

# Rapidity gap distribution in diffractive deep-inelastic scattering and parton genealogy

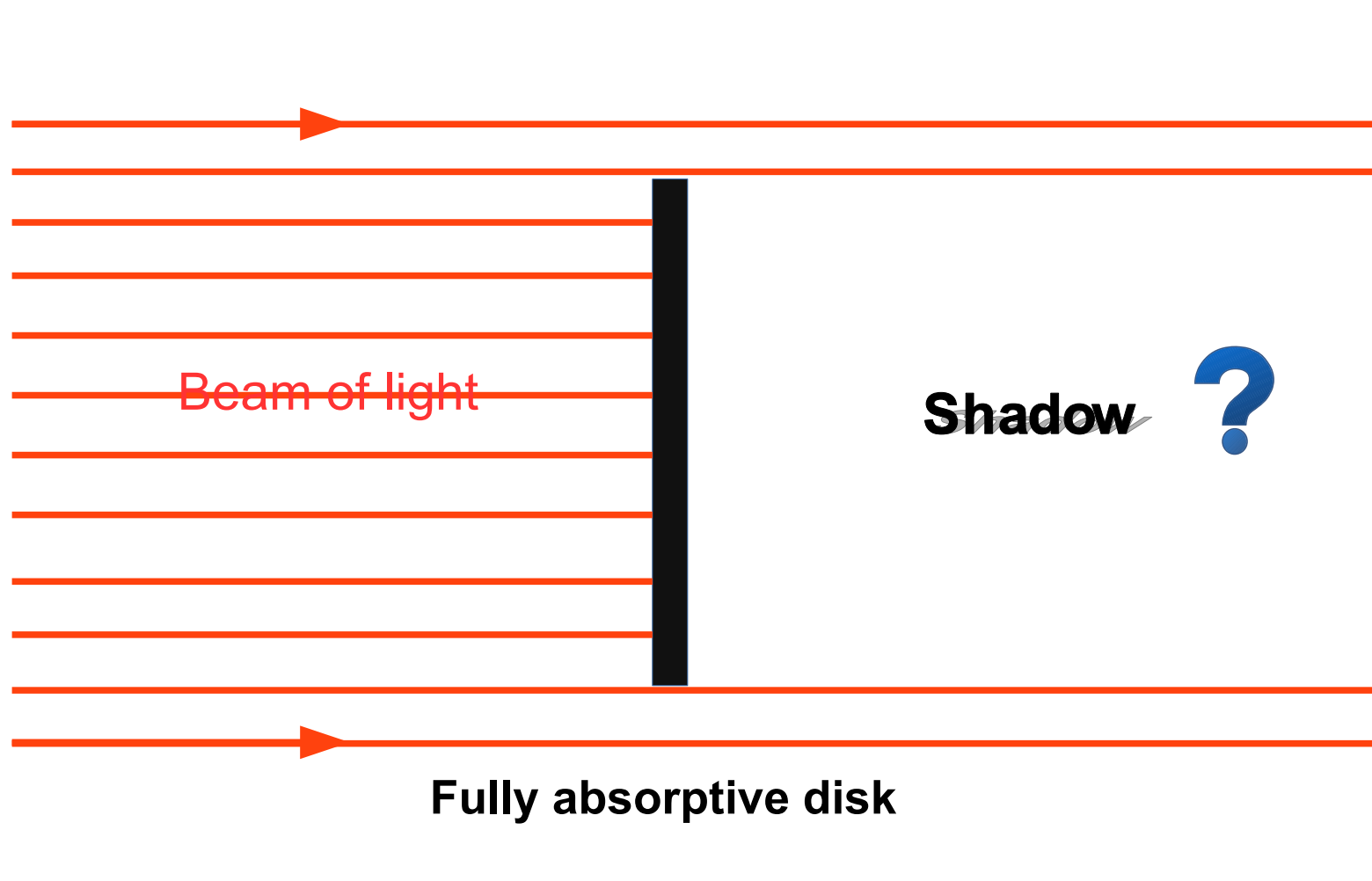
Stéphane Munier

Centre de physique théorique  
École polytechnique, CNRS, Université Paris-Saclay

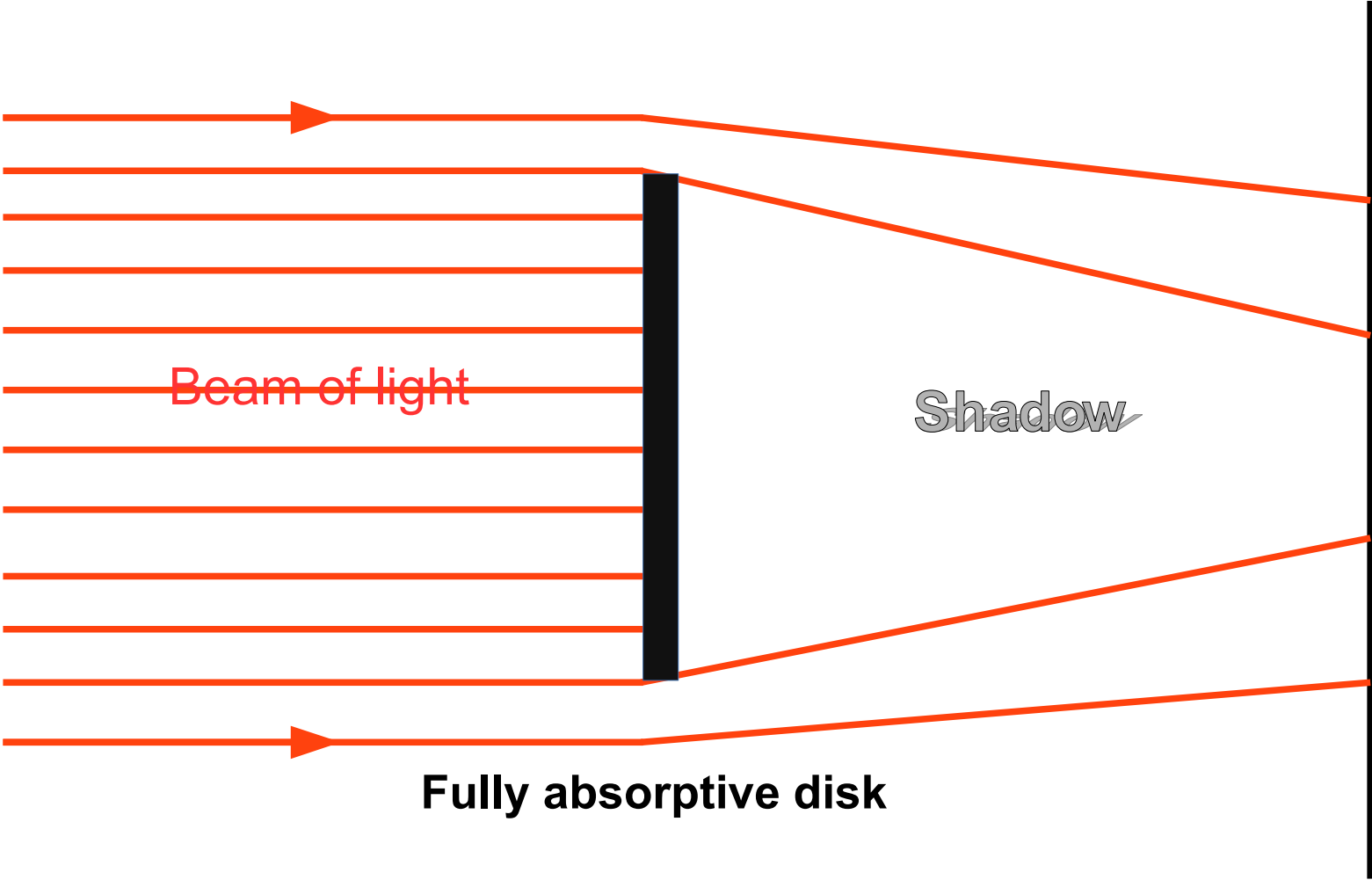
Based on work with [A.H. Mueller](#)  
arXiv:1805.02847, PRD and 1805.09417, PRL (published Aug 2018)  
and with [Dung Le](#) (in progress)



