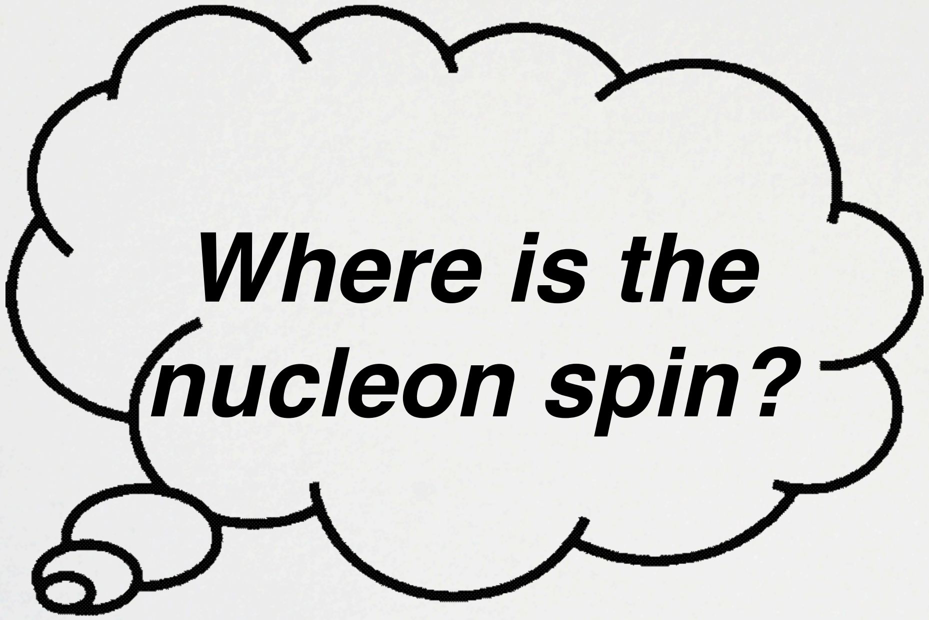
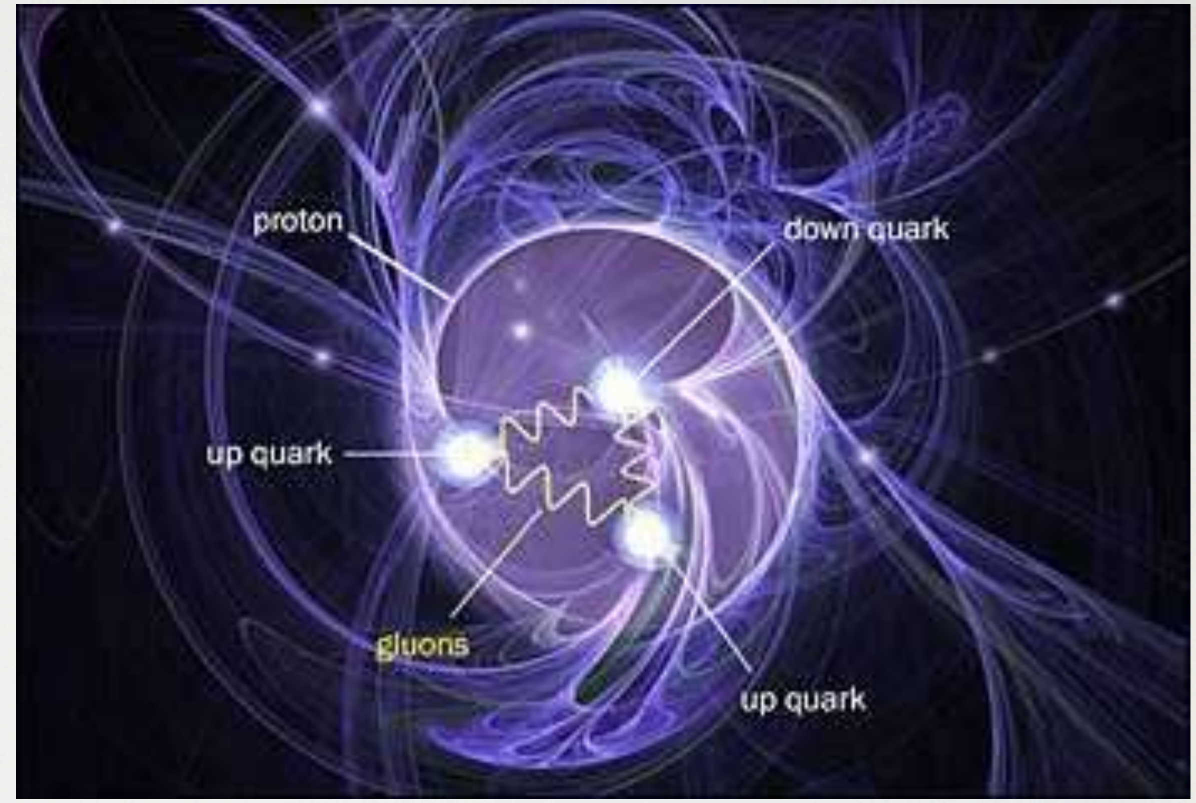
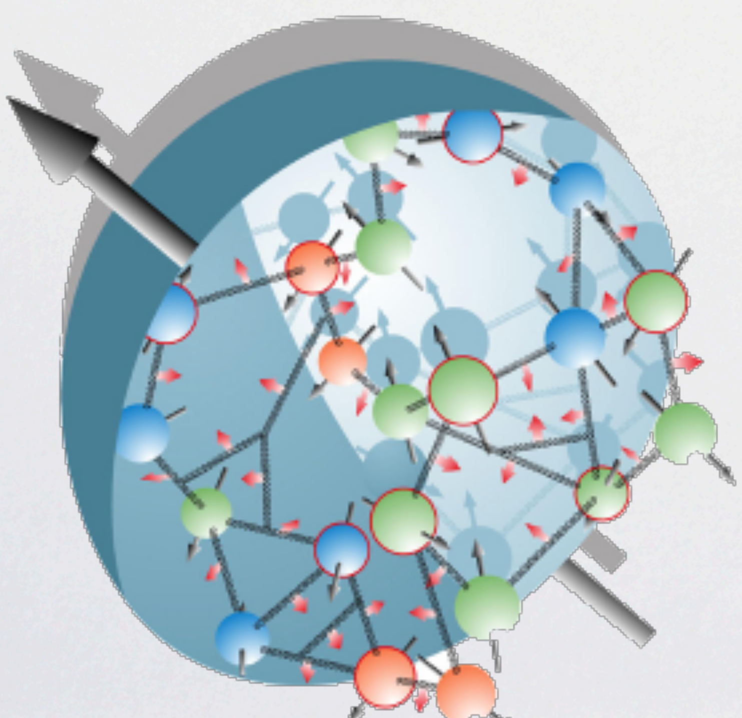
Nucleon Spin Structure from Lattice QCD







Kyriakos Hadjiyiannakou

Computation-based Science and Technology Research Centre (CaSToRC)

The Cyprus Institute

Diffraction and low-x 2018

26 August 2018 to 1 September 2018

Reggio Calabria, Italy

In collaboration with

- C. Alexandrou (University of Cyprus, The Cyprus Institute)
- M. Constantinou (Temple University)
- K. Jansen (NIC, DESY)
- C. Kallidonis (Stony Brook University)
- G. Koutsou (The Cyprus Institute)
- A. Vaquero Aviles-Casco (University of Utah)
- Krzysztof Cichy (Adam Mickiewicz University)
- Fernanda Steffens (Bonn University)

Overview

Motivation

Overview

Motivation

Lattice QCD Methodology

Overview

Motivation

Nucleon Spin decomposition

Lattice QCD Methodology

Overview

Motivation

Lattice QCD Methodology

Nucleon Spin decomposition

Few things about form factors and Parton Distributions

Overview

Motivation

Nucleon Spin decomposition

Few things about form factors and Parton Distributions

Intrinsic quark spin

Lattice QCD Methodology

Overview

Motivation

Lattice QCD Methodology

Nucleon Spin decomposition

Few things about form factors and Parton Distributions

Intrinsic quark spin

Quark & Gluon momentum fractions

Overview

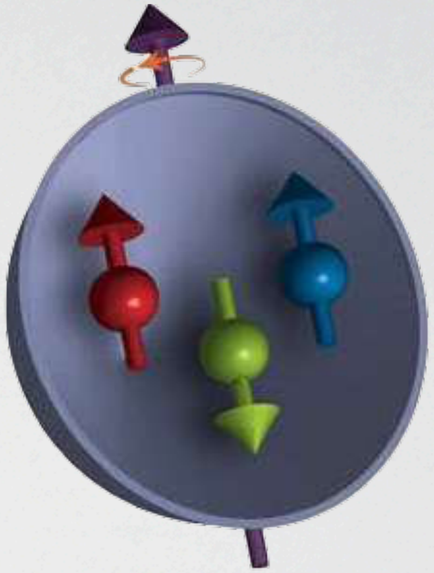
- Motivation
- Lattice QCD Methodology
- Nucleon Spin decomposition
  - Few things about form factors and Parton Distributions
  - Intrinsic quark spin
  - Quark & Gluon momentum fractions
  - Orbital and total angular momentum

Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin

Simple parton model

$$\frac{1}{2}\left(\Delta u_{\rm v}+\Delta d_{\rm v}\right)=\frac{1}{2}$$

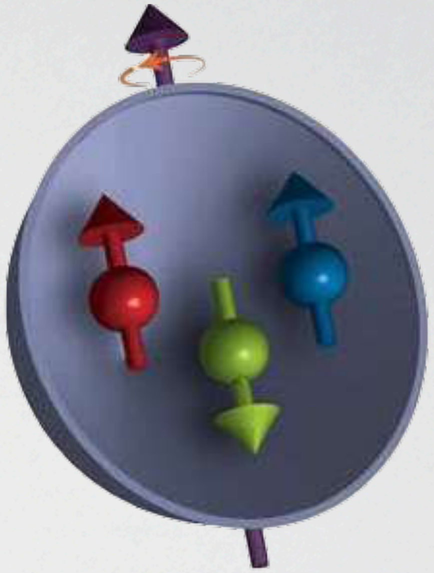


Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin
- EMC experiment in 1987 found an astonishing small contribution from the quarks, namely (4-24)% almost compatible with zero

Simple parton model

$$\frac{1}{2}(\Delta u_{\mathrm{v}} + \Delta d_{\mathrm{v}}) = \frac{1}{2}$$

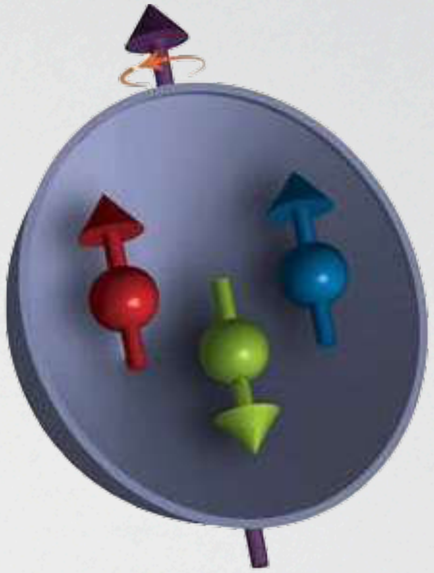


Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin
- EMC experiment in 1987 found an astonishing small contribution from the quarks, namely (4-24)% almost compatible with zero

Simple parton model

$$\frac{1}{2}\left(\Delta u_{\rm v} + \Delta d_{\rm v}\right) = \frac{1}{2}$$



$\Delta\Sigma_{q+}$: Intrinsic quark spin

L_{q+} : Quark orbital angular momentum

J_g : Gluon contribution to the nucleon spin

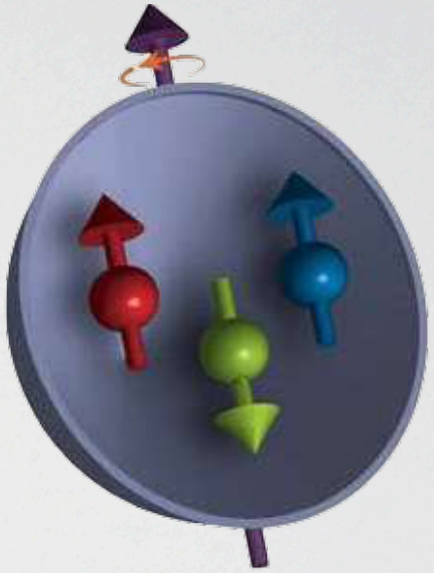
$J_{q+} = \frac{1}{2}\Delta\Sigma_{q+} + L_{q+}$: Total quark angular momentum

Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin
- EMC experiment in 1987 found an astonishing small contribution from the quarks, namely (4-24)% almost compatible with zero

Simple parton model

$$\frac{1}{2}(\Delta u_{\mathrm{v}} + \Delta d_{\mathrm{v}}) = \frac{1}{2}$$



Complete picture

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma_{q+} + L_{q+}\right) + J_g$$

Ji's sum rule

$\Delta\Sigma_{q+}$: Intrinsic quark spin

L_{q+} : Quark orbital angular momentum

J_g : Gluon contribution to the nucleon spin

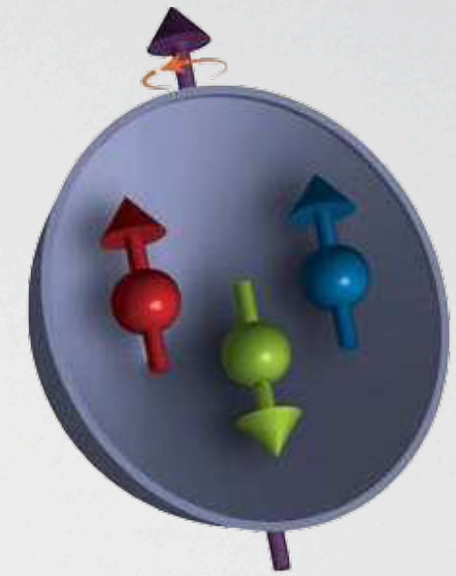
$J_{q+} = \frac{1}{2}\Delta\Sigma_{q+} + L_{q+}$: Total quark angular momentum

Proton Spin Crisis

- Up to 1980s physicists expected that quarks carry all the proton spin
- EMC experiment in 1987 found an astonishing small contribution from the quarks, namely (4-24)% almost compatible with zero

Simple parton model

$$\frac{1}{2}\left(\Delta u_{\rm v} + \Delta d_{\rm v}\right) = \frac{1}{2}$$



Complete picture

$$\frac{1}{2} = \sum_q \left(\frac{1}{2}\Delta\Sigma_{q+} + L_{q+}\right) + J_g$$

Ji's sum rule

First principle calculation is needed

$\Delta\Sigma_{q+}$: Intrinsic quark spin

L_{q+} : Quark orbital angular momentum

J_g : Gluon contribution to the nucleon spin

$J_{q+} = \frac{1}{2}\Delta\Sigma_{q+} + L_{q+}$: Total quark angular momentum

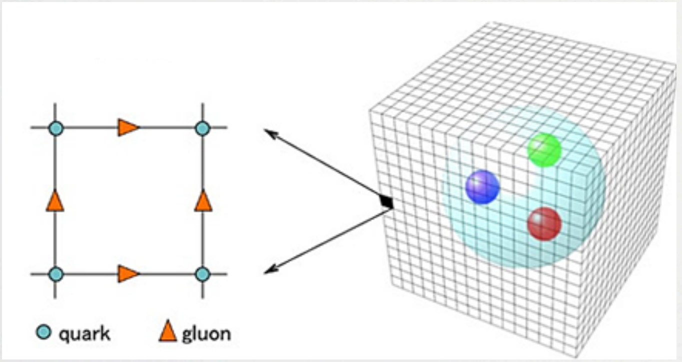
Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

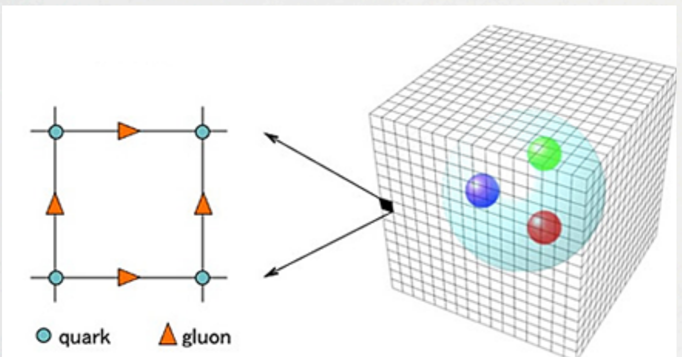
LQCD



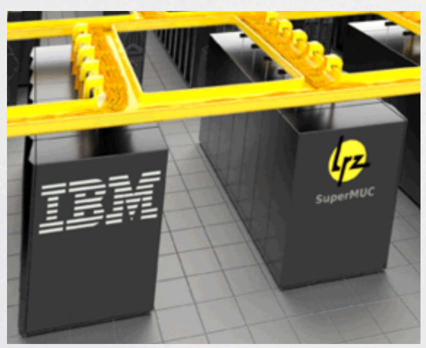
Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD



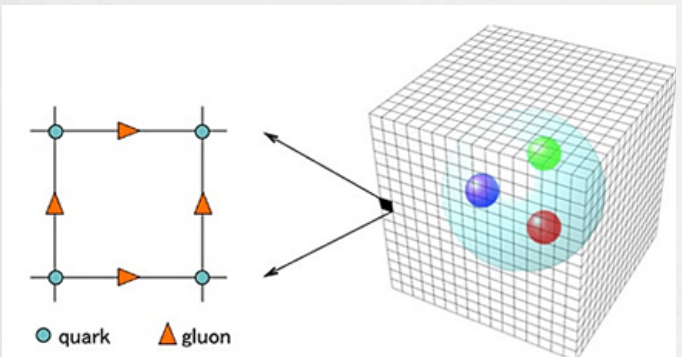
Configurations
Simulation



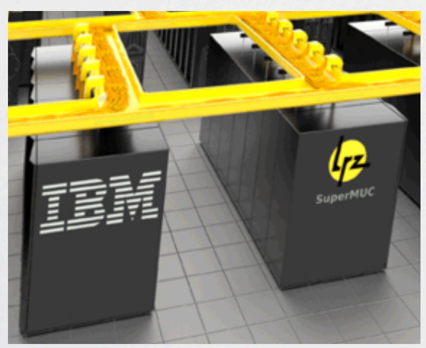
Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD



Configurations Simulation

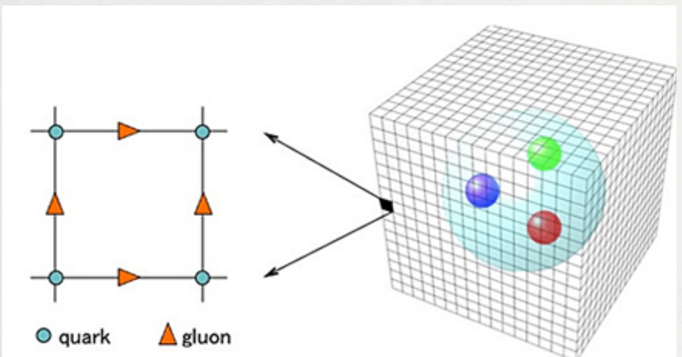


Quark Propagators

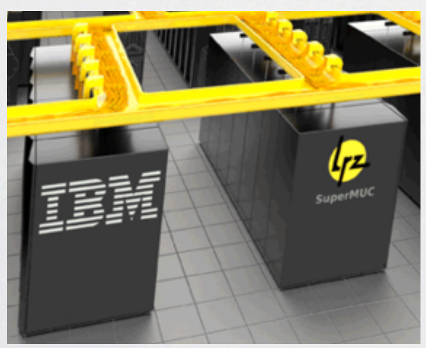
Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD

