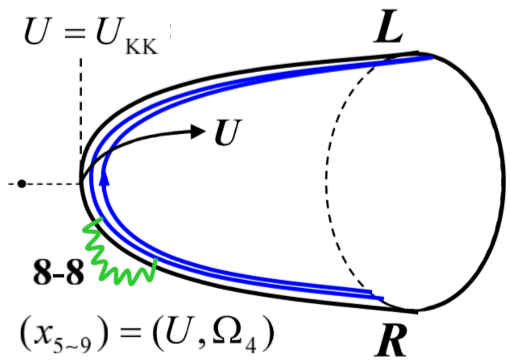
- Nf massless flavors added by means of Nf D8-anti-D8-branes.
- $U(Nf)_L$ x $U(Nf)_R$ gauge symmetry on D8 dual to classical QFT chiral symm.



Holographic QCD

Witten's SU(Nc) Yang-Mills + N_f massless fundamental quarks [Sakai,Sugimoto 2004]

- Nf massless flavors added by means of Nf D8-anti-D8-branes.
- Dual gravity description: D8 as probes (quenched approximation)



$$\epsilon_f \equiv \frac{1}{12\pi^3}\lambda^2\frac{N_f}{N_c} \ll 1$$

[Beyond quenched in FB, Cotrone 2015]

- Chiral symmetry breaking = joining of the two branches

Holographic QCD

- From D8-brane action get holographic effective action for hadrons:

$$\boxed{S_f = -\kappa \int d^4x dz \, \mathrm{Tr} \left(\frac{h(z)}{2} \mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu} + k(z) \mathcal{F}_{\mu z} \mathcal{F}^{\mu}_{z} \right) + \frac{N_c}{24\pi^2} \int \mathrm{Tr} \left(\mathcal{A} \mathcal{F}^2 - \frac{i}{2} \mathcal{A}^3 \mathcal{F} - \frac{1}{10} \mathcal{A}^5 \right)}$$

$$\kappa = \frac{N_c \lambda}{216\pi^3}, \quad h(z) = (1+z^2)^{-1/3}, \quad k(z) = (1+z^2) \qquad (U, x_4) \to (z, y)\,, \; U^3 = 1 + y^2 + z^2$$

- $\mathcal{A} = \widehat{A} \frac{\mathbb{1}}{\sqrt{2N_f}} + A^a T^a$ $U(N_f)$ gauge field in 5d

- **Fluctuations** correspond to pseudoscalars and whole tower of vector **mesons**:

$$\mathcal{U}(x^\mu) = \mathcal{P} \exp \left(-i \int_{-\infty}^{\infty} dz \, \mathcal{A}_z(x^\mu, z) \right) \qquad \eta' + N_f^2\text{-}1 \text{ PGB} \qquad f_\pi = 2\sqrt{\frac{\kappa}{\pi}}, \qquad e \sim -\frac{1}{2.5\kappa}$$

$$\mathcal{A}_\mu(x^\mu, z) = \sum_{n=1}^{\infty} B_\mu^{(n)}(x^\mu) \psi_n(z) \,.$$

$B_\mu^{(n)}$: massive (axial) vectors, n (even) odd

- **Instanton solutions** (in z, $x^{1,2,3}$) correspond to **baryons** (a' la Skyrme)
- Collective coordinate quantization ➔ baryon spectrum

Holographic QCD

- The theta term in the action:

$$\boxed{S_{\tilde{F}_2} = -\frac{1}{4\pi(2\pi l_s)^6}\int d^{10}x|\tilde{F}_2|^2}$$

- Recall : $\theta \sim \int C_1$
- Due to flavors modified field strength: $\tilde{F}_2 = dC_1 + \mathrm{Tr}\mathcal{A} \wedge \delta(y)dy$
- Gauge transformation of Tr A : $\mathrm{U(1)_A}$ rotation $\psi \to e^{i\alpha\gamma_5}\psi$
- Precisely corresponds to $\theta \to \theta - 2\alpha N_f$

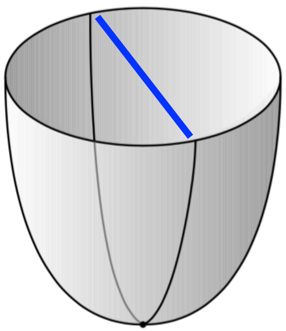
- On zero modes reduces to $S_{\tilde{F}_2} = -\frac{\chi_g}{2}\int d^4x\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta'\right)^2$ $\qquad \int dz\widehat{A}_z = \frac{2\eta'}{f_\pi}$

$$m^2_{\eta'} = m^2_{WV} \equiv \frac{2N_f}{f_\pi^2}\chi_g = \frac{1}{27\pi^2}\frac{N_f}{N_c}\lambda^2 M^2_{KK} \sim \epsilon_f M^2_{KK}$$

- Veneziano-Witten relation.

