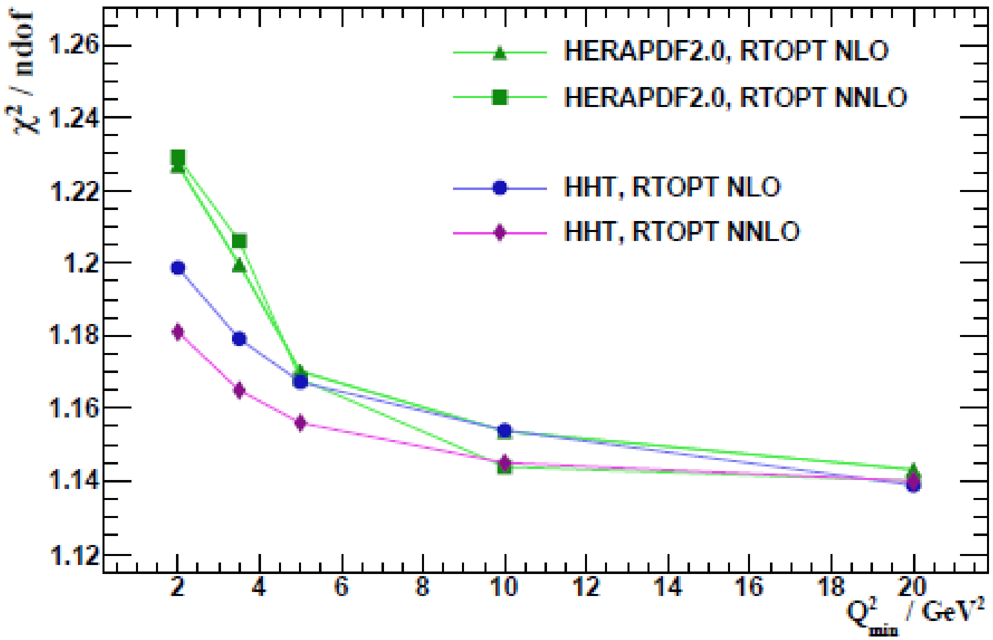
Impact of low-x resummation on QCD analysis of HERA data

xFitter developer's team
A M Cooper-Sarkar
arXIV:1802.00064

Low-x resummation:

- Improves the description of HERA data at low-x, low-Q2, high-y, without need for further parameters (as for example when adding higher twist terms)
- Results in a rising low-x gluon, which is always larger than the total Sea

The χ2/ndf of the HERAPDF2.0 NLO and NNLO fits deteriorate as the minimum value of Q^2 for data entering the fit is lowered



One way to improve this is to add higher twist terms - HHT analysis 1604.02299

But low Q2 is also low x and we have long suspected that the low-x region could require BFKL– ln(1/x) resummation. This does not require any extra parameters to fit the data (as higher twist does). It requires extending the DGLAP formalism to include ln(1/x) resummation. The tool to do this is the HELL code (Bonvini et al)

HELL has been implemented in xFitter

1. Here we explore consequences for a HERAPDF style fit

HELL implements resummation corrections to the fixed order splitting functions and coefficient functions up to NLL accuracy in ln(1/x), denoted as NLLx. The fixed order quantities are calculated by APFEL within the FONLL variable flavour number scheme.

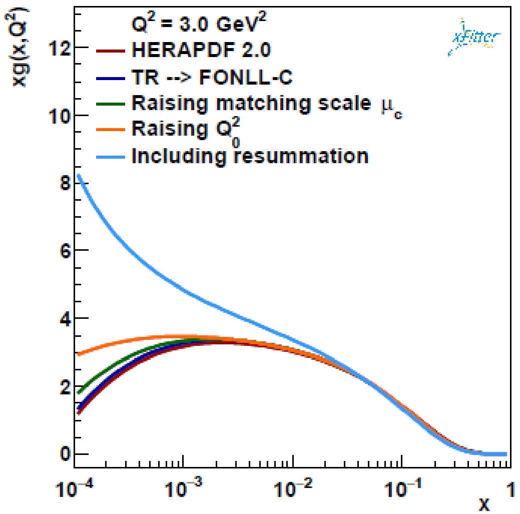
2.Thus we must use FONLL for the HERAPDF fit

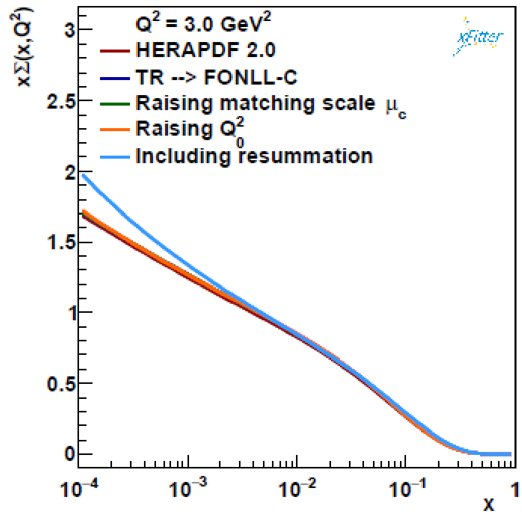
4. The computation of ln(1/x) resummation is unreliable at low scales due to the large value of α_S thus the starting scale is raised to Q^2_0=2.56GeV^2 rather than the usual HERAPDF value of Q^2_0=1.9GeV^2.

3. Consequently the charm quark threshold, μ_c, must be displaced above Q_0 while keeping the charm mass, m_c, fixed. (see 1707.05343)

5. Finally NLLx resummation can be applied

	Step-1	Step-2	Step-3	Step-4	Step-5
	HERAPDF2.0 NNLO	TR→FONLL-C	raise the charm matching scale μ_c	raise the initial scale Q_0	include NLLx resummation
HERA χ^2/d.o.f.	1363/1131	1387/1131	1390/1131	1388/1131	1316/1131





2.The increase in χ2 for FONLLC is well known and relates to the treatment of FL; terms up to $\mathcal{O}(\alpha_S^3)$ are included for RTOPT, but terms up to $\mathcal{O}(\alpha_S^2)$ are included for FONLLC. The gluon does not change shape

3. Raising the charm matching scale makes very little difference (see back-up) to χ2 or to gluon shape

4. Raising the initial scale has no effect on χ2, but does marginally change the shape of the gluon—this is a model variation which will be accounted for

5. The χ2 for the NNLO fit improves dramatically at the final step
The shape of the gluon is also changed dramatically from flattening/turning over at low-x to singular at low-x

