Scaling function for exclusive production of vector mesons and DVCS in the saturarion scheme

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Diffraction and Low x - 2018, Reggio Calabria

Based on Physical Review D https://arxiv.org/abs/1701.01141 paper by F.G. Ben, M.V.T. Machado and W. Sauter

In this episode...

- **Scaling curve** for the cross section
- ...for exclusive production of **vector mesons** and Deeply Virtual Compton Scattering (**DVCS**) in γp and γA
- **Geometrical Scaling** phenomenon

SPOILER!

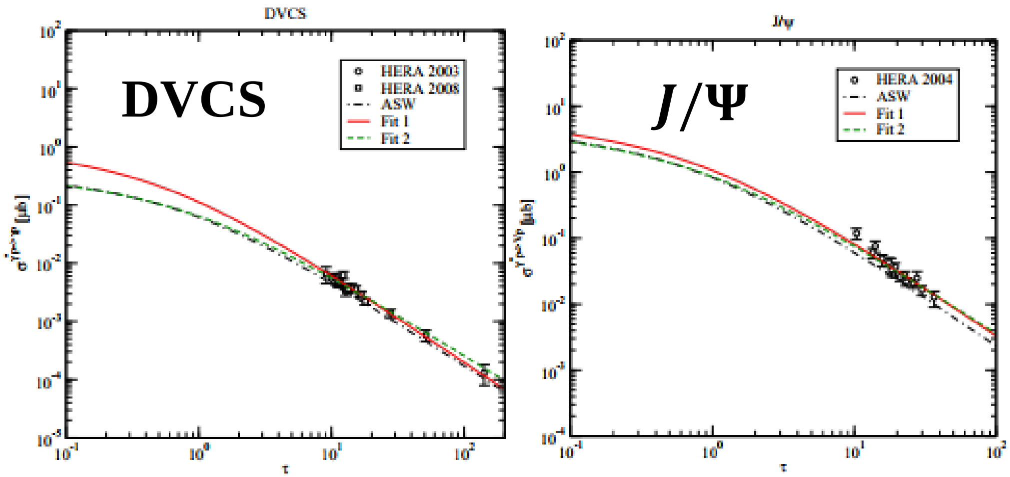


FIG. 1: The cross section for DVCS (left panel) [23] and J/ψ production (right panel) [26] as a function of the corresponding scaling variable τ. The ASW result is represented by a dot-dashed line, the Fit 1 by solid lines and Fit 2 by the dashed ones.

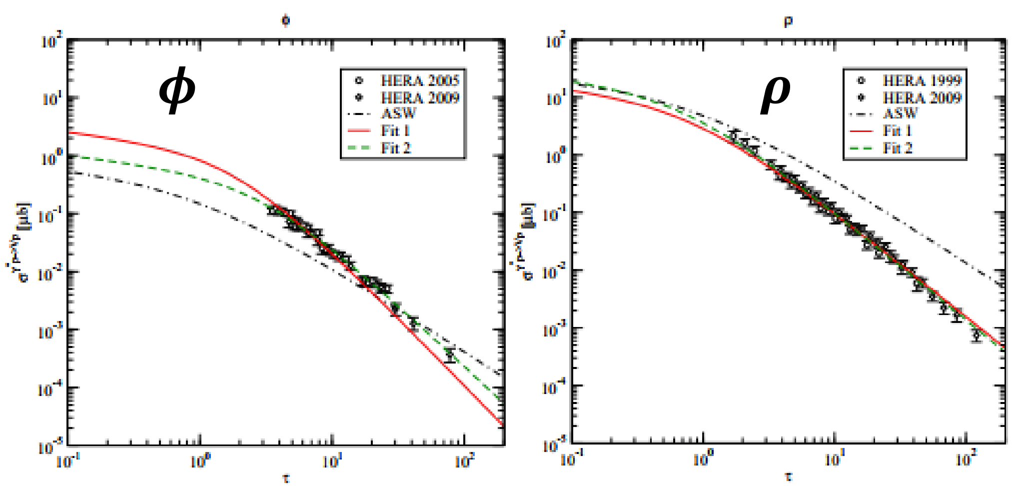
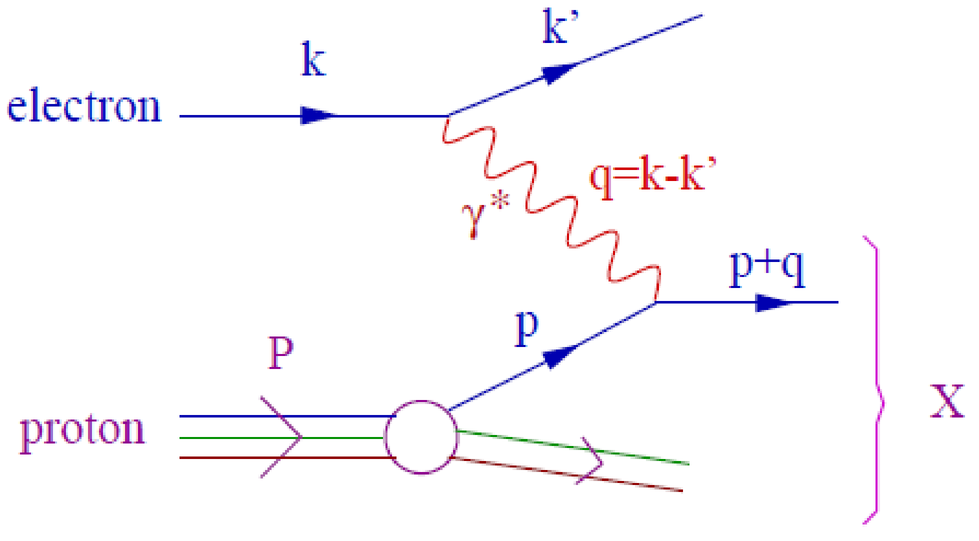


FIG. 2: The cross section for ϕ [25] (left panel) and ρ [24] production (right panel) as a function of the corresponding scaling variable τ. The ASW result is represented by a dot-dashed line, the Fit 1 by solid lines and Fit 2 by the dashed ones.