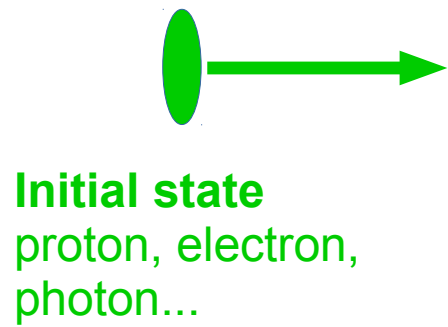
# Diffraction in optics



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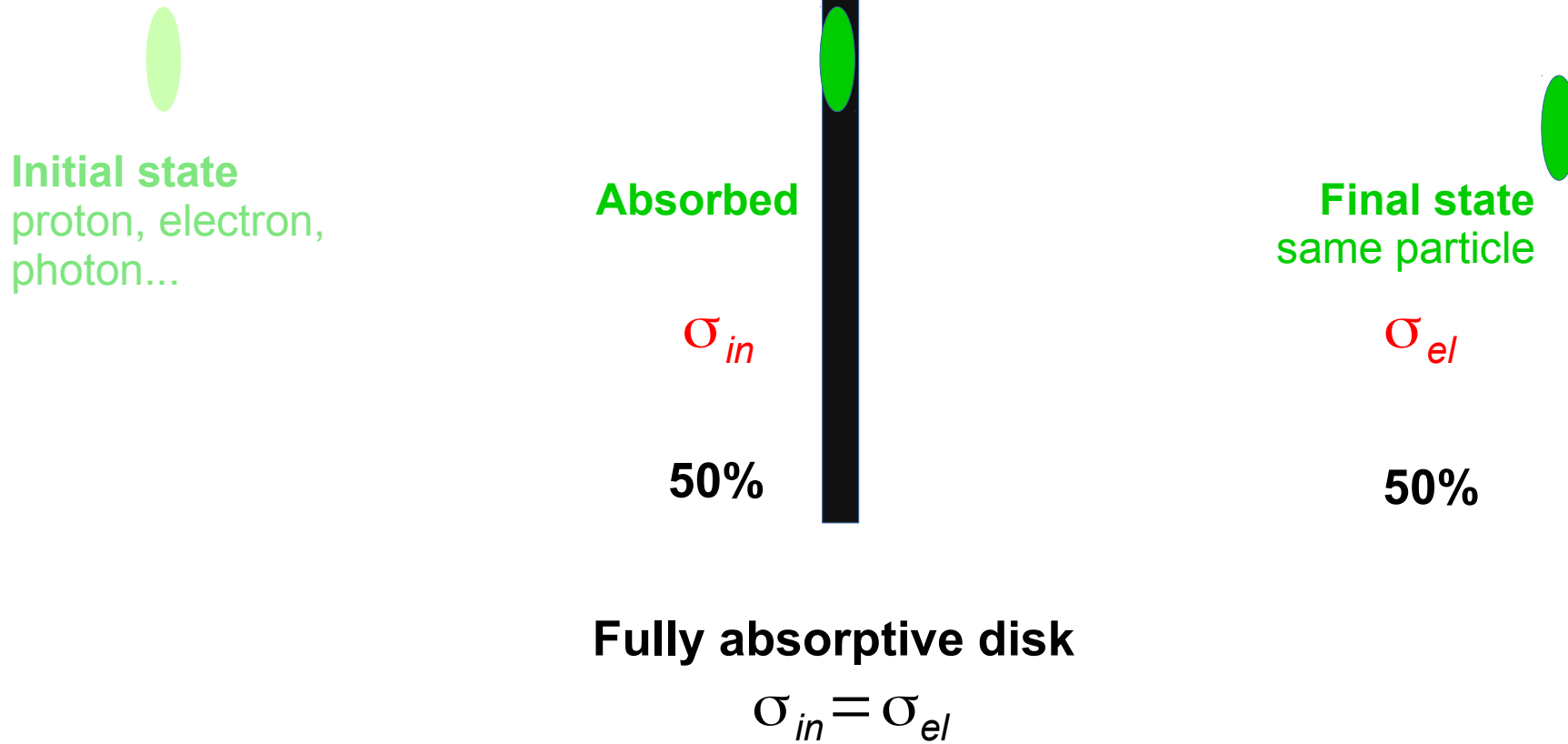
# Diffraction in quantum mechanics



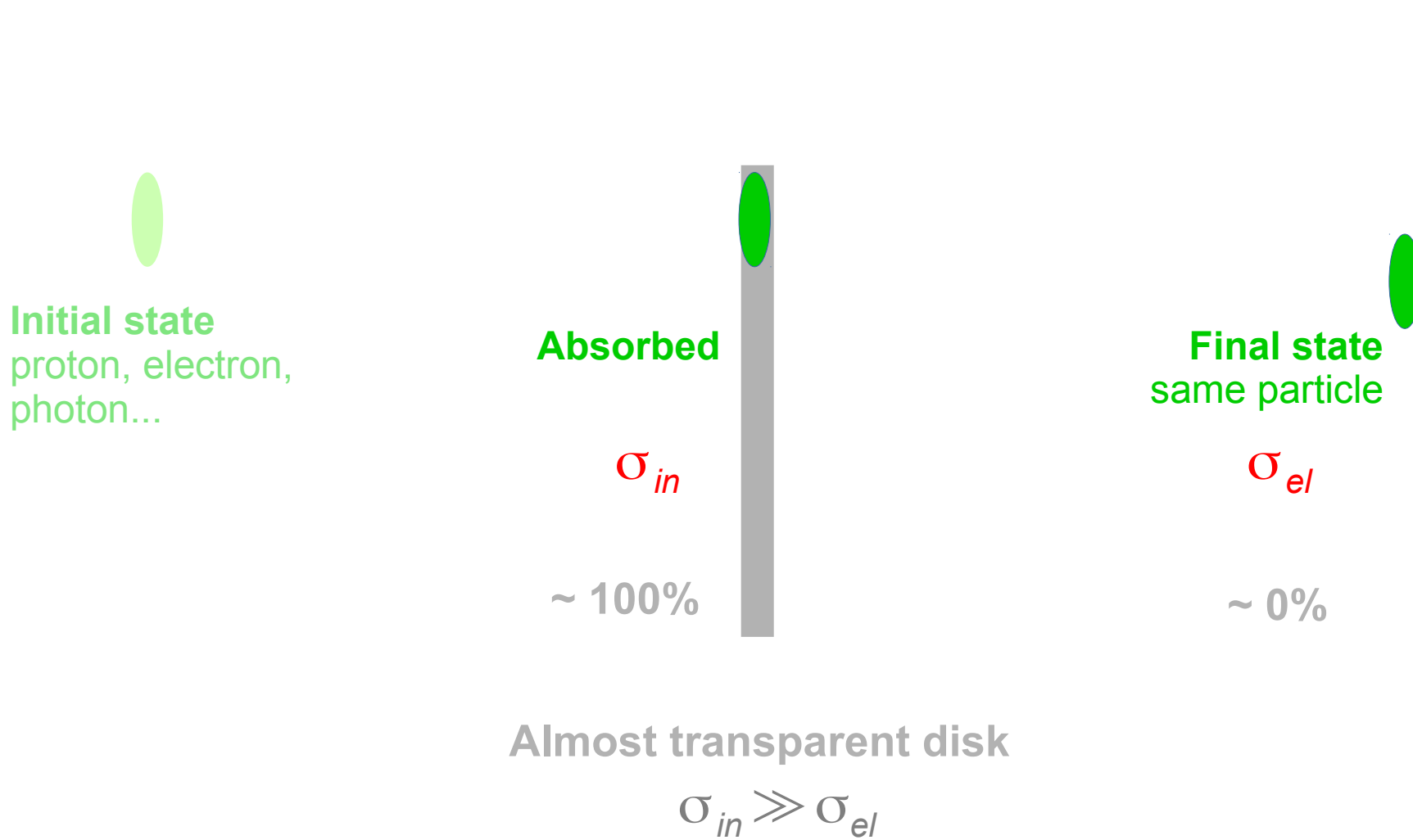
Fully absorptive disk



# Diffraction in quantum mechanics

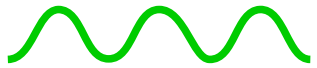


# Diffraction in quantum mechanics

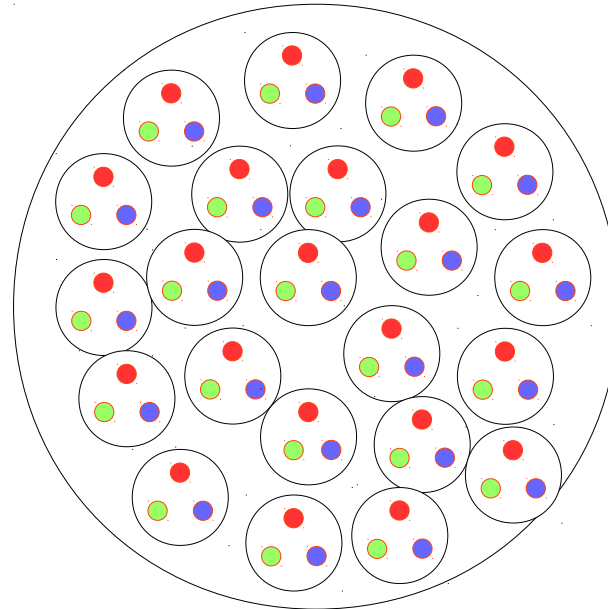


**To have diffraction, one needs strong absorption!**

# Diffraction in particle physics



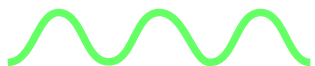
**Initial state**  
electron/virtual photon