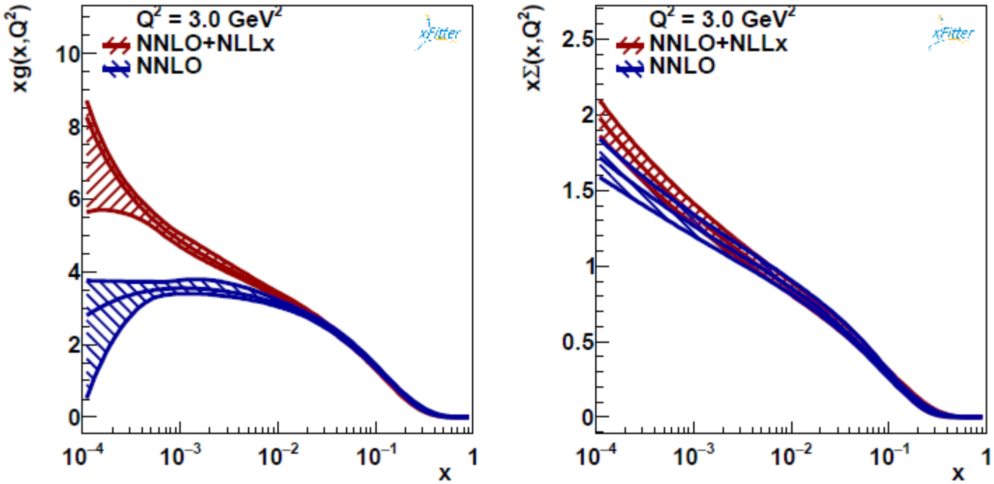
Further considerations:

Since we have change the heavy quark scheme the charm and beauty masses used may not be optimal for the new scheme. Thus charm and beauty data from HERA are included in the fit and charm and beauty mass scans are performed to determine new values m_c=1.46 and m_b=4.5GeV. Only m_c differs from that of the HERAPDF and the charm threshold is move to μ_c=1.64 correspondingly.
We include these heavy flavour data in the fits from now on since they are potentially sensitive to low x resummation.

Since we have a very different shape of the gluon PDF a parametrisation scan is performed at NNLO+NLLx to determine if the HERAPDF parametrisation is adequate. The form of the parametrisation is confirmed, however the negative term in the gluon is now small ~3σ from zero. In fact this is also the case for the NNLO fit due to the raised starting scale Q^2_0=2.56GeV^2
Nevertheless the resulting gluon shapes for NNLO and NNLO+NLLx are very different.
The form of the common parametrisation used for both NNLO and NLLO+NLLx is

$$\begin{aligned}
xg(x) &= A_g x^{B_g}(1-x)^{C_g} - A'_g x^{B'_g}(1-x)^{C'_g}, \\
xu_v(x) &= A_{u_v} x^{B_{u_v}}(1-x)^{C_{u_v}}\left(1+E_{u_v}x^2\right), \\
xd_v(x) &= A_{d_v} x^{B_{d_v}}(1-x)^{C_{d_v}}, \\
x\bar{U}(x) &= A_{\bar{U}} x^{B_{\bar{U}}}(1-x)^{C_{\bar{U}}}\left(1+D_{\bar{U}}x\right), \\
x\bar{D}(x) &= A_{\bar{D}} x^{B_{\bar{D}}}(1-x)^{C_{\bar{D}}}.
\end{aligned}$$

After these adjustments – and adding PDF uncertainties we have



A decrease in χ2 of 74 using ln(1/x) resummation

Largely due to the NC e+p 920 data

But also less need for shifts of systematic uncertainties

$$\chi^2 = \sum_i \frac{\left[D_i - T_i\left(1 - \sum_j \gamma_j^i b_j\right)\right]^2}{\delta_{i,\mathrm{unc}}^2 T_i^2 + \delta_{i,\mathrm{stat}}^2 D_i T_i} + \sum_j b_j^2 + \sum_i \ln \frac{\delta_{i,\mathrm{unc}}^2 T_i^2 + \delta_{i,\mathrm{stat}}^2 D_i T_i}{\delta_{i,\mathrm{unc}}^2 D_i^2 + \delta_{i,\mathrm{stat}}^2 D_i^2},$$

	NNLO fit with new settings	NNLO+NLLx fit with new settings
Total $\chi^2 (= \tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{d.o.f.}$	$1468\,(1327 + 119 + 22)/1207$	$1394\,(1305 + 91 - 2)/1207$
dataset inclusive $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(1264 + 103 + 21)/1145$	$(1239 + 78 - 4)/1145$
- subset NC 920 $\tilde{\chi}^2/\mathrm{n.d.p.}$	$447/377$	$413/377$
- subset NC 820 $\tilde{\chi}^2/\mathrm{n.d.p.}$	$67/70$	$65/70$
dataset charm $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(47 + 12 - 1)/47$	$(50 + 11 - 1)/47$
dataset beauty $(\tilde{\chi}^2 + \mathrm{corr} + \log)/\mathrm{n.d.p.}$	$(16 + 2 + 3)/29$	$(16 + 2 + 3)/29$

What is included in PDF uncertainties?

Experimental uncertainties for sure, but also model and parametrisation uncertainties according to the usual HERAPDF procedure

$\Delta m_c = \pm 0.05$, $\Delta m_b = 0.25$, $\Delta\alpha_s = 0.001$ around 0.118

$Q^2_0 = 2.88$ rather than 2.56 GeV^2

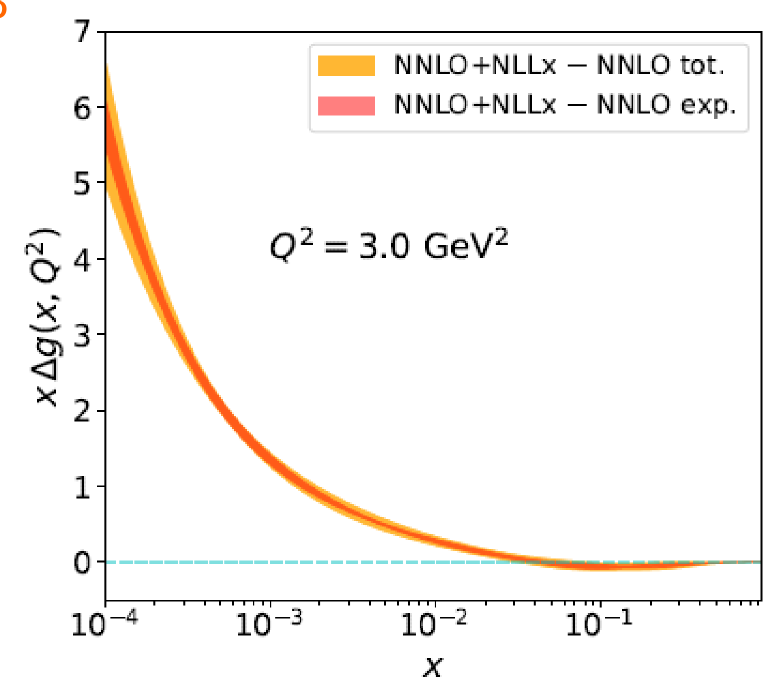
$Q^2_{min} = 2.5, 5.0$ rather than 3.5 GeV^2