Holographic QCD

Mass term for the flavors [Aharony,Kutasov 2008; Hashimoto, Hirayama, Lin, Yee 2008]



$$\boxed{S_{\rm mass} = c \int \mathrm{d}^4x \, \mathrm{Tr}\, \mathcal{P} \left[M \exp\left(-i \int_{-\infty}^{\infty} \mathcal{A}_z \mathrm{d}z \right) + \mathrm{c.c.} \right]}$$

c ~ chiral condensate; M= m_q **1** : mass matrix
Valid for $m_q \ll M_{KK}$

WSS effective action and chiral Lagrangian

S = S$_{WSS}$+S$_{mass}$+S$_\theta$

$$S_{WSS} = -\kappa \int d^4x dz \left(\frac{1}{2} k(z)^{-1/3} \, \mathrm{Tr}\, \mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} + k(z) \mathrm{Tr}\, \mathcal{F}_{\mu z}\mathcal{F}^{\mu}{}_{z} \right) + \frac{N}{24\pi^2} \int \omega_5(\mathcal{A})$$

$$S_{\mathrm{mass}} = c \int \mathrm{d}^4x \, \mathrm{Tr}\, \mathcal{P} \left[M \exp\left(-i \int_{-\infty}^{\infty} \mathcal{A}_z \mathrm{d}z \right) + \mathrm{c.c.} \right] \qquad S_\theta = -\frac{\chi_g}{2} \int d^4x \left[\theta + \int dz \mathrm{Tr} \mathcal{A}_z \right]^2$$

- At low energy and integrating over z: the chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \left[\mathrm{Tr}\left(\partial_\mu U \partial^\mu U^\dagger \right) + 2B \mathrm{Tr}\left(MU + U^\dagger M^\dagger \right) - \frac{k}{N} (i \log \det U + \theta)^2 \right]$$

- Vacuum solution: $m_\pi^2 \sin\varphi = m_{WV}^2 \left(\frac{\theta}{N_f} - \varphi \right) \qquad \varphi \equiv -\frac{1}{\sqrt{2N_f}} \int_{-\infty}^{\infty} \widehat{A}_z dz$
- Relations $c m_q = \frac{1}{4} f_\pi^2 m_\pi^2$ (GMOR) $\qquad m_{WV}^2 \equiv \frac{2N_f}{f_\pi^2} \chi_g$ (Witten-Veneziano)
- Energy density: $f(\theta) - f(0) = \frac{N_f}{2} m_\pi^2 f_\pi^2 \left[1 - \cos\left(\frac{\theta}{N_f} \right) \right] + \mathcal{O}(m_\pi^4) \qquad \chi = \frac{\partial^2 f(\theta)}{\partial \theta^2}\Big|_{\theta=0} = \frac{m_\pi^2 f_\pi^2}{2N_f}$
- Masses: $m_{\eta'}^2(\varphi) = m_{WV}^2 + m_\pi^2(\varphi) \,, \quad m_\pi^2(\varphi) = m_\pi^2 \cos\varphi$

The neutron electric dipole moment

- Coefficient d_n in:

$$\langle n, s | \int d^3x\, \vec{x}\, J^0_{e.m.} | n, s \rangle = d_n \langle s | \vec{\sigma} | s \rangle$$

- Holographic data:

  - **Neutron state |n>** : (quantized) instanton in 5d total action $S_f + S_{F2} + S_m$

  - **Electric current J^μ_{em}** : from 5d field $F^{\mu z}|_{z=\pm\infty}$: $J_{\mu\, em} = -\kappa \left[k(z) \mathrm{Tr}(F_{\mu z} \tau^3) + \frac{k(z)}{N_c} \widehat{F}_{\mu z} \right]_{z \to -\infty}^{z \to \infty}$

