TECHNISCHE UNIVERSITÄT WIEN

Quantum Null Energy Condition

Analytic, Perturbative and Numerical Results

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based on work with: C. Ecker, D. Grumiller, W. van der Schee, S. Sheikh-Jabbari

Doktoratskolleg Particles and Interactions

1 Introduction

Energy conditions (EC) constrain the energy momentum tensor $T_{\mu\nu}$. Example:

$$\text{Null EC}: \quad T_{\mu\nu}k^\mu k^\nu \geq 0 \quad \forall k_\mu k^\mu = 0$$

- Physical motivation: energy/energy fluxes expected to be non-negative (classically)
- Mathematical consequence: convexity condition for focussing-/singularity theorems
- **Problem: all classical ECs violated by reasonable quantum fields**

Therefore, need quantum EC. Example:

$$\text{Averaged NEC}: \quad \int dx^\lambda k_\lambda \langle T_{\mu\nu}k^\mu k^\nu\rangle \geq 0$$

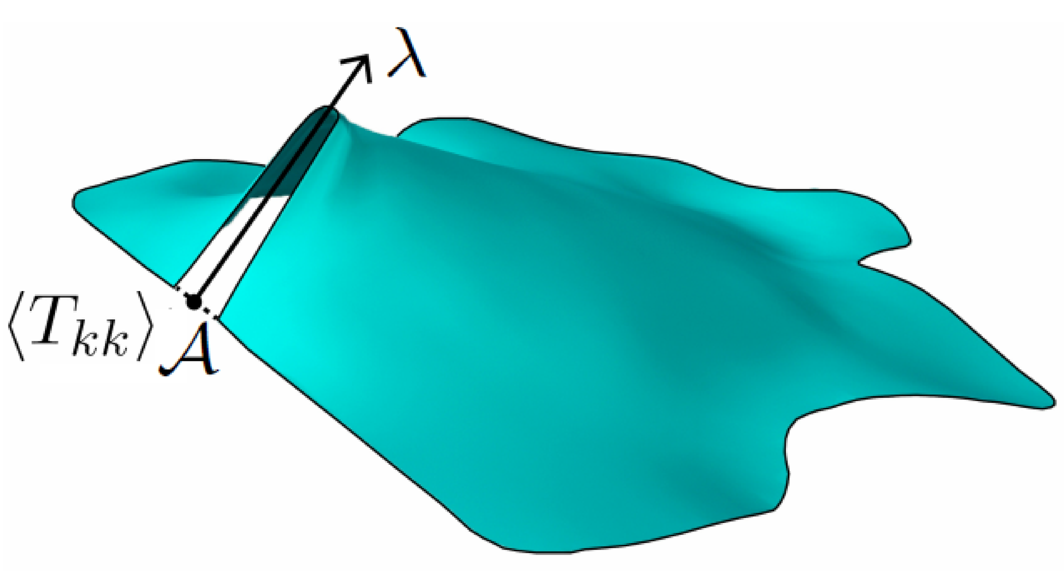
- Proofs of ANEC exist
- Sufficient for proving focussing theorems
- Physical relation to quantum interest conjecture
- **Problem: ANEC non-local**

Is there a local quantum energy condition?

2 QNEC

- **QNEC**, proposed by Bousso, Fisher, Leichenauer and Wall [1] is the following inequality:

$$\langle T_{kk}\rangle \geq \frac{\hbar}{2\pi\sqrt{\gamma}}S''$$



$T_{kk} = T_{\mu\nu}k^\mu k^\nu \quad \forall k_\mu k^\mu = 0.$

γ: induced metric at the boundary of entangling region
S'': second variation of entanglement entropy (EE) w.r.t. surface deformations along k^μ

- QNEC proven for free QFTs [2] holographic CFTs [3] and general CFTs [4]
- QNEC stronger in 2D:

$$\langle T_{kk}\rangle \geq \frac{\hbar}{2\pi}\left(S'' + \frac{6}{c}S'^2\right)$$