The largest difference comes from changing the Q^2_{min} to 5 GeV^2

Parametrisation uncertainties are evaluate by adding extra terms D,E,F to the polynomials

$P_i(x) = (1 + D_i x + E_i x^2) e^{F_i x}$, which describe the PDFs $xq_i(x) = A_i x^{B_i} (1-x)^{C_i} P_i(x)$,

This can give different shapes to the PDFs even when the χ2 of the fit is barely different. The largest difference comes from a D_{uv} term.



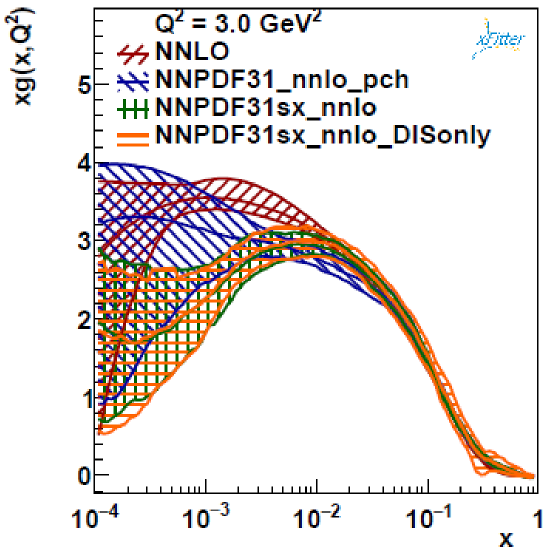
Clearly since the data used in the NNLO and NNLO+NLLx fits are the same the uncertainties on the fits are highly correlated. Thus to evaluate the real difference in the gluon shapes we must account for correlations.

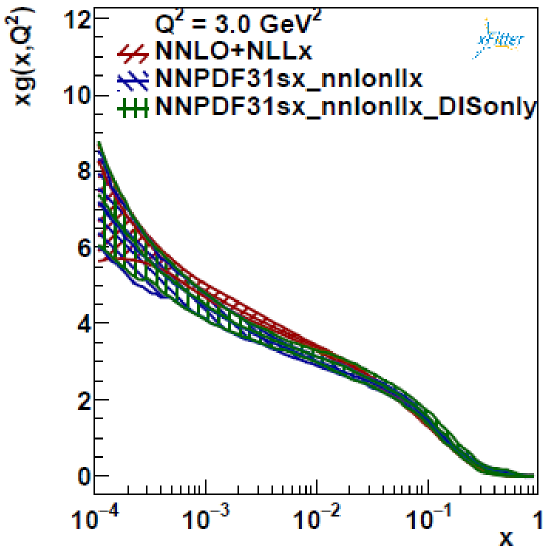
Uncertainties are evaluated by MC replicas of the data using the same random number sequence for both fits to evaluate the spread of the synchronised differences- and this is shown above as $x\Delta g(x,Q^2)$

NNPDF: arXiv:1710.05935, see similar trends in the gluon shape and improvements in χ2

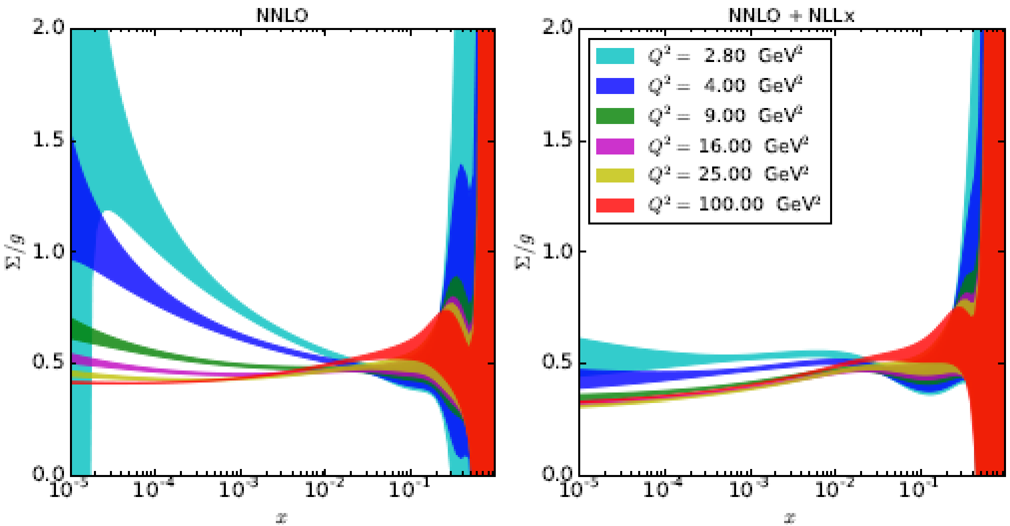
However because the treatment of charm in NNPDF is different—fitted charm, rather than perturbative charm– we show several variants of their analysis in order to aid comparison, with our work

NOTE how all gluons are more consistent after ln(1/x) resummation is included





Further study of the shapes of the gluon and the sea



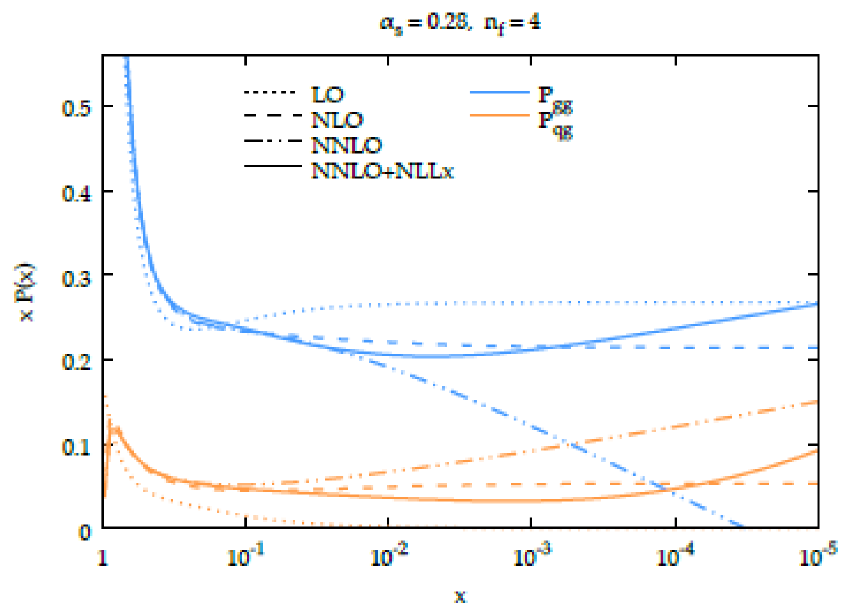
In fixed order fits at NNLO the gluon dips below the sea at low-x as Q^2 is reduced.
This is a general feature of NNLO fits- not just of HERAPDF.

