

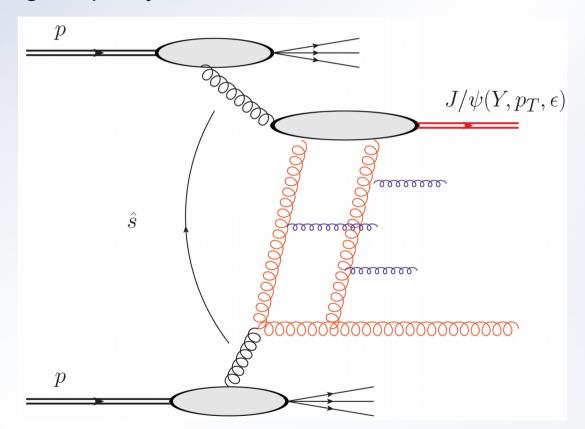
Outline

Work in progress in collaboration with Piotr Kotko, Anna Stasto and Mariusz Sadzikowski

- Motivation and phenomenological context
- Theoretical small-x evolution context:
 non-forward BFKL equation and BKP states
- Evaluation of the lowest order amplitudes
- Two BFKL pomeron evolution
- Results, comparison to data
- Discussion

Definition of the process

 Consider high pT vector meson production with a jet, with large rapidity distance



- Integrate out the jet to get contribution to inclusive production
- Proposed by Khoze, Martin, Ryskin, Stirling

Heavy quarkonia hadroproduction

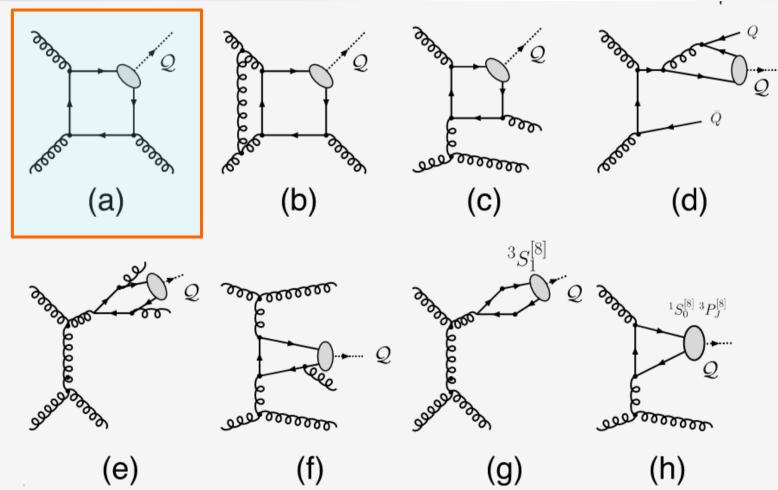
Production mechanisms in QCD:

- Color singlet, collinear: LO, NLO
- Color octet
- Color singlet NNLO*
- Color singlet kT-factorization
- Color singlet with double scattering without correlations

This study:

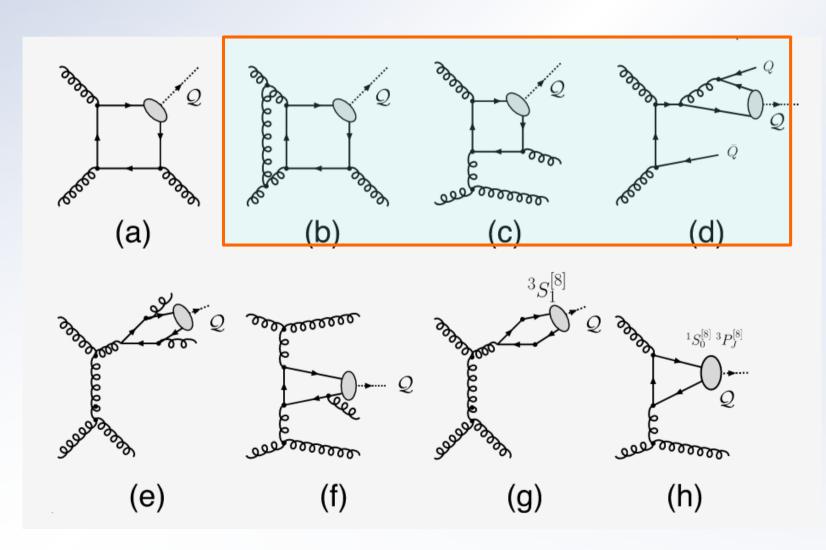
Color singlet, double scattering with correlations