- Holographic calculation:

  - Work at linear order in θ, m_q , with N_f=2 degenerate quarks

  - Linear-in-θ corrections to |n> : NONE

  - F^{0z} as instanton solution of equations of motion of $S_f + S_{C1} + S_m$

The neutron electric dipole moment

Fitting parameters with f_π = 92 MeV, m_π= 135 MeV, m_ρ=776 MeV yields

d_n= 1.8 10^{-16} θ e cm

Compatible with other estimates

Year	Approach/model	$c = d_n/(\theta \cdot 10^{-16} e \cdot \mathrm{cm})$
1979	bag model	2.7
1980	ChPT	3.6
1981	ChPT	1
1981	ChPT	5.5
1982	ChPT	20
1984	chiral bag model	3.0
1984	soft pion Skyrme model	1.2
1984	single nucleon contribution	11
1990 [20]	Skyrme model $N_f = 3$	2
1991 [19]	Skyrme model $N_f = 2$	1.4
1991	ChPT	3.3(1.8)
1991	ChPT	4.8
1992	ChPT	−7.2, −3.9
1999	sum rules	2.4(1.0)
2000	heavy baryon ChPT	7.5(3.2)
2004	instanton liquid	10(4)
2007	holographic QCD "hard–wall"	1.08
2015	1502.02295 [hep-lat]	- 3.8 (2)

Table partially taken from Panagopoulos, Vicari 2008

QCD Domain Walls and Holography

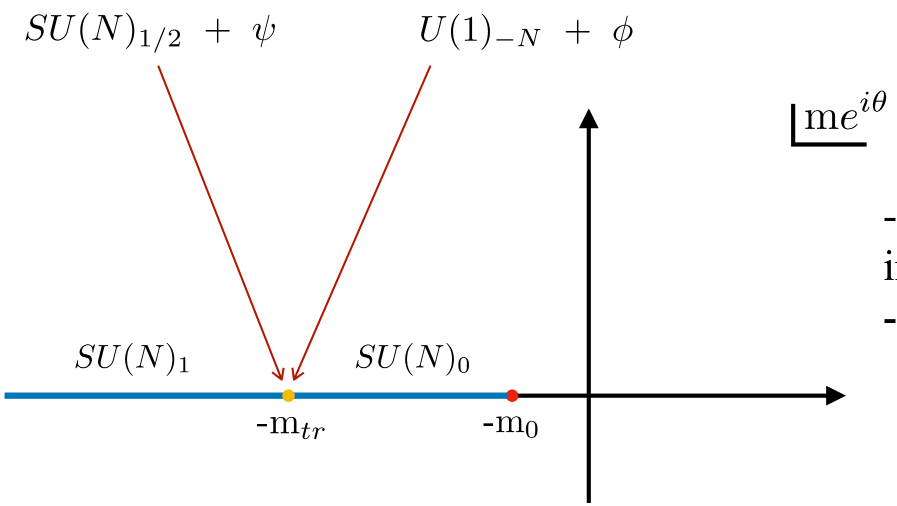
- At $\theta = \pi$ CP spontaneously broken: 2 degenerate gapped vacua
- Large N Yang-Mills [Witten 1980]
- Small N Yang-Mills [Gaiotto, Kapustin, Komargodski, Seiberg 2017]
- Massive QCD [Gaiotto, Komardgoski, Seiberg 2017]

- There is a 3d Domain Wall separating the 2 vacua
- What is the 3d QFT on the DW?

- YM: $SU(N)_1$ Chern-Simons theory [Gaiotto, Kapustin, Komardgoski, Seiberg 2017]
- QCD: depends on quark mass [Gaiotto, Komardgoski, Seiberg 2017]