Small x physics

- Shall use Deep Inelastic Scattering (DIS) to motivate physical framework

electron (k) + proton (P) $\longrightarrow$ electron (k') + X (P_X)



- Useful kinematic variables (2 independent ones):

$$Q^2 \equiv -q^2 \geq 0, \qquad s \equiv (P+q)^2, \qquad x \equiv \frac{Q^2}{2(P\cdot q)} \approx \frac{Q^2}{s+Q^2}$$

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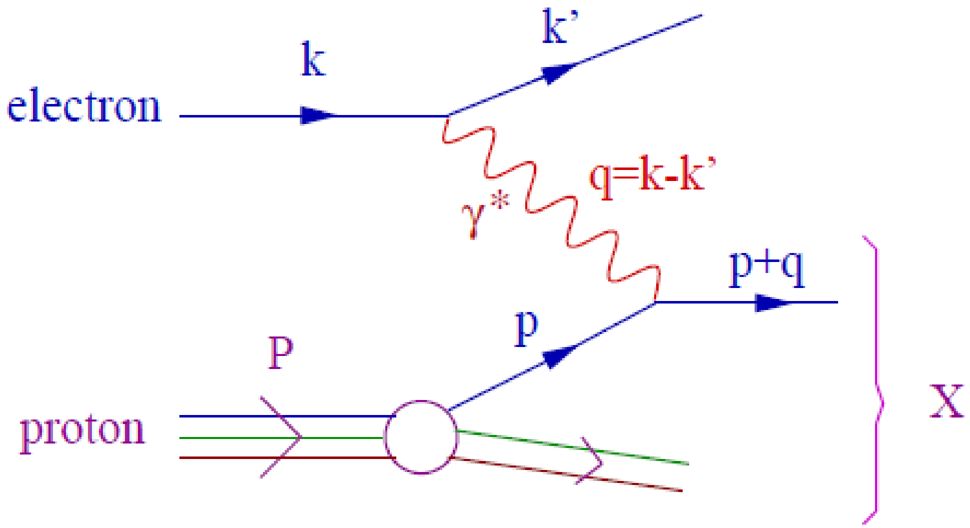

- High energy: $s \gg Q^2 \leftrightarrow small\ x \approx \frac{Q^2}{s} \ll 1$

$$Q^2 \equiv -q^2 \geq 0, \qquad s \equiv (P+q)^2, \qquad x \equiv \frac{Q^2}{2(P \cdot q)} \approx \frac{Q^2}{s+Q^2}$$

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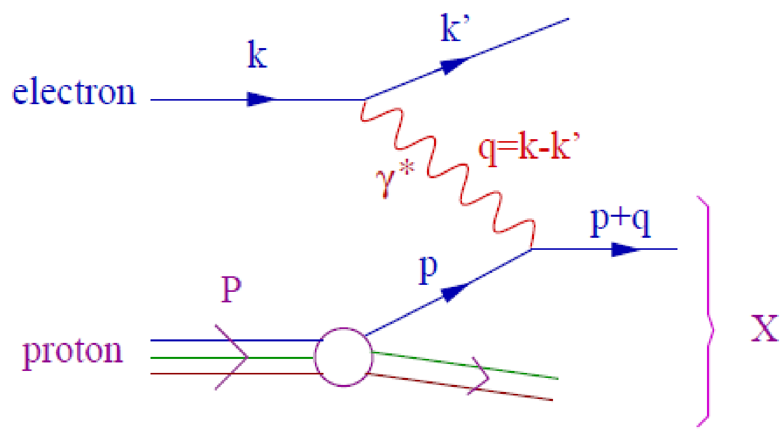
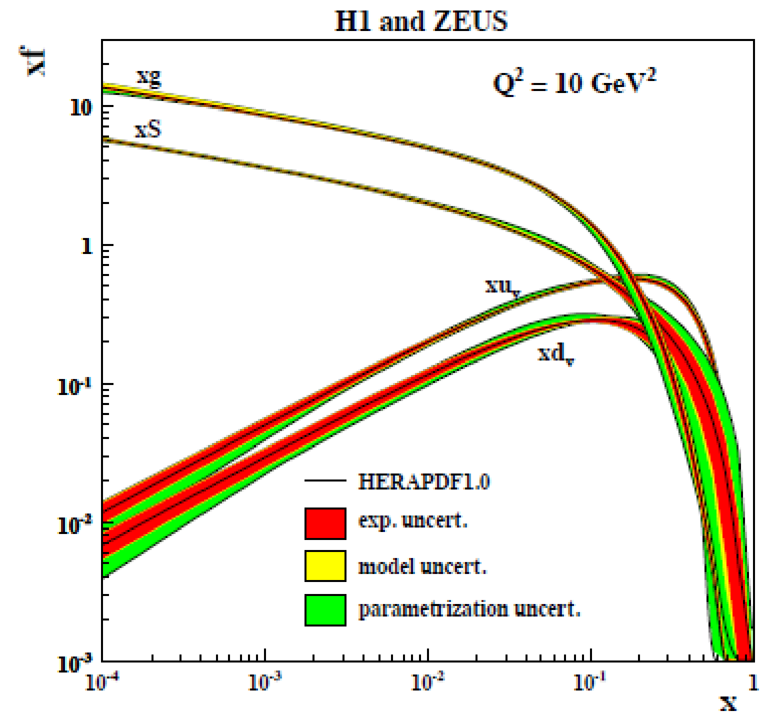
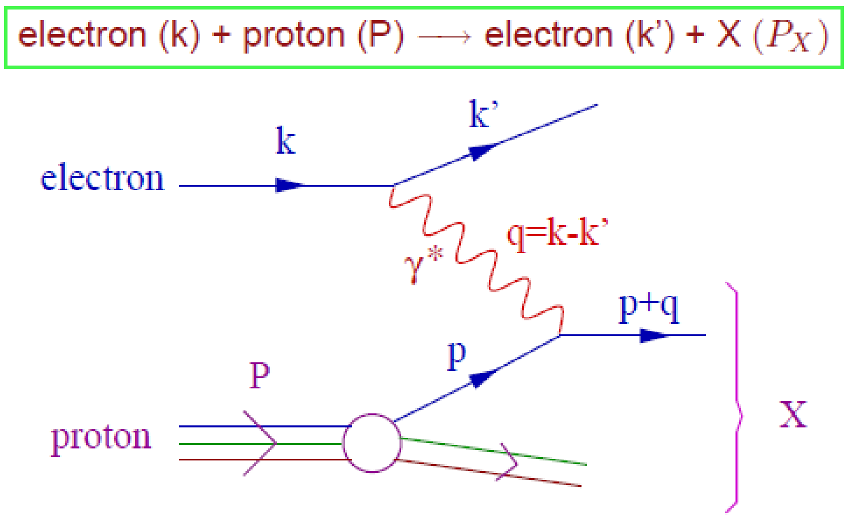