Color singlet LO



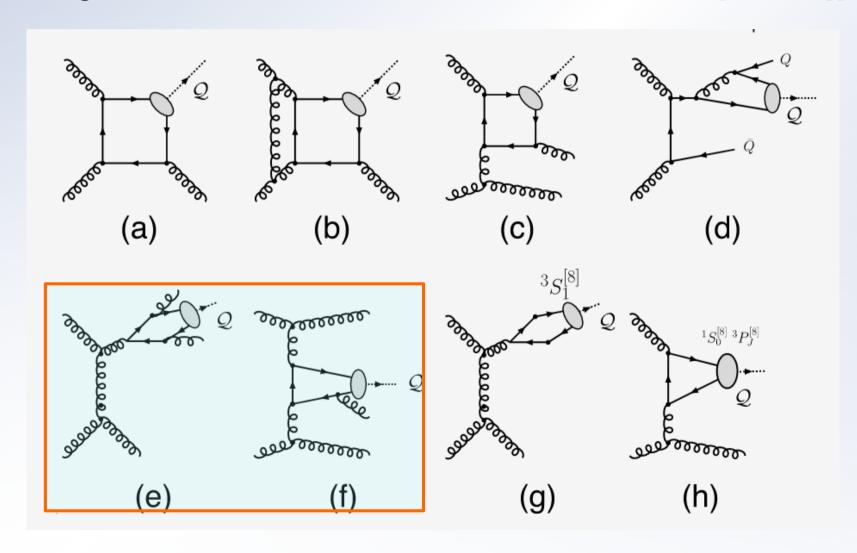
Color singlet NLO

[Lansberg]



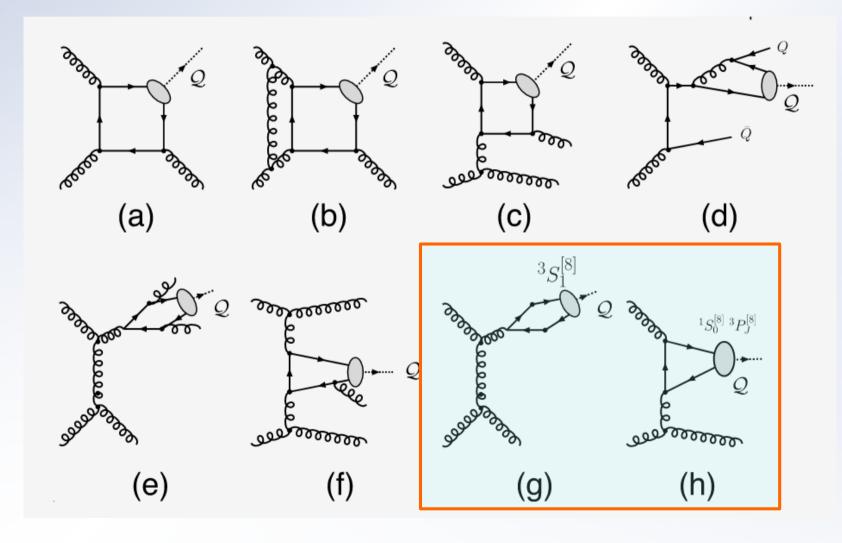
Color singlet NNLO*

[Lansberg]



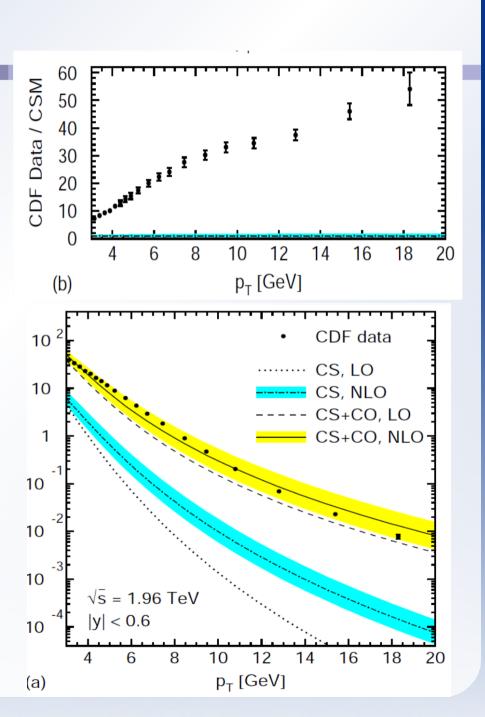
Color octet

[Lansberg]