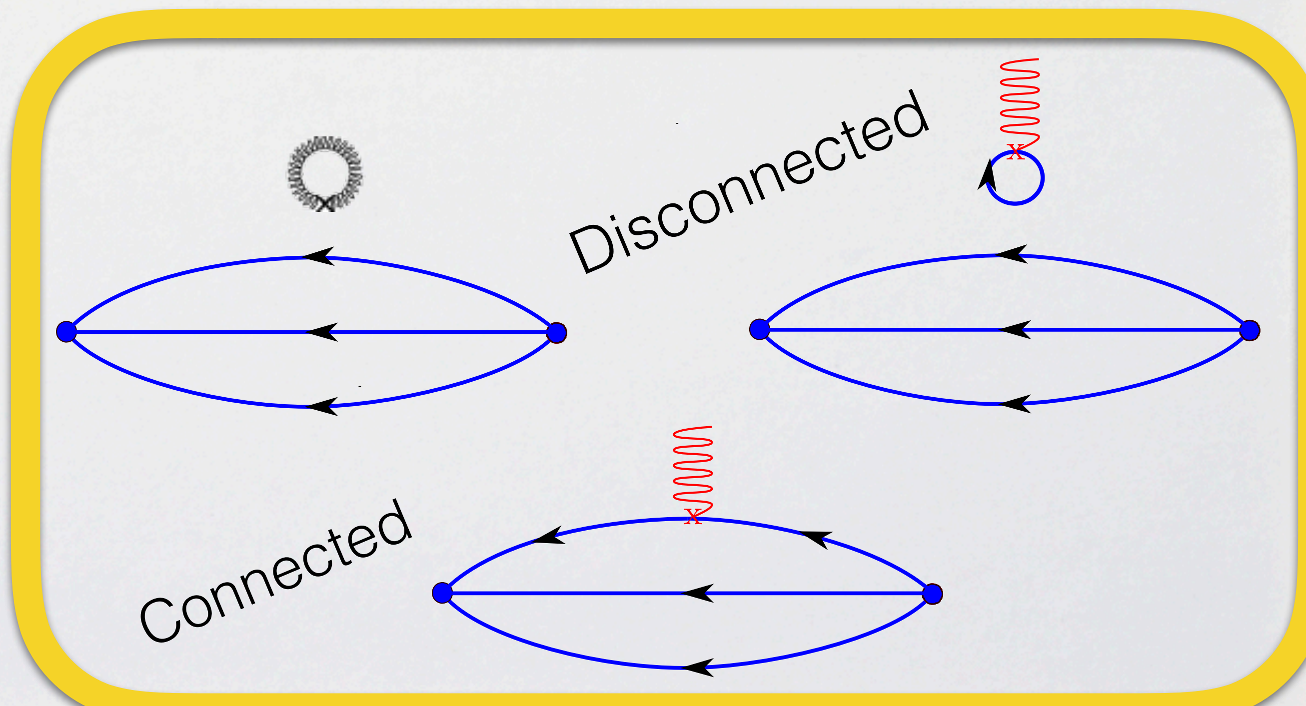


Configurations
Simulation



Quark Propagators

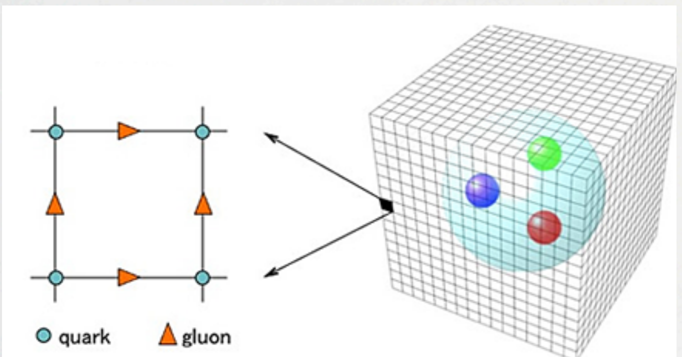
Contractions



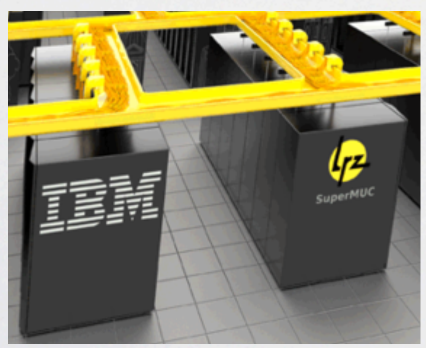
Computation of observables on the Lattice

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U) \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD

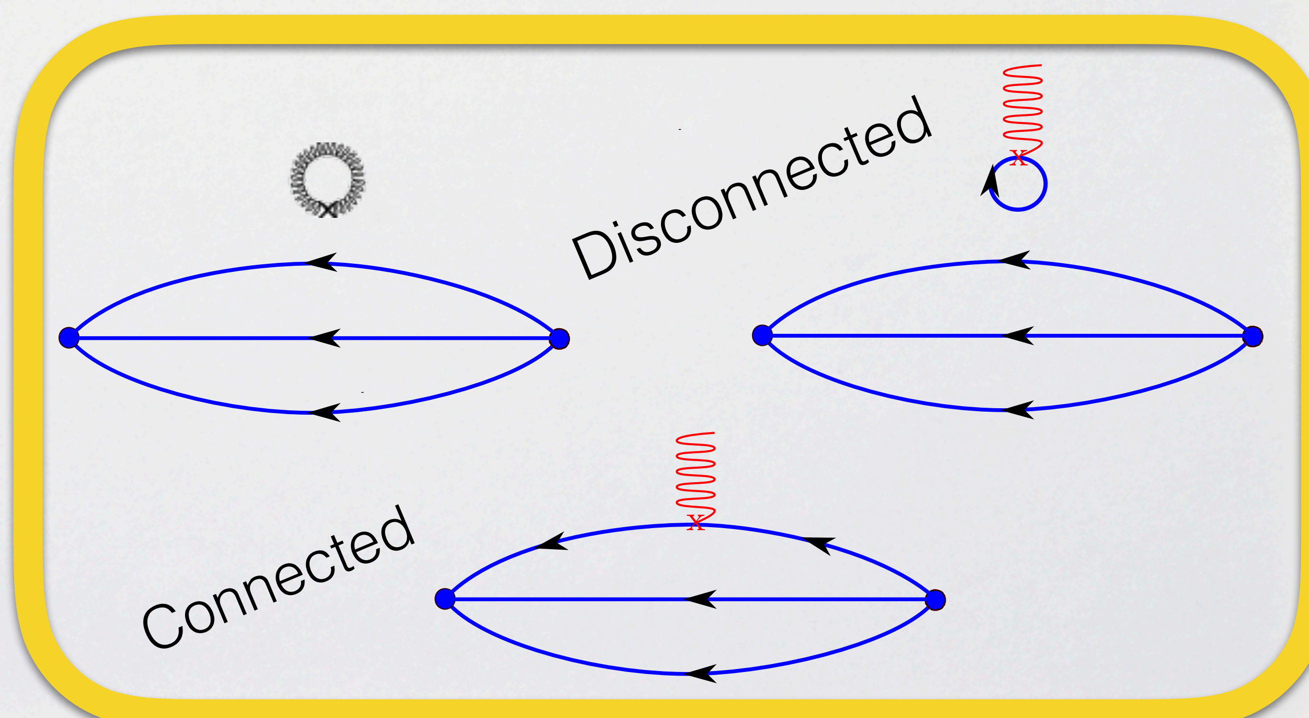


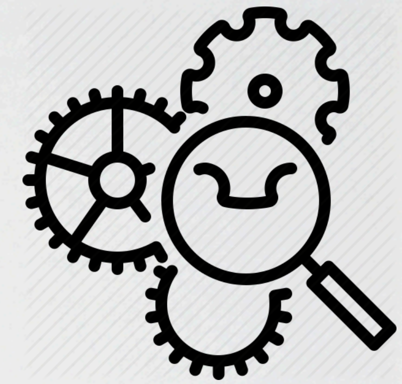
Configurations Simulation



Quark Propagators

Contractions



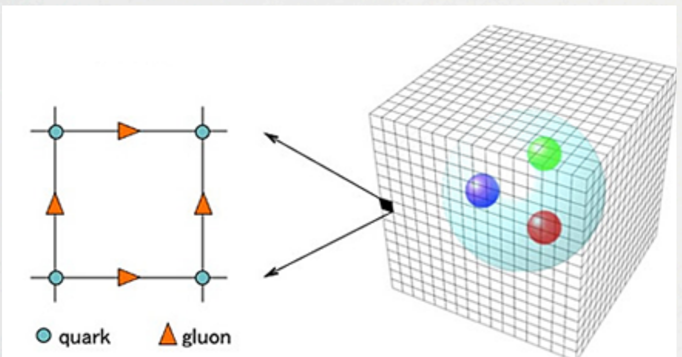


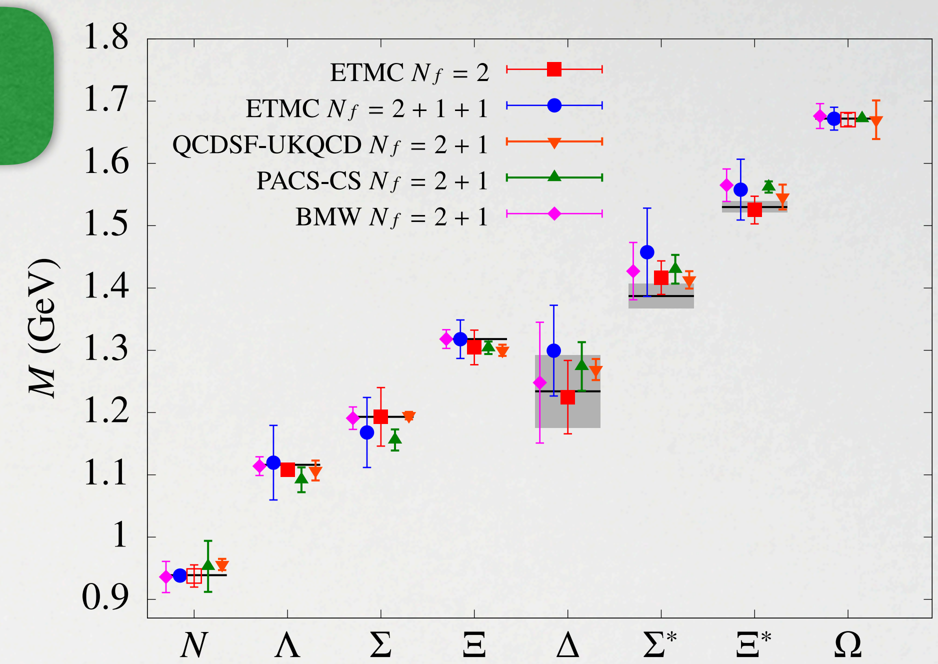
Data Analysis

Computation of observables on the Lattice

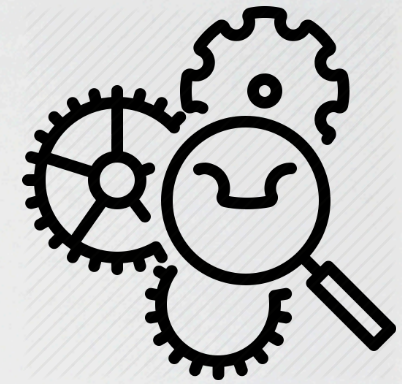
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U)\, \det(D[U]^{N_f}) e^{-S[U]}$$

LQCD



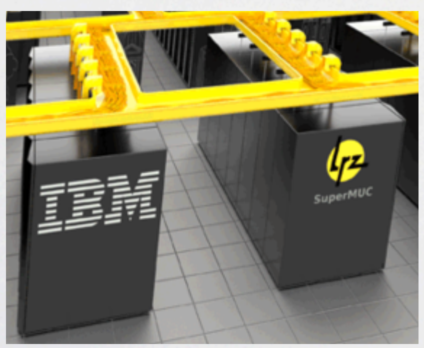


Phys. Rev. D96 (2017) no.3, 034511 1704.02647



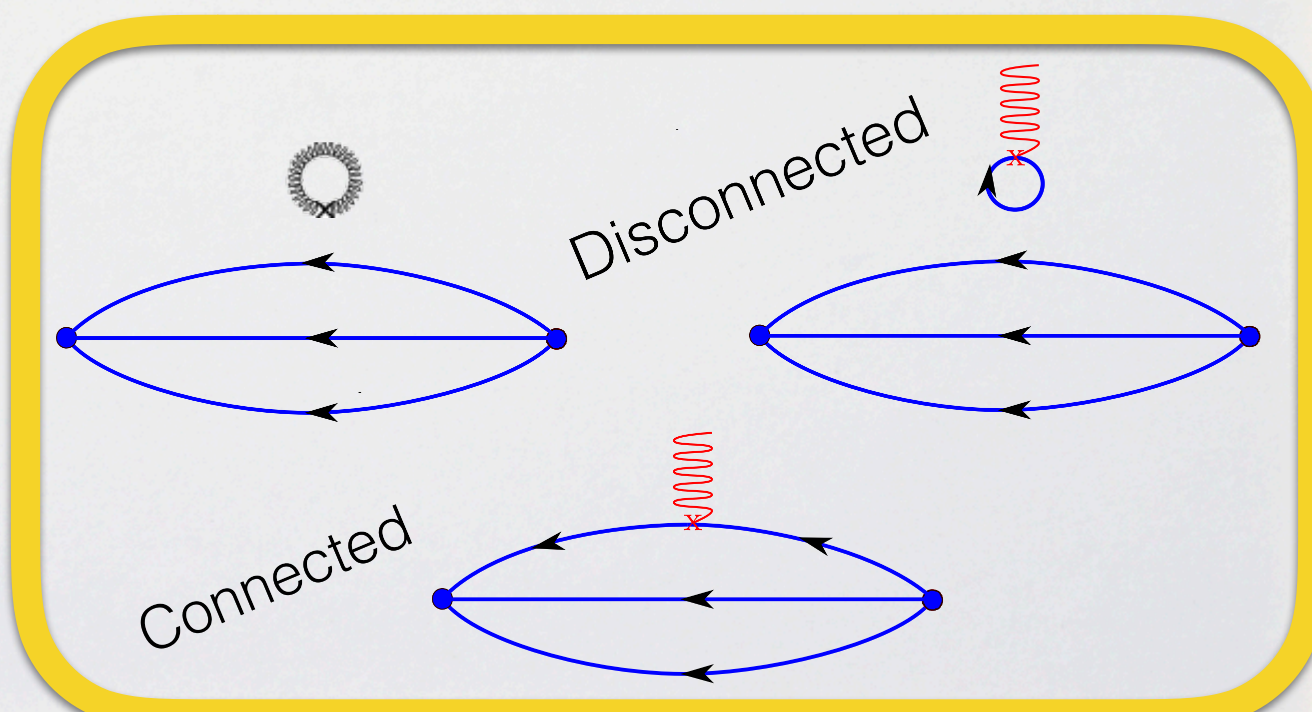
Data Analysis

Configurations Simulation



Quark Propagators

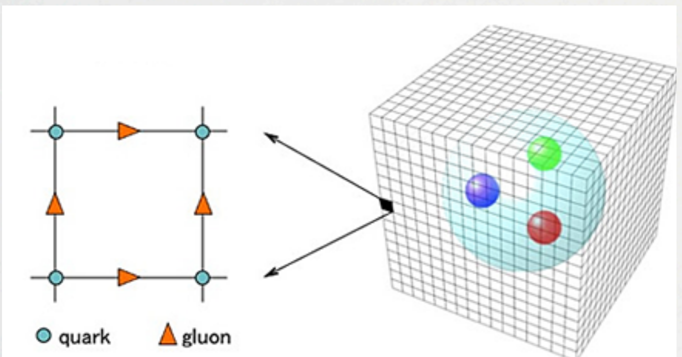
Contractions

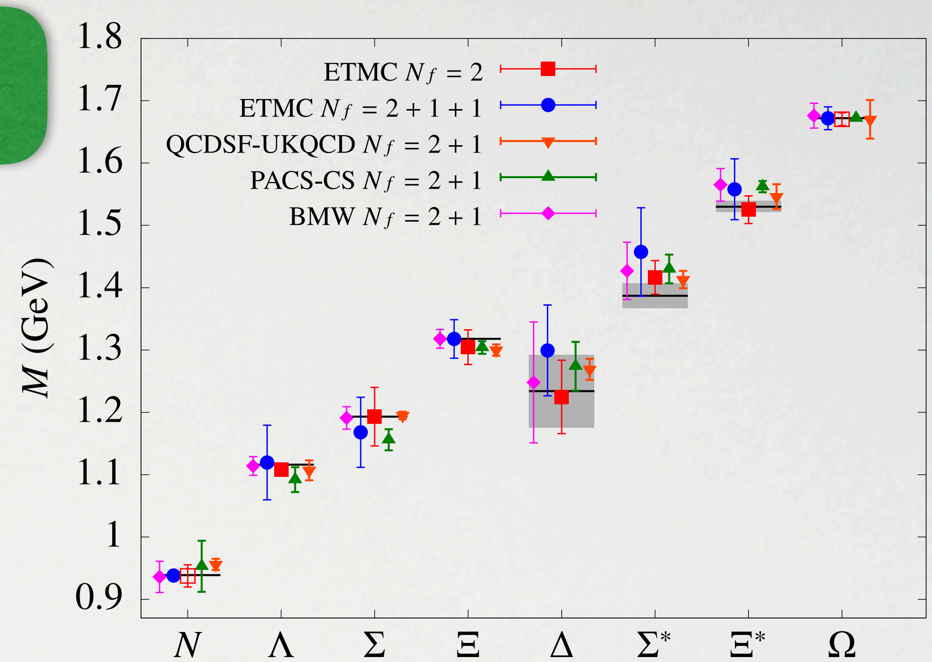


Computation of observables on the Lattice

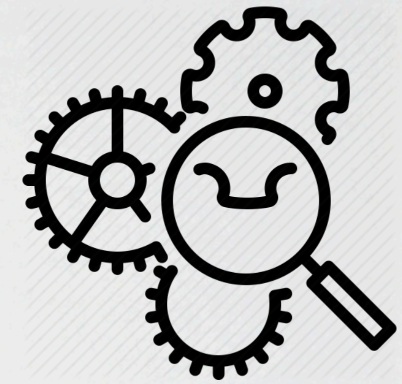
$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int_U \mathcal{O}(D^{-1}[U], U)\, \det(D[U]^{N_f})\, e^{-S[U]}$$