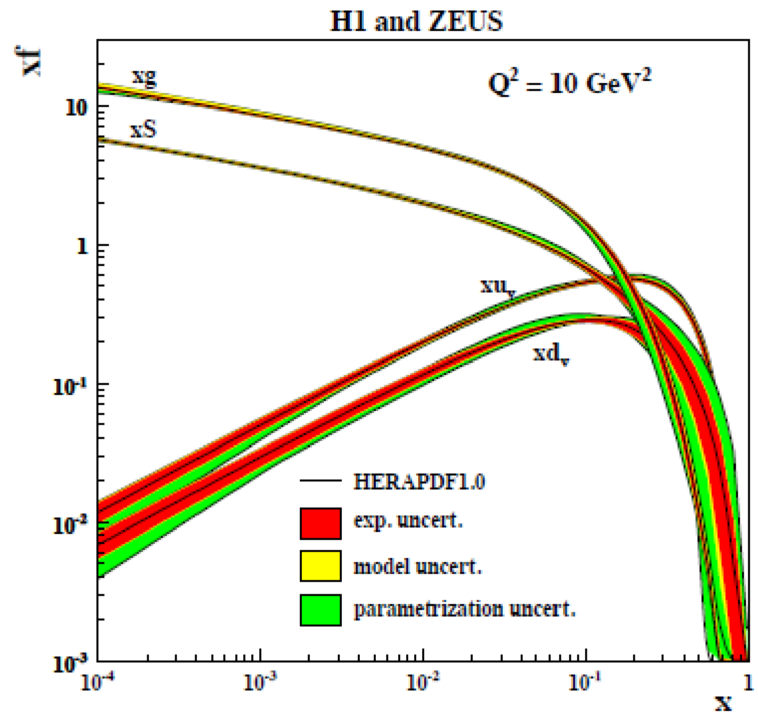


- Parton distribution functions $xq(x, Q^2)$ and $xG(x, Q^2)$

➢ Number of partons (quarks, gluons) with transverse area $A \sim \frac{1}{Q^2}$ and $p_z = xP_z$

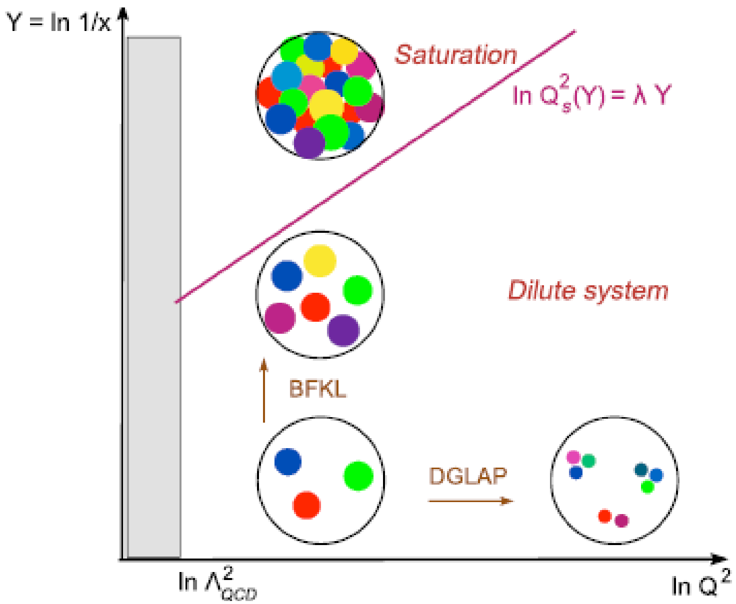
Small x physics

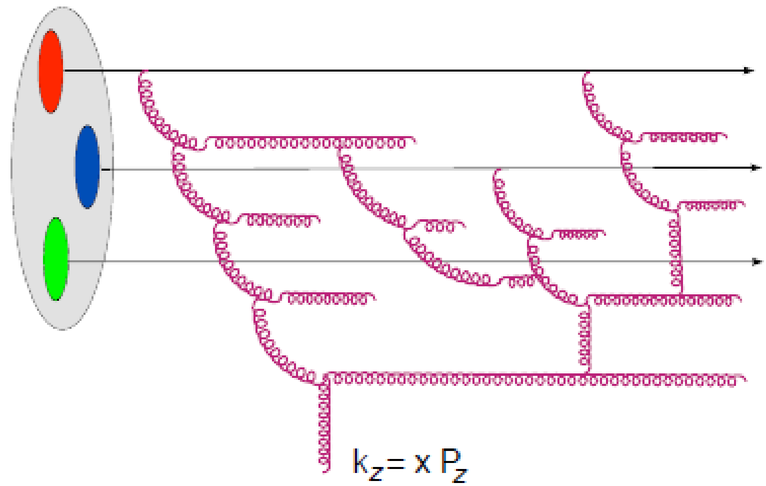




- At small x hadron wavefunction is mostly gluons
- Partonic content changes when changing x or Q^2 due to additional radiation