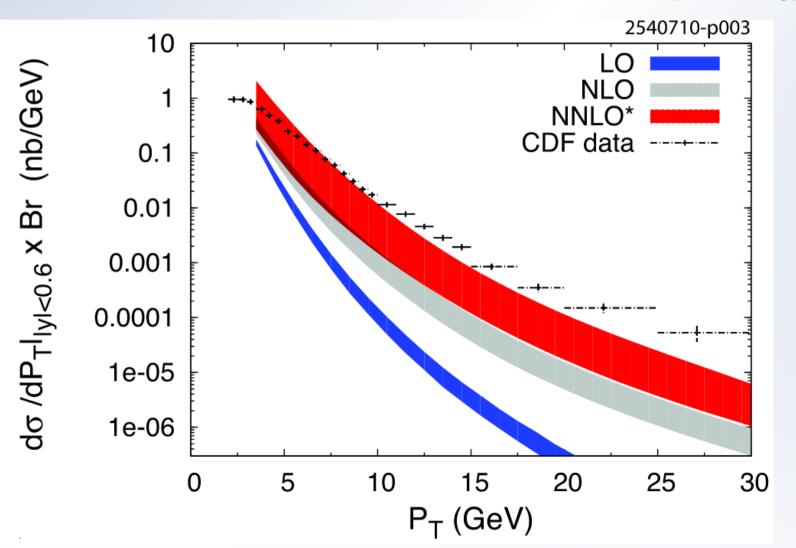
Color singlet vs color octet

- Conventional color singlet mechanism: simple and straightforward, but badly underestimates the data
- Color octet mechanism is able to describe the data but the successful description relies on several multiplicative parameters that are fitted
- There is still room for alternative approaches



Example: color singlet beyond NLO: NNLO*

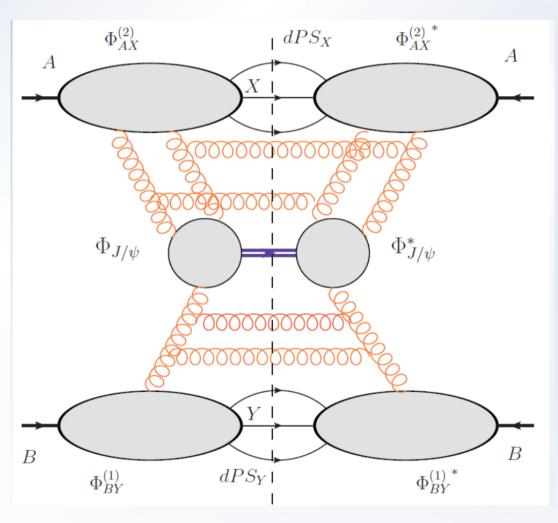




Color singlet – beyond NLO: three gluon fusion contribution (higher twist)

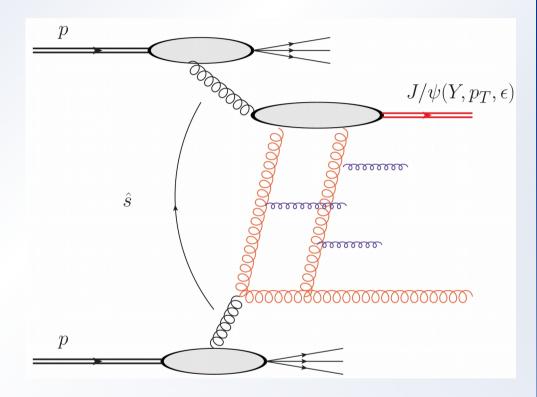
- Conventional color singlet mechanism relies on two gluon fusion followed by gluon emission
- Alternative: fusion of two gluons from the beam and one gluon from the target
- Higher twist suppression but enhancement by double gluon density
- Found to lead to a ~25%
 contribution to data at
 moderate pT, but irrelevant at
 large pT

[Khoze, Martin, Ryskin, Stirling; M. Sadzikowski, LM]



At NNLO: heavy quarkonium + jet with sizable rapidity distance

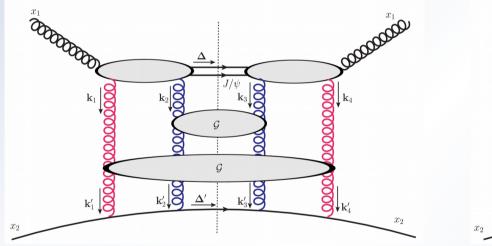
- Vector meson vertex: fusion of three gluons
- Two gluons come from a single parton – no higher twist suppression!
- In the cross-section enhancement factor appears from the double hard pomeron and the jet

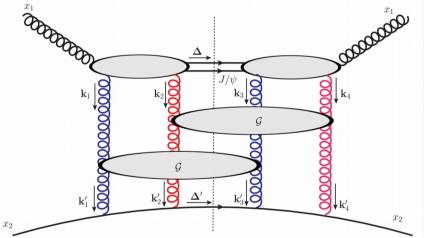


- Enters as a part of NNLO correction to color singlet
- It is a gauge invariant contribution in the high energy limit

The correlated double pomeron contribution

- In partonic cross section one finds four gluon t-channel evolution
- In high energy limit, in the LL1/x approximation the evolution described by Bartels-Kwieciński-Praszałowicz (BKP) equation

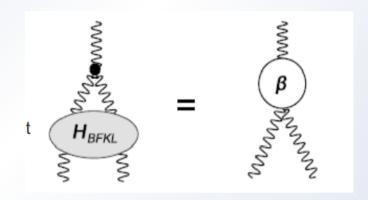


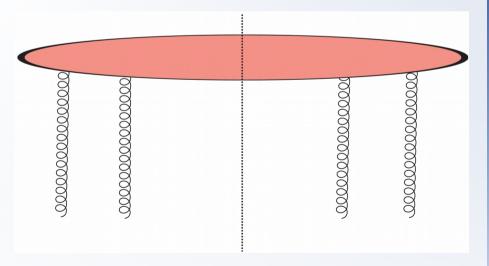


- Leading singularity at high energies in large Nc limit: the double pomeron exchange
- The pomerons originate from a single parton ~ correlated double parton density