LQCD



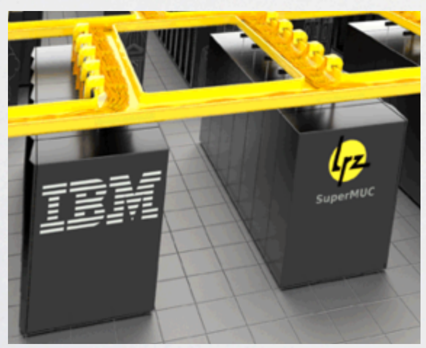


Phys. Rev. D96 (2017) no.3, 034511 1704.02647



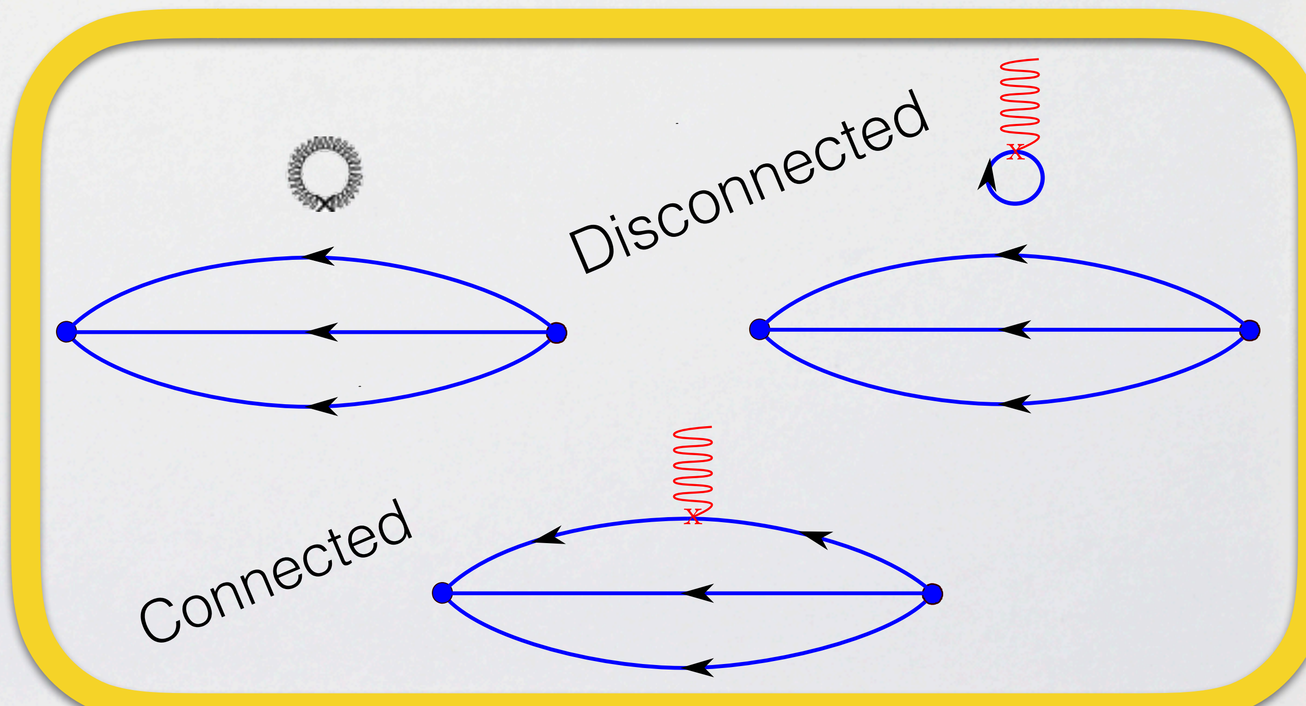
Data Analysis

Configurations Simulation



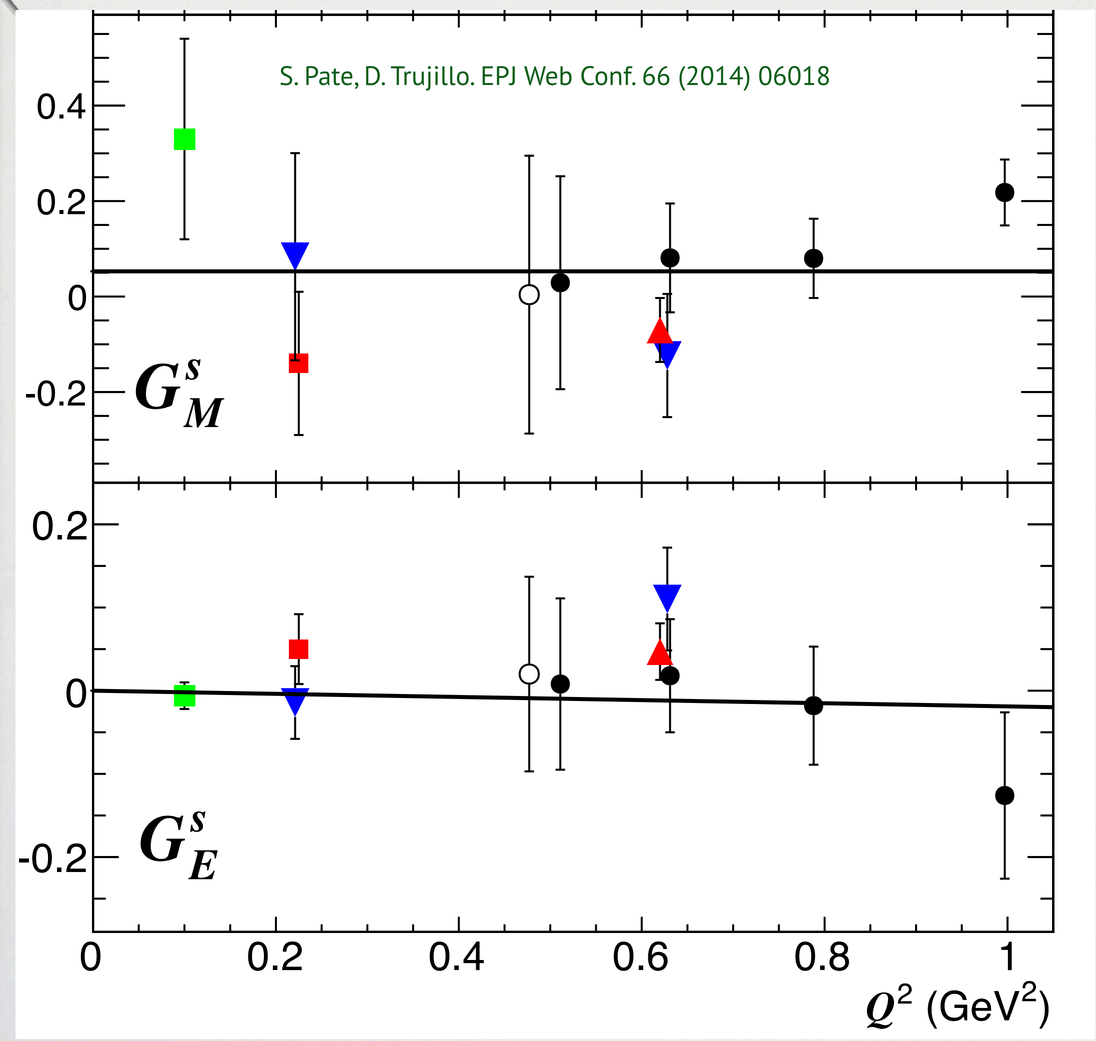
Quark Propagators

Contractions



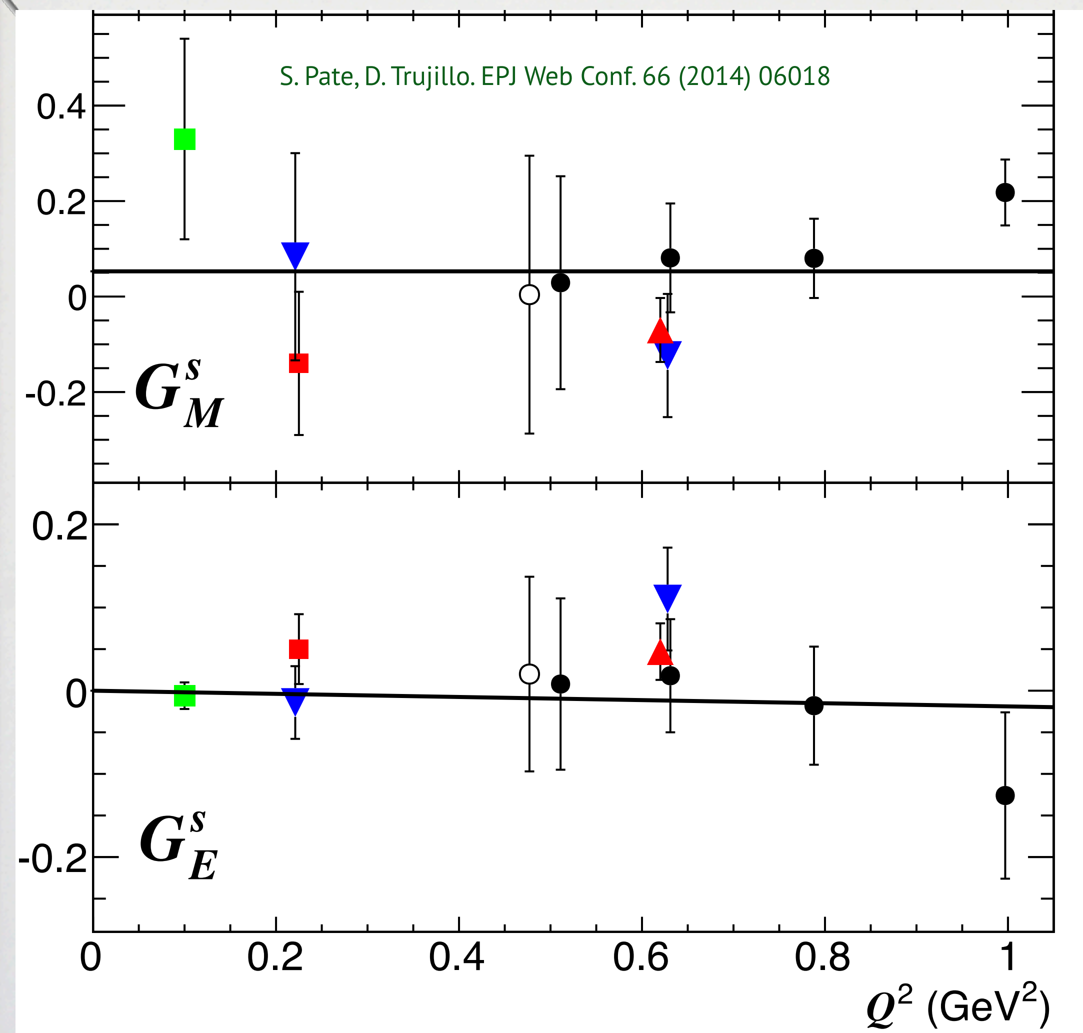
Strangeness of the Nucleon

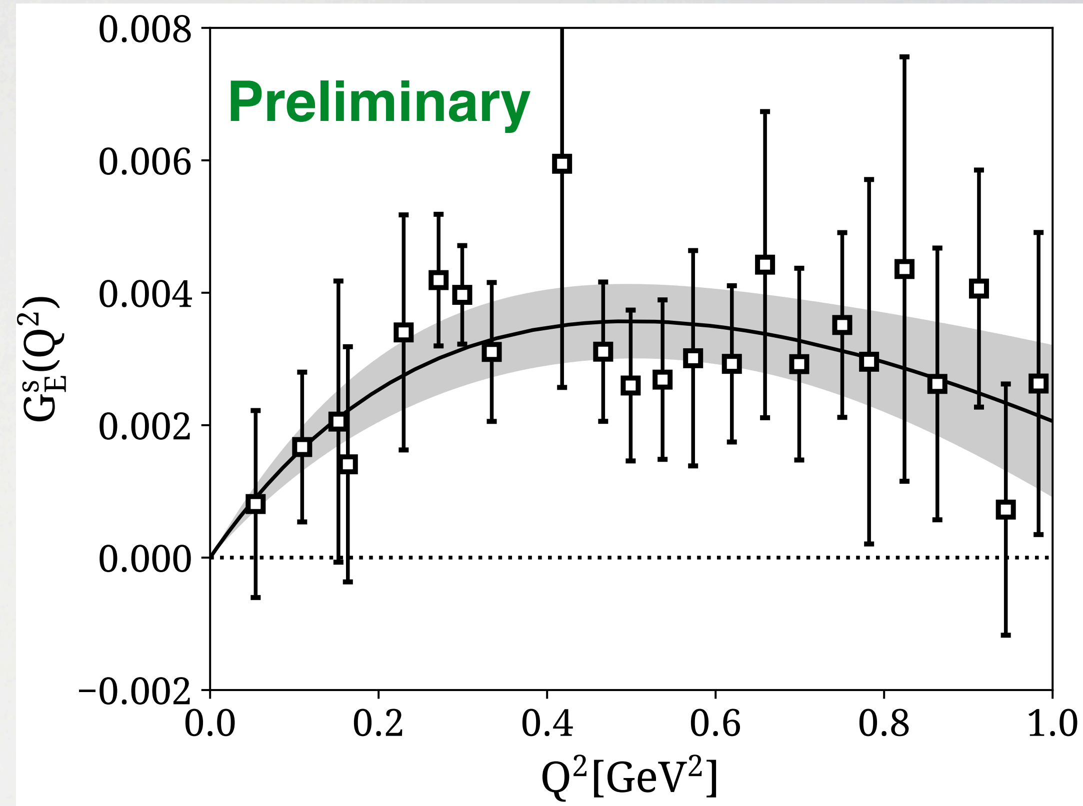
Strangeness of the Nucleon



- Strange Electric and Magnetic FFs are poorly know experimentally

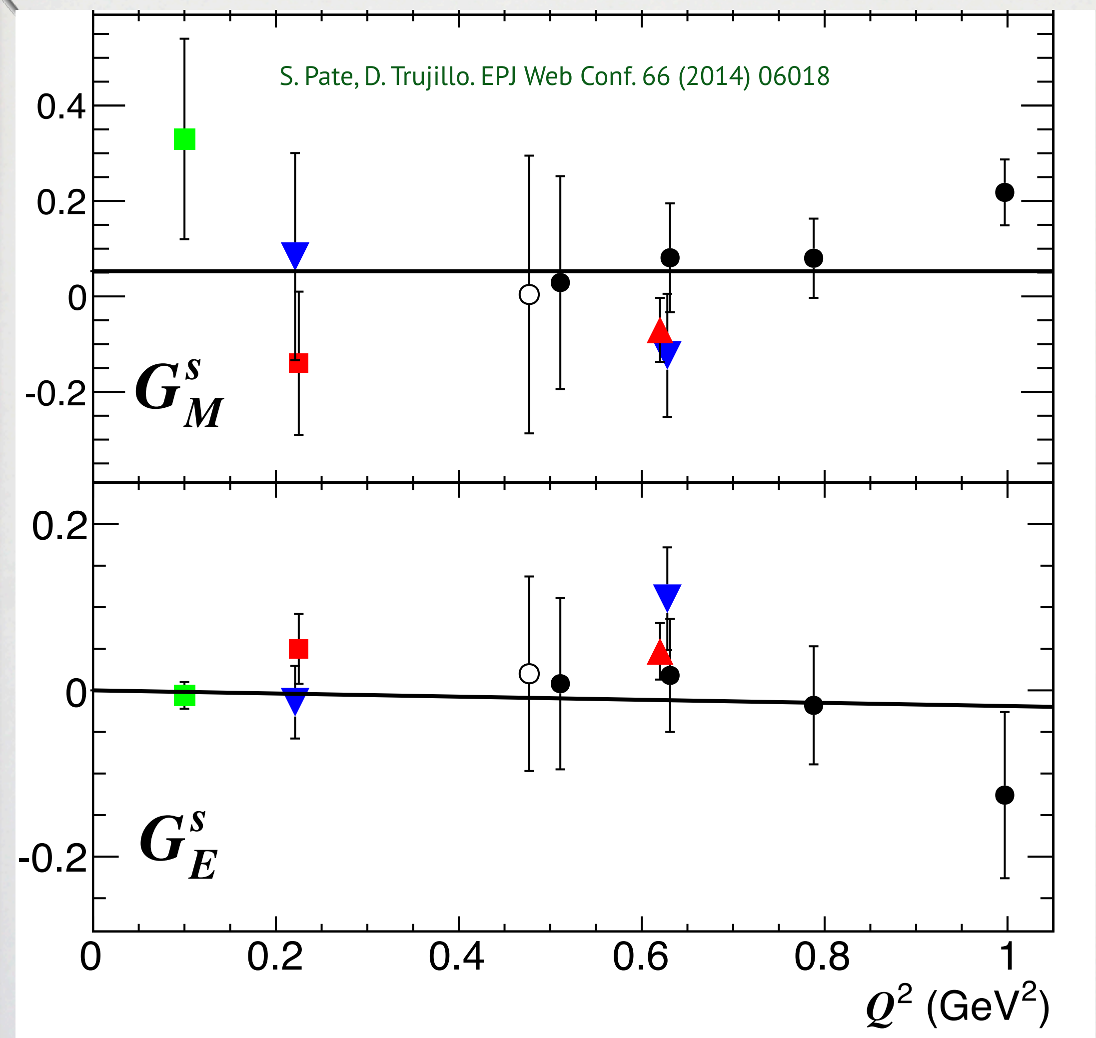
Strangeness of the Nucleon

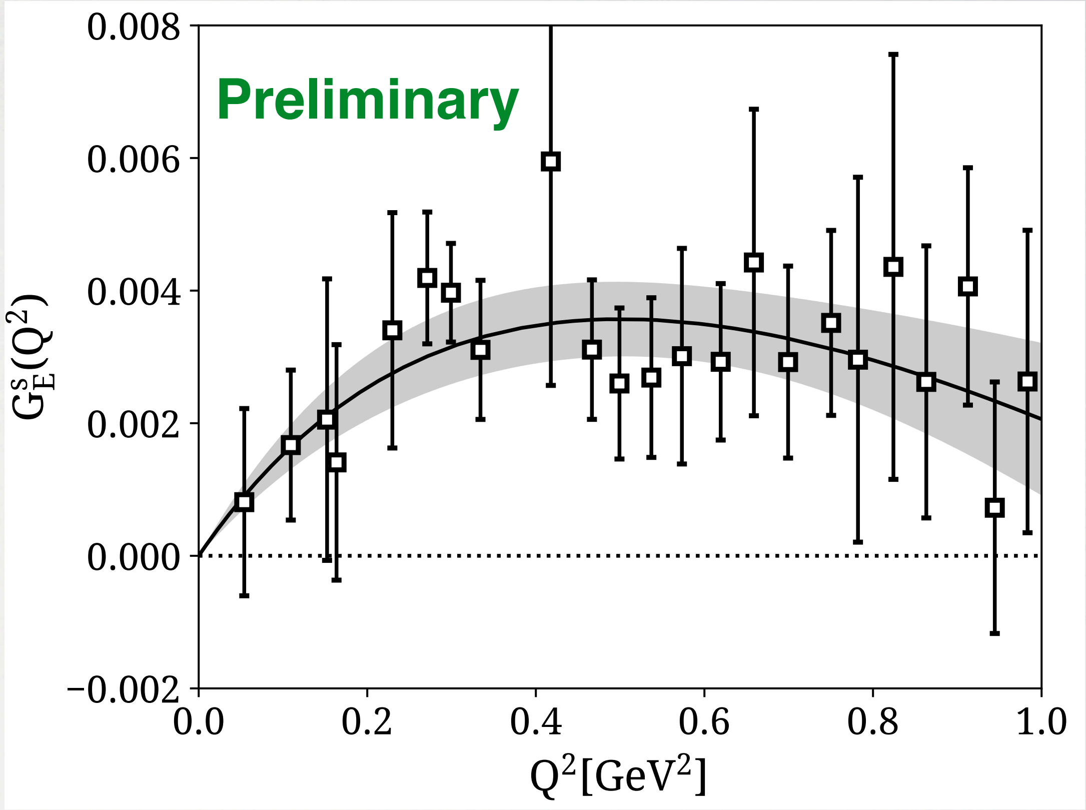


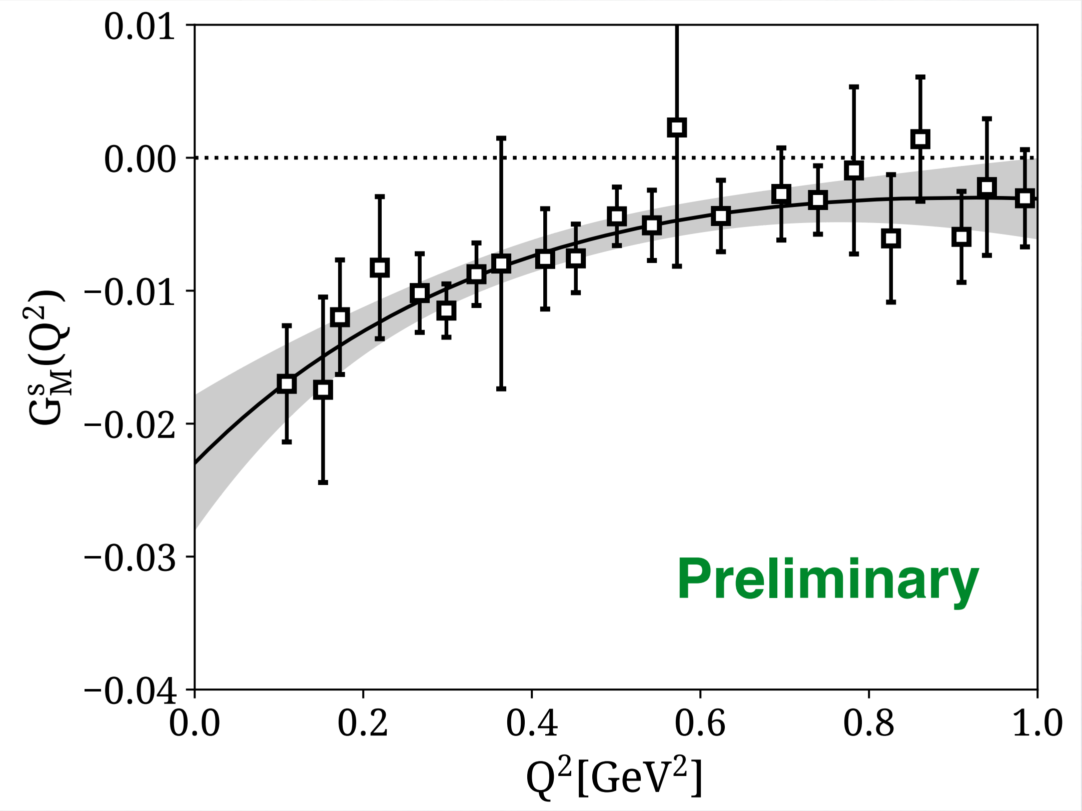


- Strange Electric and Magnetic FFs are poorly know experimentally
- LQCD: strange electric FF, clear signal but higher accuracy is needed

Strangeness of the Nucleon







- Strange Electric and Magnetic FFs are poorly know experimentally
- LQCD: strange electric FF, clear signal but higher accuracy is needed
- LQCD: Strange magnetic FF is more precise

First moments of PDFs (Mellin moments)

- First moments are readily accessible on the lattice

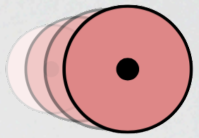
First moments of PDFs (Mellin moments)

- First moments are readily accessible on the lattice

✱ Unpolarized

$$\mathcal{O}_V^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$$\langle 1\rangle_{u_+-d_+} = g_{\rm v},\quad \langle x\rangle_{u_+-d_+},\quad \langle x\rangle_{u_++d_+}$$



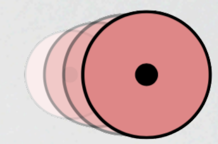
First moments of PDFs (Mellin moments)

- First moments are readily accessible on the lattice

✱ Unpolarized

$$\mathcal{O}_V^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$

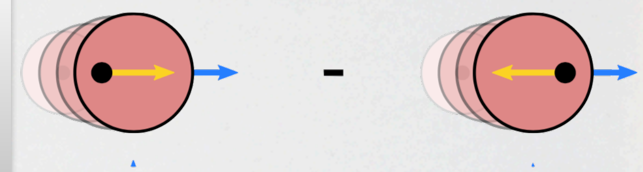
$\langle 1\rangle_{u_+-d_+} = g_{\rm v},\ \ \langle x\rangle_{u_+-d_+},\ \ \langle x\rangle_{u_++d_+}$



✱ Helicity

$$\mathcal{O}_A^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu} iD^{\mu_1} iD^{\mu_2} \ldots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{\Delta u_+-\Delta d_+} = g_{\rm A},\ \ \langle x\rangle_{\Delta u_+-\Delta d_+},\ \ \langle x\rangle_{\Delta u_++\Delta d_+}$



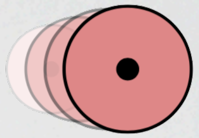
First moments of PDFs (Mellin moments)

- First moments are readily accessible on the lattice

✱ Unpolarized

$$\mathcal{O}_V^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{u_+-d_+} = g_{\rm v}, \;\; \langle x\rangle_{u_+-d_+}, \;\; \langle x\rangle_{u_++d_+}$



✱ Helicity

$$\mathcal{O}_A^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{\Delta u_+-\Delta d_+} = g_{\rm A}, \;\; \langle x\rangle_{\Delta u_+-\Delta d_+}, \;\; \langle x\rangle_{\Delta u_++\Delta d_+}$

✱ Transversity

$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\ldots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{\delta u_+-\delta d_+} = g_{\rm T}, \;\; \langle x\rangle_{\delta u_+-\delta d_+}, \;\; \langle x\rangle_{\delta u_++\delta d_+}$

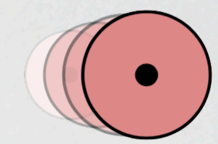
First moments of PDFs (Mellin moments)

- First moments are readily accessible on the lattice

✱ Unpolarized

$$\mathcal{O}_V^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\cdots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{u_+-d_+} = g_{\rm V},\ \ \langle x\rangle_{u_+-d_+},\ \ \langle x\rangle_{u_++d_+}$



✱ Helicity

$$\mathcal{O}_A^{\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu}iD^{\mu_1}iD^{\mu_2}\cdots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{\Delta u_+-\Delta d_+} = g_{\rm A},\ \ \langle x\rangle_{\Delta u_+-\Delta d_+},\ \ \langle x\rangle_{\Delta u_++\Delta d_+}$