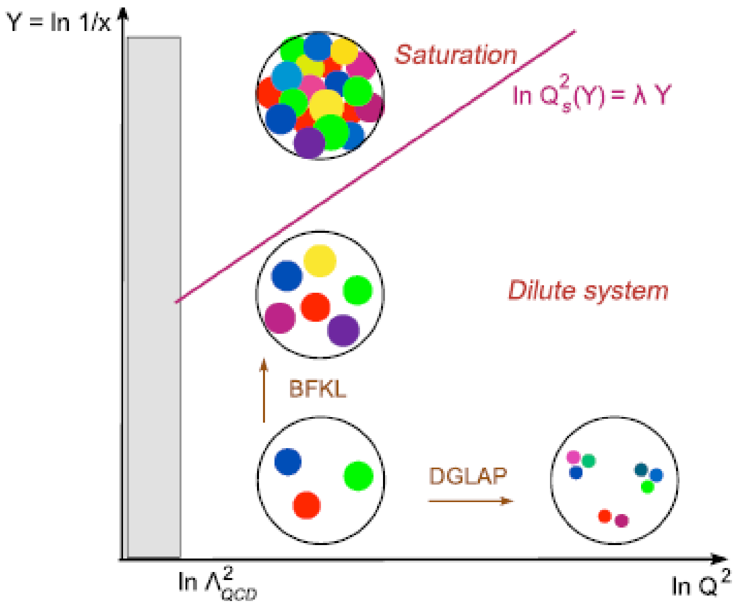
Parton Evolution

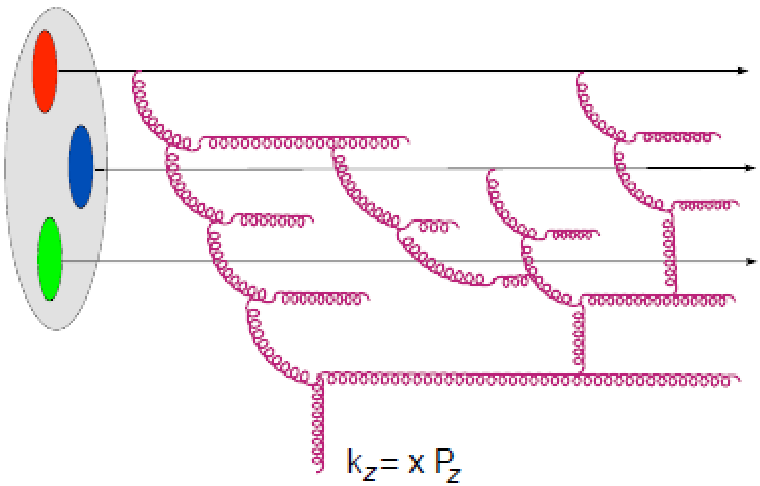




- Gluon occupation number $n(x, Q^2) = xg(x, Q^2) \times \frac{1}{Q^2} \times \frac{1}{R^2}$

Parton Evolution





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- Small x evolution (fixed Q, increasing s):

➢ There's an increasing number of gluons that have similar transverse area $A \sim \frac{1}{Q^2}$

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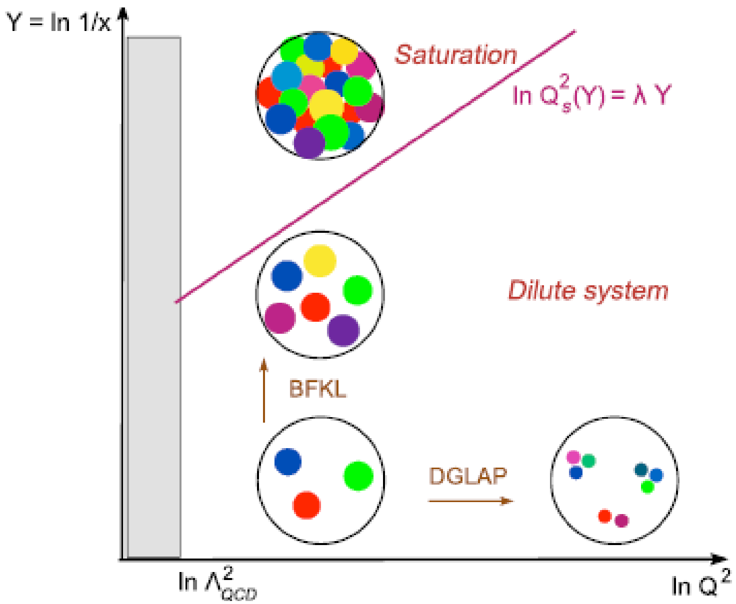