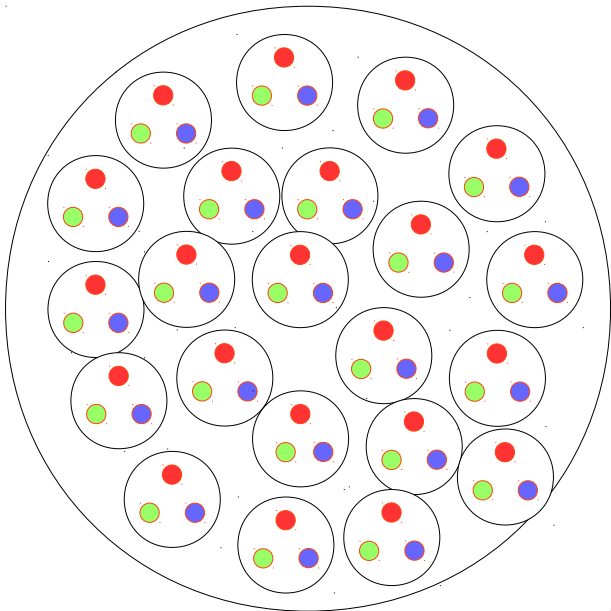
**Nucleus**



# Diffraction in particle physics



Initial state  
electron/virtual photon



**Nucleus**

**Intact in the final state**

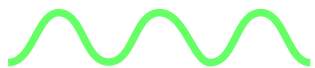
Final state  
meson

**quasi-elastic**

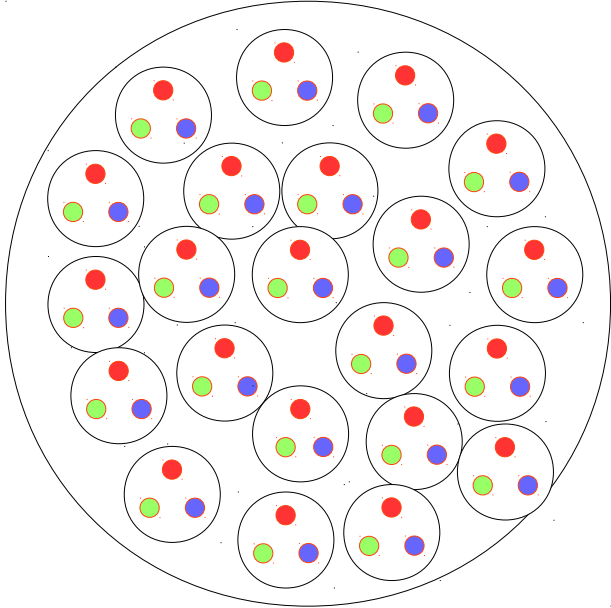




# Diffraction in particle physics



**Initial state**  
electron/virtual photon



**Nucleus**

**Intact in the final state**

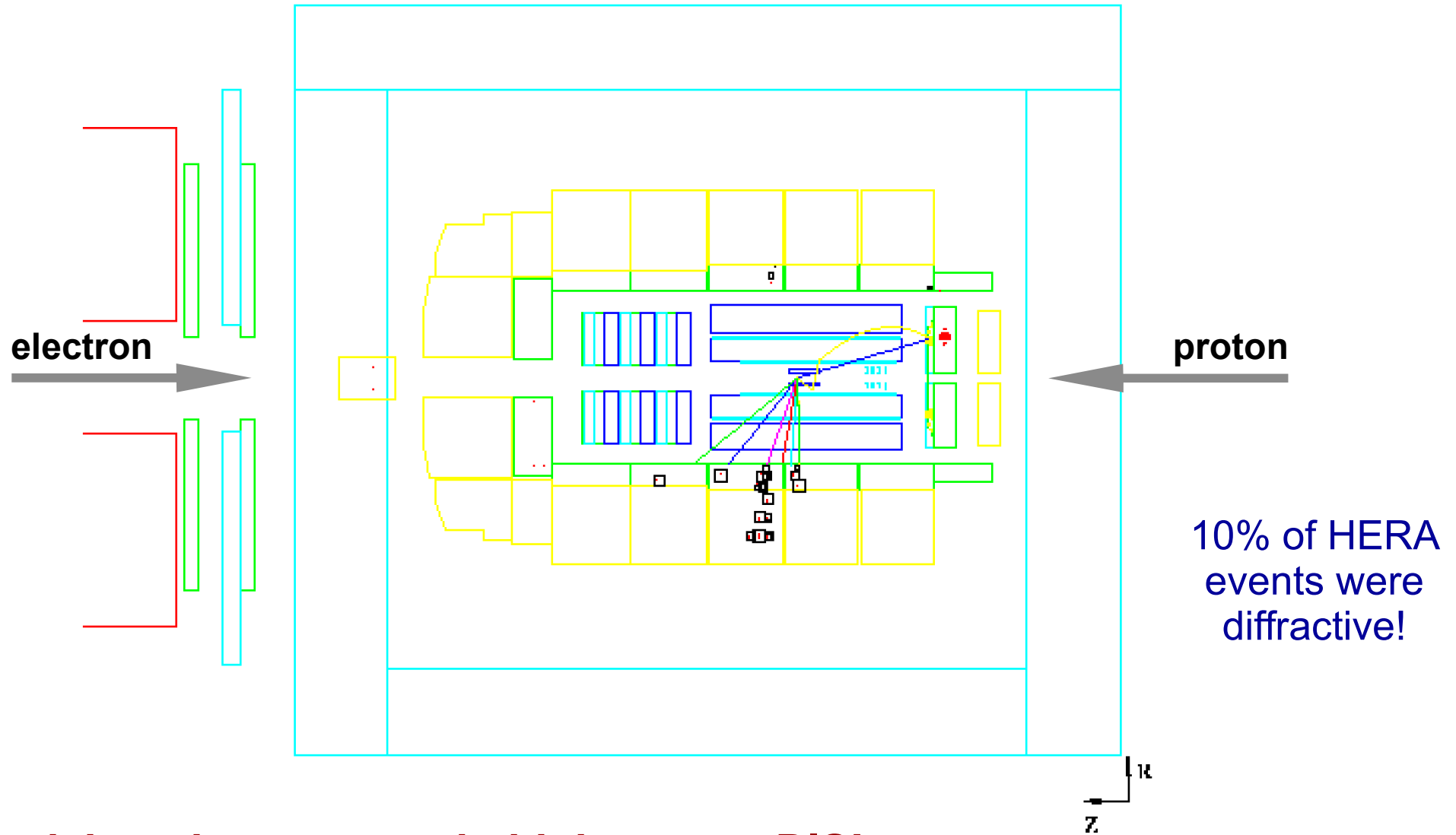
**Final state**  
several hadrons



**dissociative**

# Diffraction in deep-inelastic scattering

A major highlight of HERA!



**Surprising phenomenon in high-energy DIS!**

Its existence seems almost contradictory with the parton model...

*Its observation boosted saturation physics!*

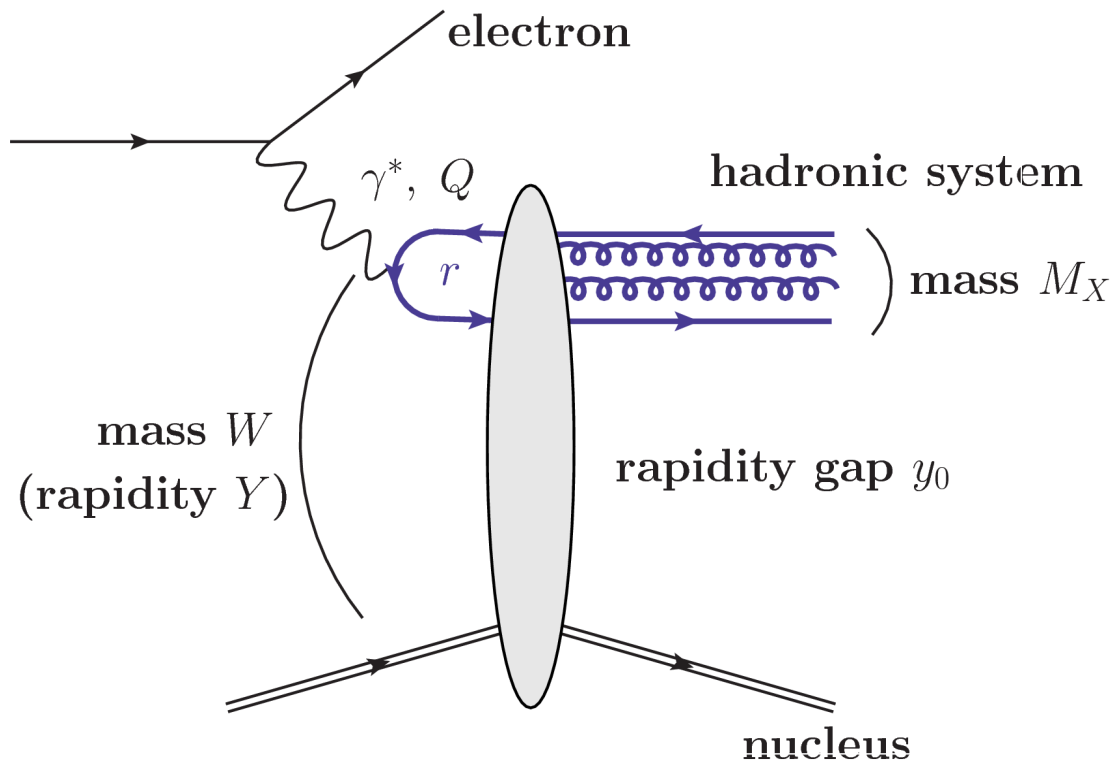
[Golec-Biernat, Wüsthoff 1998]