✱ Transversity

$$\mathcal{O}_T^{\nu\mu\mu_1\mu_2\cdots\mu_n} = \bar{\psi}\sigma^{\nu\{\mu}iD^{\mu_1}iD^{\mu_2}\cdots iD^{\mu_n\}}\psi$$

$\langle 1\rangle_{\delta u_+-\delta d_+} = g_{\rm T},\ \ \langle x\rangle_{\delta u_+-\delta d_+},\ \ \langle x\rangle_{\delta u_++\delta d_+}$

- A lot of activity to compute Parton Distribution Functions directly on the lattice

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$\xi^- = (t-z)/\sqrt(2)$

Wilson line connection two points

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$\xi^- = (t - z)/\sqrt(2)$

Wilson line connection two points

- Cannot compute on the lattice because since is light-cone dominated $\xi^2 = t^2 - z^2 \sim 0$

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$\xi^- = (t-z)/\sqrt(2)$

Wilson line connection two points

- Cannot compute on the lattice because since is light-cone dominated $\xi^2 = t^2 - z^2 \sim 0$
- Only spatial correlators can be computed on the lattice

$$\tilde{q}(x,P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(0,z)\gamma_3 W(z)\psi(0,0)|P\rangle$$

X. Ji Phys.Rev. Lett. 110 (2013) 262002

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$\xi^- = (t-z)/\sqrt(2)$

Wilson line connection two points

- Cannot compute on the lattice because since is light-cone dominated $\xi^2 = t^2 - z^2 \sim 0$
- Only spatial correlators can be computed on the lattice

$$\tilde{q}(x,P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(0,z)\gamma_3 W(z)\psi(0,0)|P\rangle$$

X. Ji Phys.Rev. Lett. 110 (2013) 262002

- Connection between the two definitions in the IMF

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$$\xi^- = (t-z)/\sqrt{(2)}$$

Wilson line connection two points

- Cannot compute on the lattice because since is light-cone dominated $\xi^2 = t^2 - z^2 \sim 0$
- Only spatial correlators can be computed on the lattice

$$\tilde{q}(x,P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(0,z)\gamma_3 W(z)\psi(0,0)|P\rangle$$

X. Ji Phys.Rev. Lett. 110 (2013) 262002

- Connection between the two definitions in the IMF
- Challenges we face on the lattice

PDFs on the lattice

- Parton distribution functions are given by light correlators as

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^+ W(\xi^-,0)\psi(0)|P\rangle$$

$$\xi^- = (t-z)/\sqrt{(2)}$$

Wilson line connection two points

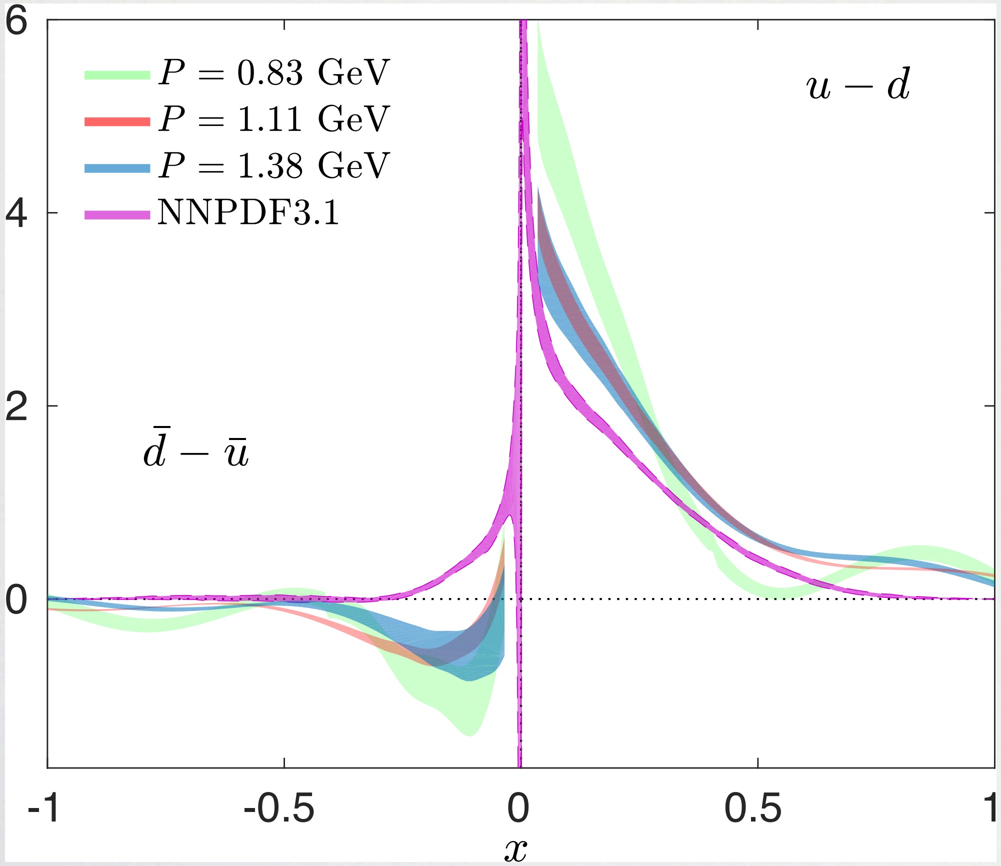
- Cannot compute on the lattice because since is light-cone dominated $\xi^2 = t^2 - z^2 \sim 0$
- Only spatial correlators can be computed on the lattice

$$\tilde{q}(x,P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izxP_3} \langle P|\bar{\psi}(0,z)\gamma_3 W(z)\psi(0,0)|P\rangle$$

X. Ji Phys.Rev. Lett. 110 (2013) 262002

- Connection between the two definitions in the IMF
- Challenges we face on the lattice
  - Finite momenta available (statistical noise increase with momentum)
  - Excited state effects increase for large momenta
  - Complicated renormalization plan (divergence that need to be treated)
  - Continuum extrapolation should be understood

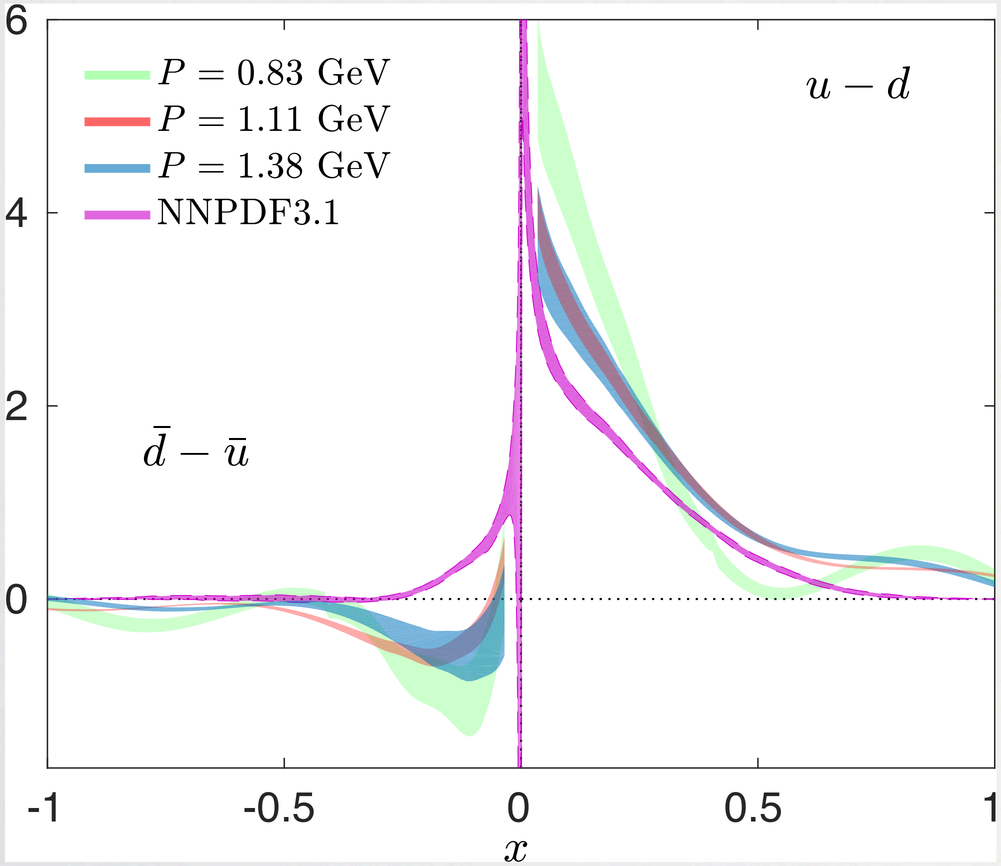
Unpolarized



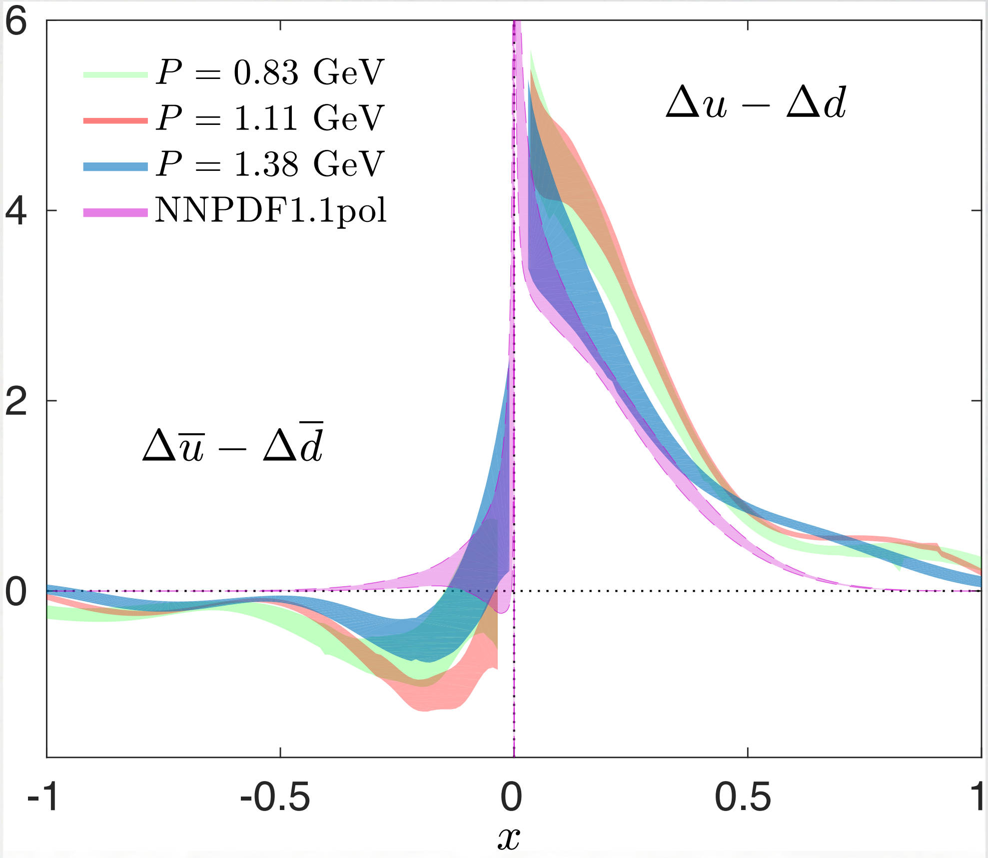
- Lattice results shift towards NNPDF result as the momentum increase
- Lattice results are still higher
- Problematic negative x region

Unpolarized, Helicity & Transversity

Unpolarized



Helicity

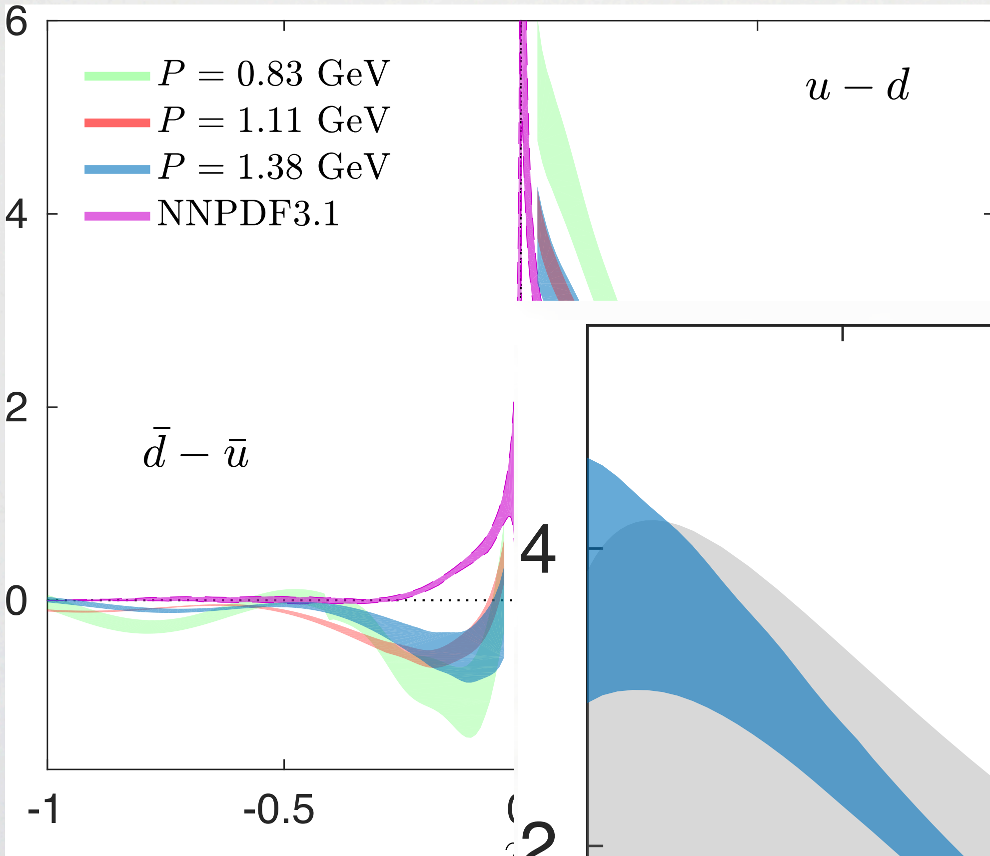


- Lattice results shift towards NNPDF result as the momentum increase
- Lattice results are still higher
- Problematic negative x region

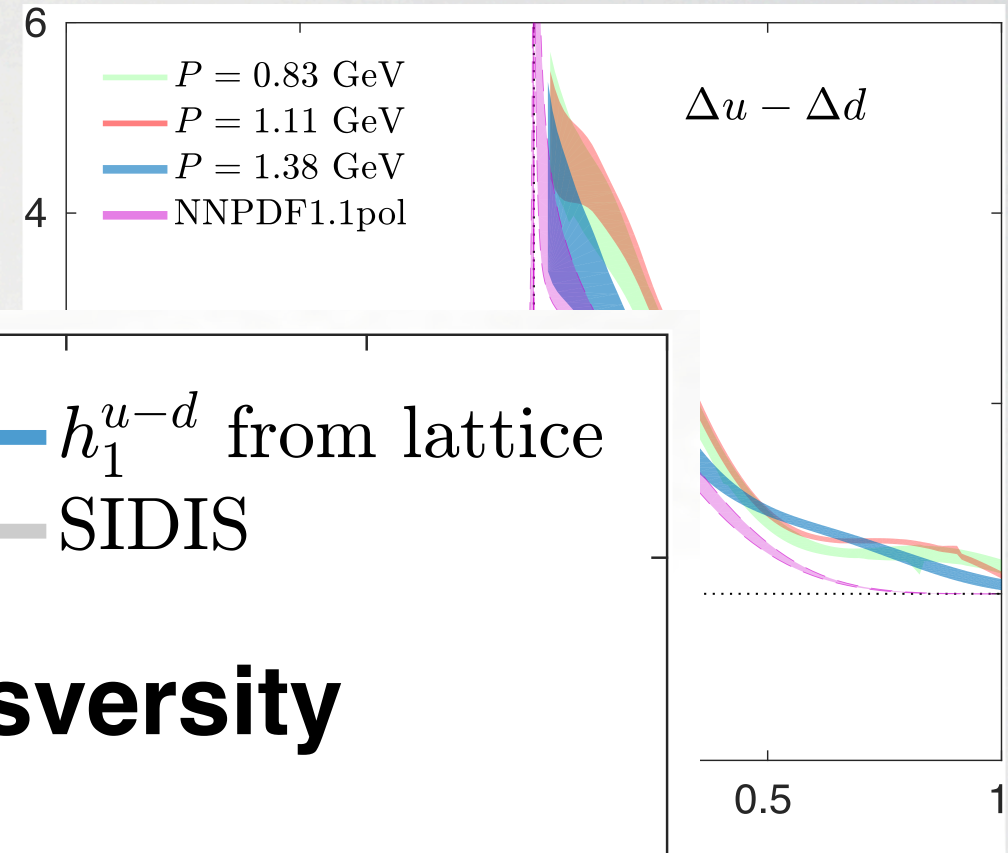
- Less accurate results for the helicity distribution
- Agreement for $x<0.4$ but disagreement for the $x>0.4$
- Problematic negative x region

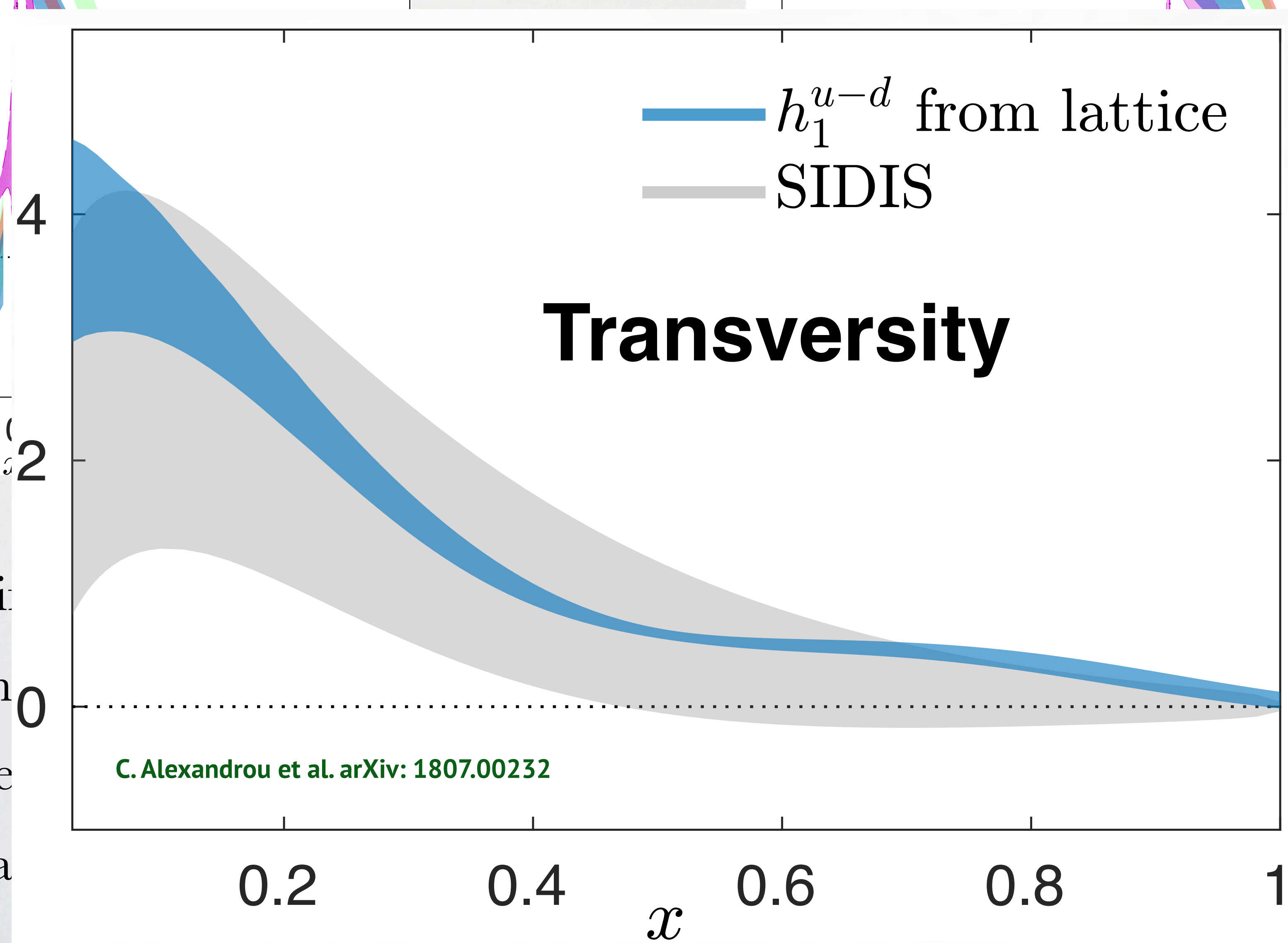
Unpolarized, Helicity & Transversity

Unpolarized



Helicity





- Lattice results shi … helicity
result as the momen … isagreement
- Lattice results are
- Problematic nega … on