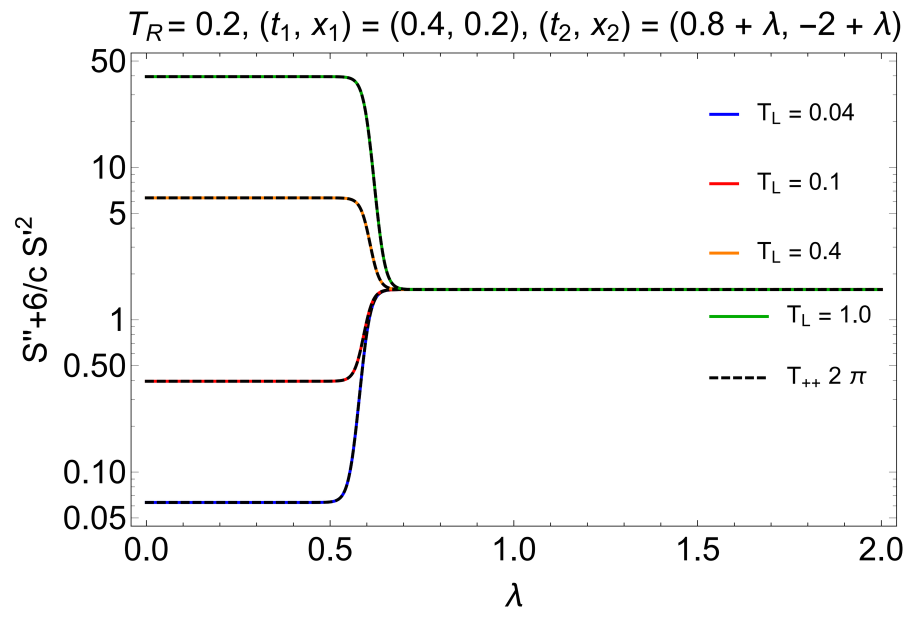
c: central charge

3 Analytic Calculation

- AdS$_3$/CFT$_2$: holographic proof of QNEC saturation for all states dual to Bañados geometries (vacuum, particles in AdS$_3$, BTZ black holes and their Virasoro descendants)
- QNEC saturation even for far-from equilibrium transport in strongly coupled CFT like the hot-cold bath critical quantum system studied in [5]

Sketch of proof:

EE transforms like anomalous weight $(0,0)$ operator.
Define vertex operator: $V := exp(\frac{6}{c}S)$.
QNEC saturation equivalent to vertex operator solving Hill's equation: $V'' - \mathcal{L}V = 0$.
Conformal symmetries show that for states dual to Bañados geometries vertex operator always solves Hill's equation.



QNEC saturation for hot-cold bath.

4 Perturbative Calculation

- erturbative QNEC calculation for AdS_5 Schwarzschild black brane for small and large entangling regions; numerically in between [6].
- Finite central charge corrections in AdS$_3$ backreacted by massive scalar field with conformal weights (h,h) (work in progress [9]).

Requires corrections to RT-surface and from bulk EE [8]:

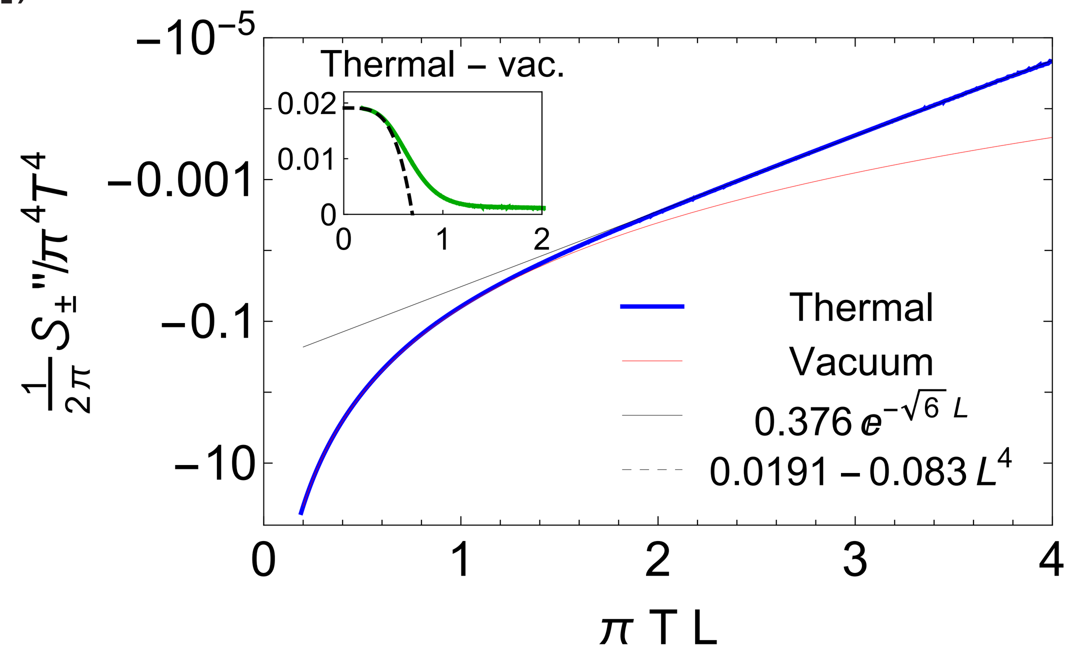
$$S = \frac{\mathcal{A}}{4G_N} + \frac{\delta\mathcal{A}}{4G_N} + S_{\text{bulk}}$$

QNEC satisfied for all conformal weights $h \geq 1/2$.
Small interval:
QNEC saturation up to small corrections:

$$2\pi\langle T_{\pm\pm}\rangle - S'' - \frac{6}{c}\left(S'\right)^2 = +\mathcal{O}(\Delta\varphi^{4h})$$

Half-interval:
QNEC satisfied but gapped by h/4 in large-h limit:

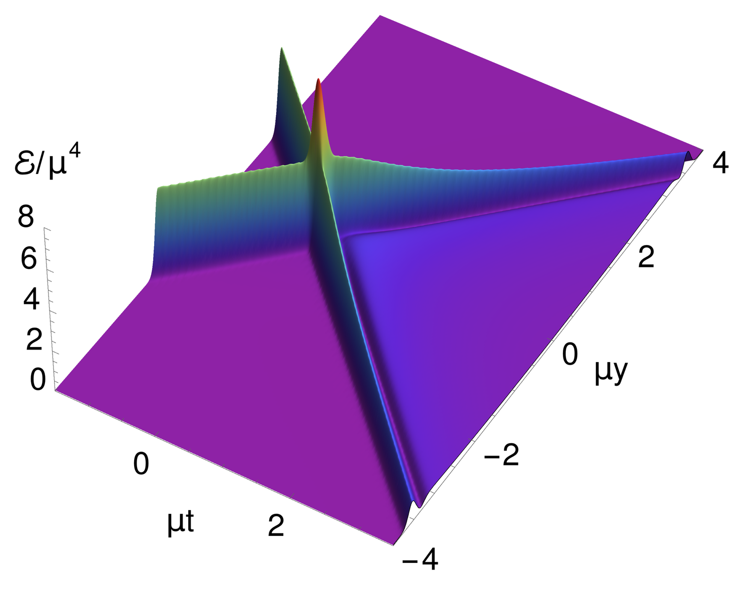
$$2\pi\langle T_{\pm\pm}\rangle - S'' - \frac{6}{c}\left(S'\right)^2\Big|_{\Delta\varphi=\pi} = \frac{h}{4} + \dots$$