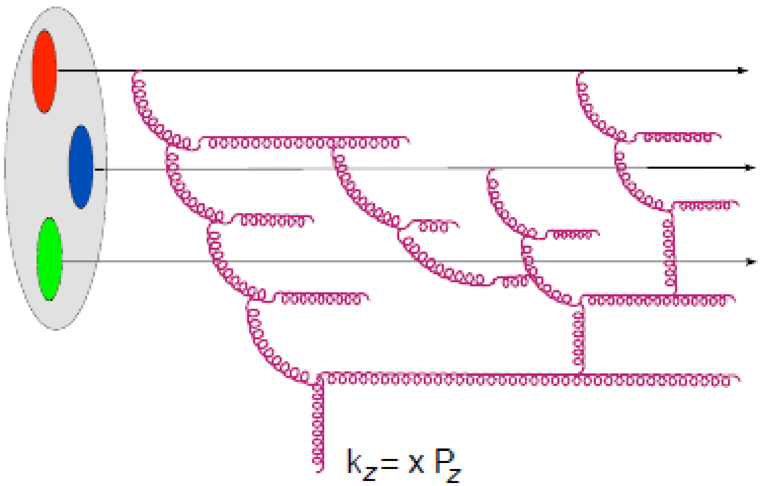

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- Small x evolution (fixed Q, increasing s):

➢ There's an increasing number of gluons that have similar transverse area $A \sim \frac{1}{Q^2} \rightarrow$ overlap leads to nonlinear recombination and saturation

Geometrical Scaling

- Stasto, et. al (2001)

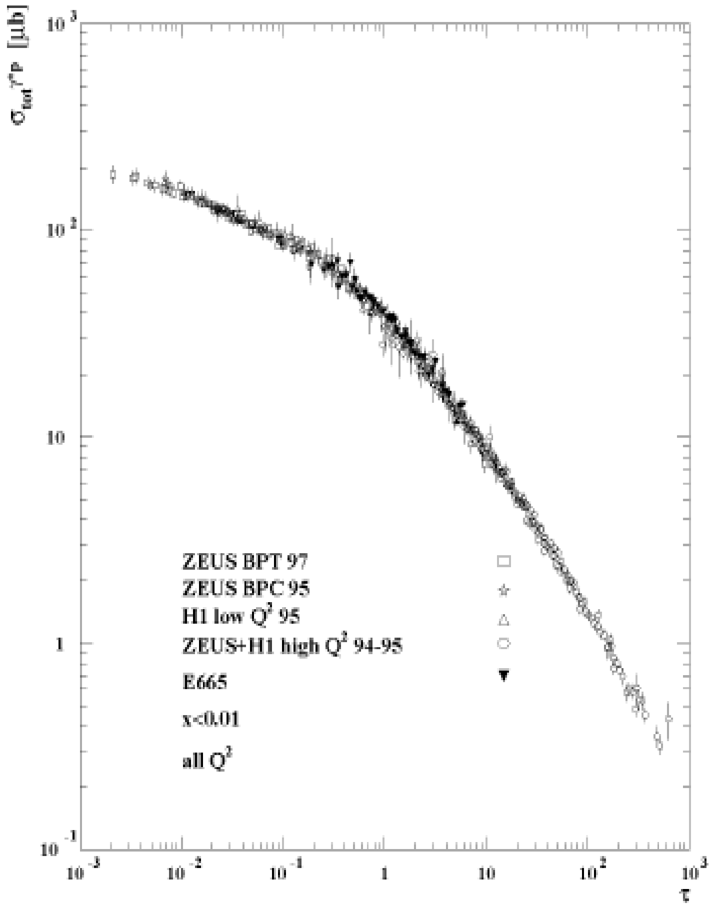
Saturation → Inclusive ep cross section is a function of a single variable τ

$$\sigma_{\gamma^* p}(x, Q^2) = \sigma_{\gamma^* p}(\tau)$$

Where $\tau = Q^2/Q_s^2(x)$

And $Q_s(x) = Q_o x^{-\lambda}$ defines the saturation scale

- HERA confirmation:
  $x < 0.01\ and\ Q^2 \leq 400 GeV^2$

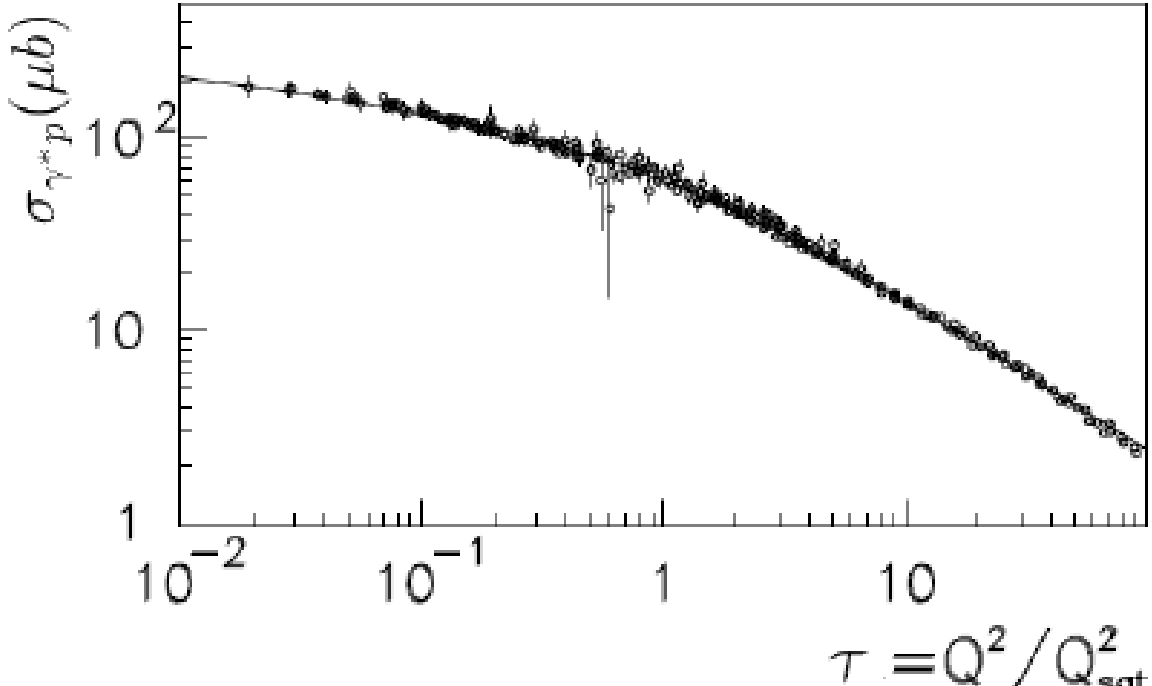


Inclusive Scaling Curve

Armesto, et. al (2009): data are well described by

$$\sigma_{\gamma^* p}(\tau) = \overline{\sigma_0}[\gamma_E + \Gamma(0,\xi) + \ln \xi]$$

where $\xi = a/\tau^b$, with $a = 1.868, b = 0.746$ and $\overline{\sigma_0} = 40.56\ \mu b$ (data fit)