These plots show the ratio of total sea Σ to gluon vs x for various Q2 for NNLO and NNLO+NLLx fits.
The ratio is much more stable with Low x resummation

This arises from the behaviour of the P_{qg} and P_{gg} splitting functions.
At NNLO $xP_{qg}(x) > xP_{gg}(x)$ for $x \lesssim 10^{-3}$,

Whereas for NNLO+NLLx

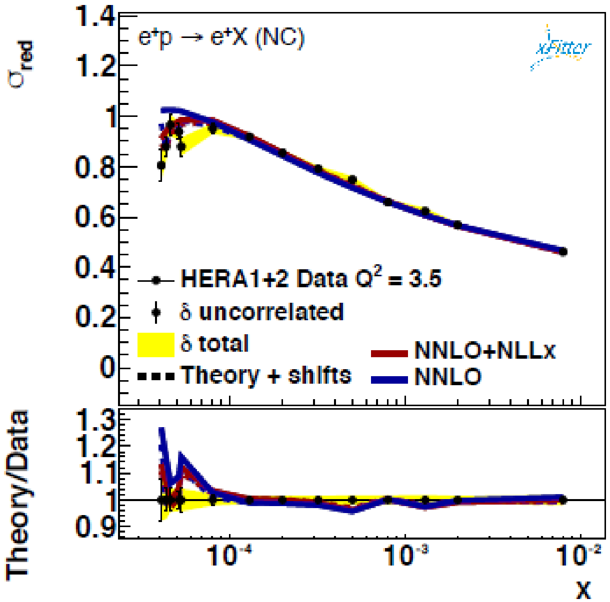
$$xP_{qg}(x) < xP_{gg}(x)$$

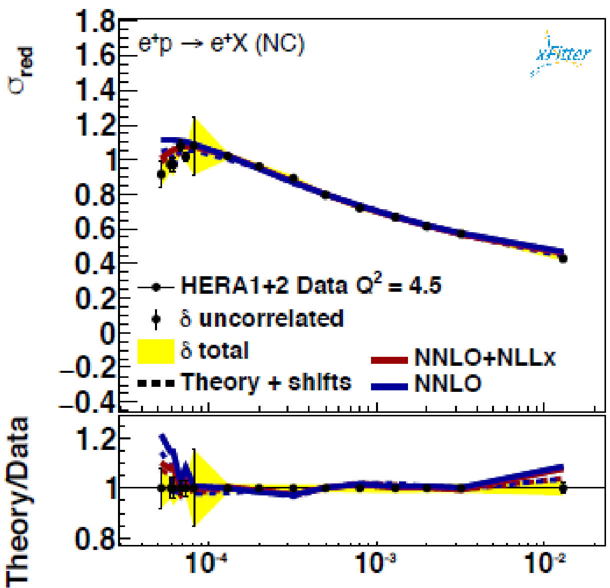


Comparison to data

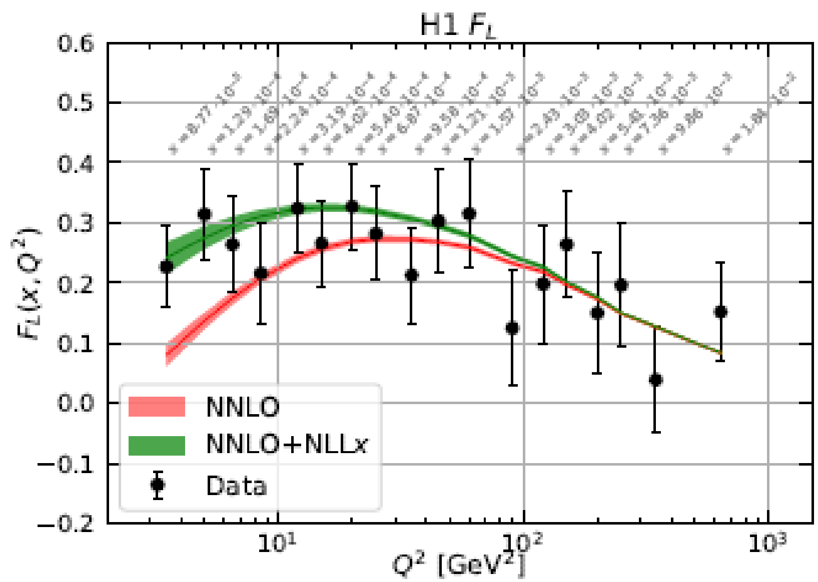
Comparison to data in the lowest Q^2 bins shows that the fit with low x resummation is much better able to follow the turn over of the data that happens at low-x, low Q^2, high-y due to the F_L term in the reduced cross section