Intrinsic spin contributions

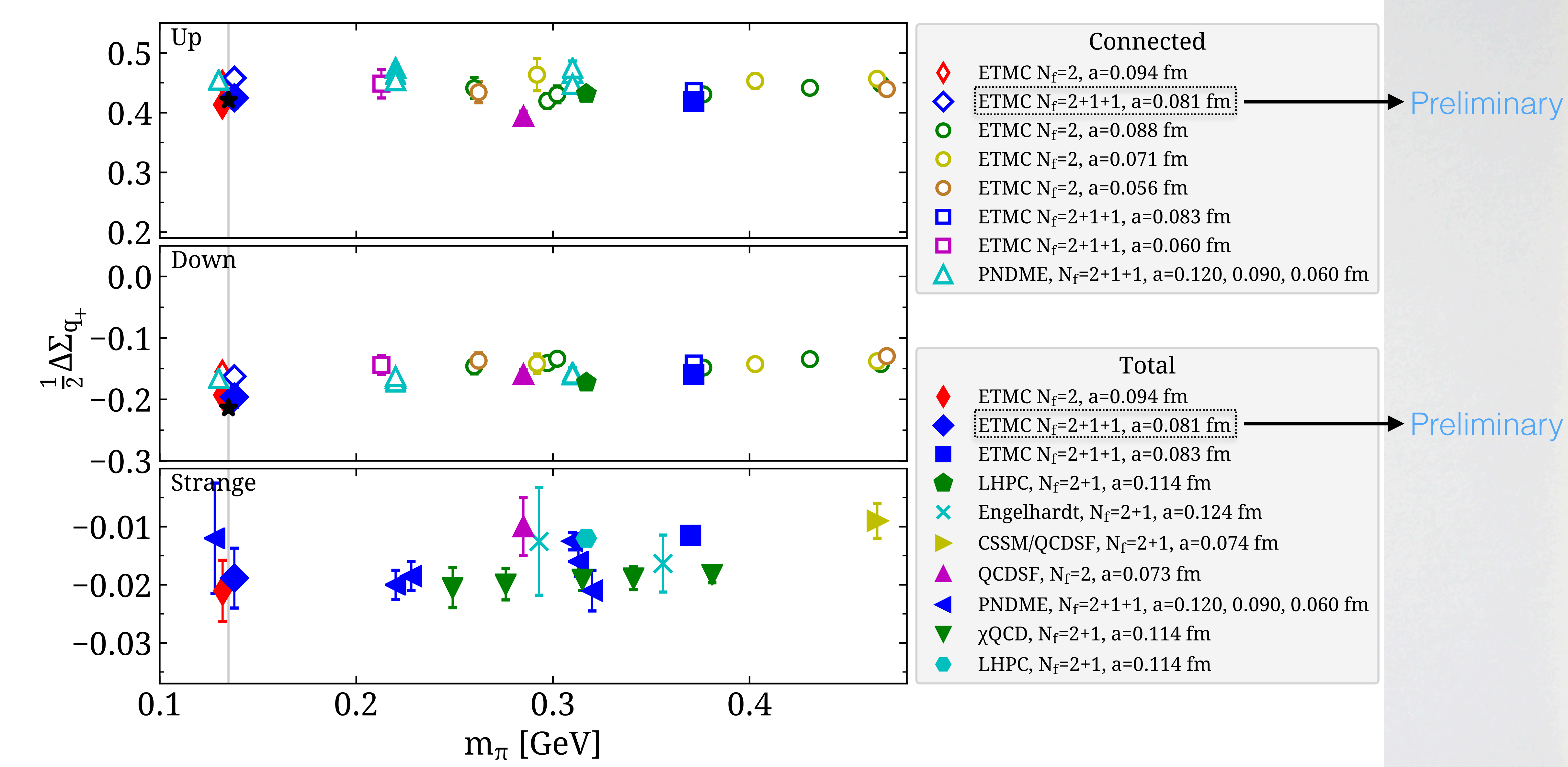
$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,\left[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)\right] = g_A^q$$

Quantities are given in $\overline{\mathrm{MS}}$ scheme at $\mu^2 = 4\ \mathrm{GeV}^2$

Intrinsic spin contributions

$$\Delta\Sigma_{q_+}(\mu^2) = \int_0^1 dx\,[\Delta q(x,\mu^2) + \Delta\bar{q}(x,\mu^2)] = g_A^q$$

Quantities are given in $\overline{\text{MS}}$ scheme at $\mu^2 = 4\ \text{GeV}^2$



- Mild discretization effects
- Mild pion mass dependence
- Sea quarks contribution is crucial to find agreement with the experiment
- Up, down and strange contributions are up to around 40% of 1/2 at the physical point

Quark momentum fraction

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\rm DV}|N(p,s)\rangle = \bar{u}_N(p',s') \Big[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big] u_N(p,s)$$

$$\mathcal{O}^{\mu\nu}_{\rm DV} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$$

Quark momentum fraction

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\rm DV}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

$$\mathcal{O}^{\mu\nu}_{\rm DV} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$$

$$J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

$$\langle x\rangle = A_{20}(0)$$

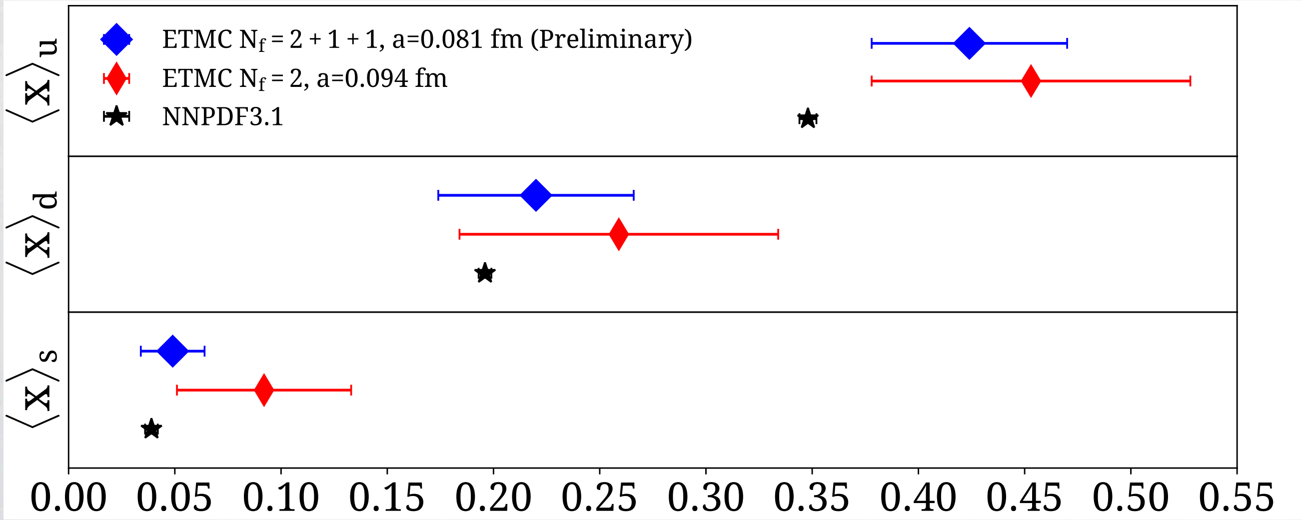
Quark momentum fraction

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\rm DV}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

$$\mathcal{O}^{\mu\nu}_{\rm DV} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$$

$$J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

$$\langle x\rangle = A_{20}(0)$$



- Lattice results have large errors compared to the phenomenological results
- Individual contributions receive contributions from disconnected diagrams
- Improved techniques for the disconnected needed

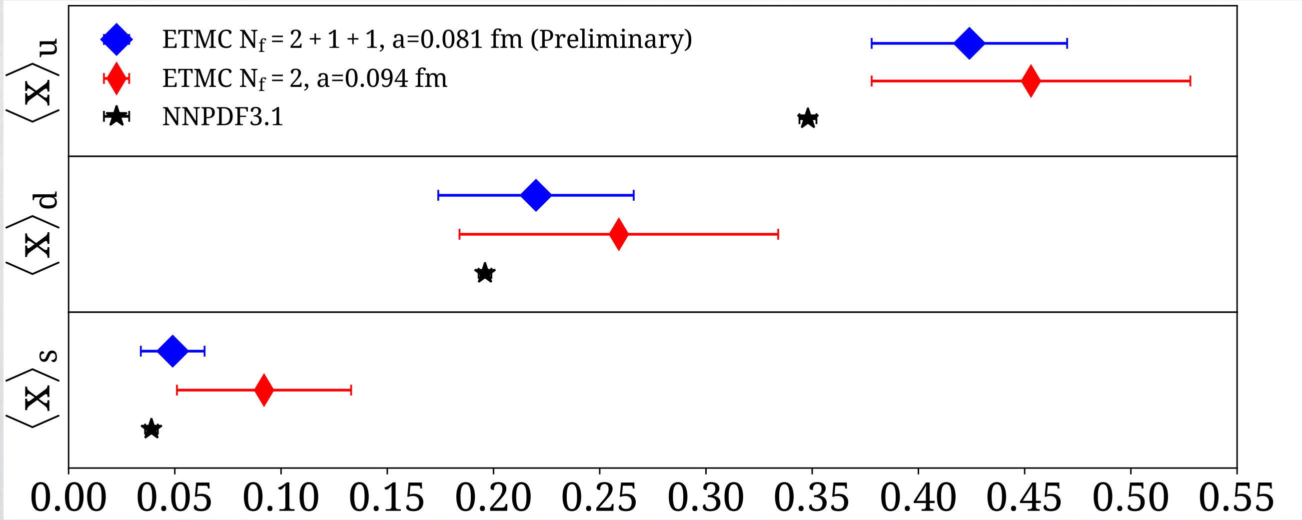
Quark momentum fraction

$$\langle N(p',s')|\mathcal{O}^{\mu\nu}_{\rm DV}|N(p,s)\rangle = \bar{u}_N(p',s')\Big[A_{20}(q^2)\gamma^{\{\mu}P^{\nu\}} + B_{20}(q^2)\frac{i\sigma^{\{\mu\alpha}q_\alpha P^{\nu\}}}{2m} + C_{20}(q^2)\frac{q^{\{\mu}q^{\nu\}}}{m}\Big]u_N(p,s)$$

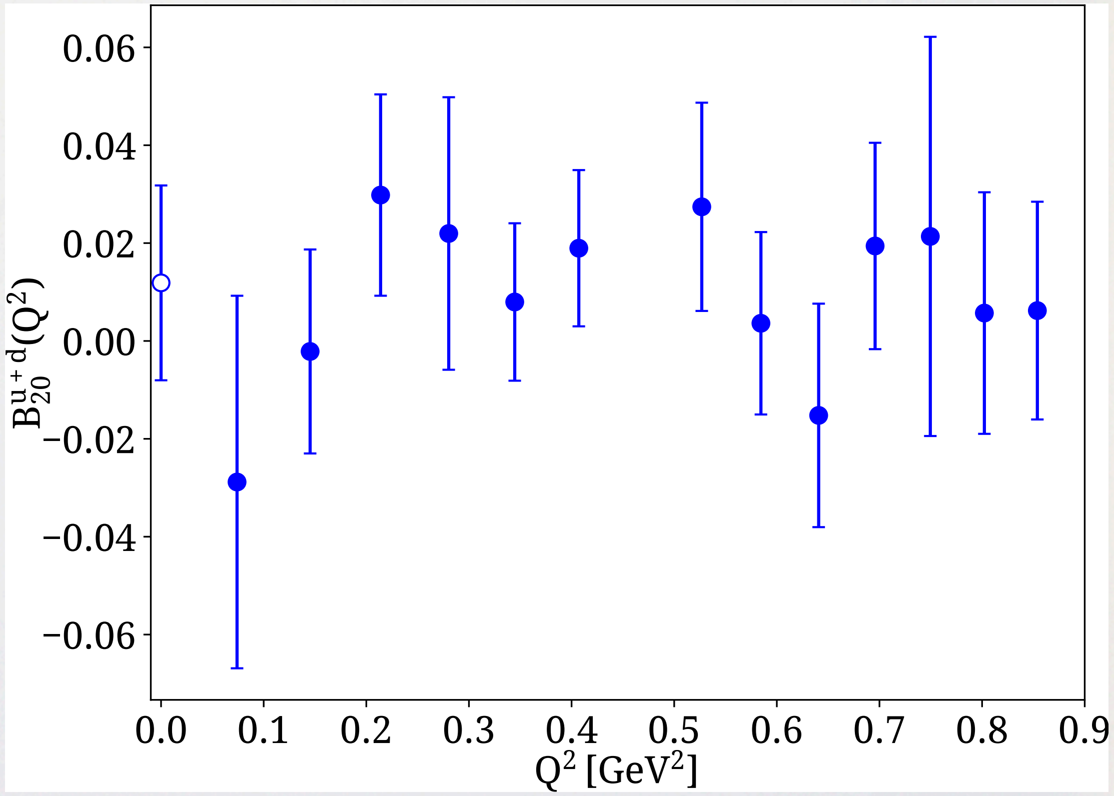
$$\mathcal{O}^{\mu\nu}_{\rm DV} = \bar{q}\gamma^{\{\mu}iD^{\nu\}}q$$

$$J_q = \frac{1}{2}\left[A^q_{20}(0) + B^q_{20}(0)\right]$$

$$\langle x\rangle = A_{20}(0)$$



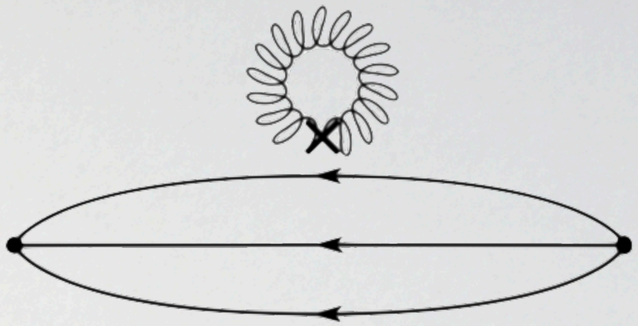
$B_{20}^{u+d,s,c}$ are found to be small and compatible with zero



- Lattice results have large errors compared to the phenomenological results
- Individual contributions receive contributions from disconnected diagrams
- Improved techniques for the disconnected needed

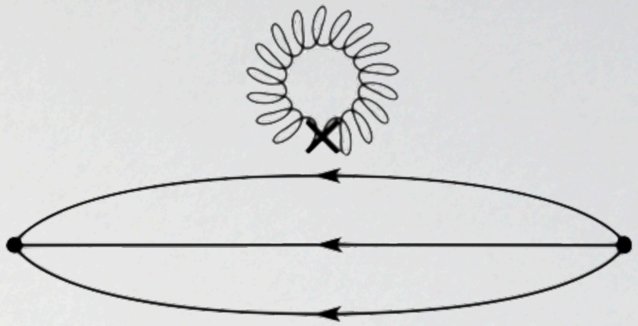
Gluon momentum fraction

Direct Calculation: $\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$



Gluon momentum fraction

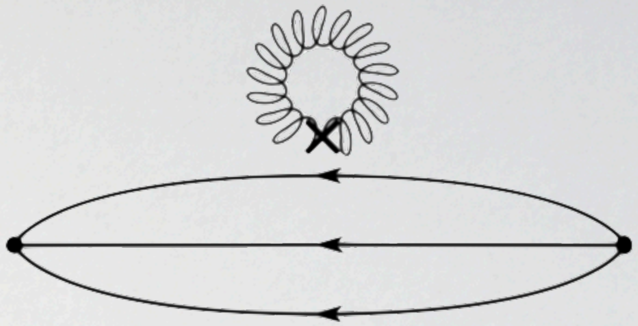
Direct Calculation: $\mathcal{O}^{g}_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$



$$J_g = \frac{1}{2}[A^{g}_{20}(0) + B^{g}_{20}(0)]$$

Gluon momentum fraction

Direct Calculation: $\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\,[G_{\mu\rho}G_{\nu\rho}]$



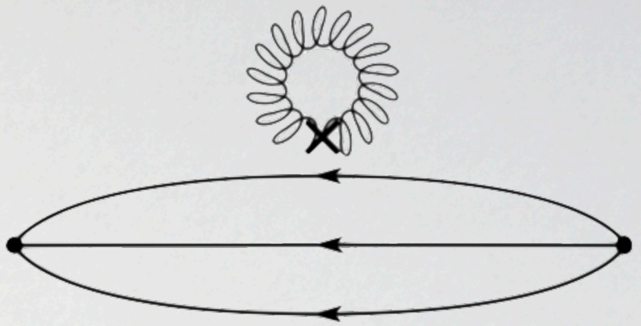
$$J_g = \frac{1}{2}[A^g_{20}(0) + B^g_{20}(0)]$$

- Not included in this study
- But we know that $\sum_q B^q_{20}(0) = -B^g_{20}(0)$

Gluon momentum fraction

Direct Calculation:

$$\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$$



$$J_g = \frac{1}{2}[A^g_{20}(0) + B^g_{20}(0)]$$

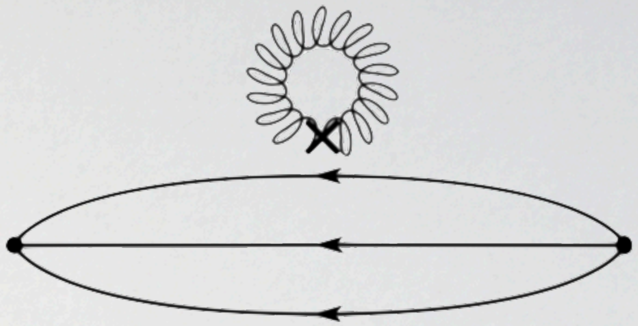
- Not included in this study
- But we know that $\sum_q B^q_{20}(0) = -B^g_{20}(0)$

- Disconnected diagram:
  - Noisy quantity
  - Requires special techniques (smearing)

Gluon momentum fraction

Direct Calculation:

$$\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$$



$$J_g = \frac{1}{2}[A^g_{20}(0) + B^g_{20}(0)]$$

- Not included in this study
- But we know that $\sum_q B^q_{20}(0) = -B^g_{20}(0)$

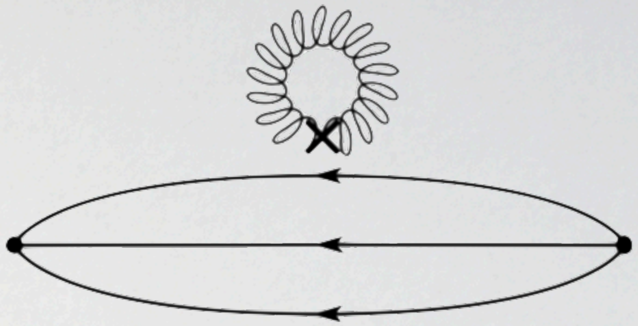
- Disconnected diagram:
  - Noisy quantity
  - Requires special techniques (smearing)

- Renormalization:
  - Mixing with quark momentum fraction
  - Perturbative renormalization

$$\langle x\rangle^R_g = Z_{gg}\langle x\rangle^B_g + Z_{gq}\sum_q \langle x\rangle^B_q$$

Gluon momentum fraction

Direct Calculation: $\mathcal{O}^g_{\mu\nu} = -\mathrm{Tr}\left[G_{\mu\rho}G_{\nu\rho}\right]$



$$J_g = \frac{1}{2}[A^g_{20}(0) + B^g_{20}(0)]$$

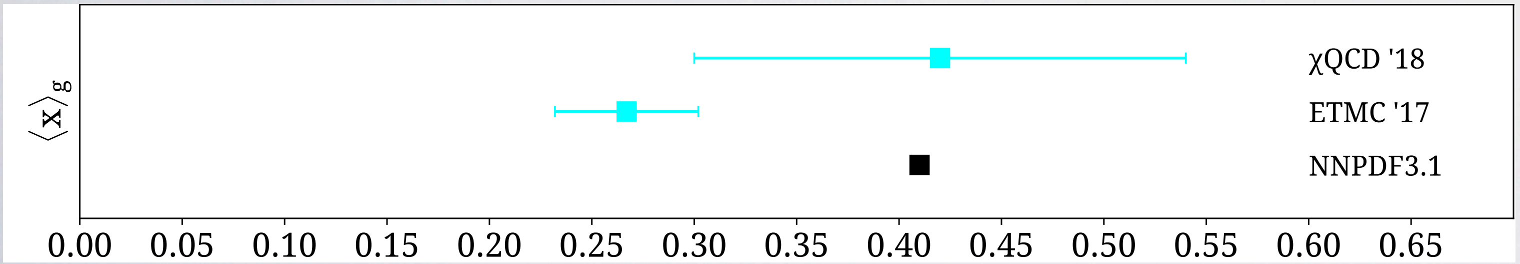
- Not included in this study
- But we know that $\sum_q B^q_{20}(0) = -B^g_{20}(0)$

- Disconnected diagram:
  - Noisy quantity
  - Requires special techniques (smearing)

- Renormalization:
  - Mixing with quark momentum fraction
  - Perturbative renormalization

$$\langle x\rangle^R_g = Z_{gg}\langle x\rangle^B_g + Z_{gq}\sum_q \langle x\rangle^B_q$$