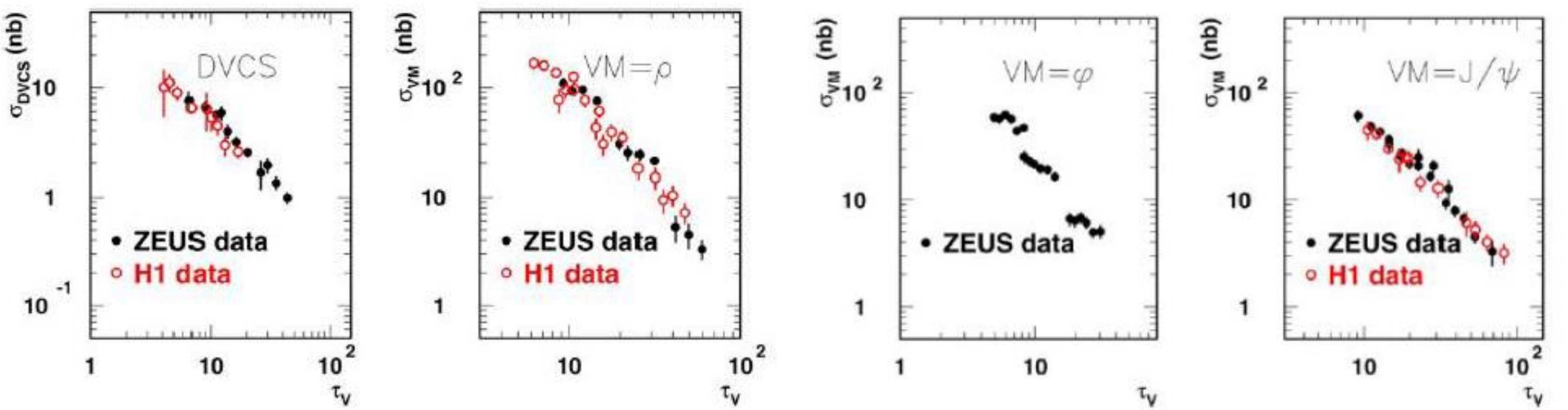
Vector Meson Production

Marquet, et. al (2006) → geometrical scaling in vector meson production and DVCS

For vector meson production

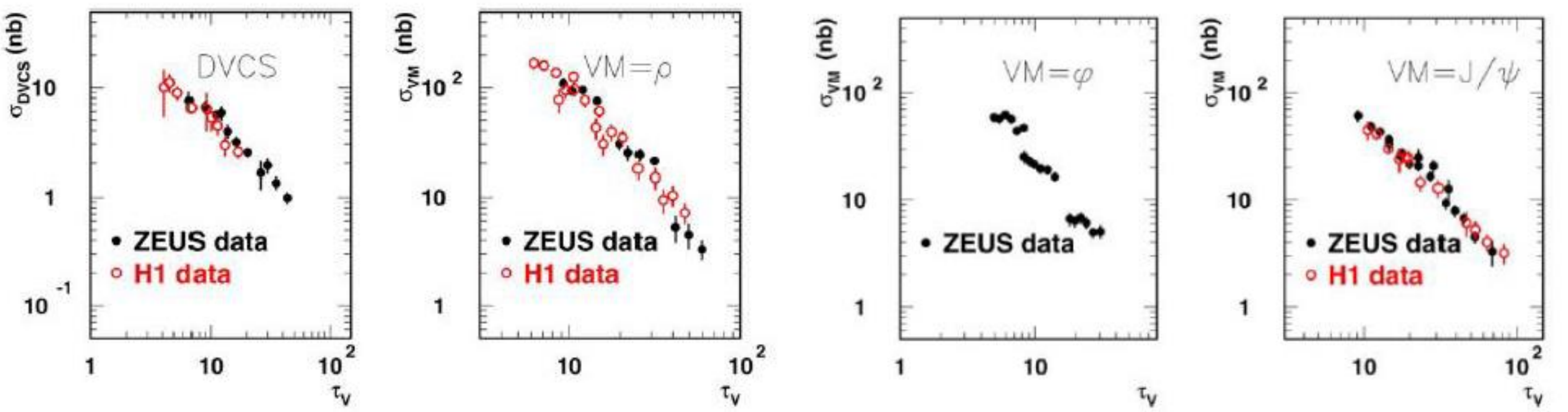
$$\sigma_{\gamma^* p \to Vp}(x, Q^2, M_V^2) = \sigma_{\gamma^* p \to Vp}(\tau_V)$$

where $\tau_V = Q^2/Q_S^2(x_P)$ e $x_P = \frac{Q^2+M_V^2}{Q^2+W^2}$



In this episode...