$$\sigma_{\rm red} = F_2 - \frac{y^2}{Y_+} F_L$$

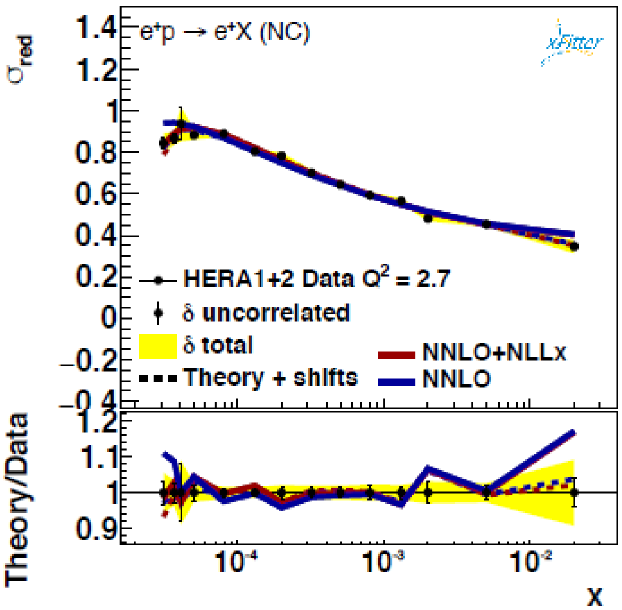




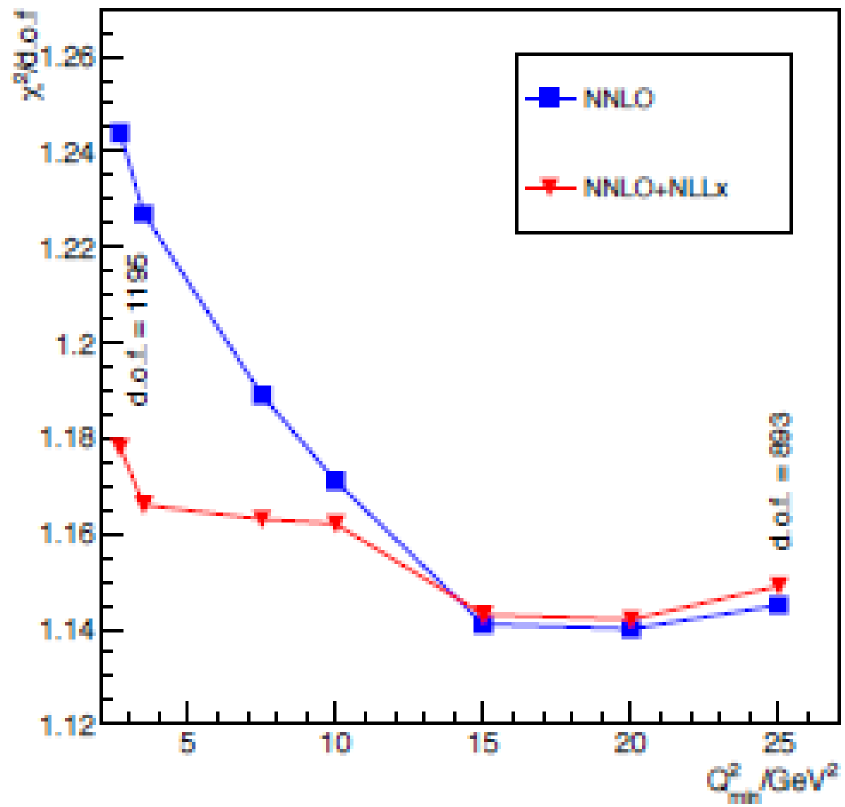
Looking at H1 F_L data directly shows that FL is larger at low Q^2/x for the NLLx fit



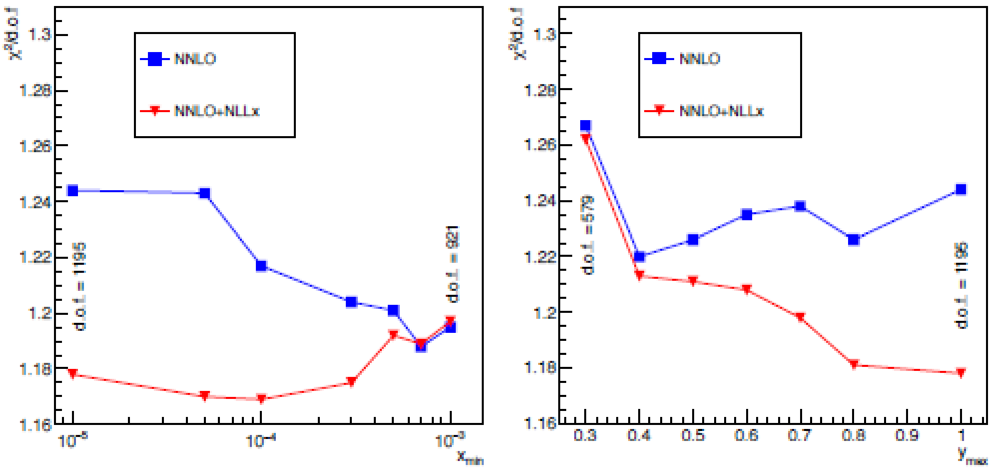
Theory +shifts means shifts due to experimental systematics- the term Σγb in the χ2



We also extend below the usual low Q2 cut –off for the data, down to Q2=2.7GeV2.
The NNLO+NLLx fit including these data gives very similar PDFs to the standard fit



To dileneate the kinematic region for which NLLX resummation improves the fits we perform a χ2 scan in Q^2_{min} for NNLO and NNLO+NLLx fits
This shows an improvement at low Q^2, $Q^2_{min} < 15$ GeV^2



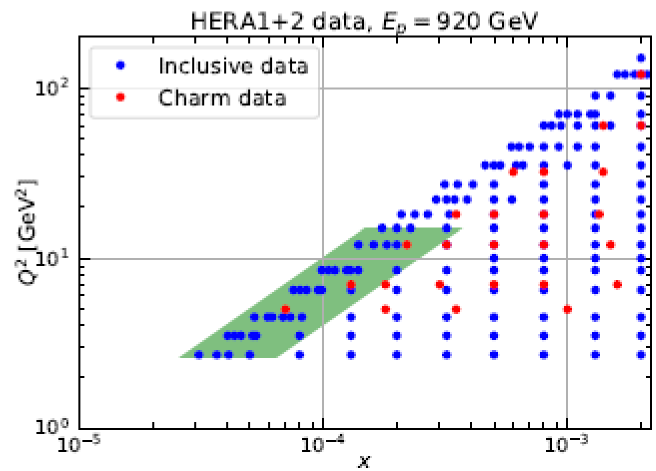
We also scan vs x_{min} seeing improvement for $x_{min} < 5.10^{-4}$

And against y^{max} seeing improvement for $y_{max}>0.4$

This emphasizes the importance of low x resummation at high-y for the DIS data because of the role of the FL term

$$\sigma_{red} = F_2 - \frac{y^2}{Y_+} F_L$$

The scans shown here were done refitting the PDFs at each step—thus they dileneate a region where the fixed order calculation is poor –even with refitting –as illustrated on the x, Q^2 plane here.



Summary

Low-x resummation:

- Improves the description of HERA data at low-x, low-Q2, high-y, without need for further parameters (as for example when adding higher twist terms)
- Results in a rising low-x gluon, which is always larger than the total Sea

Back-up

Technical aside on charm matching scale

The slight change in shape of the gluon when the charm threshold is displaced is partly due to the PDF matching conditions . Moving up the charm threshold using fixed order NNLO depresses the charm PDF at larger scales. Since charm is generated by gluon splitting this requires a slightly larger gluon to compensate. However PDF matching conditions are also affected by logs of 1/x, which are resummed in HELL such that the spread cause by matching is significantly reduced for NNLO+NLLx.