# Diffraction in deep-inelastic scattering

**This talk:** Diffractive dissociative electron-nucleus scattering  
 **$\sim \bar{q} q$  dipole-nucleus**

[Nikolaev, Zakharov '90  
Golec-Biernat, Wüsthoff 1998]



**What is the distribution of the rapidity gap  $y_0$ ?**

$$\frac{d\sigma_{\text{diff}}}{dy_0}$$

# Outline

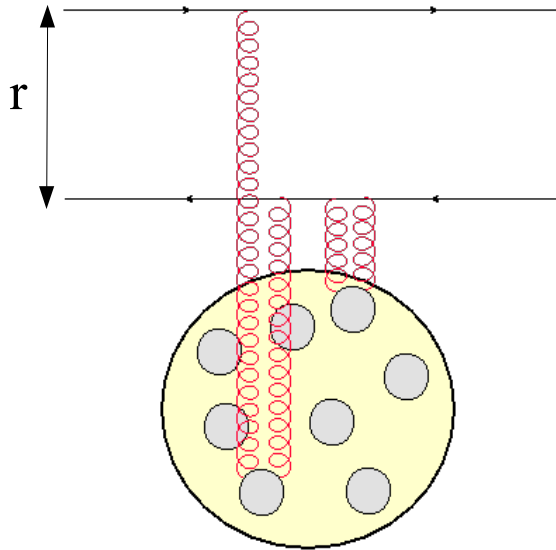
- ★ Equation for the gap distribution
- ★ Picture of onium-nucleus scattering
- ★ Ancestry in branching random walks
- ★ Numerical checks

# Equation for the gap distribution

## 1) Total amplitude: the Balitsky-Kovchegov equation

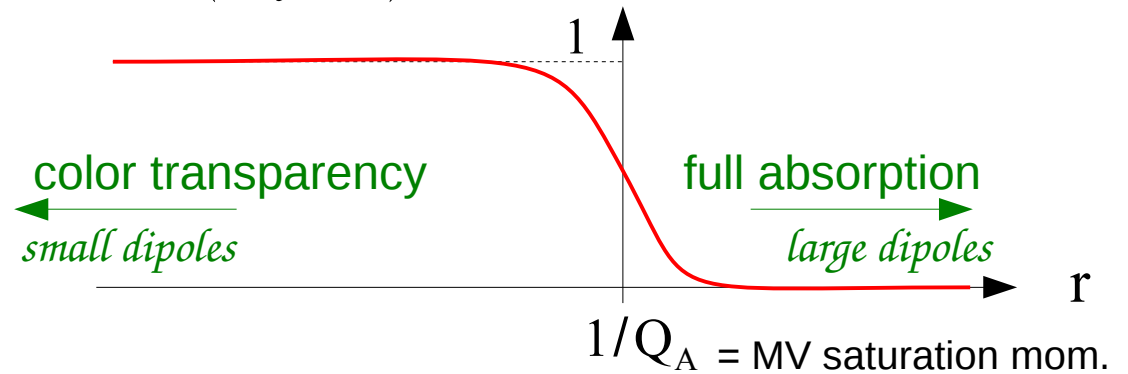
Forward elastic S-matrix element for dipole-nucleus scattering:

$$\sigma_{tot} = 2(1 - S)$$



$$S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

[McLerran, Venugopalan]

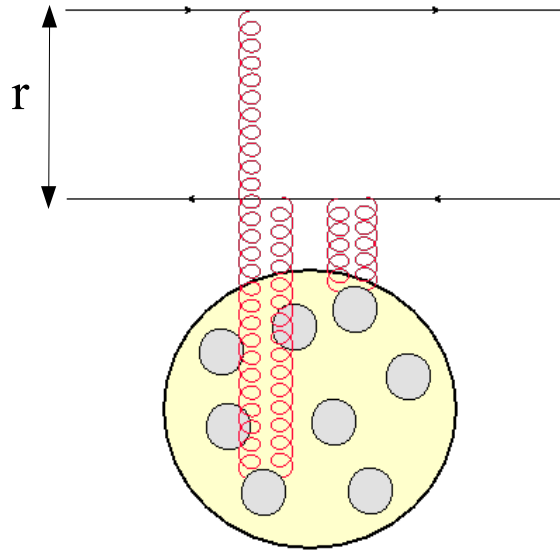


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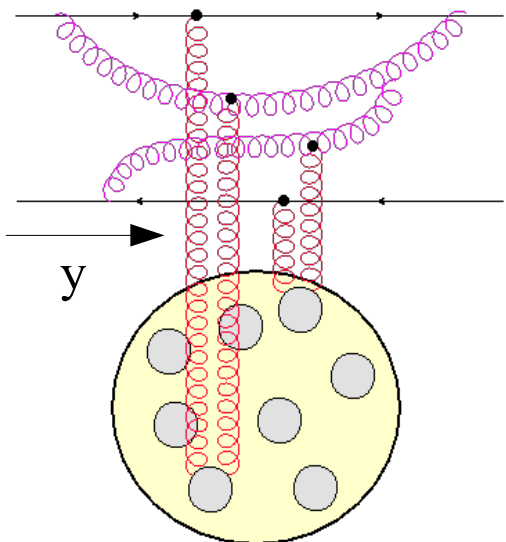
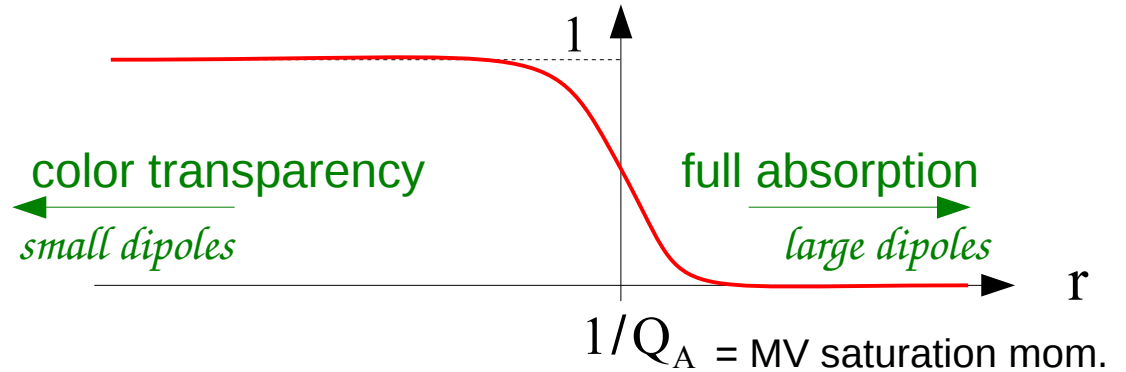
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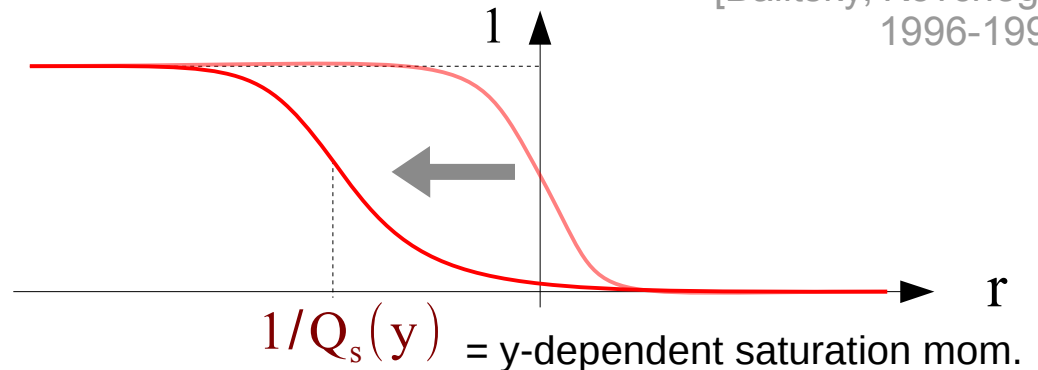
$$S(r, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

[McLerran, Venugopalan]



$$\partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} [S(r', y) S(r-r', y) - S(r, y)]$$

[Balitsky, Kovchegov  
1996-1999]