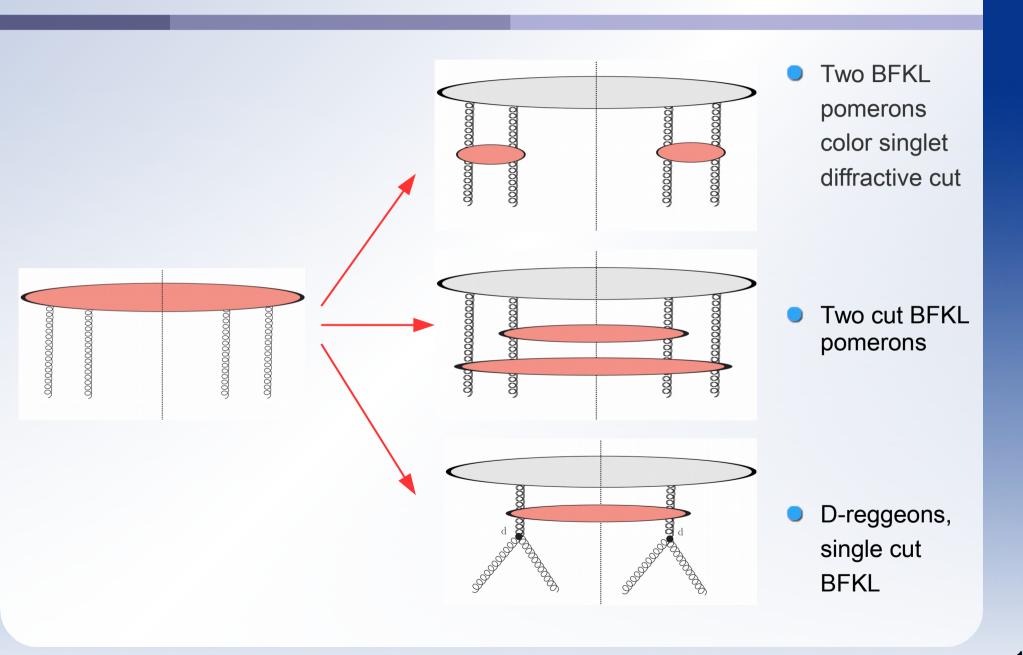
BKP states in t-channel

- Analysis of four gluon state in the t-channel in high energy limit must take into account:
- Gluon reggeization
- Symmetry of the 4-gluon state
- BKP 4-gluon amplitude with central cut has symmetries: for exchanges of gluons (12), (34) and (12) with (34)
- Decomposition into eigenstates of BKP evolution



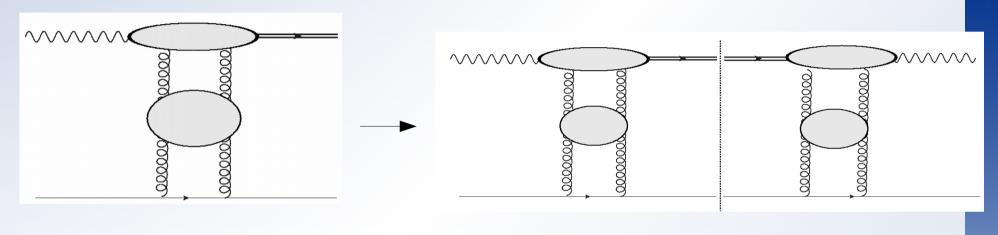


BKP states in t-channel

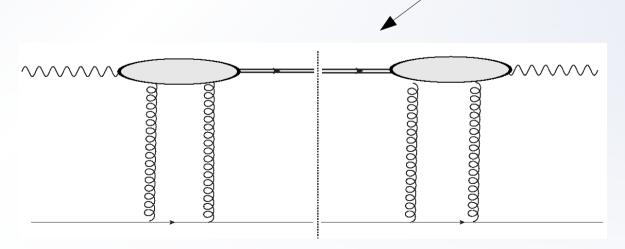


The correlated double pomeron contribution: how to compute? Step 1

 Very well known starting point: proton-dissociative heavy vector meson photoproduction at high pT with rapidity gap