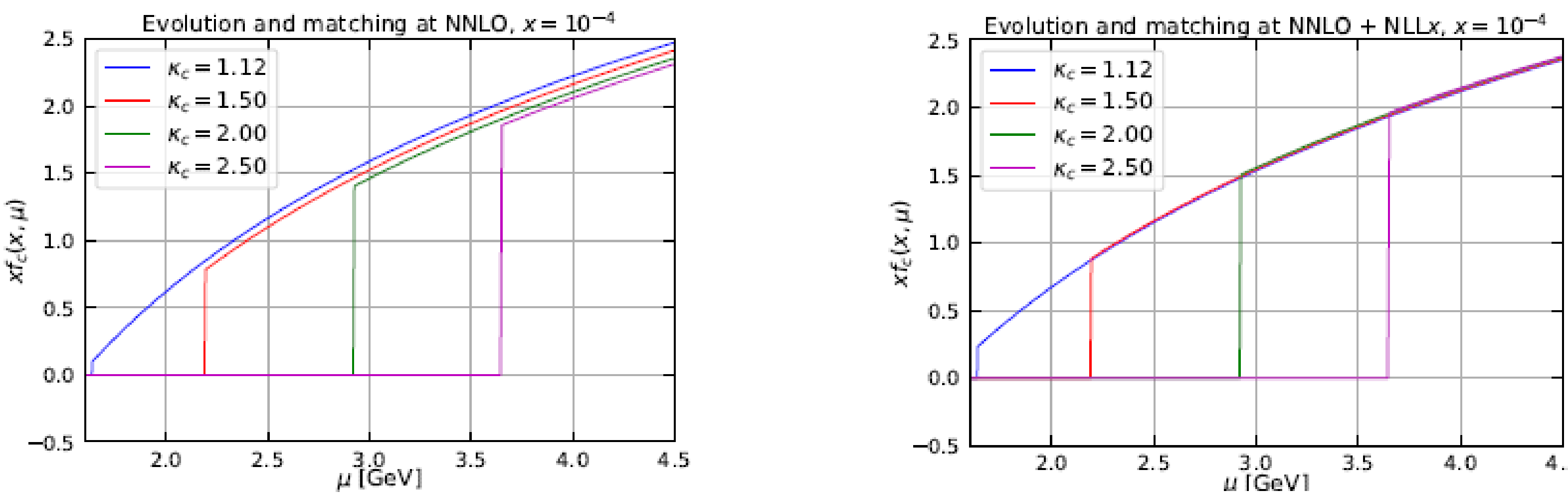
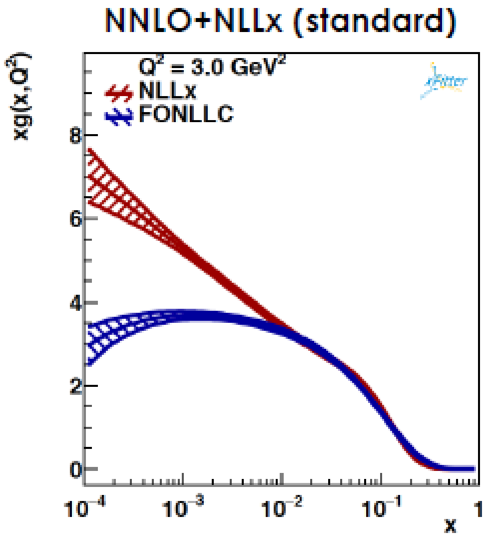
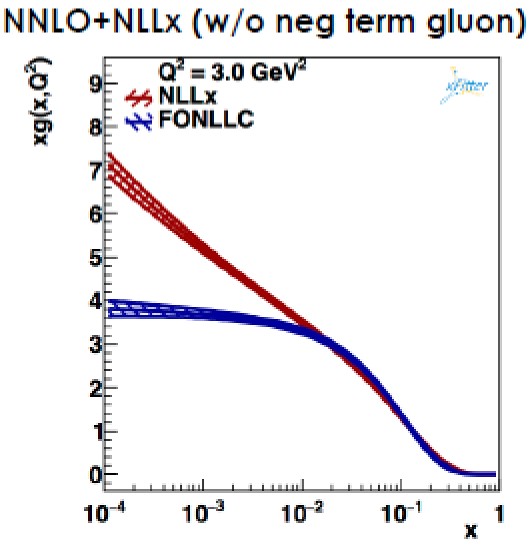


Figure 1 The charm PDF at $x = 10^{-4}$ as a function of the factorisation scale μ for different values of the charm threshold $\mu_c = \kappa_c m_c$, with $\kappa_c = 1.12, 1.5, 2, 2.5$. The plots show the effect of the matching at NNLO (upper plot) and at NNLO+NLLx (lower plot).

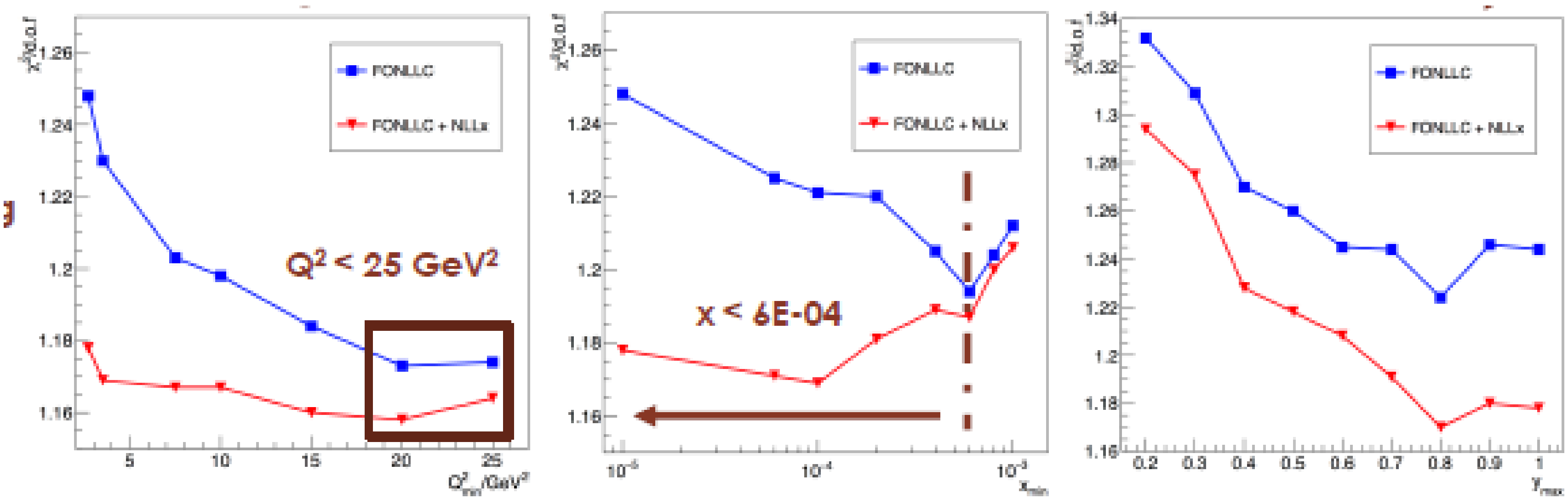
Adding the negative gluon term

Do we really need the negative term of gluon? → We produced a version of the **final NNLO+NLLx and NNLO fits without the negative term** just to check this