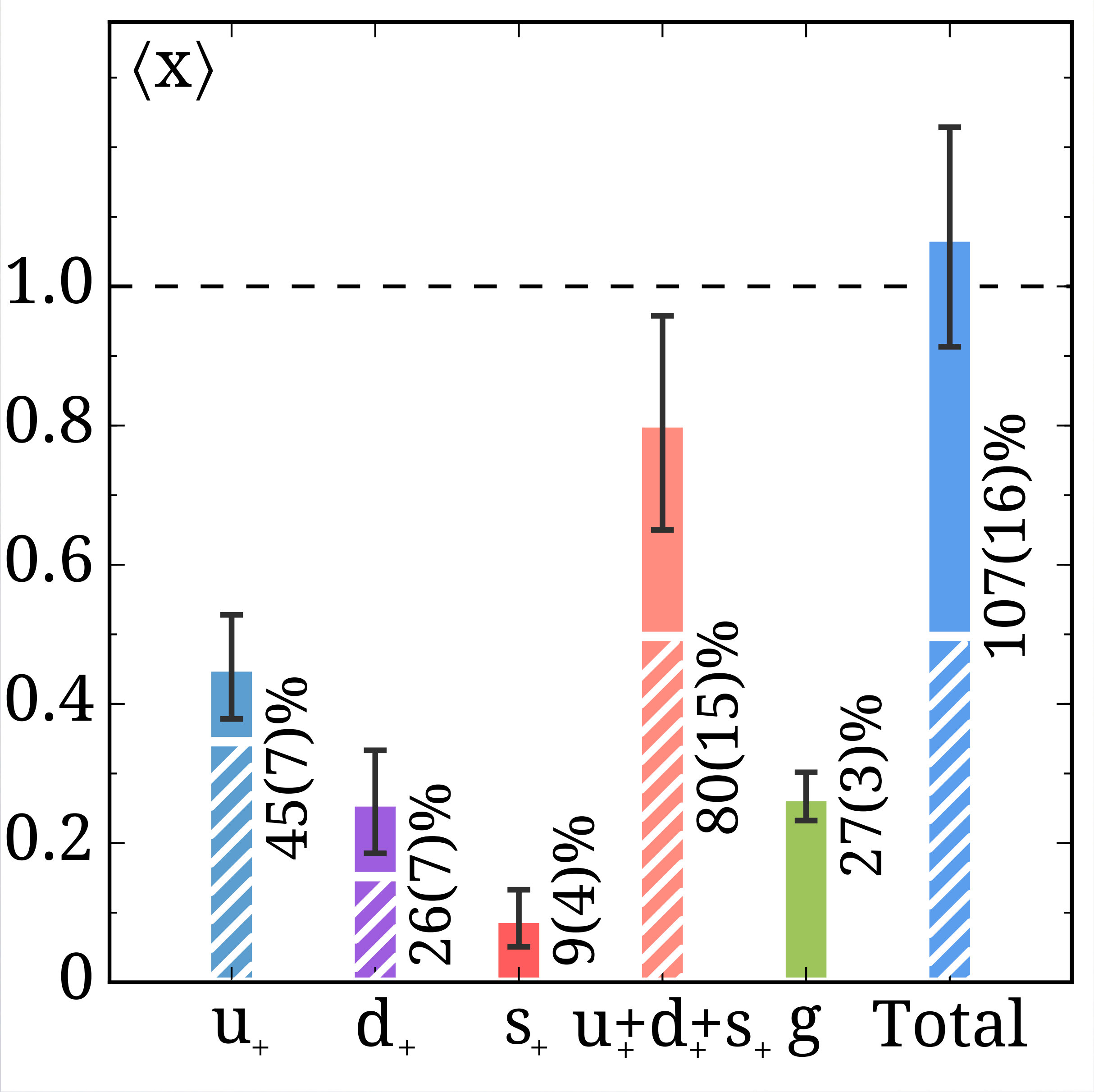
C. Alexandrou et al. ,Phys. Rev. D96, 054503 (2017),



$$\langle x\rangle^R_g = 0.267(22)(19)(24)$$

Momentum decomposition

$$\langle x \rangle_{u_+ + d_+ + s_+ + g} = 1.07(12)(10)$$



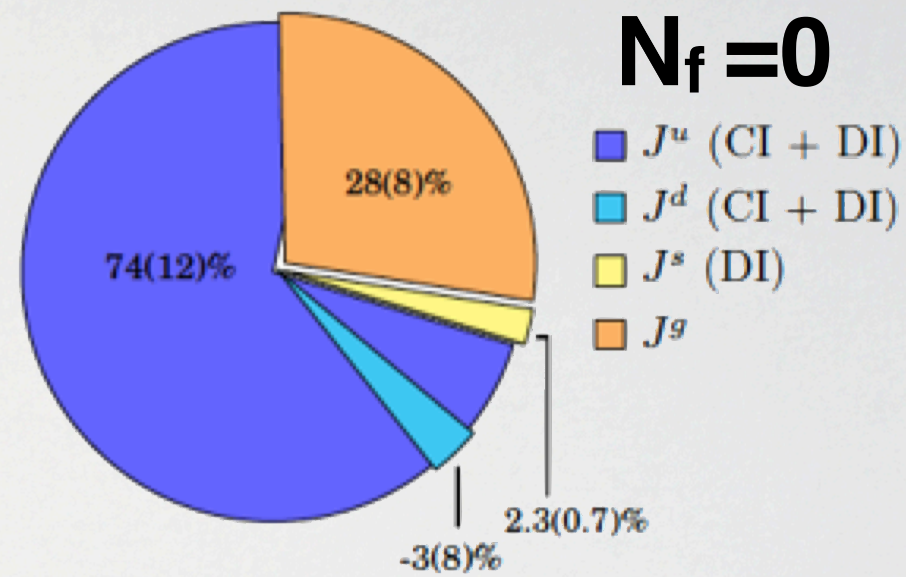
C. Alexandrou et al. ,Phys. Rev. Lett. 119, 142002 (2017), 1706.02973

- Includes up, down, strange and gluons simulated at the the physical pion mass
- Momentum sum satisfied within errors
- Crucial disconnected contribution (solid) compare to connected (hatched)
- Uncertainties are about 10% in component contributions

Spin decomposition

Spin decomposition

χQCD collaboration

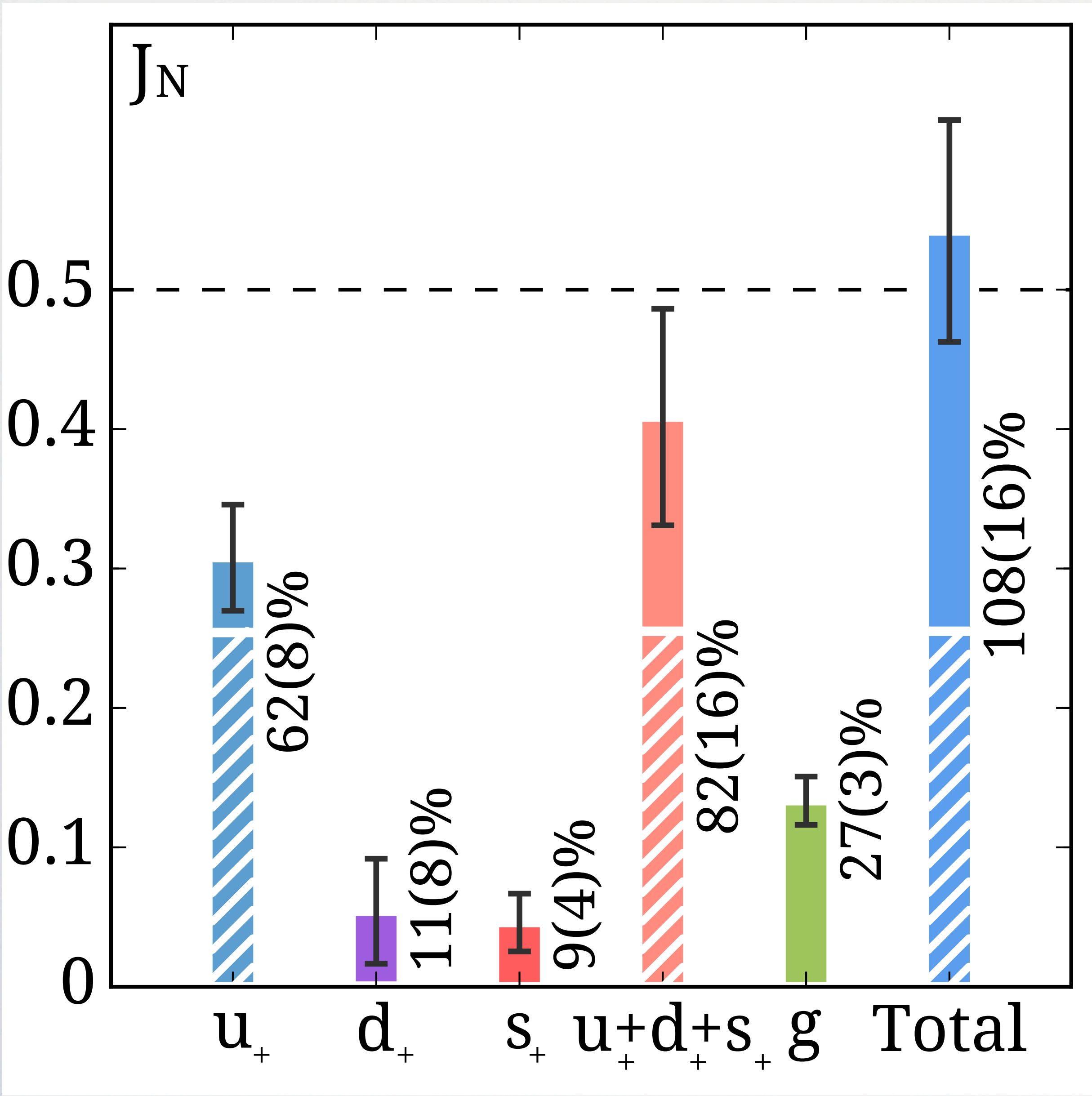


M. Deka et al. ,Phys. Rev. D. 91, 014505 (2015), 1312.4816

Spin decomposition

ETMC

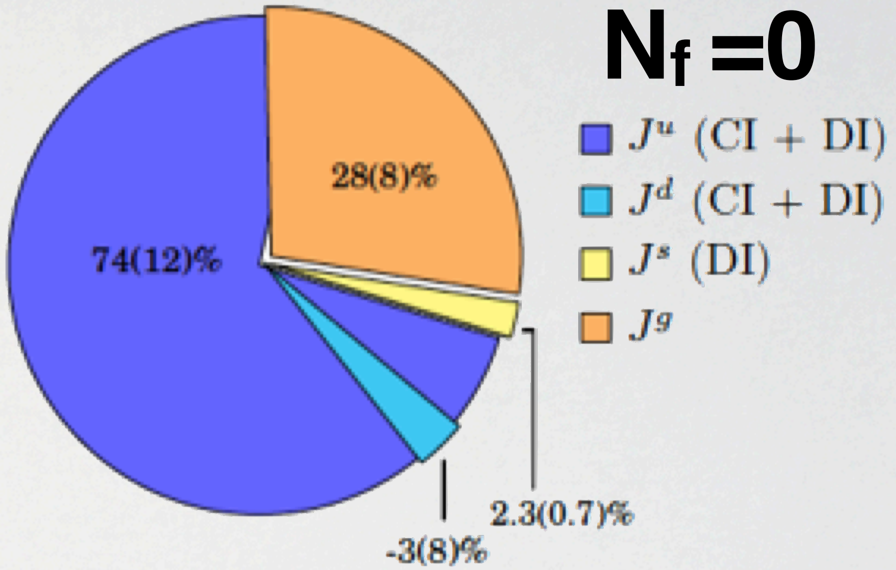
$N_f = 2$ $J^N_{u_+ + d_+ + s_+ + g} = 0.541(62)(49)$



C. Alexandrou et al. ,Phys. Rev. Lett. 119, 142002 (2017), 1706.02973

χQCD collaboration

$N_f = 0$

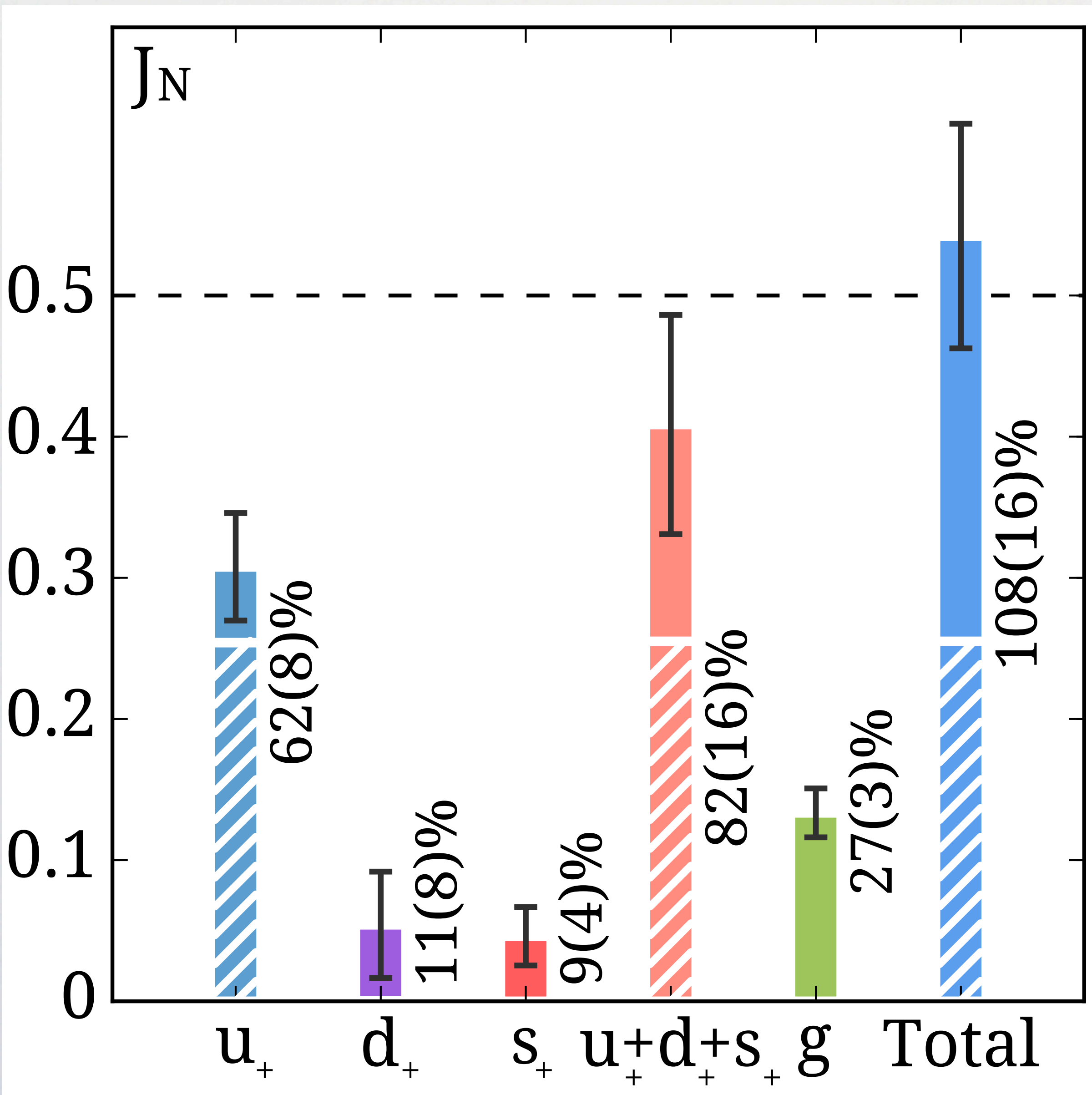


M. Deka et al. ,Phys. Rev. D. 91, 014505 (2015), 1312.4816

Spin decomposition

ETMC

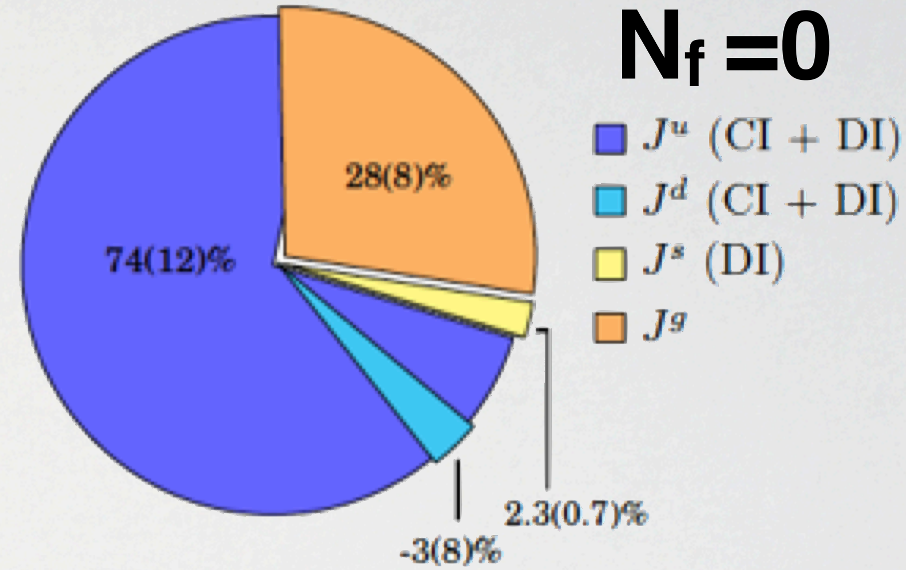
$N_f = 2$ $J^N_{u_+ + d_+ + s_+ + g} = 0.541(62)(49)$



C. Alexandrou et al.,Phys. Rev. Lett. 119, 142002 (2017), 1706.02973

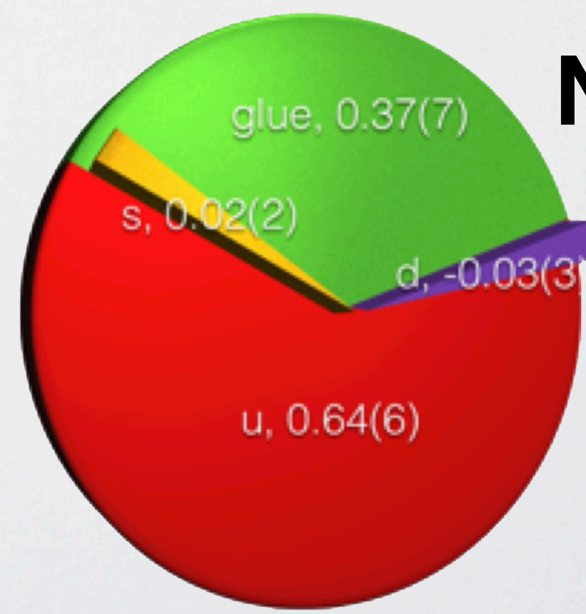
χQCD collaboration

$N_f = 0$



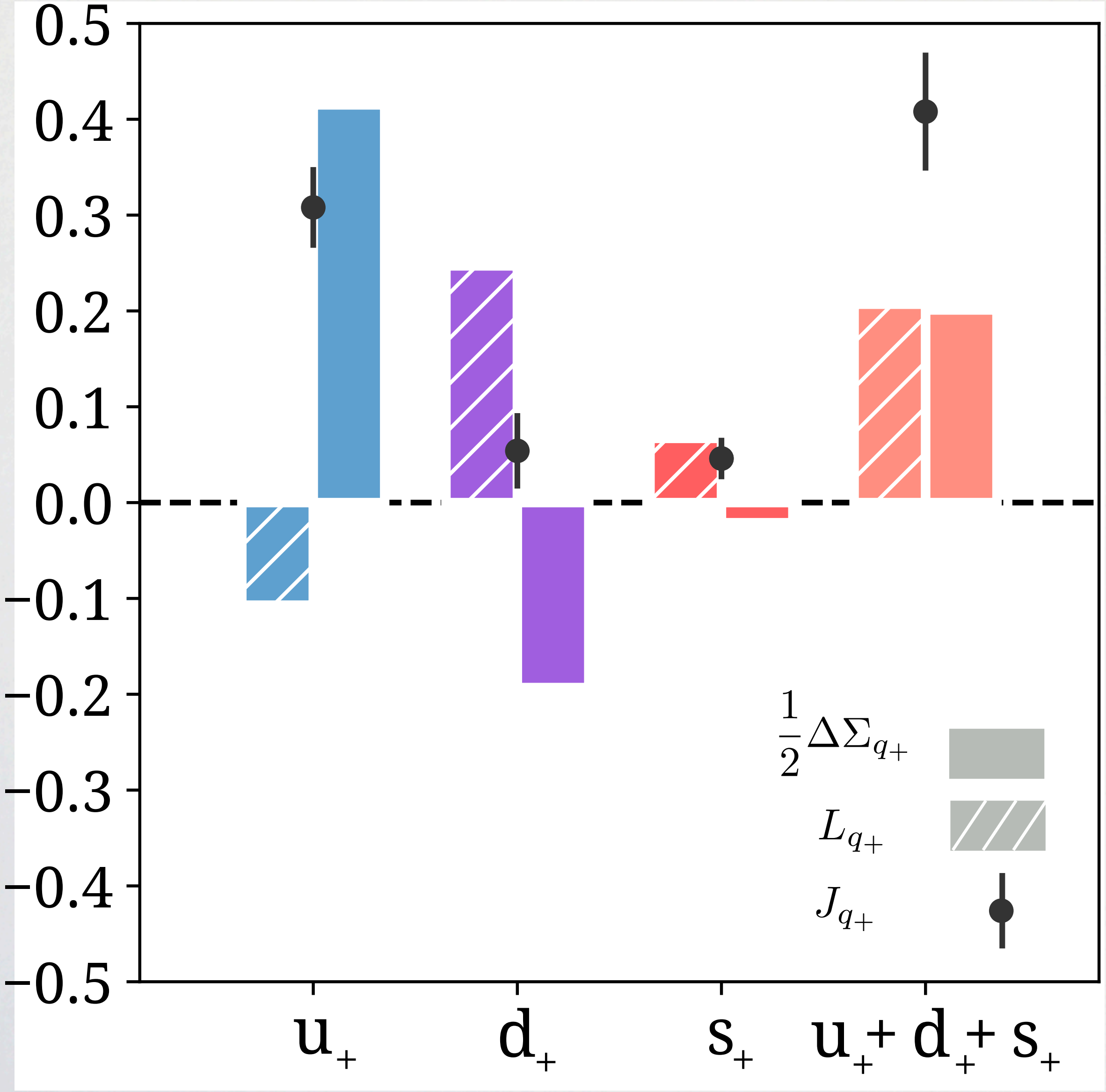
M. Deka et al.,Phys. Rev. D. 91, 014505 (2015), 1312.4816