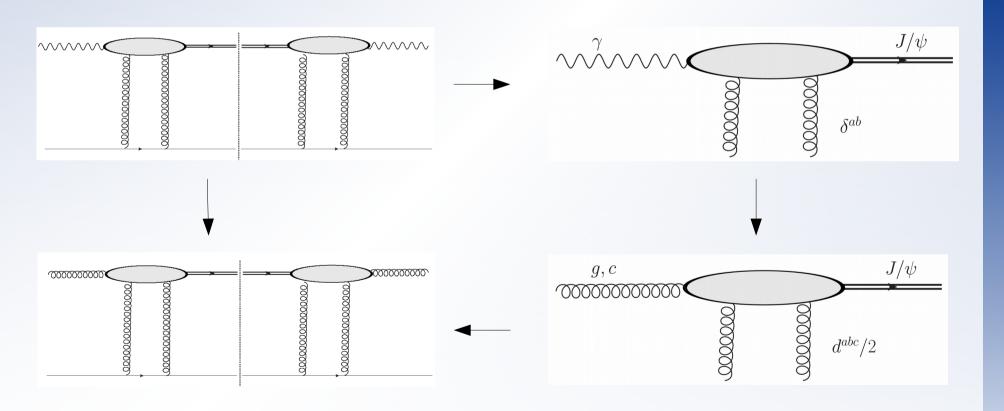


Go to the lowest order



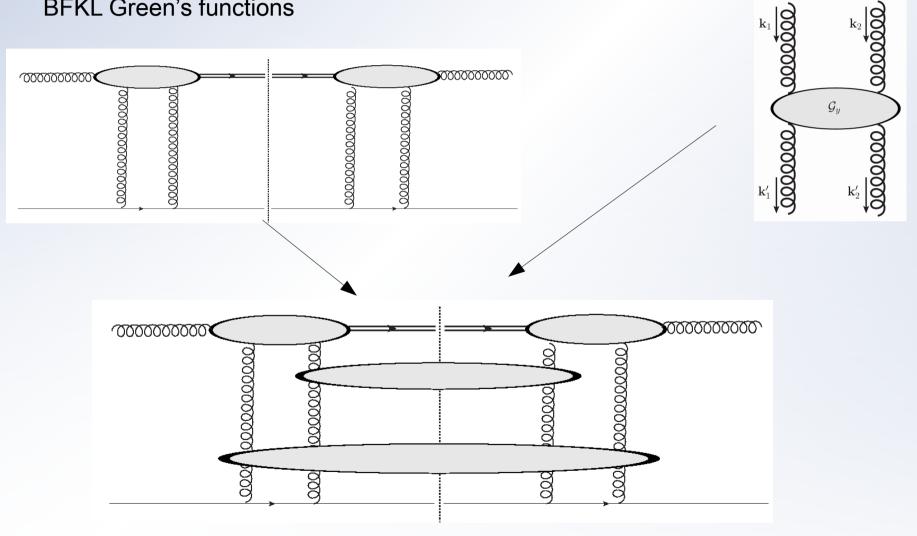
The correlated double pomeron contribution: how? Step 2

 The lowest order amplitudes for quasi-diffractive production and the 3 gluon fusion differ only by coupling constants and color factors, due to symmetry of the color part, the kinematical parts are the same

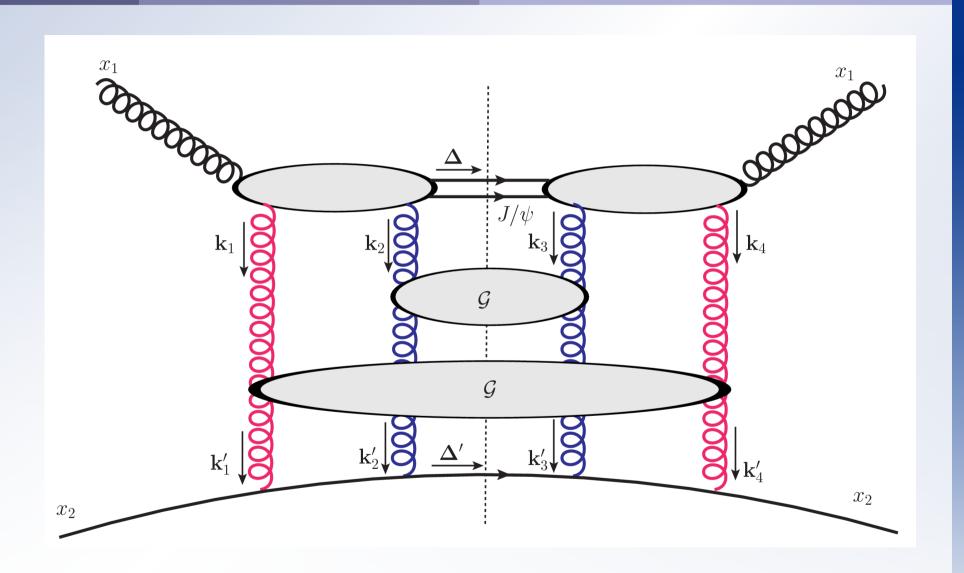


The correlated double pomeron contribution: how? Step 3

 Dress-up the lowest order amplitude with the BFKL evolution / BFKL Green's functions



Problem to resolve: double non-forward BFKL evolution and integrate over the BFKL pomeron loop