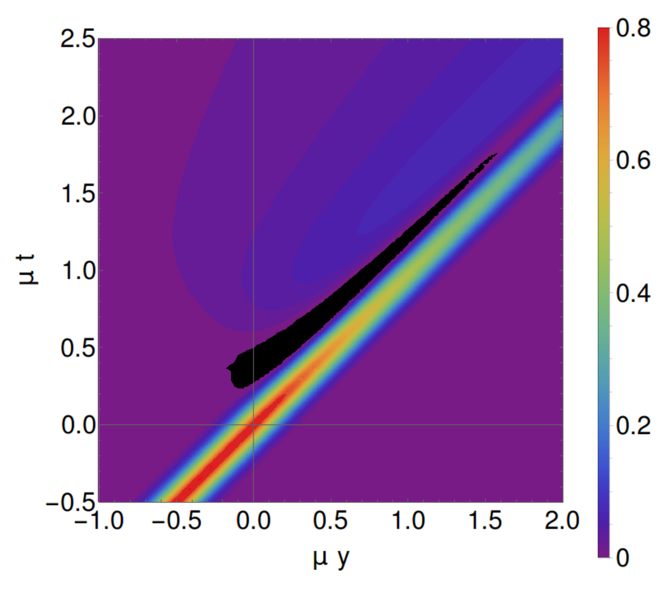
QNEC as function of size of entangling region. Black lines are perturbative, colored lines numerical results.

5 Numerical Calculation

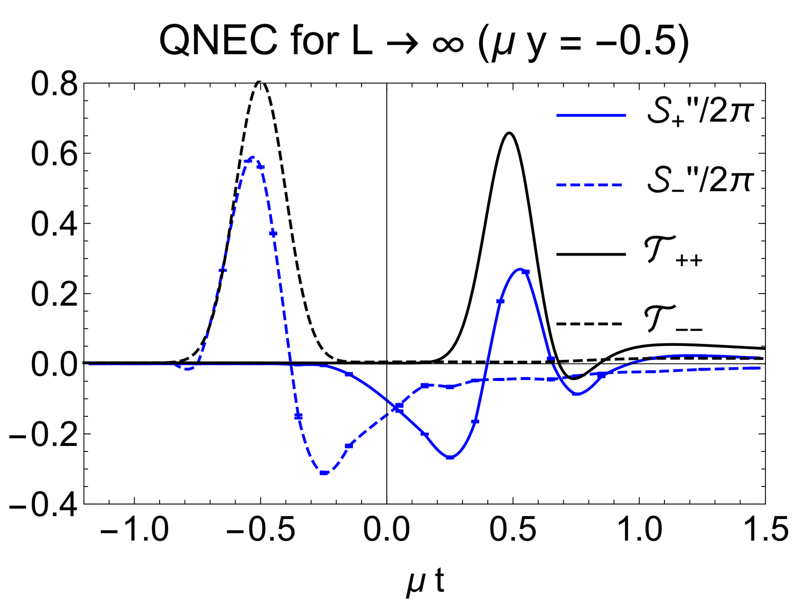
- Holographic model for non-abelian plasma formation (toy model for heavy ion collisions): colliding gravitational shock waves in AdS_5.
- Null EC violated, but QNEC satisifed and can saturate in far from equilibrium state [7].



Energy density of colliding shock waves.



NEC violation in shock wave scenario.



Time evolution of QNEC.

6 Summary & References

- QNEC only known local EC [1], proven for large class of unitary QFTs [2, 3, 4].
- Provided first numerical studies of QNEC in AdS$_5$/CFT$_4$ [7].
- Gravitational shockwaves violate Null EC but satisfy QNEC [7].
- QNEC saturation possible in far from equilibrium regime - "quantum equilibrium"? [7, 9].
- Finite central charge corrections to QNEC in AdS_3/CFT_2: saturation at small interval, gap at large interval [9].

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