# Equation for the gap distribution

## 2) Diffractive cross section: the Kovchegov-Levin equation

[Kovchegov, Levin 2000

Hatta, Iancu, Marquet, Soyez, Triantafyllopoulos 2006]

$$S(\mathbf{r}, y=0) = e^{-\frac{r^2 Q_A^2}{4}}$$

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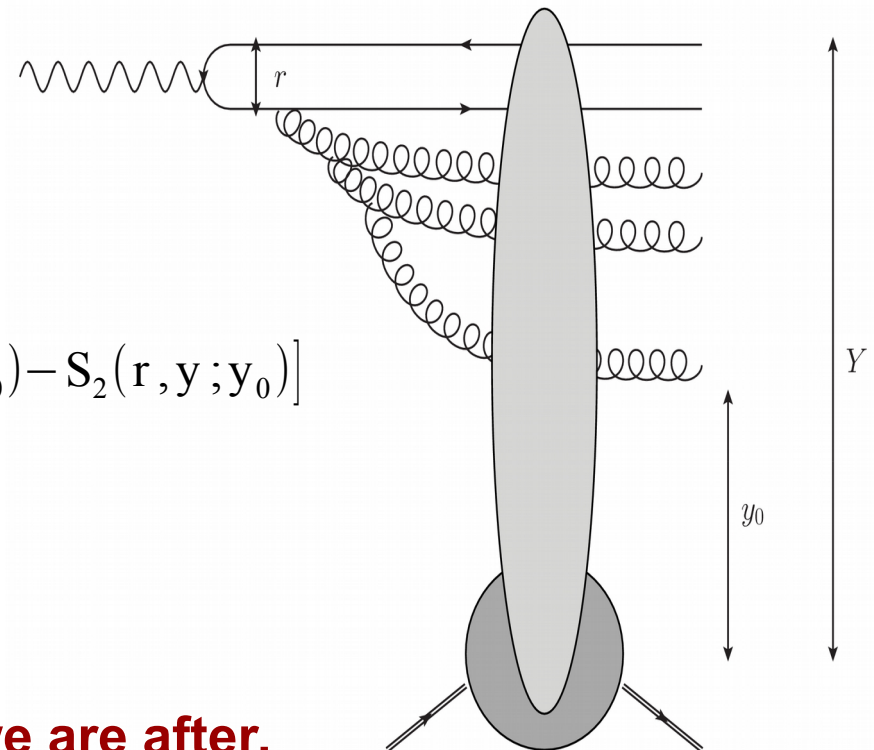
Define  $S_2$  as

$$S_2(\mathbf{r}, y_0; y_0) = [S(\mathbf{r}, y_0)]^2$$

$$\text{for } y > y_0: \quad \partial_y S_2(\mathbf{r}, y; y_0) = \bar{\alpha} \int \frac{d^2 \mathbf{r}'}{2\pi} \frac{r^2}{r'^2 (\mathbf{r} - \mathbf{r}')^2} \times [S_2(\mathbf{r}', y; y_0) S_2(\mathbf{r} - \mathbf{r}', y; y_0) - S_2(\mathbf{r}, y; y_0)]$$

$$\frac{d \sigma_{diff}}{dy_0} = - \frac{\partial}{\partial y_0} S_2(\mathbf{r}, Y; y_0)$$

**The solution to these equations is what we are after, but they are difficult to solve!**



# Outline

- ★ Equation for the gap distribution
- ★ Picture of onium-nucleus scattering
- ★ Ancestry in branching random walks
- ★ Numerical checks

# Picture of onium-nucleus scattering

## Total cross section

$$\sigma_{tot}(r, Y) = 2[1 - S(r, Y)] \quad \partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} [S(r', y)S(r-r', y) - S(r, y)]$$

Solution:

$$Q_s^2(Y) \simeq Q_A^2 \frac{e^{\text{const} \times \bar{\alpha} Y}}{(\bar{\alpha} Y)^{3/2 \gamma_0}}$$

$$1 - S_{r Q_s(\tilde{Y}) \ll 1} \ln \frac{1}{r^2 Q_s^2(Y)} [r^2 Q_s^2(Y)]^{\gamma_0} \ll 1$$

( $\gamma_0 \simeq 0.63$ )

# Picture of onium-nucleus scattering

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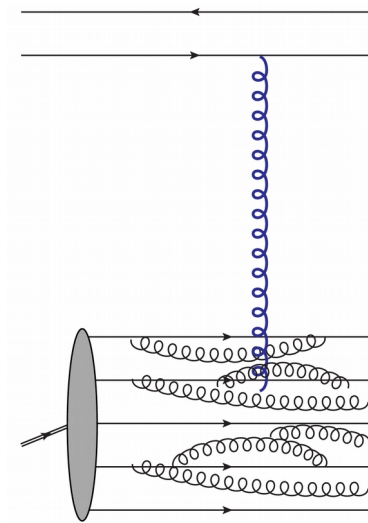
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## Onium restframe



1-S is like the transparency  
of the boosted nucleus

# Picture of onium-nucleus scattering

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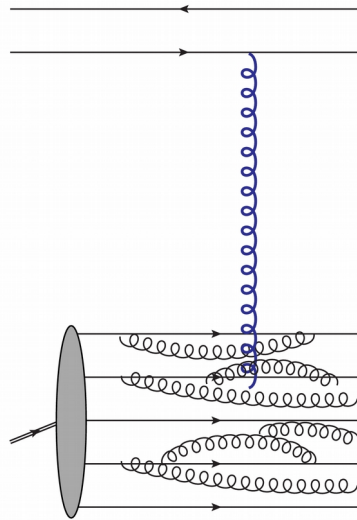
$$\partial_y S(r, y) = \bar{\alpha} \int \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r-r')^2} [S(r', y) S(r-r', y) - S(r, y)]$$

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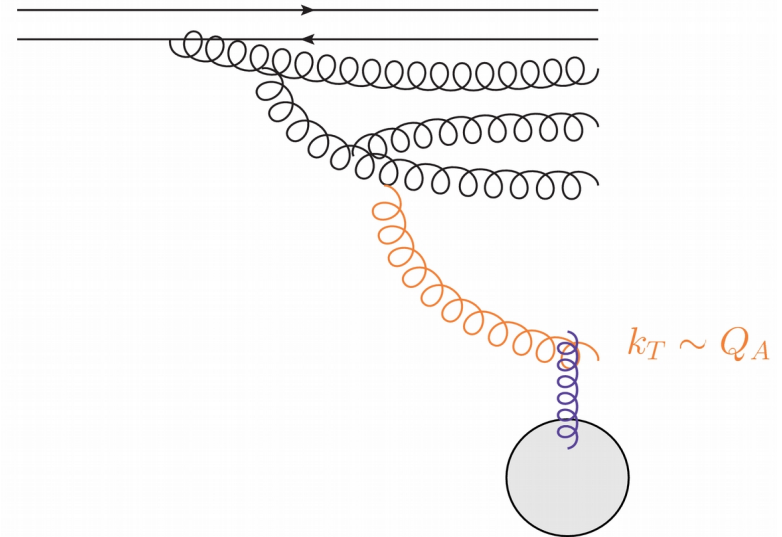
( $\gamma_0 \approx 0.63$ )

### Onium restframe



1-S is like the transparency of the boosted nucleus

### Nucleus restframe



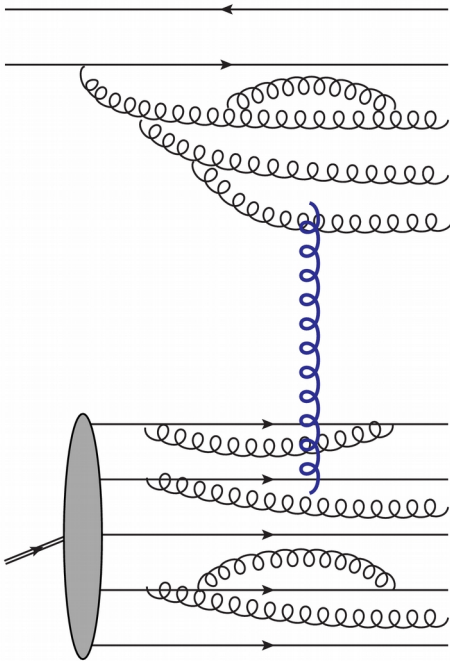
1-S is like the probability to have a gluon of transverse momentum larger than the nuclear saturation momentum in the boosted onium Fock state

# Picture of onium-nucleus scattering

Total and diffractive cross section in the  $y_0$  frame

in which the nucleus has rapidity  $y_0$

**Total**

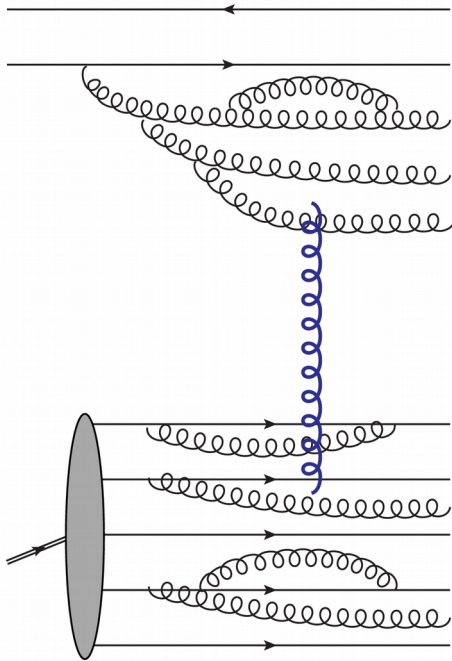


# Picture of onium-nucleus scattering

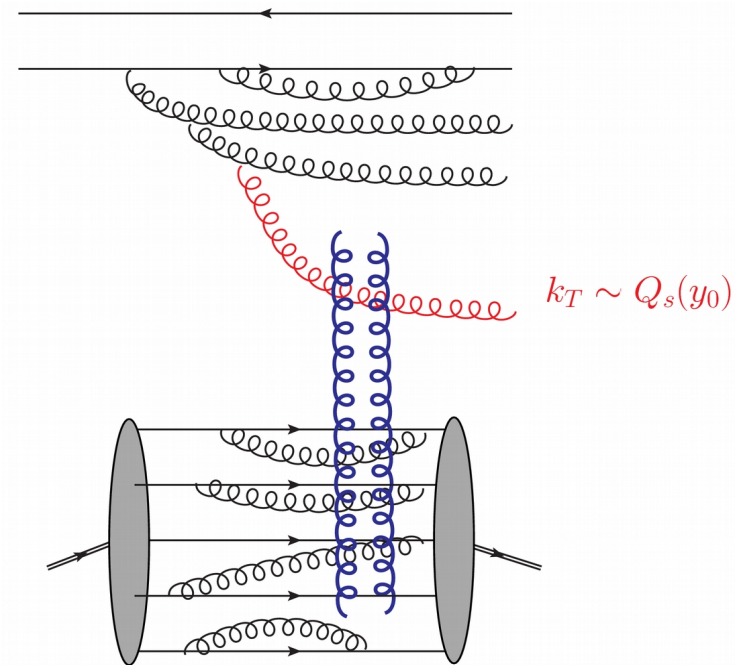
Total and diffractive cross section in the  $y_0$  frame

in which the nucleus has rapidity  $y_0$

**Total**



**Diffractive**



The scattering between the state of the onium and the evolved nucleus needs to be elastic