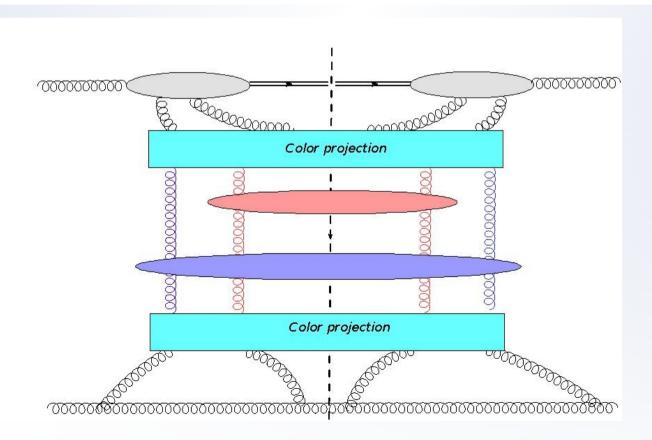


Projection on the two pomeron state

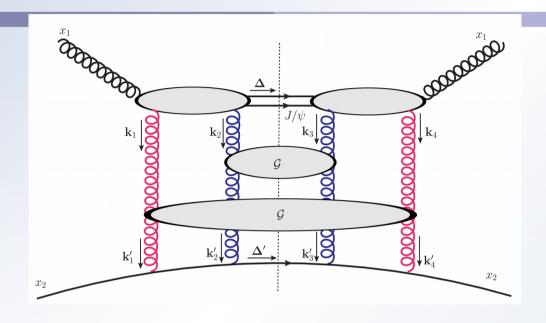
Project on the BKP state using color basis with adequate symmetries

$$|1\rangle = N_1 \delta^{ab} \delta^{cd}, \qquad |d_R\rangle = N_d d^{rab} d^{cdr}, \quad |2P\rangle = N_{2P} (\delta^{ac} \delta^{bd} + \delta^{ad} \delta^{bc})$$

$$\langle A|A'\rangle = \delta_{AA'} + \mathcal{O}(1/N_c^2),$$
 Projection operator on $2P = |2P\rangle\langle 2P|$



Solving the double non-forward BFKL exchange problem



$$|\mathcal{M}|^{2} = \mathcal{N} \int d^{2}\boldsymbol{q} \int d^{2}\boldsymbol{k}_{1}d^{2}\boldsymbol{k}_{2} \int d^{2}\boldsymbol{k}_{1}'d^{2}\boldsymbol{k}_{2}' \frac{1}{\boldsymbol{k}_{1}^{2}(\boldsymbol{q} - \boldsymbol{k}_{1})^{2}\boldsymbol{k}_{2}^{2}(\boldsymbol{q} - \boldsymbol{k}_{2})^{2}}$$

$$\times \Phi_{J/\Psi}(\boldsymbol{k}_{1}, \boldsymbol{k}_{2}) \Phi_{J/\Psi}^{*}(\boldsymbol{q} - \boldsymbol{k}_{1}, -\boldsymbol{q} - \boldsymbol{k}_{2}) \delta^{(2)}(\boldsymbol{k}_{1} + \boldsymbol{k}_{2} - \boldsymbol{p}_{T})$$

$$\times \mathcal{G}(\boldsymbol{k}_{1}, \boldsymbol{k}_{1}'; \boldsymbol{q}, Y) \mathcal{G}(\boldsymbol{k}_{2}, \boldsymbol{k}_{2}'; -\boldsymbol{q}, Y)$$

$$\times \Phi_{q}^{2P}(\boldsymbol{k}_{1}', \boldsymbol{k}_{2}', \boldsymbol{q} - \boldsymbol{k}_{1}', -\boldsymbol{q} - \boldsymbol{k}_{2}')$$

Solving the double non-forward BFKL exchange problem

 The LL BFKL Green's function with conformal eigenfunctions E (n,v) in position space [Lev Lipatov]

$$\tilde{\mathcal{G}}(r_1, r_2, r_1', r_2') = \sum_n \int d\nu \, w(n, \nu) \, \int d^2 r_0 E_{n, \nu}^*(r_{01}^*, r_{02}^*) \exp(\bar{\alpha}_s Y \chi_n(\nu)) E_{n, \nu}(r_{01}', r_{02}')$$

Momentum representation of the Green's function

$$\hat{\mathcal{G}}(k, k'; q, Y) = \sum \int d\nu \, w(n, \nu) \, \langle k | E(q, n, \nu) \rangle \, \exp(\bar{\alpha}_s Y \chi_n(\nu)) \, \langle E(q, n, \nu) | k' \rangle$$

BFKL exchange amplitude with impact factors of particles A and B

$$\mathcal{M}(q,Y) \sim \langle \Phi_A | \hat{\mathcal{G}}(q,Y) | \Phi_B \rangle$$

$$\mathcal{M}(q,Y) \sim \sum_{n} \int d\nu \langle \Phi_A | E(q,n,\nu) \rangle \exp(\bar{\alpha}_s Y \chi_n(\nu)) \langle \Phi_B | E(q,n,\nu) \rangle$$