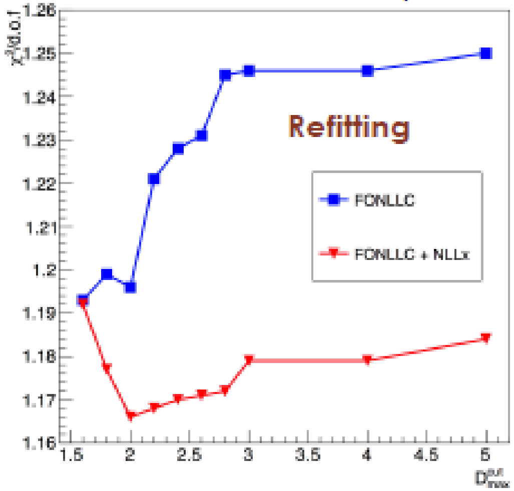
The point is that even without the negative term the gluon for NLLO likes to take a flattish shape at low-x, whereas for NNLO+NLLx it takes a singular shape

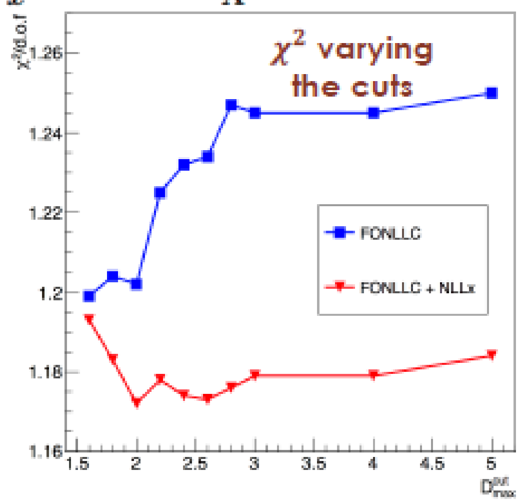


IF we do not refit at every step of the scan but we use the same PDFs as determined for the minimal cuts, we get these results.
The improvement of NLLx is still at low-x and low Q2 but is not so obviously concentrated at high-y. The refitting takes into account the need to fit DIS data for which the effect of F_{lL} is concentrated at high-y

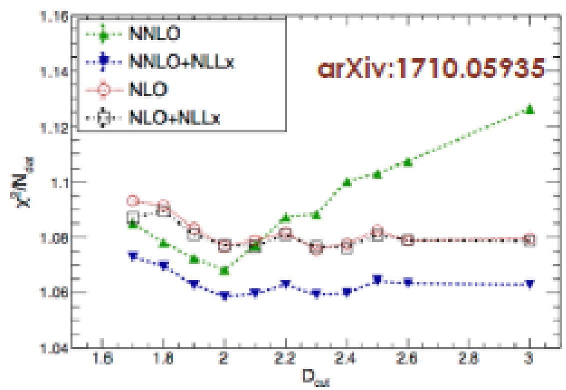
$$\sigma_{\rm red} = F_2 - \frac{y^2}{Y_+} F_L$$

Simultaneous cut on Q^2 and x implemented: $\ln\frac{1}{x} \geq \beta_0 H_{\rm cut} \ln\frac{Q^2}{\Lambda^2}$ $\Lambda \simeq 88$ MeV $\beta_0 \simeq 0.61$





NNPDF3.1sx, HERA NC inclusive data



Consistent with what has been found in the NNPDF paper:

- $D_{cut} > 2$ defines the region where resummation is important
- Flat-ish χ^2 distribution for NNLO+NLLx
- Above $D_{cut} = 3$ few data points added even if with huge steps

We could also perform a cut jointly in x and Q^2 which follows the dependence of resummation terms on $\alpha_S(Q^2)\ln(1/x)$ ie on $\ln(1/x)/\ln(Q^2)$

$H = \ln(1/x)/\ln Q^2/\Lambda^2$ with $\Lambda = 88$ MeV.

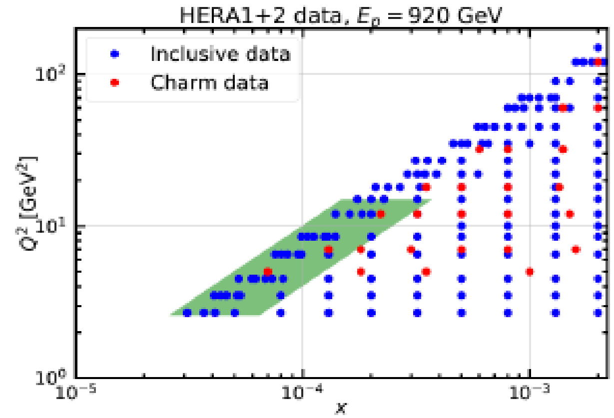


Figure 11 Scatter plot of the low-x and low-Q^2 kinematic region covered by the HERA1+2 inclusive data and charm data at E_p = 920 GeV. The green shaded area indicates the region in which $\ln(1/x)$ resummation has a significant effect.

Defined by:

- $x < 5 \cdot 10^{-4}$
- $2.7\ GeV^2 < Q^2 < 15\ GeV^2$
- $0.4 < y < 1.0$

Since the χ^2 scans have been obtained independently from one another, one may wonder whether our estimate is reliable.

In order to check this, we have performed two additional fits, one with and one without resummation, excluding only the data points for which Q^2 < 15 GeV^2 and y > 0.4.