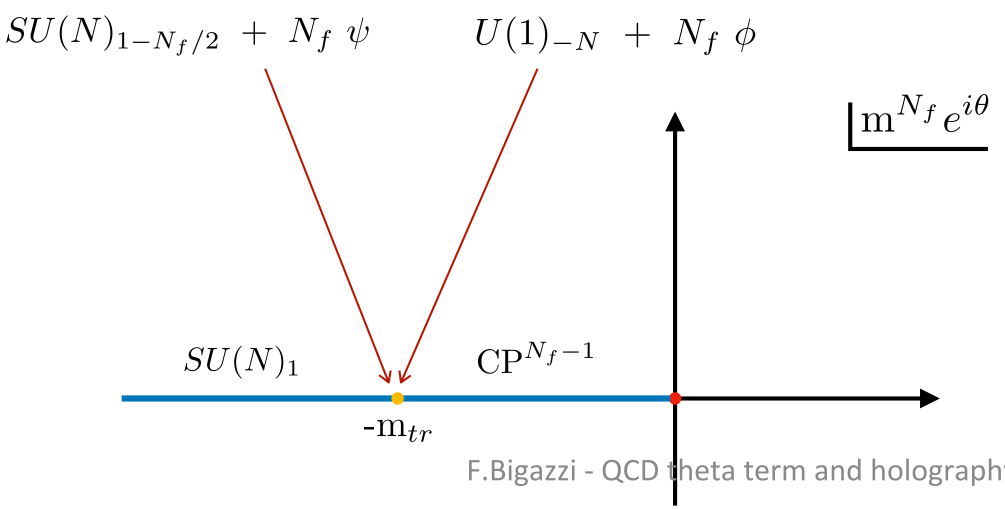
QCD Domain Walls and Holography

- Nf=1, mass m, θ = π [Gaiotto, Komargodski, Seiberg 2007]



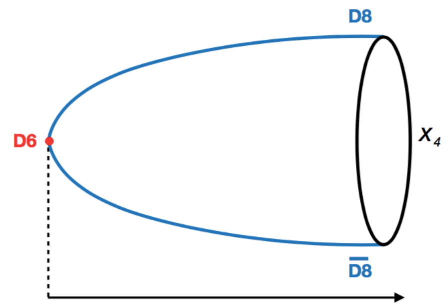
- $m=m_0$ II order phase transition in 4d (η' becomes massless)
- $m = m_t$ 3d transition (??)

- Nf>1, mass m, θ = π [Gaiotto, Komargodsky, Seiberg 2007]



QCD Domain Walls and Holography

- In WSS model, DW = D6-brane along x^0, x^1, x^2, S^4 at cigar tip [Witten 1998]
- In fact D6 couples to C_1 which in turn is related to θ
- Crossing a D6 shifts θ by 2π



- D6-brane action contains a Chern-Simons term

$$\tau_6 \frac{(2\pi\alpha')^2}{2} \int_{M_7} C_3 \wedge \mathcal{F} \wedge \mathcal{F} = \tau_6 \frac{(2\pi\alpha')^2}{2} \int_{S^4 \times M_3} F_4 \wedge \mathcal{A} \wedge \mathcal{F} = \frac{N}{4\pi} \int_{M_3} \mathcal{A} \wedge \mathcal{F}$$

- A $U(1)_N$ CS theory level rank dual to $SU(N)_{-1}$ [Aharony; Hsin, Seiberg; Karch, Tong]
- Without flavors (no D8-branes) this is precisely what you expect from QFT.

QCD Domain Walls and Holography

- On D6 DW + Nf D8: $U(1)_N$ CS theory + Nf fundamental scalars ϕ
- In flat space they are tachyons
- In the WSS background their mass M depends on quark mass m
- Increasing quark mass, gauge field on D6-D8 increases
- T-dual to separating the branes: at large separation no tachyon

$$m_T^2 \sim -\frac{\pi}{2} T_s|_{(z=0)} + M_{KK}^2 (\delta \hat{A}_z)^2|_{z=0} \sim -\lambda M_{KK}^2 + \eta_0^2(m) M_{KK}^2$$

- $m < m_{tr}$, $M^2<0$: D6 dissolved into D8 flux
- $m > m_{tr}$, $M^2>0$: can integrate out, DW theory is $U(1)_N$
- $m = m_{tr}$, $M^2 = 0$: DW theory is $U(1)_N + N_f\, \phi$ at criticality

- Qualitative picture: a second order phase transition at $m=m_t$

Conclusions and perspectives

- Holography gives insights on theta dependence in large N YM and QCD
- Qualitative agreement with Lattice Yang-Mills at small theta
- Novel predictions/benchmarks
- Novel holographic estimate of the Neutron Electric Dipole Moment.
- Novel insights on 3d Domain Wall effective theory, highly non trivial cross checks of 3d dualities, QFT expectations and predictions