$N_f = 2+1$



In preparation

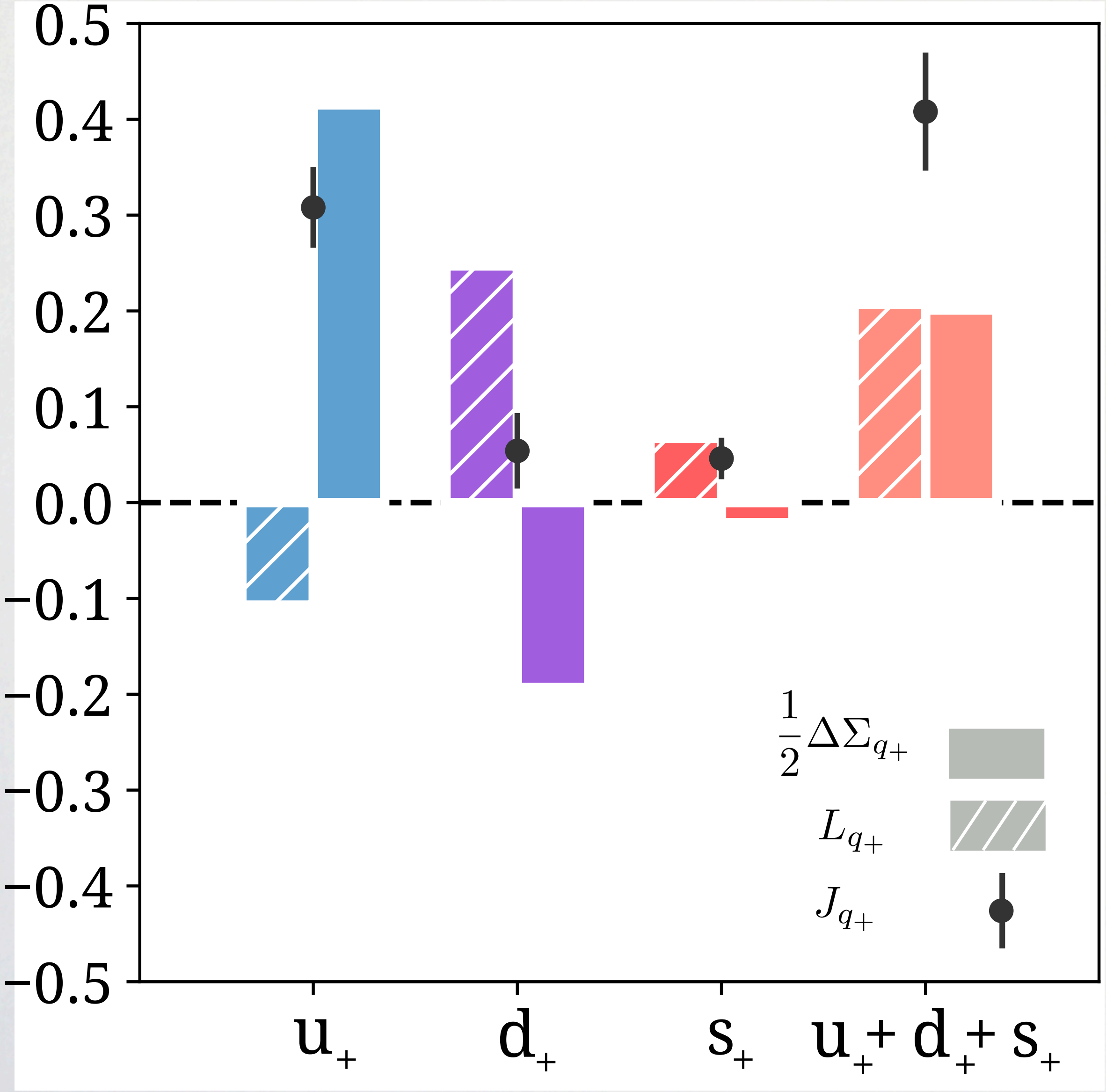
Orbital angular momentum



L_{q+} : indirectly from

$$L_{q+} = J_{q+} - \frac{1}{2}\Delta\Sigma_{q+}$$

Orbital angular momentum



L_{q+} : indirectly from

$$L_{q+} = J_{q+} - \frac{1}{2}\Delta\Sigma_{q+}$$

C. Alexandrou et al. ,Phys. Rev. Lett. 119, 142002 (2017), 1706.02973

	$\frac{1}{2}\Delta\Sigma$	J	L	$\langle x \rangle$
u_+	0.415(13)(2)	0.308(30)(24)	-0.107(32)(24)	0.453(57)(48)
d_+	-0.193(8)(3)	0.054(29)(24)	0.247(30)(24)	0.259(57)(47)
s_+	-0.021(5)(1)	0.046(21)(0)	0.067(21)(1)	0.092(41)(0)
g	-	0.133(11)(14)	-	0.267(22)(27)
tot.	0.201(17)(5)	0.541(62)(49)	0.207(64)(45)	1.07(12)(10)

Summary

With Simulations directly at the physical point LQCD enters a new era

Summary

With Simulations directly at the physical point LQCD enters a new era

- A complete LQCD study about the origin of the nucleon spin
  - Spin sum satisfied: $J^N_{u_++d_++s_++g} = 0.541(62)(49)$
  - Momentum sum satisfied: $\langle x \rangle_{u_++d_++s_++g} = 1.07(12)(10)$

Summary

With Simulations directly at the physical point LQCD enters a new era

- A complete LQCD study about the origin of the nucleon spin
  - Spin sum satisfied: $J^N_{u_++d_++s_++g} = 0.541(62)(49)$
  - Momentum sum satisfied: $\langle x \rangle_{u_++d_++s_++g} = 1.07(12)(10)$
- Large effort on addressing systematic effects
  - Volume effects: $64^3 \times 128$
  - Quenching effects from the heavy quarks: $N_f = 2+1+1$

Summary

With Simulations directly at the physical point LQCD enters a new era

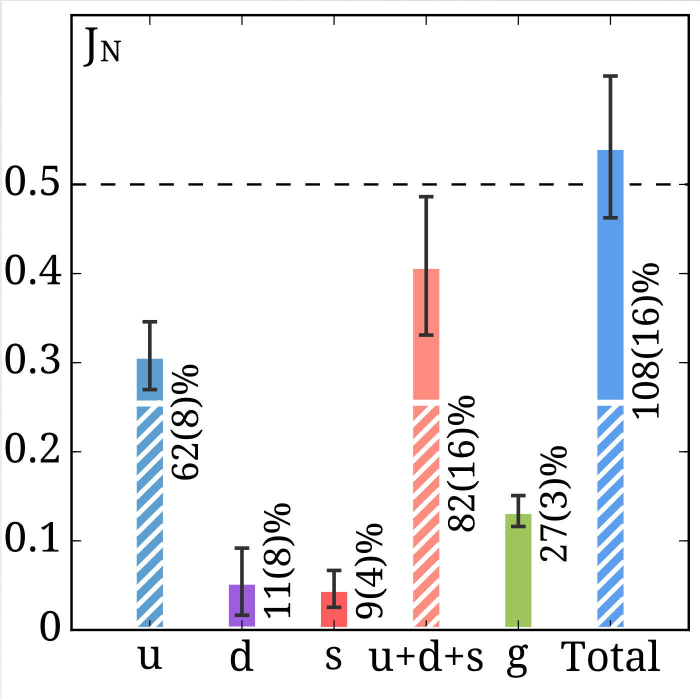
- A complete LQCD study about the origin of the nucleon spin
  - Spin sum satisfied: $J^N_{u_+ + d_+ + s_+ + g} = 0.541(62)(49)$
  - Momentum sum satisfied: $\langle x \rangle_{u_+ + d_+ + s_+ + g} = 1.07(12)(10)$
- Large effort on addressing systematic effects
  - Volume effects: $64^3 \times 128$
  - Quenching effects from the heavy quarks: $N_f = 2 + 1 + 1$
- Results of the Parton Distribution Functions on the lattice
  - Better understanding of the systematic effects
  - More efficient ways to reach large momenta

Summary

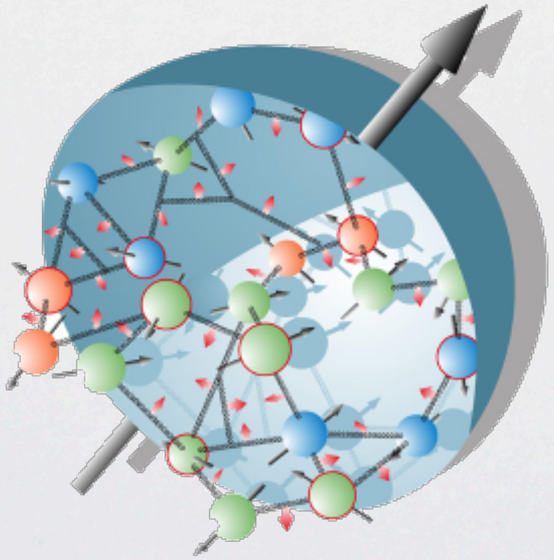
With Simulations directly at the physical point LQCD enters a new era

- A complete LQCD study about the origin of the nucleon spin
  - Spin sum satisfied: $J^N_{u_+ + d_+ + s_+ + g} = 0.541(62)(49)$
  - Momentum sum satisfied: $\langle x \rangle_{u_+ + d_+ + s_+ + g} = 1.07(12)(10)$
- Large effort on addressing systematic effects
  - Volume effects: $64^3 \times 128$
  - Quenching effects from the heavy quarks: $N_f = 2 + 1 + 1$
- Results of the Parton Distribution Functions on the lattice
  - Better understanding of the systematic effects
  - More efficient ways to reach large momenta
- Further improvement
  - More efficient techniques in the determination of the disconnected contributions
  - Better assessment of the excited states contamination

THANK YOU!



C. Alexandrou et al. ,Phys. Rev. Lett. 119, 142002 (2017), 1706.02973



Backup Slides

Systematic effects on the lattice

- **Discretization effects**
  - Extrapolation to the continuum limit
  - Simulations for several lattice spacing

Systematic effects on the lattice

- **Discretization effects**
  - Extrapolation to the continuum limit
  - Simulations for several lattice spacing
- **Finite volume effects**
  - Infinite volume limit
  - Simulations on larger volumes

Systematic effects on the lattice

- **Discretization effects**
  - Extrapolation to the continuum limit
  - Simulations for several lattice spacing
- **Finite volume effects**
  - Infinite volume limit
  - Simulations on larger volumes
- **Simulations directly at the physical point**
  - Systematic effects from chiral extrapolation are eliminated

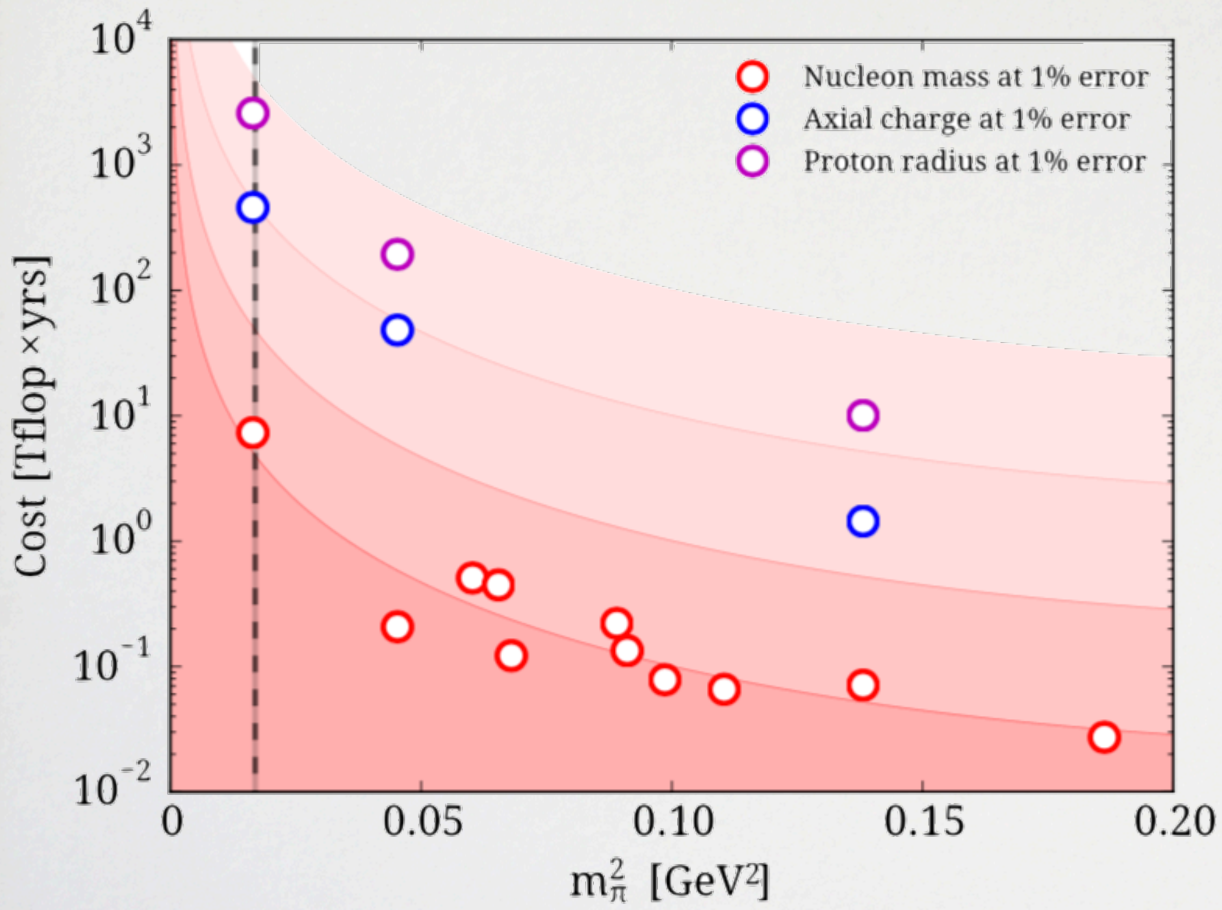
Systematic effects on the lattice

- **Discretization effects**
  - Extrapolation to the continuum limit
  - Simulations for several lattice spacing
- **Finite volume effects**
  - Infinite volume limit
  - Simulations on larger volumes
- **Simulations directly at the physical point**
  - Systematic effects from chiral extrapolation are eliminated
- **Excited States contamination**
  - Several methods (Plateau, Summation, Two-state fit) to isolate ground state
  - Noise-to-signal ratio increases rapidly at the physical point

Systematic effects on the lattice

- **Discretization effects**
  - Extrapolation to the continuum limit
  - Simulations for several lattice spacing
- **Finite volume effects**
  - Infinite volume limit
  - Simulations on larger volumes
- **Simulations directly at the physical point**
  - Systematic effects from chiral extrapolation are eliminated
- **Excited States contamination**
  - Several methods (Plateau, Summation, Two-state fit) to isolate ground state
  - Noise-to-signal ratio increases rapidly at the physical point
- **Renormalization**
  - Improved estimation using perturbative subtraction of lattice artifacts
  - Mixing is done perturbatively

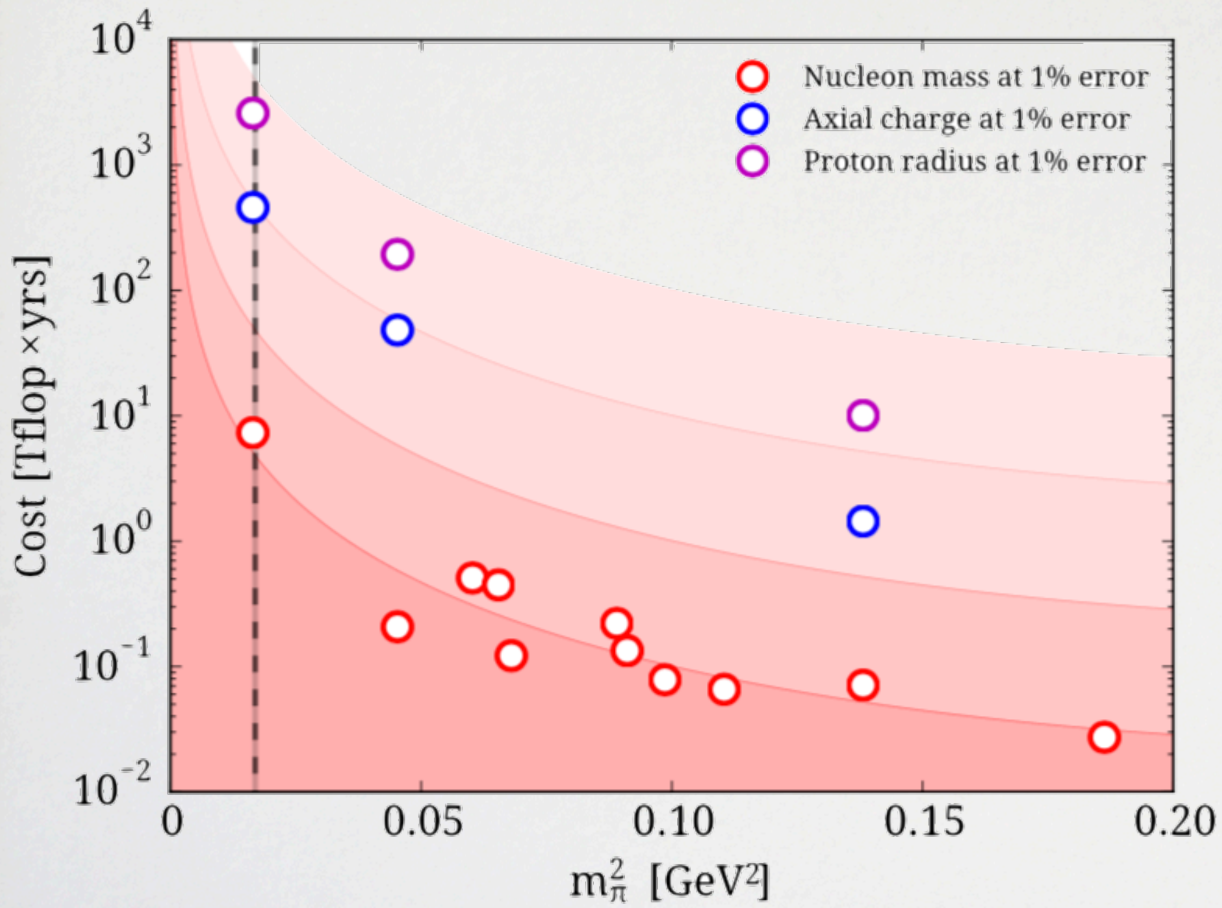
LQCD progress



Going to the physical point is a big challenge

- Multi-grid solvers yielding around 100x improvement of computer time at the physical point
- Improved stochastic methods for suppressing noise in disconnected diagrams

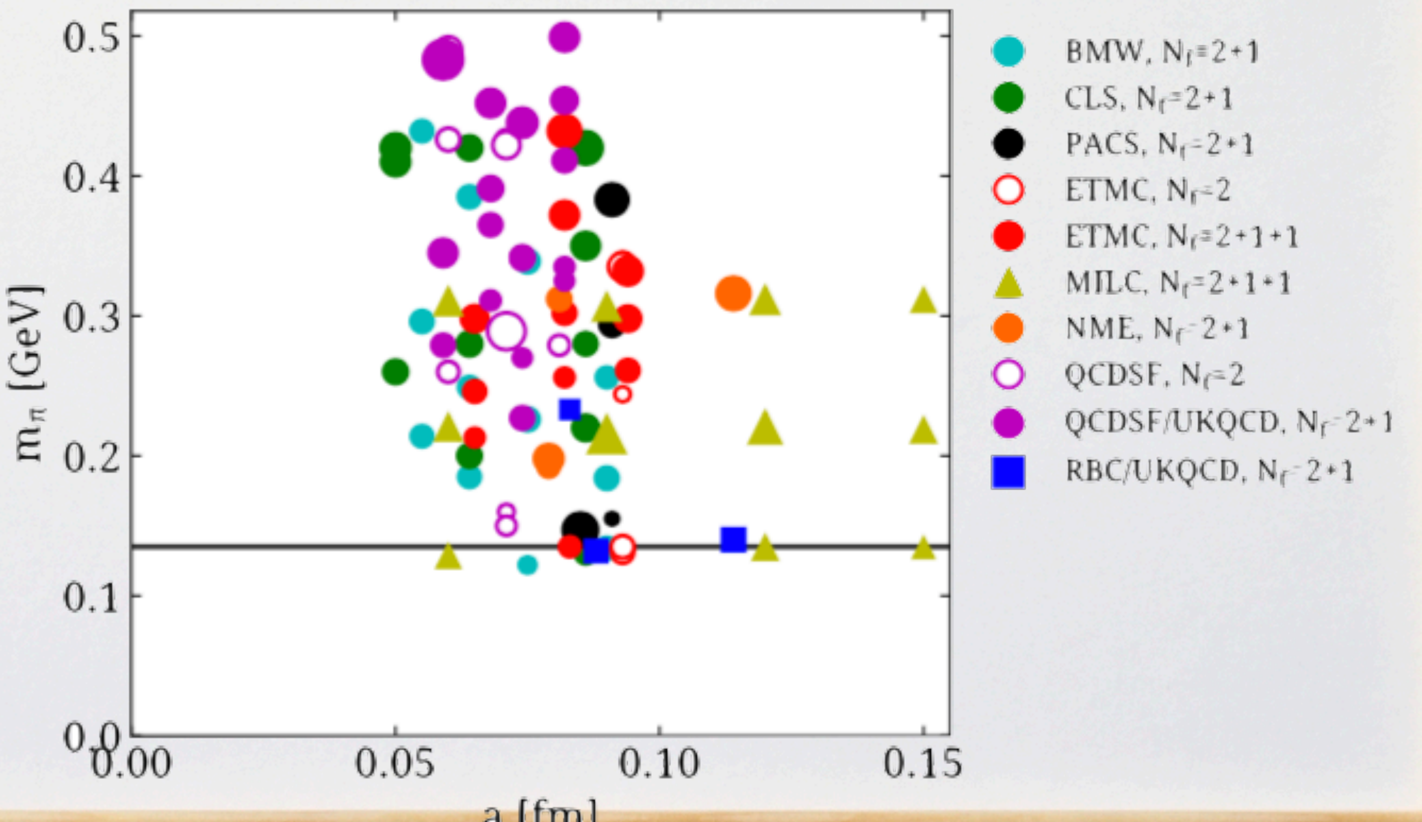
LQCD progress



Going to the physical point is a big challenge

- Multi-grid solvers yielding around 100x improvement of computer time at the physical point
- Improved stochastic methods for suppressing noise in disconnected diagrams