**This requires an unusually low- $k_T$  gluon at rapidity  $y_0$**

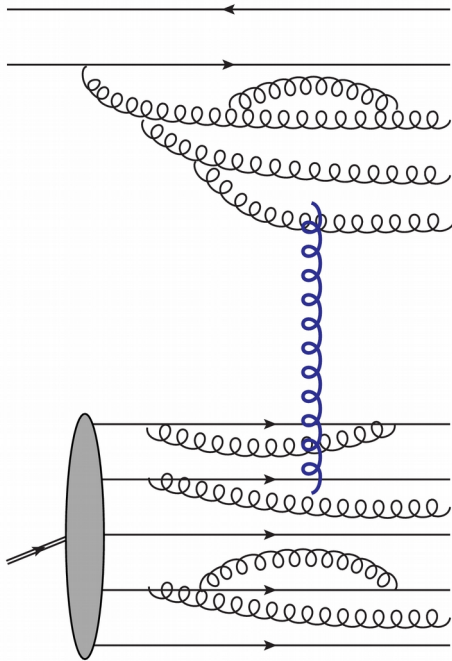


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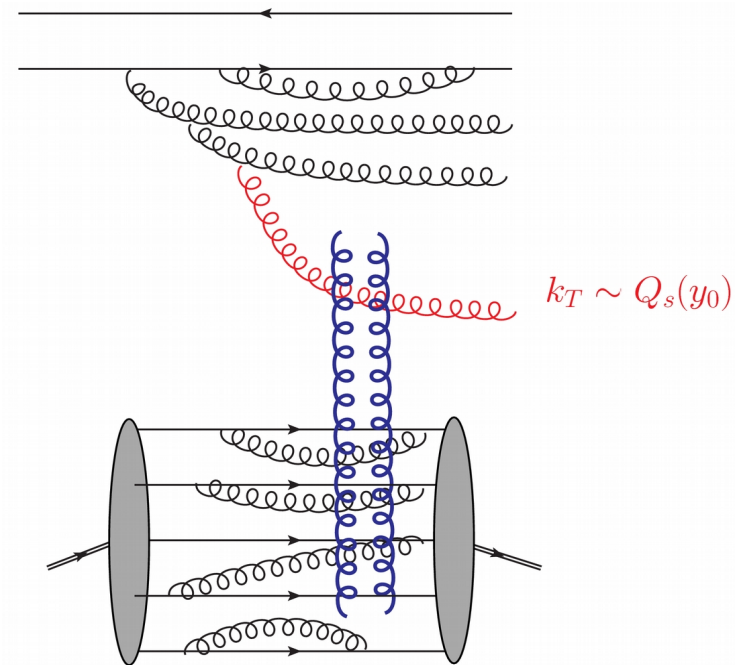
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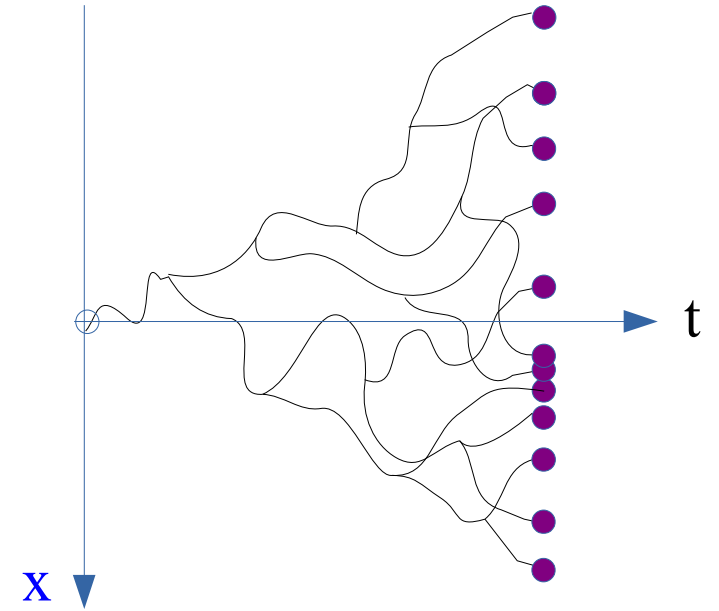
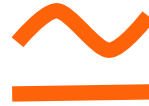
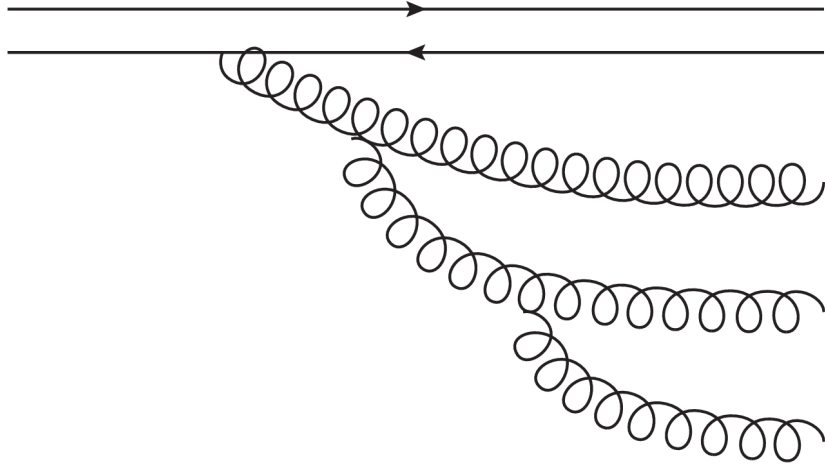
Probability given by the BK equation!

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}$$

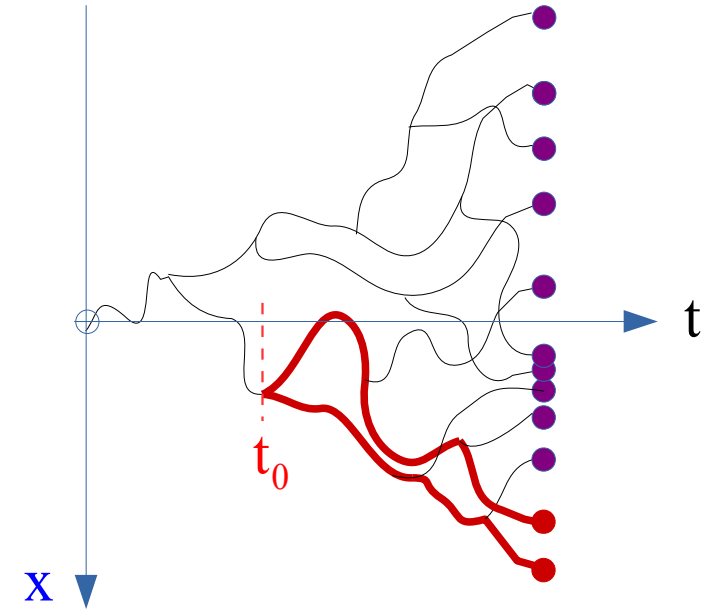
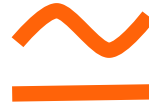
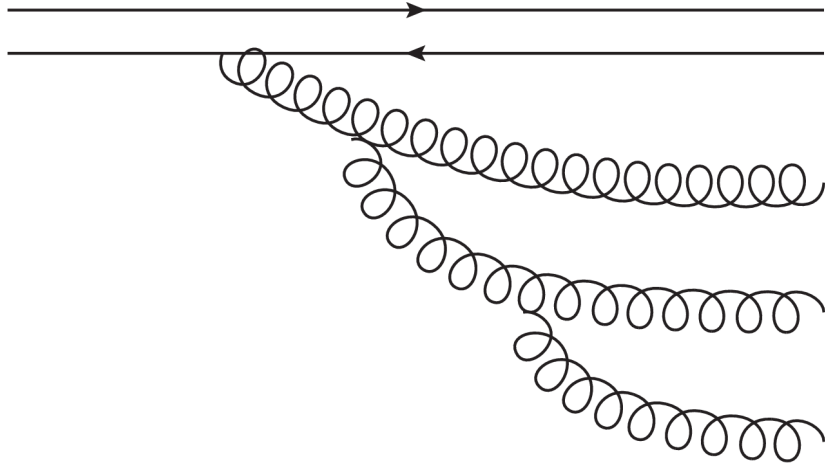
# Outline