Thank you

Coupling with pions

From old-fashioned pion-nucleon effective Lagrangian

$$\mathcal{L}_{\pi NN} = \boldsymbol{\pi} \cdot \bar{N} \boldsymbol{\tau} (i\gamma_5 g_{\pi NN} + \bar{g}_{\pi NN}) N$$

$$\langle \pi^a(x) \rangle = -\frac{g_{\pi NN}}{8\pi M_N} \frac{x_i}{r^3} \langle \sigma_i \tau^a \rangle$$ + CP breaking term

$$\langle \pi^a \rangle = A^a{}_z$$ Large r limit of our instanton solution

$$g_{\pi NN} \sim N_c^{\frac{3}{2}} \qquad \bar{g}_{\pi NN} \sim \theta N_c^{??}$$

No linear in theta term found in Az : $\bar{g}_{\pi NN} = 0$ at leading order in 1/N

Holographic baryons

- At large N, baryons as solitons of chiral Lagrangian (M≈N): Skyrmions
- In WSS model they are instantons on D8-branes action [Hata,Sakai,Sugimoto,Yamato. 2007]
- Baryon number = instanton number

$$n_B = \frac{1}{8\pi^2}\int_B \mathrm{Tr}\,\mathcal{F}\wedge\mathcal{F} \qquad B = \{\vec{x}, z\} \text{ 4d subspace}$$

- Look for $n_B=1$ solutions from effective action $S_{D8}=S_{YM}+S_{CS}$ for holographic mesons

- Decompose $U(N_f)$ field as $\mathcal{A} = \widehat{A}\frac{\mathbb{1}}{\sqrt{2N_f}} + A^a T^a$. Take $N_f=2$

- Solution around z=0 (flat space limit): charged BPST-like instanton

$$A_M^{\rm cl} = -i f(\xi) g \partial_M g^{-1}\,, \quad \widehat{A}_0^{\rm cl} = \frac{1}{8\pi^2 a}\frac{1}{\xi^2}\left[1 - \frac{\rho^4}{(\rho^2+\xi^2)^2}\right]\,, \quad A_0^{\rm cl} = \widehat{A}_M = 0$$

$$f(\xi) = \frac{\xi^2}{\xi^2+\rho^2}\,, \quad g(x) = \frac{(z-Z)\mathbb{1} - i(\vec{x}-\vec{X})\cdot\vec{\tau}}{\xi} \qquad \xi^2 \equiv (\vec{x}-\vec{X})^2 + (z-Z)^2$$

- Instanton (pseudo) moduli: size (ρ), position (X^i, Z) in 4d space

Note on instanton quantization

- In non-rel limit Hamiltonian of instanton moduli. Time dependence induced by

$$A_0^{\rm cl} \longmapsto A_0' = 0\,, \qquad A_M^{\rm cl} \longmapsto A_M' = V A_M^{\rm cl} V^{-1} - iV\partial_M V^{-1}$$

$V(t,\vec{x},z)$
SU(2) matrix

- Fermionic states (baryons) selected by $\Psi[V] = -\Psi[-V]$

$$\mathcal{L} = \frac{M_0}{2}\left(\dot{\vec{X}}^2 + \dot{Z}^2 + 2(\dot{y}^I)^2\right) - M_0\left[1 + \frac{\rho^2}{6} + \frac{N_c^2}{320\pi^4\kappa^2}\frac{1}{\rho^2} + \frac{Z^2}{3}\right]$$

$\rho^2 = y^I y^I \qquad a^I = y^I \rho^{-1}$ (quantizing these, get spin and isospin)

$$\boxed{\begin{array}{ll} |p\uparrow\rangle \propto R(\rho)\psi_Z(Z)(a_1 + ia_2)\,, & |p\downarrow\rangle \propto R(\rho)\psi_Z(Z)(a_4 - ia_3) \\ |n\uparrow\rangle \propto R(\rho)\psi_Z(Z)(a_4 + ia_3)\,, & |n\downarrow\rangle \propto R(\rho)\psi_Z(Z)(a_1 - ia_2) \end{array}}$$

$$R(\rho) = \rho^{-1+2\sqrt{1+N_c^2/5}}\, e^{-\frac{M_0}{\sqrt{6}}\rho^2}\,, \qquad \psi_Z(Z) = e^{-\frac{M_0}{\sqrt{6}}Z^2} \qquad M_0 = \frac{N_c\lambda}{27\pi}$$

- Classically: small size, $\rho^2 \sim 1/\lambda$
- Neutron: state with s=1/2, I_3 = -1/2.