- Multiple collaborations simulating at the physical pion mass
- Simulations at bigger volumes and smaller lattice spacings are desirable



Extraction of the axial charge

Maximally twisted fermions:

→ **Automatic $\mathcal{O}(a)$ improvement**
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,

→ **Very attractive for hadron structure**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)

Extraction of the axial charge

Maximally twisted fermions:

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

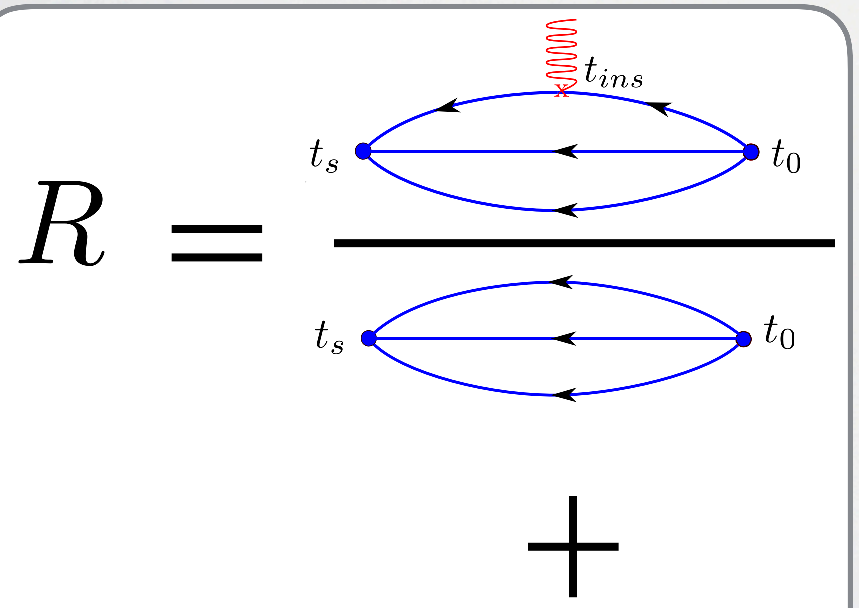
(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

→ **Automatic $\mathcal{O}(a)$ improvement**

R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,

→ **Very attractive for hadron structure**

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)



Extraction of the axial charge

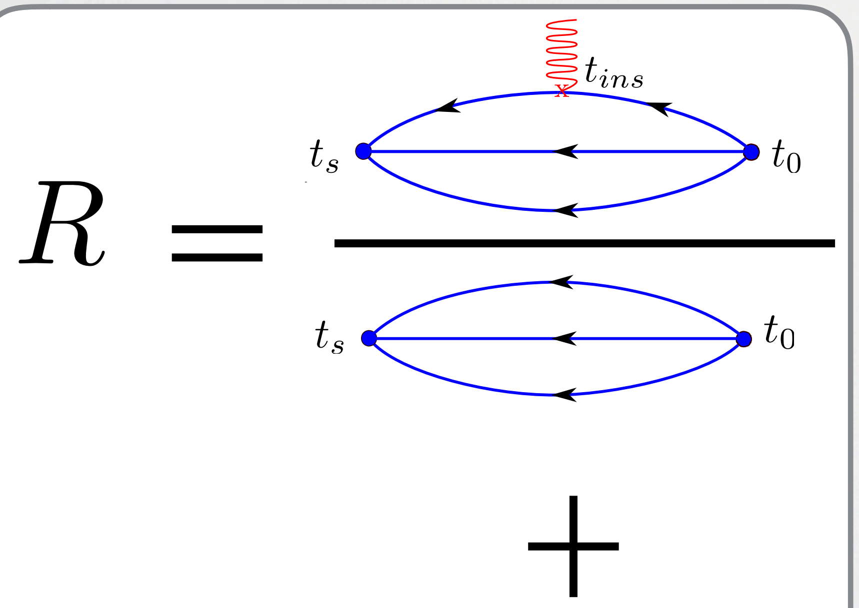
Maximally twisted fermions:

- **Automatic $\mathcal{O}(a)$ improvement**
  R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,
- **Very attractive for hadron structure**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)



Plateau method

$$R(t_s, t_{ins}, t_0) \xrightarrow[t_s - t_{ins} \gg 1]{t_{ins} - t_0 \gg 1} \mathcal{M}\left[1 + \mathcal{O}\left(e^{-\Delta E(t_{ins}-t_0)}, e^{-\Delta E(t_s - t_{ins})}\right)\right]$$

Extraction of the axial charge

Maximally twisted fermions:

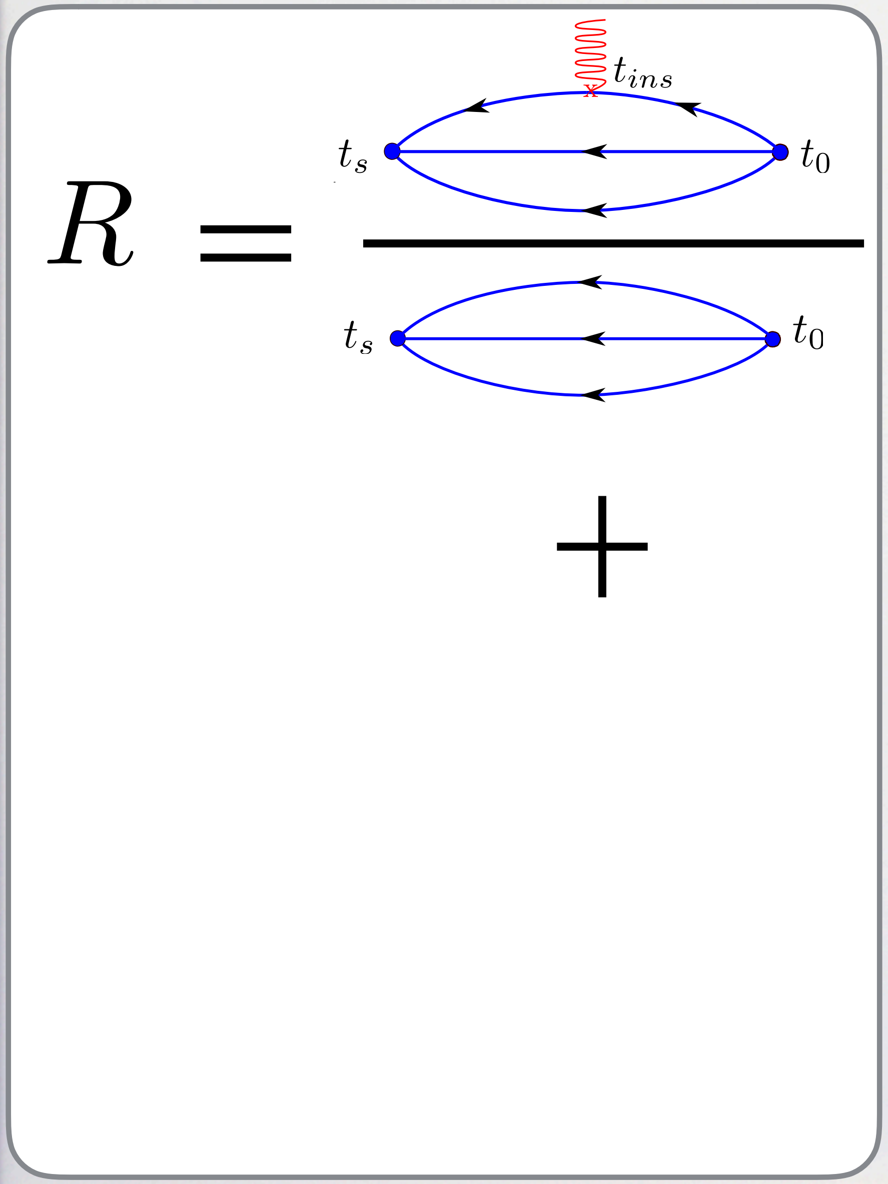
→ **Automatic $\mathcal{O}(a)$ improvement**
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,

→ **Very attractive for hadron structure**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f=2$
- Lattice size: $48^3 \times 96$

(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)



Plateau method

$$R(t_s,t_{ins},t_0) \xrightarrow[t_s-t_{ins}\gg 1]{t_{ins}-t_0\gg 1} \mathcal{M}\left[1+\mathcal{O}\left(e^{-\Delta E(t_{ins}-t_0)}, e^{-\Delta E(t_s-t_{ins})}\right)\right]$$

Summation method

$$\sum_{t_{ins}} R(t_s,t_{ins},t_0) \xrightarrow{t_s-t_0\gg 1} C + \mathcal{M}(t_s-t_0) + \mathcal{O}\left(e^{-\Delta E t_s}\right)$$

Extraction of the axial charge

Maximally twisted fermions:

→ **Automatic $\mathcal{O}(a)$ improvement**
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,

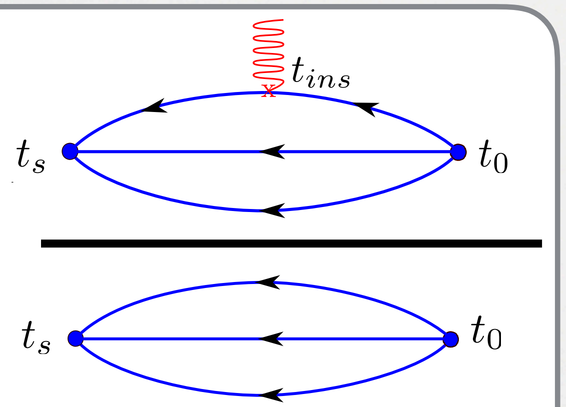
→ **Very attractive for hadron structure**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)

$R =$



$+$

Plateau method

$$R(t_s, t_{ins}, t_0) \xrightarrow[t_s - t_{ins} \gg 1]{t_{ins} - t_0 \gg 1} \mathcal{M}\left[1 + \mathcal{O}\left(e^{-\Delta E(t_{ins}-t_0)}, e^{-\Delta E(t_s - t_{ins})}\right)\right]$$

Summation method

$$\sum_{t_{ins}} R(t_s, t_{ins}, t_0) \xrightarrow{t_s - t_0 \gg 1} C + \mathcal{M}(t_s - t_0) + \mathcal{O}\left(e^{-\Delta E t_s}\right)$$

Two-state fit method

$$\begin{aligned} G^{3pt} &= A_{00} e^{-E_0(t_s - t_0)} + A_{01} e^{-E_0(t_s - t_{ins})} e^{E_1(t_{ins} - t_0)} \\ &+ A_{10} e^{-E_1(t_s - t_{ins})} e^{-E_0(t_{ins} - t_0)} + A_{11} e^{-E_1(t_s - t_0)} \end{aligned}$$

$$G^{2pt} = c_0 e^{-E_0(t_s - t_0)} + c_1 e^{-E_1(t_s - t_0)} \qquad \mathcal{M} = \frac{A_{00}}{c_0}$$

Extraction of the axial charge

Maximally twisted → **Automatic $\mathcal{O}(a)$ improvement**

R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,

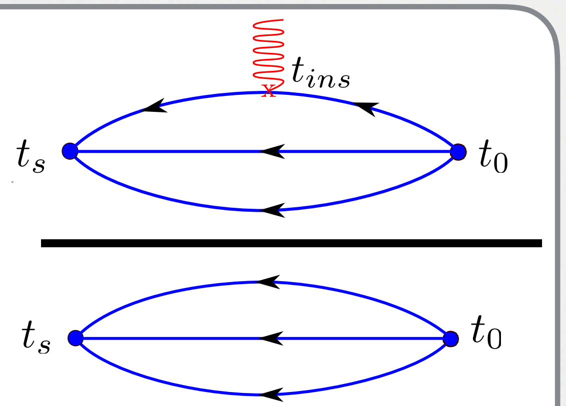
→ **Very attractive for hadron**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

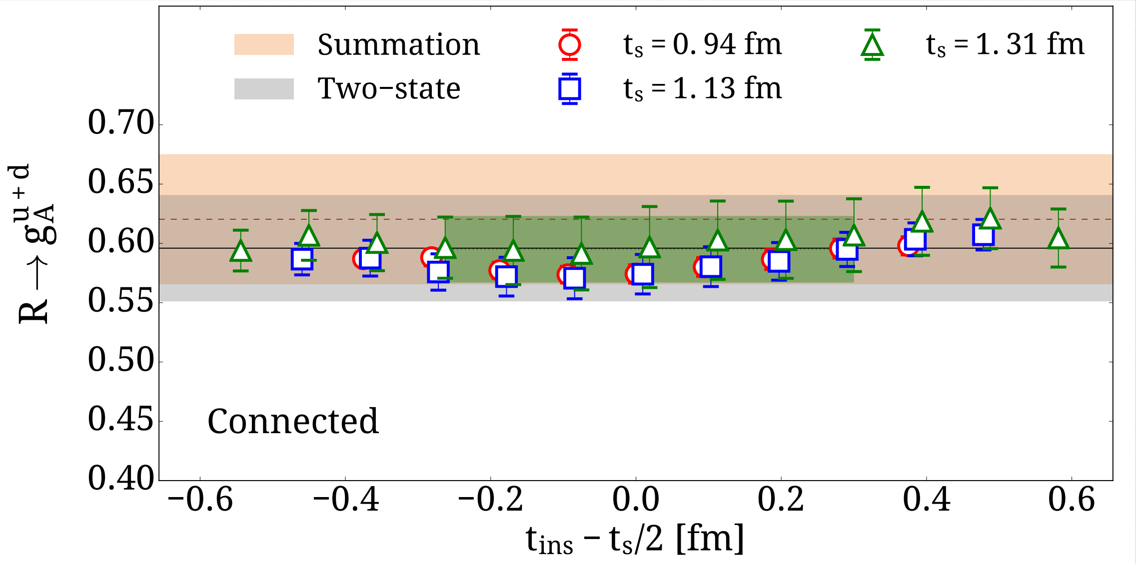
(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)

$R =$



+



C. Alexandrou et al., Phys. Rev. D96 (2017) no.5,

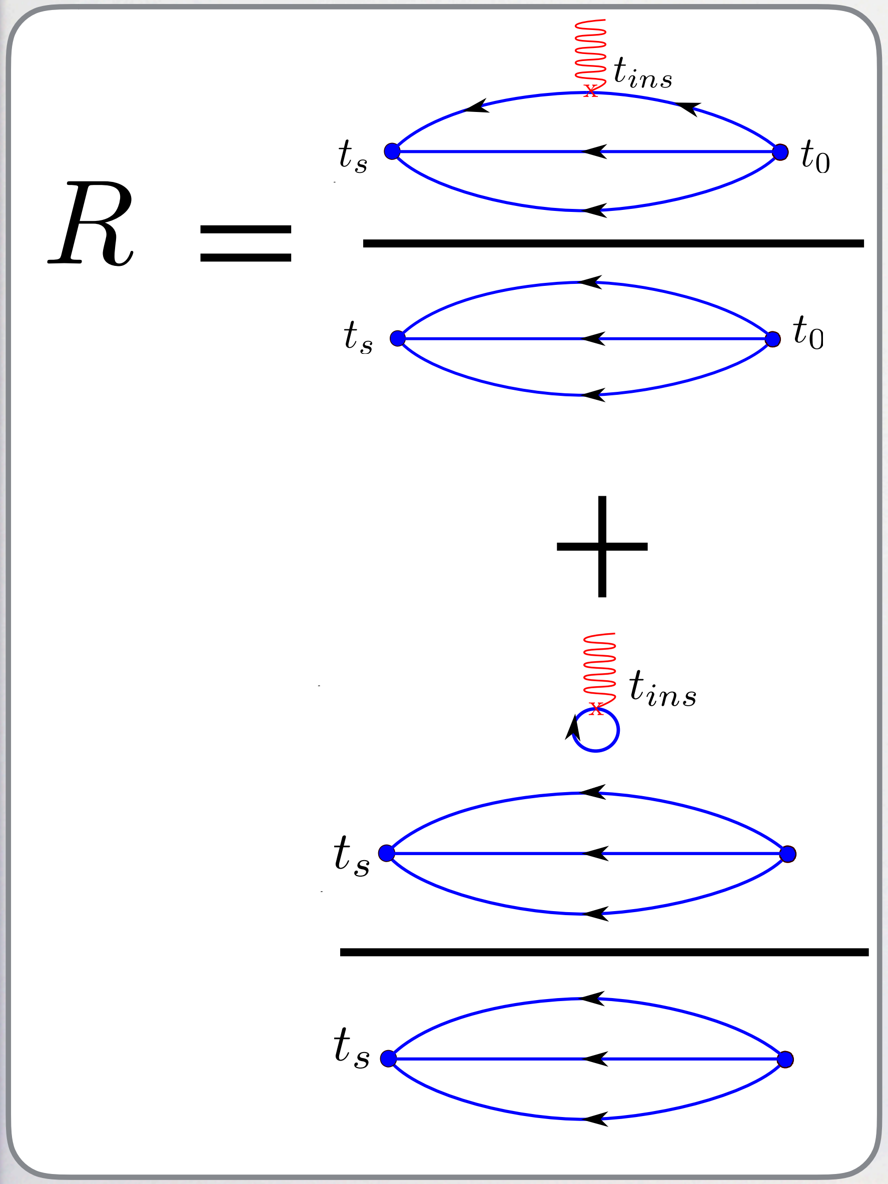
Extraction of the axial charge

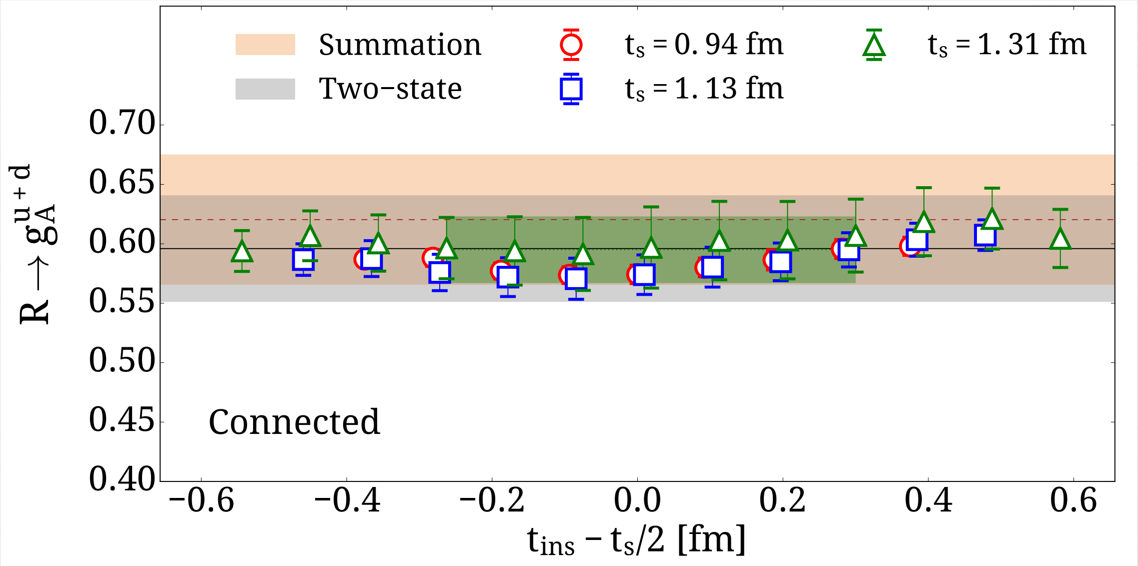
Maximally twisted → **Automatic $\mathcal{O}(a)$ improvement**
R. Frezzotti, G. C. Rossi, JHEP 0408 (2004) 007,
→ **Very attractive for hadron**

- Configurations Simulation by ETMC
- Dynamical quarks: $N_f = 2$
- Lattice size: $48^3 \times 96$

(ETMC) A. Abdel-Rehim et al. Phys. Rev. D95 094525 (2017),

- Lattice spacing: a = 0.0938(3)(2) fm
- m_π = 130 MeV
- $m_\pi L$ = 2.98(1)





C. Alexandrou et al., Phys. Rev. D96 (2017) no.5,