Solving the double non-forward BFKL exchange problem

Extension to double pomeron exchange amplitude

$$|\mathcal{M}|^2 \sim \sum_{n} \sum_{n'} \int d\nu \int d\nu' \, w(n,\nu) w(n,\nu') \, \exp(\bar{\alpha}_s Y \chi_n(\nu)) \, \exp(\bar{\alpha}_s Y \chi_{n'}(\nu'))$$

$$\times \int d^2q \langle \Phi_Q^{2P}[|E(q,n,\nu)\rangle \otimes |E(-q,n',\nu')\rangle] [\langle E(q,n,\nu)| \otimes \langle E(-q,n',\nu')|] \Phi_{J/\psi^2}^{2P} \rangle$$

$$\Phi_{J/\psi^2}^{2P}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{q}-\boldsymbol{k}_1,-\boldsymbol{q}-\boldsymbol{k}_2) = \Phi_{J/\Psi}(\boldsymbol{k}_1,\boldsymbol{k}_2)\,\Phi_{J/\Psi}^*(\boldsymbol{q}-\boldsymbol{k}_1,-\boldsymbol{q}-\boldsymbol{k}_2)\,\delta^{(2)}(\boldsymbol{k}_1+\boldsymbol{k}_2-\boldsymbol{p}_T)$$

 The pointlike parton (q or g) impact factor: Mueller-Tang prescription generalized to two pomerons

$$\Phi_Q(k,q) \sim \operatorname{const}(k,q) \longrightarrow \langle E(q,n,\nu) | \Phi_Q \rangle = \Phi_{M-T}(q,n,\nu)$$

$$\Phi_Q^{2P}(\{k_i\},q) \sim \operatorname{const}(\{k_i\},q)$$

$$[\langle E(q,n,\nu)| \otimes \langle E(-q,n',\nu')|]\Phi_Q^{2P}\rangle \sim \Phi_{M-T}(q,n,\nu)\Phi_{M-T}(-q,n',\nu')$$

Solving the double non-forward BFKL exchange problem: numerical approach

- We also developed fully numerical approach to solve non-forward LL BFKL equation
- Integro-differential (w.r.t. rapidity Y) equation with two dimensional integral kernel
- Currently, in numerical approach we use an infrared cut-off s0 on gluon virtuality. Running coupling and other NLL BFKL effects not included yet
- The double pomeron exchange amplitude is obtained by numerical integration over the loop of the non-forward BFKL pomerons
- The semi-analytic and numerical approaches agree

Results: the lowest order

Analytic results known for diffractive amplitude [Ginzburg, Ivanov]

for
$$p_T \gg M$$
: $M_{diff} \sim \frac{C_1 e q_c g_s^4}{p_T^4} \log(p_T^2/M^2)(\varepsilon_V^* \varepsilon_\gamma)$

Diffractive cross section at high pT → note enhancement factor from polarizations

$$\frac{d\sigma_{diff}}{dp_T^2} \sim \sum_{\varepsilon_V, \varepsilon_\gamma} |M_{diff}|^2 \sim \frac{C_1^2 q_c^2 \alpha_{em} \alpha_s^4}{p_T^8} \log^2(p_T^2/M^2) \left(1 + \frac{p_T^2}{2M^2}\right)$$

 Suitable modification of coupling constants and color factor leads to the lowest order two pomeron cross section: notice hard scaling!

$$\frac{d\sigma_{2P}}{dp_T} \sim \frac{C_{2P}\alpha_s^5}{p_T^7}\log^2(p_T^2/M^2)\left(1 + \frac{p_T^2}{2M^2}\right) \sim \frac{1}{p_T^5}$$

Results: double BFKL amplitudes at parton level

Main features of the double BFKL pomeron amplitude at high pT >> M

- Dominance of low pomeron qT < M, qT << pT in the pomeron loop
- The pT-dependence even harder than for the lowest order amplitude due to the BFKL anomalous dimension ~ 1/2
- Energy dependence of two BFKL pomerons leading to steep rise
 of partonic cross section with partonic invariant mass squared ~ s^{0.6}

From partonic to hadronic level

It is straightforward to get the pp inclusive cross section from partonic cross sections