A scaling curve expression that describes DVCS and vector meson cross sections



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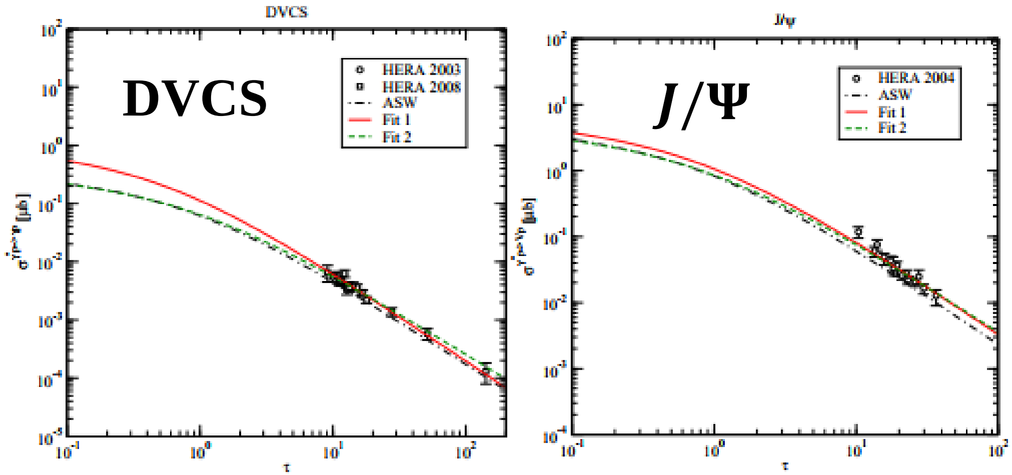

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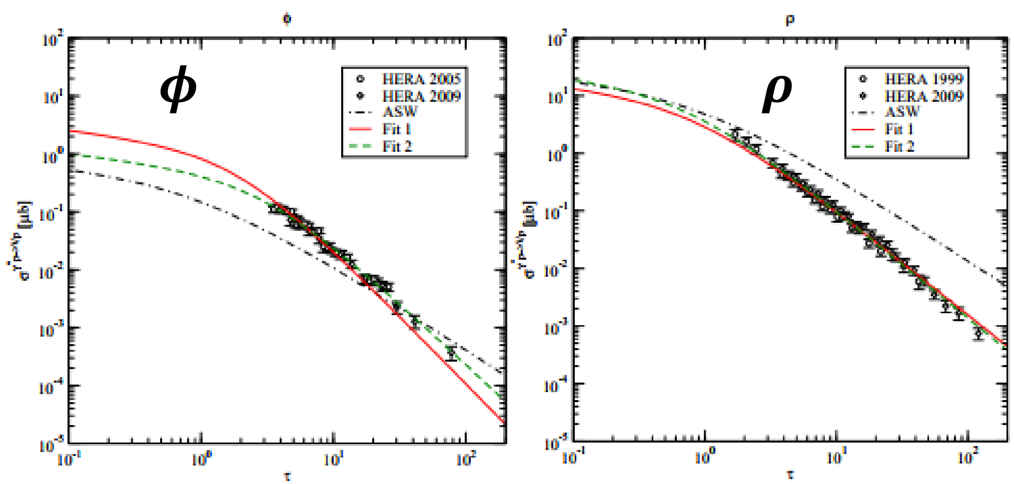

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Exclusive Cross Section

- Armesto et. al → expression for $\gamma^* p \to X$ **(total cross section)**

$$\sigma_{\gamma^* p}(\tau) = \overline{\sigma_0}[\gamma_E + \Gamma(0,\xi) + \ln \xi]$$

- Similar expression for $\gamma^* p \to \gamma^* p$ **(elastic cross section)** comes from Optical Theorem + Eikonal approach

$$\sigma_{el}^{\gamma^* p \to \gamma^* p} = \frac{\overline{\sigma_0}}{2}[\ln\left(\frac{\xi}{2}\right) + \gamma_E - \Gamma(0,2\xi) + 2\Gamma(0,\xi)]$$

- **WANT:** $\gamma^* p \to Vp$ **(exclusive vector meson cross section)**

Exclusive Cross Section

- Armesto et. al → expression for $\gamma^* p \to X$ **(total cross section)**

$$\sigma_{\gamma^* p}(\tau) = \overline{\sigma_0}[\gamma_E + \Gamma(0,\xi) + \ln \xi]$$

- Eikonal approach → $a_{el}(s,b) = i(1 - e^{-\frac{\Omega(s,b)}{2}})$, where $\Omega(s,b) = \nu(s) e^{-\frac{b^2}{R^2}}$

Optical theorem leads to

$$\sigma_{tot} = 2\int Im\, a_{el}(s,b) d^2b = 2\pi R^2 \left[\gamma_E + \ln\left(\frac{\nu}{2}\right) + \Gamma\left(0,\frac{\nu}{2}\right)\right]$$

Similar to $\sigma_{\gamma^* p}(\tau) = \overline{\sigma_0}[\gamma_E + \Gamma(0,\xi) + \ln \xi]$, where $2\pi R \to \overline{\sigma_0}$ e $\nu/2 \to \xi$

In those variables

$$\sigma_{el}^{\gamma^* p \to \gamma^* p} = \int |a_{el}(s,b)|^2 d^2b = \frac{\overline{\sigma_0}}{2}[\ln\left(\frac{\xi}{2}\right) + \gamma_E - \Gamma(0,2\xi) + 2\Gamma(0,\xi)]$$