––– NNLO+NLLx –––

```
After minimisation    1249.201064 1.174

Partial chi2s
395.95( +3.95)   354  HERA1+2 NCep 920
 51.32( -0.64)    56  HERA1+2 NCep 820
179.52( -1.09)   214  HERA1+2 NCep 575
179.12( -2.25)   170  HERA1+2 NCep 460
222.78( -0.82)   159  HERA1+2 NCem
 45.59( +0.57)    39  HERA1+2 CCep
 53.88( -2.45)    42  HERA1+2 CCem
 44.53( -1.11)    44  Charm cross section

 Correlated Chi2     80.329061352348674
 Log penalty Chi2    -3.8395890369565198
```

––– NNLO –––

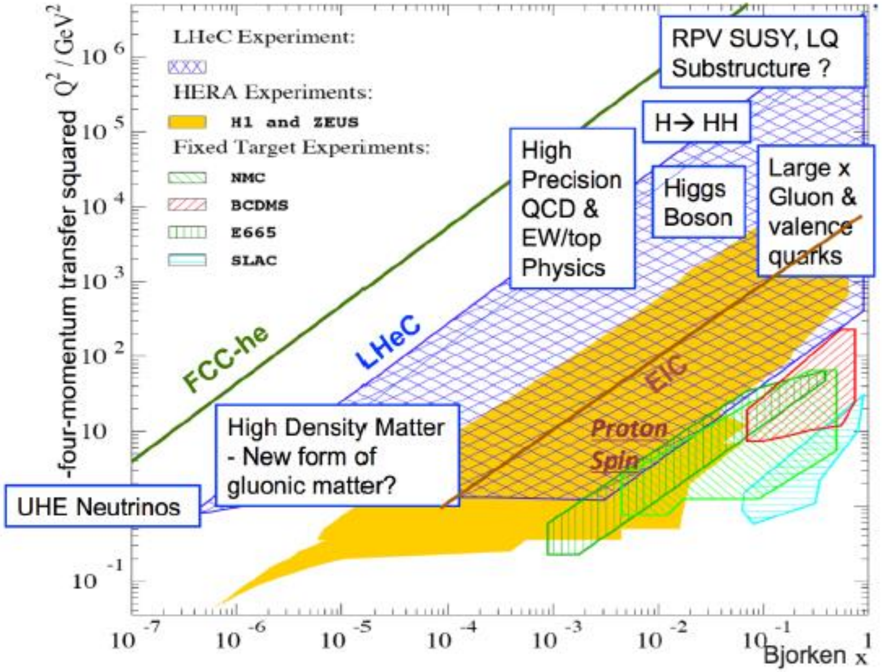
```
After minimisation    1264.22   1064 1.188

Partial chi2s
402.82( +7.25)   354  HERA1+2 NCep 920
 52.23( -0.10)    56  HERA1+2 NCep 820
177.53( +1.15)   214  HERA1+2 NCep 575
176.67( -0.31)   170  HERA1+2 NCep 460
215.44( +1.04)   159  HERA1+2 NCem
 44.30( +0.35)    39  HERA1+2 CCep
 54.93( -1.58)    42  HERA1+2 CCem
 45.39( -1.31)    44  Charm cross

 Correlated Chi2    88.418716117383113
 Log penalty Chi2    6.4854418695532452
```

- The total χ^2's of these fits differ by around 15 units in favour of the resummed fit, mostly due to the correlated and logarithmic terms, to be compared to the 73 units of when the shaded area is instead included.
- This confirms that, the context of DIS, the shaded area in Fig. 11 does provide a reliable estimate of the kinematic region in which resummation works significantly better than fixed order.

LHeC and FCC-eh



LHeC kinematic reach:

Q^2 up to 10^6 GeV^2

x down to 10^{-6}

FCC-eh extends further,

Q^2 to 10^7 GeV^2, x to 10^{-7}

- **outline of this talk:**
- PDFs at FCC-eh
- strong coupling (α_s)

The relevance of this kinematic region to the LHeC is obvious

Another striking difference with respect to our analysis is that a significant reduction (by more than 50 units for 47 datapoints) of the χ^2 of charm production data when including $\ln(1/x)$ resummation was found in Ref. [9]. The origin of such a huge effect can be traced back to the poor quality of the description of charm data at fixed NNLO in the NNPDF3.1sx study. Indeed, the NNPDF3.1sx χ^2 of this dataset when resummation is included is very similar to that of the present study, differing by just 2 units. The reason of this difference in the quality of the description of charm data at fixed order is related to the treatment of the charm PDFs. However, the discrepancy cannot be ascribed to the fact that the charm PDFs are fitted in Ref. [9]. In fact, fitting the charm PDFs should give more flexibility to better describe the data. Rather, it is the actual construction of the FONLL-C prediction which differs when the charm PDFs are fitted. Specifically, when fitting the charm PDFs, it has been pointed out that an extra contribution, denoted by $\Delta_{\rm IC}$,

The introduction of this extra term has the consequence that the phenomenological damping factor usually introduced in the FONLL scheme with perturbative charm to suppress subleading higher-order terms in the vicinity of the charm threshold [38], becomes ineffective. Indeed, when the charm PDFs are fitted, and thus a non-perturbative (or intrinsic) component is allowed, the contributions multiplied by the damping are no longer subleading, and cannot therefore be suppressed. The bad description of the charm data at fixed order in Ref. [9] is thus the consequence of three concurring effects: (1) the absence of damping, (2) the presence of the extra contribution $\Delta_{\rm IC}$ to the FONLL formula, and (3) the fitted charm PDFs which makes this $\Delta_{\rm IC}$ contribution sizeable. Since our charm PDFs are generated perturbatively, the $\Delta_{\rm IC}$ contribution is subleading and does not affect our results significantly. Specifically, the effect of adding such $\Delta_{\rm IC}$ term would effectively be equivalent to removing the damping factor. We have thus performed a fixed-order fit without the damping in the FONLL formula and found that, as expected, the results are not significantly affected (in particular, the χ^2 of the charm dataset remains unchanged).

We have also recomputed the χ^2 of the charm dataset using FONLL without damping and the NNLO PDFs of Ref. [9], which contain fitted charm, and found indeed that the description of the data is worsened, even though not at the level of the results of Ref. [9] (which additionally include the extra $\Delta_{\rm IC}$ contribution to FONLL). Note that the deterioration of χ^2 in this case comes mostly from the correlated contribution to the χ^2, second term in Eq. (5). We have also performed the same exercise activating the damping, which effectively suppresses all contributions due to the fitted-charm PDFs in the vicinity of the charm threshold making the result closer to what one obtains in the perturbative charm case. By doing so we find that the description improves significantly, bringing it at the level of our results. Note that similar tests have been performed in the NNPDF3.1sx study (see the discussion in Sect. 4.1 of Ref. [9]), finding compatible results.