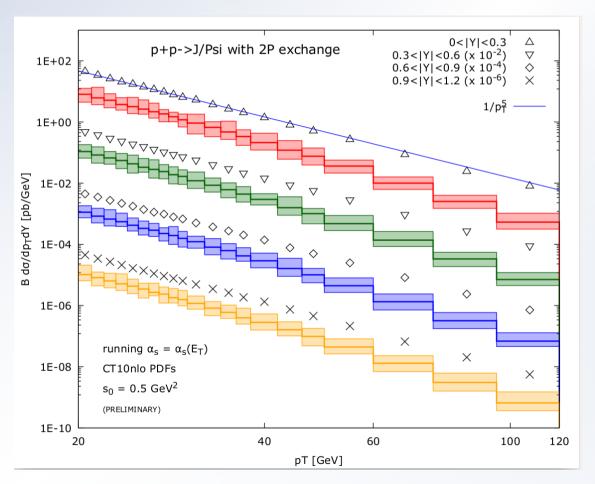
$$\frac{d\sigma(pp \to J/\psi X)}{dp_T dY} = \int dx_1 \int dx_2 \, \delta(Y - \log(x_1 \sqrt{S_{pp}}/E_T))$$

$$\times g(x_1, \mu) \left[C_q \sum_q q(x_2, \mu) + C_g g(x_2, \mu) \right] \frac{d\hat{\sigma}_0}{dp_T} (x_1 x_2 S_{pp})$$

- Non-trivial color coefficients C_q and Cg for quark and gluon partonic targets
- The default choice of parton and strong coupling constant is E_T

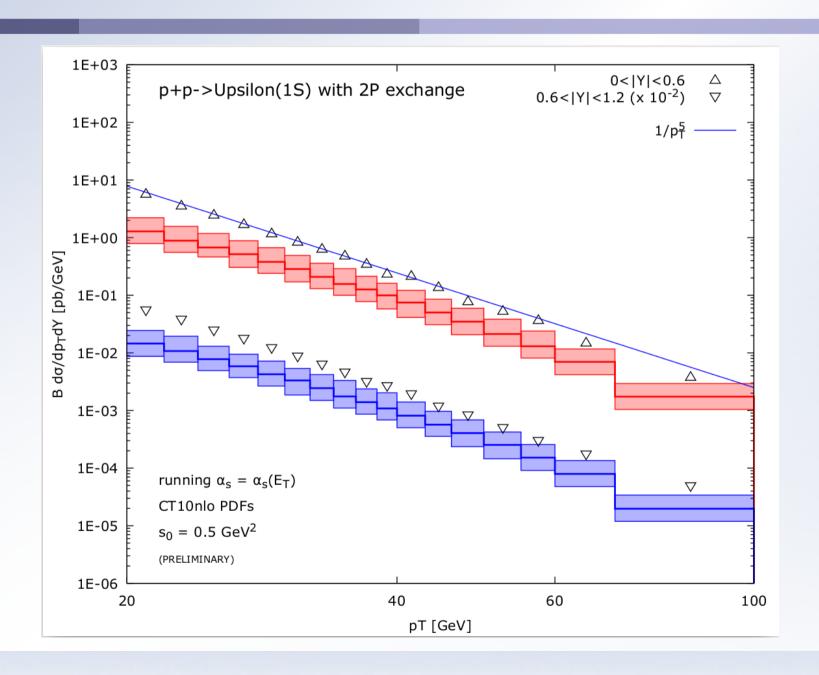
Comparison to data: J/ψ

Results compared to CMS data for prompt J/ψ at 13 TeV



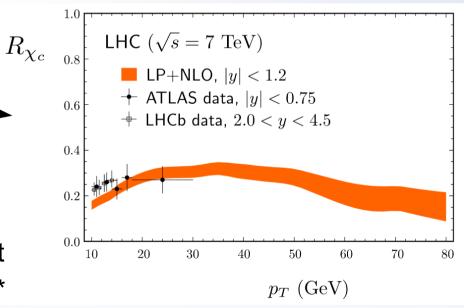
Dependence on pT well described, normalization 1/4 of prompt J/ψ, hence
 1/3 of direct J/ψ

Comparison to CMS data: Upsilon



Discussion

- Sizable contribution was found to inclusive heavy vector meson production
- The computation was done with natural / conservative choice of parameters: BFKL pomeron intercept ~0.27, infra-red cutoff s0 =0.5 GeV²
- We did not include subleading BKP states like the pair od d-reggeons
- We did not include possible enhancement factors coming from off-forward evolution of two pomerons (Shuvaev factor)²
- The striking feature of results is pT-hardness of the cross sections, consistent with data
- Recall also that relative feed-down contribution to prompt J/ψ is ~1/4
- At the NNLO order for color singlet there are also other possibly relevant
- Contributions found earlier → NNLO*



Conclusions

- We obtained for the first time the full solution for two non-forward BFKL pomeron amplitude, with the pomerons correlated by common origin at a point-like parton
- The amplitude was applied to estimate partonic cross-section for associated J/psi plus jet production with large rapidity separation.
 The BFKL pomeron loop was evaluated for this configuration
- Contribution to inclusive vector meson hadroproduction was found by integration over the jet phase space
- Conservative estimates give ~1/3 of the J/psi cross section at large pT, and about ~1/2 of the Upsilon cross section at large pT at the LHC, with correct pT dependence
- The question of how big are color singlet / contributions seems to be still open



Relation to Bartels triple pomeron vertex with double Pomeron cut