- ★ Equation for the gap distribution
- ★ Picture of onium-nucleus scattering
- ★ Ancestry in branching random walks
- ★ Numerical checks

# Branching random walks

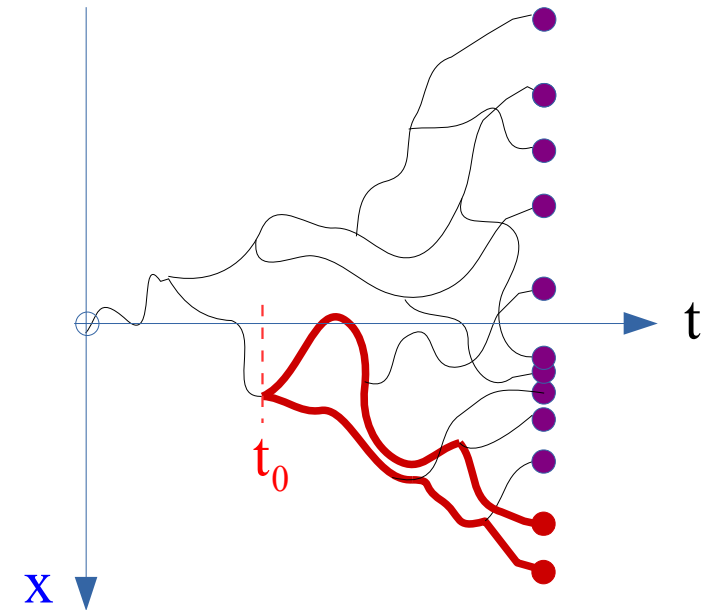
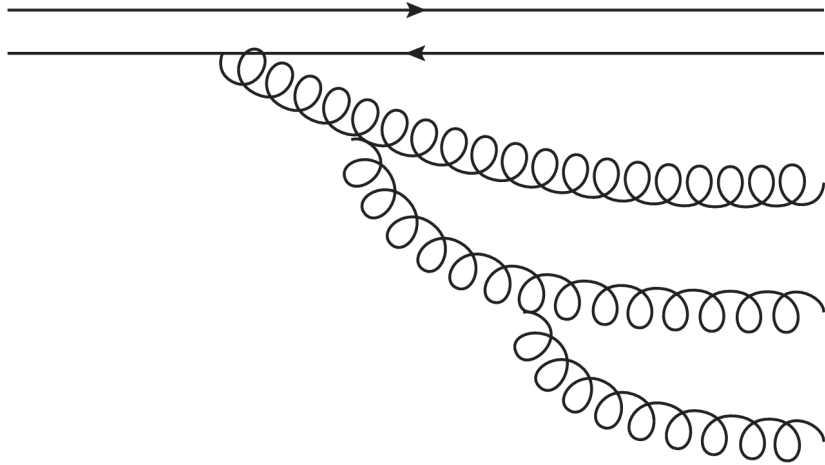


# Branching random walks



**Distribution of decay time of most recent common ancestor?**

# Branching random walks



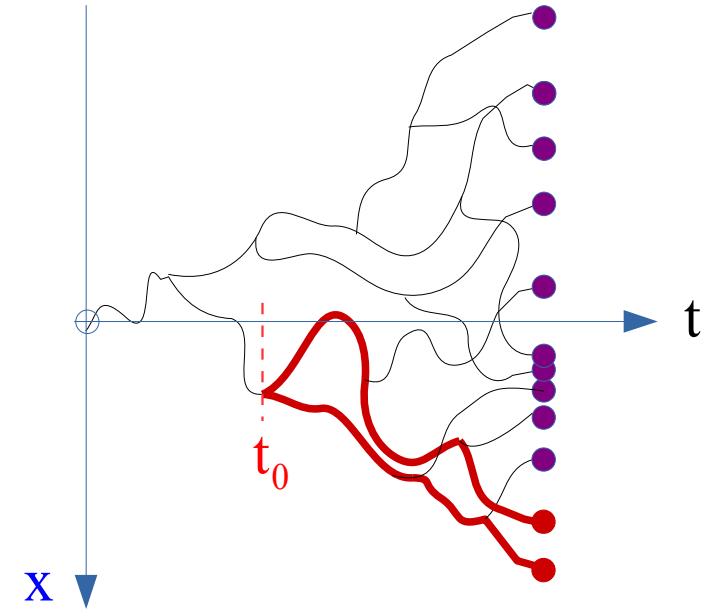
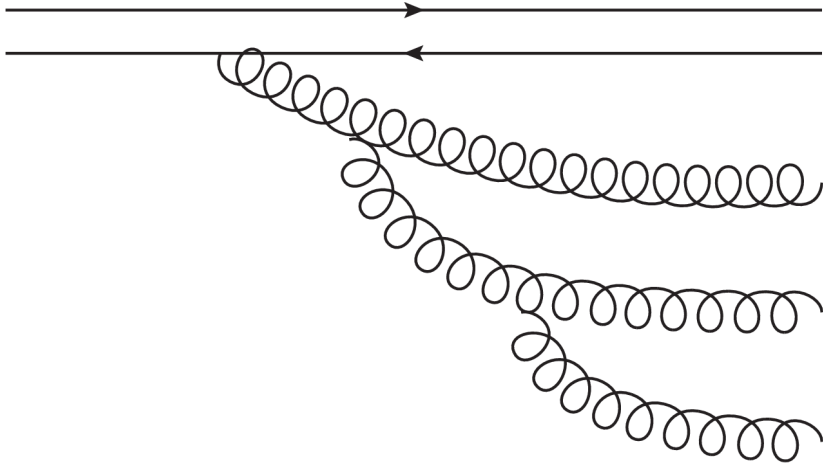
**Distribution of decay time of most recent common ancestor?**

$$\frac{dp}{dt_0} = \frac{1}{\sqrt{4\pi}} \times \left[ \frac{t}{t_0(t-t_0)} \right]^{3/2}$$

[Derrida, Mottishaw 2016]

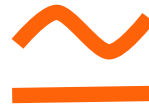
(for branching Brownian motion with diffusion constant 2 and splitting rate 1)

# Branching random walks



**Distribution of rapidity gaps:**

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}$$



$$\begin{aligned} \bar{\alpha} Y &\Leftrightarrow t \\ \bar{\alpha} y_0 &\Leftrightarrow t_0 \end{aligned}$$

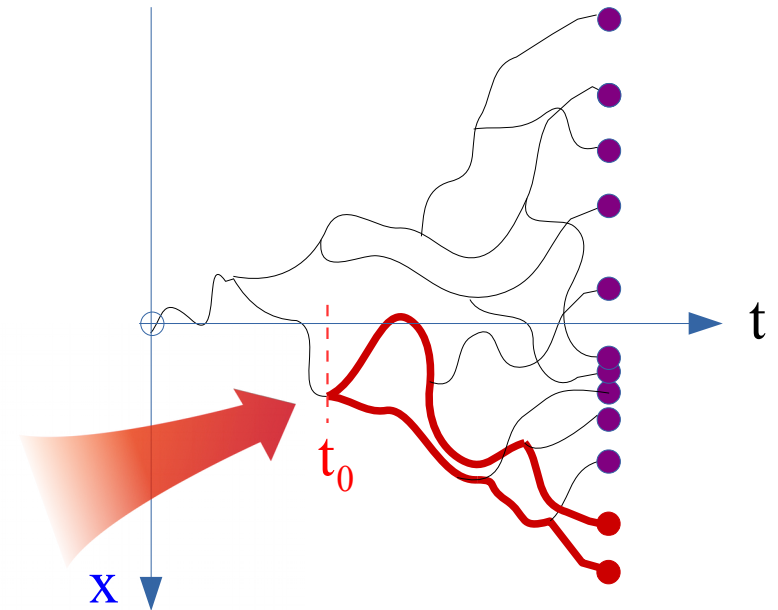
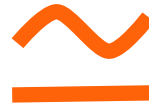
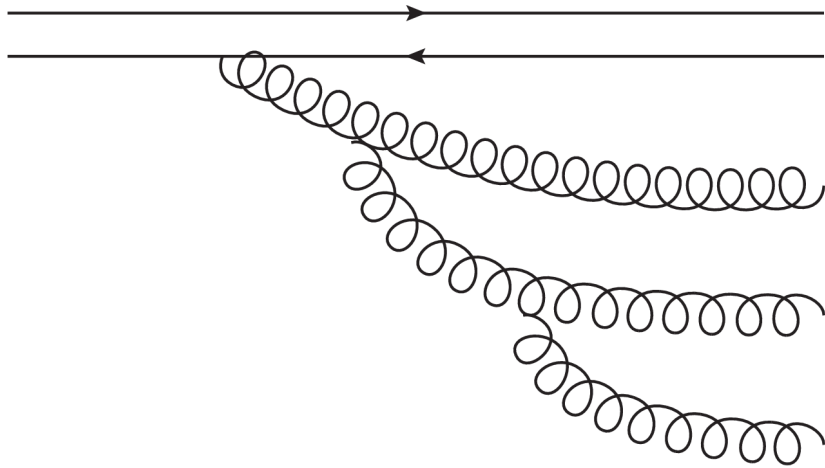
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[Derrida, Mottishaw 2016]