Holographic baryons at small θ, m_q

- Look for instanton solution of original WSS D8 action plus mass and theta term

$$S_{\rm mass} = c \int \mathrm{d}^4x \, \mathrm{Tr}\, \mathcal{P} \left[M \exp\left(-i \int_{-\infty}^{\infty} \mathcal{A}_z \mathrm{d}z \right) + \mathrm{c.c.} \right]$$

$$S_\theta = -\frac{\chi_g}{2} \int d^4x \left[\theta + \int dz \mathrm{Tr} \mathcal{A}_z \right]^2$$

- To first order in mass, baryon Hamiltonian does not receive O(θ) corrections.
- New instanton solution to leading order in mass and theta

$$\boxed{\hat{A}_z^{\rm mass} = \frac{1}{1+z^2} u(r)}$$

$$u(r) = \frac{c m_q \theta}{\kappa} \int_0^\infty \mathrm{d}r' \, u_G(r, r') \left(1 + \cos \frac{\pi}{\sqrt{1+\rho^2/r'^2}} \right)$$

$$\boxed{A^0_{\rm mass} = W(r,z)(\vec{x} - \vec{X}) \cdot \vec{\tau}}$$

$$h(z) \left(\partial_r^2 W(r,z) + \frac{4}{r} \partial_r W(r,z) + \frac{8\rho^2}{(\xi^2+\rho^2)^2} W(r,z) \right) + \partial_z (k(z) \partial_z W(r,z)) =$$
$$= \frac{27\pi}{\lambda} \frac{\rho^2}{(\xi^2+\rho^2)^2} \frac{1}{r} \frac{u'(r)}{1+z^2} \equiv \mathscr{F}(r,z)$$

- A dipole potential induced by the η' vev.

NEDM in Witten-Sakai-Sugimoto

$$\vec{D}_{n,s} = \int \mathrm{d}^3x\, \vec{x}\, \langle n, s | J^0_{\mathrm{em}} | n, s \rangle \qquad J_{\mu\, em} = -\kappa \left[k(z) \mathrm{Tr}(F_{\mu z} \tau^3) + \frac{k(z)}{N_c} \widehat{F}_{\mu z} \right]_{z \to -\infty}^{z \to \infty}$$

$$\vec{\mathcal{D}}^{s.c.}_{n,s} = \frac{8\pi}{9} \int_0^\infty dr\, r^4\, \kappa [k(z)\, \partial_z W(r,z)]^{z\to\infty}_{z\to-\infty} \langle s | \vec{\sigma} | s \rangle = -\vec{\mathcal{D}}^{s.c.}_{p,s}$$

$$d^{s.c.}_n = \frac{8\pi}{9} \int_0^\infty dr\, r^4\, \kappa [k(z)\, \partial_z W(r,z)]^{z\to\infty}_{z\to-\infty}$$

- Skyrme-like behavior $\quad d^{s.c.}_n \simeq 82 \dfrac{N_c m_\pi^2 \theta}{\lambda^2 M^3_{KK}}$ $\quad$ at $\quad N_c \gg 1\,, \lambda \gg 1$

- Holographic result takes into account contribution from whole meson tower

$$d^{s.c.}_n = -\frac{8\pi}{9} \sum_{n=1}^{\infty} g_{v^n} \int_0^\infty dr\, r^4 R_{2n-1}(r)$$

- Notice only odd indices: complete vector-meson dominance
- The first mode gives only 40% result. Need contribution of higher modes

NEDM in Witten-Sakai-Sugimoto

- Taking full wave function into account

$$\langle d_n \rangle = \frac{\int \rho^3 R(\rho)^2 \psi_Z^2(Z) d_n(\rho, Z) d\rho dZ}{\int \rho^3 R(\rho)^2 \psi_Z^2(Z) d\rho dZ}$$

- Fitting parameters with f_π = 92 MeV, m_π= 135 MeV, m_ρ=776 MeV yields

d_n= 1.8 10^{-16} θ e cm