(for branching Brownian motion with diffusion constant 2 and splitting rate 1)

**Physical reason for the correspondence:**

**The common ancestor is an unusually large fluctuation! (here: large value of x)**

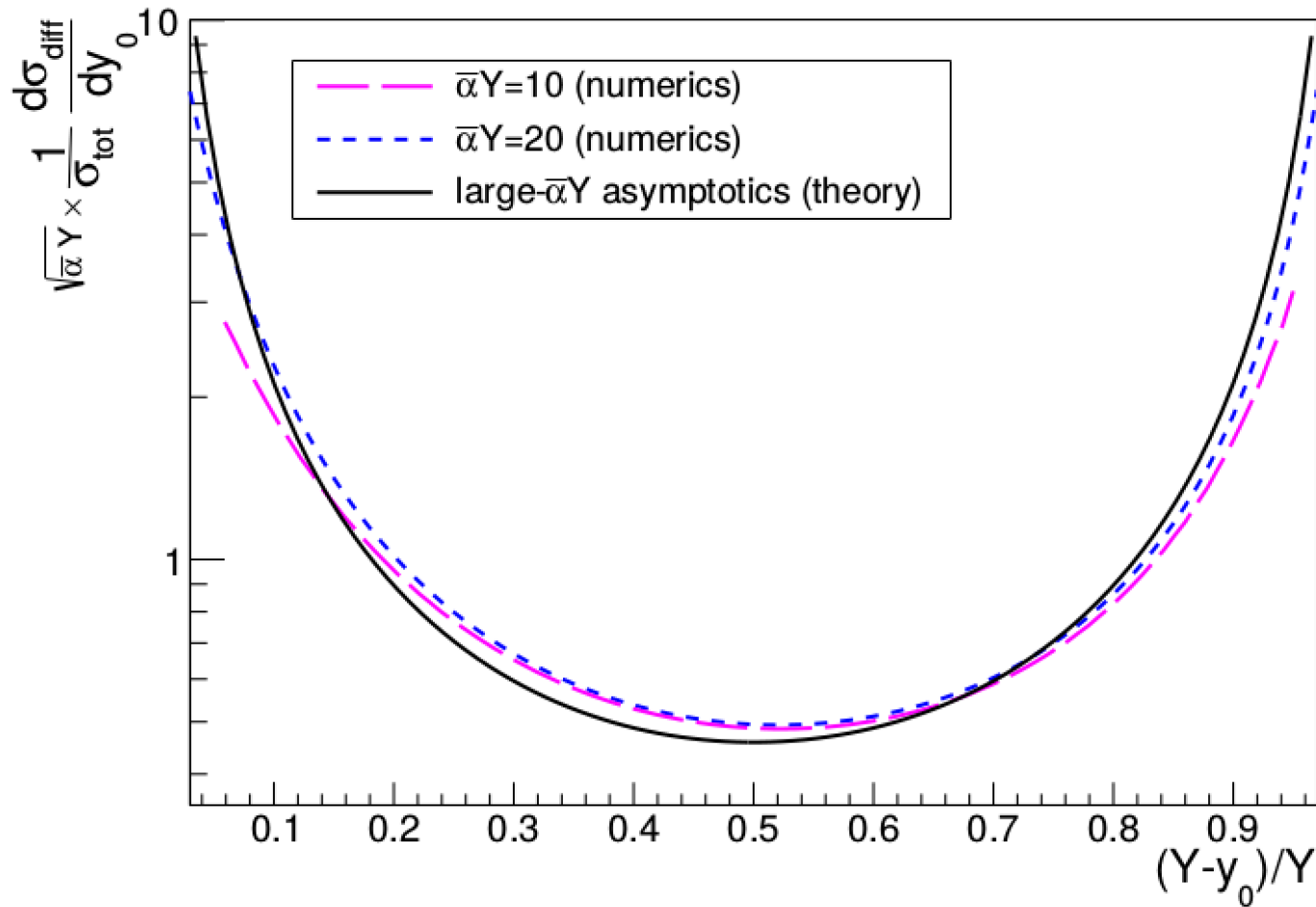
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# Comparison to numerics

Very large- $Y$  asymptotics

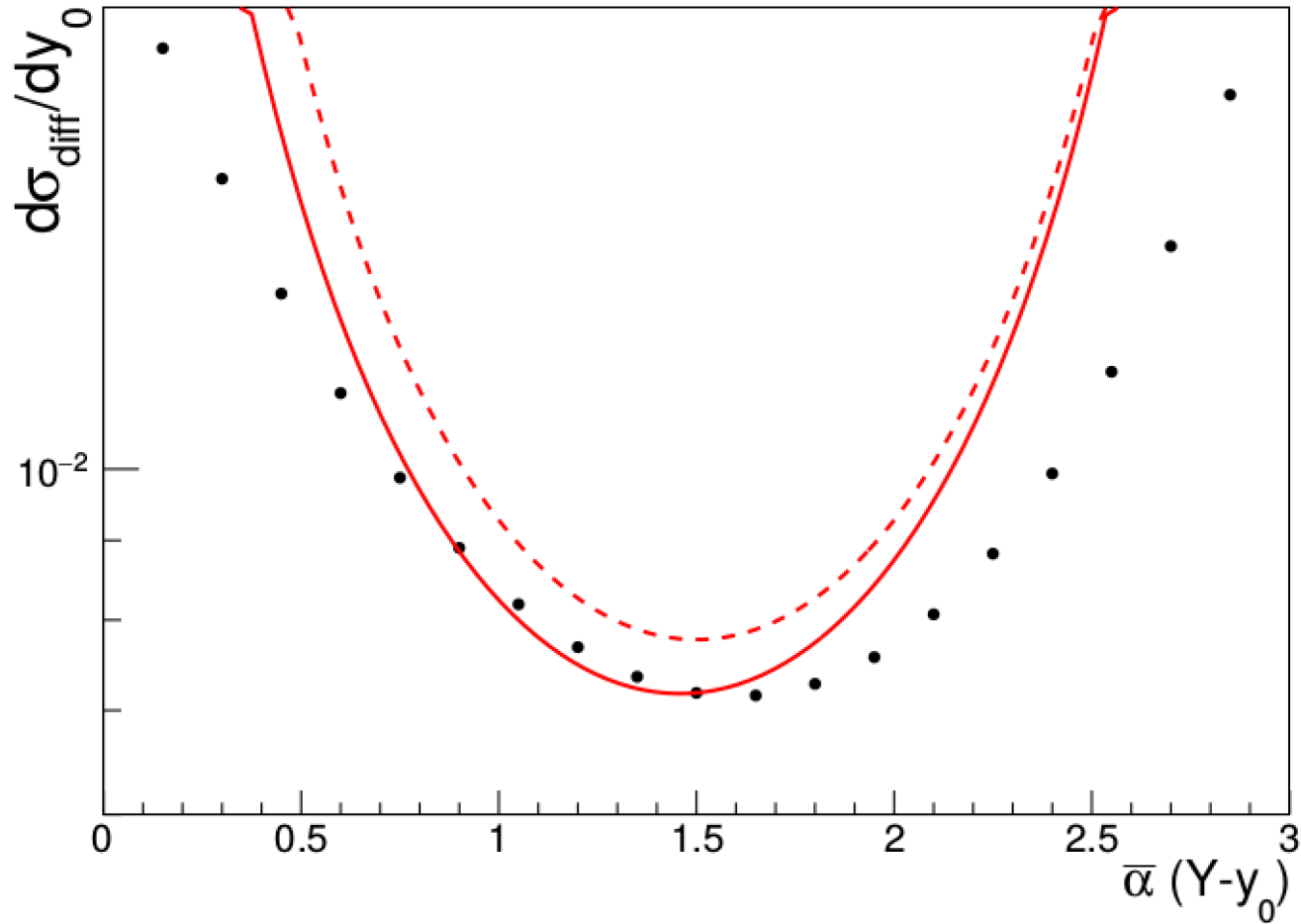


Theory: 
$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}$$

Numerics: Solution to the Kovchegov-Levin eq.

# Comparison to numerics

Realistic rapidities (EIC)  $\bar{\alpha} Y = 3$



Theory:  $\frac{d\sigma_{\text{diff}}}{dy_0} \sim \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}$

Numerics: Solution to the Kovchegov-Levin eq.

# Summary

- We have predicted, up to an overall constant, the rapidity gap distribution in diffractive dissociation of a virtual photon off a nucleus:

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma_{\text{diff}}}{dy_0} = \text{const} \times \left[ \frac{\bar{\alpha} Y}{\bar{\alpha} y_0 (\bar{\alpha} (Y - y_0))} \right]^{3/2}$$

Partonic interpretation: the rapidity gap is due to a **large fluctuation** (unusually low-transverse momentum gluon) in the course of the QCD evolution of the onium Fock state

- There is a deep analogy between this distribution and the distribution of the time at which common ancestors of extreme particles in branching random walks decay

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## Outlook

- We may have an even closer analogy between diffraction and genealogy: The overall constant in the distributions may be the same!

What about the finite-rapidity corrections?

*Work in progress...*