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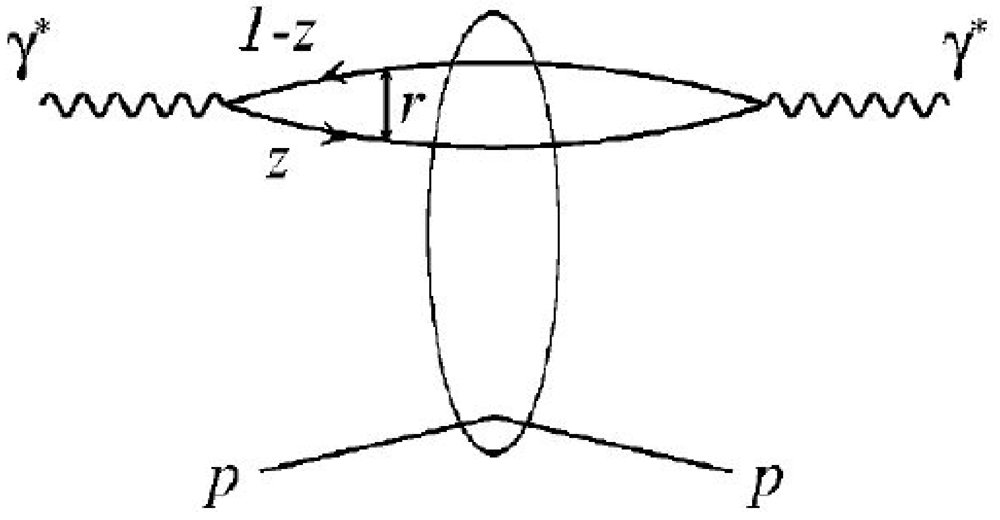
- Will treat $\boldsymbol{\gamma^* p \to V p}$ as **quasielastic** and modify elastic cross section $\gamma^* p \to \gamma^* p$ to obtain expression for vector meson

Quasielastic cross section



Quasielastic cross section

- In the dipole framework one can write

$$\sigma_{\gamma^* p} \propto \int dz \int d^2 r \psi^*(z,r,Q^2)\hat{\sigma}(x,r^2)\psi(z,r,Q)$$

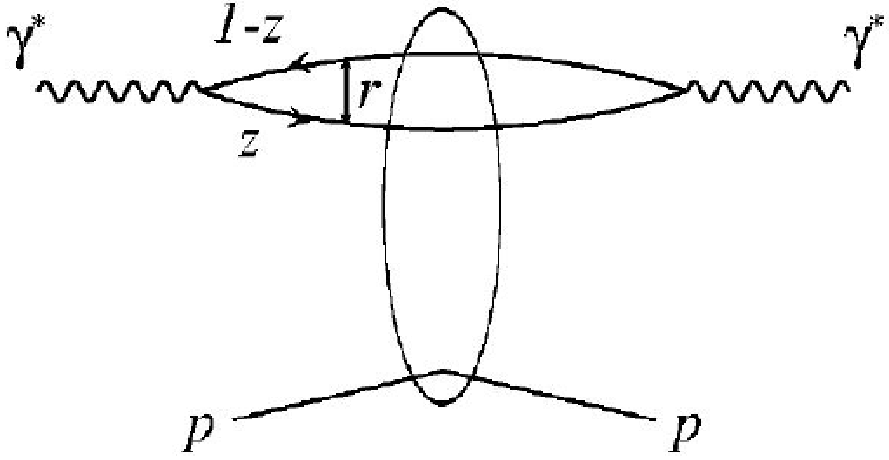
$$\frac{d\sigma_{VM}}{dt} = \frac{1}{16\pi}\left|\int dz \int d^2 r \psi_V^*(z,r,Q^2)\hat{\sigma}\left(x_p,r^2\right)\psi(z,r,Q)\right|^2$$

Vector meson wavefunction ψ_V is model dependent! (Motyka, et. al)

- In essence:

$$\sigma_{\gamma^* p} \propto \psi^*\psi \propto \alpha_{EM}\sum_F e_f^2$$

$$\sigma_{VM} \propto |\psi_V^*\psi|^2 \propto 4\pi\alpha_{EM}\hat{e}_f^2\frac{f_V^2}{M_V^2}$$



Quasielastic cross section

Therefore

$$\frac{\sigma_{VM}}{\sigma_{\gamma^* p}} \propto \frac{\overline{\sigma_V}}{\overline{\sigma_0}} = \frac{4\pi \widehat{e_f}^{\,2} f_V^2}{\sum e_f^2 M_V^2}$$

$\overline{\sigma_0} \rightarrow \overline{\sigma_V}$ will be assumed to incorporate $\psi \rightarrow \psi_V$

Quasielastic cross section

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$$\sigma_{VM}^{\gamma^* p \rightarrow V p} = \frac{\overline{\sigma_V}}{2} [\ln\left(\frac{\xi}{2}\right) + \gamma_E - \Gamma(0,2\xi) + 2\Gamma(0,\xi)]$$

where $\xi = a/\tau^b$, with $a = 1.868, b = 0.746$ and $\overline{\sigma_0} = 40.56\ \mu b$ (fit values by Armesto et. al)

Results and Fitting

	a	b	$\bar{\sigma}_0(\mu\text{b})$	χ^2/d.o.f.
DVCS				
ASW	1.868	0.746	40.56	3.248
Fit 1	1.313	0.769	114.610	0.768
Fit 2	1.938	0.710	40.56	0.754
J/ψ				
ASW	1.868	0.746	40.56	4.567
Fit 1	1.851	0.733	52.524	1.083
Fit 2	1.919	0.704	40.56	1.183
ϕ				
ASW	1.868	0.746	40.56	21.706
Fit 1	1.936	0.750	72.717	8.843
Fit 2	2.061	0.695	40.56	14.419
ρ				
ASW	1.868	0.746	40.56	529.004
Fit 1	1.684	0.916	27.333	1.266
Fit 2	1.467	0.943	40.56	1.011

TABLE II: Summary of fitting procedure. ASW is the result using the original parameters from the fit to *ep* HERA data [9]. Fit 1 adjusts parameters a, b and normalization $\bar{\sigma}_0$. Fit 2 adjusts a and b keeping fixed $\bar{\sigma}_0 = 40.56\,\mu$b (as for the inclusive case).

Results and Fitting

Original parameters from Armesto et. al fit for ep data for total cross section

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Fit 1 ajusts a, b and σ_0

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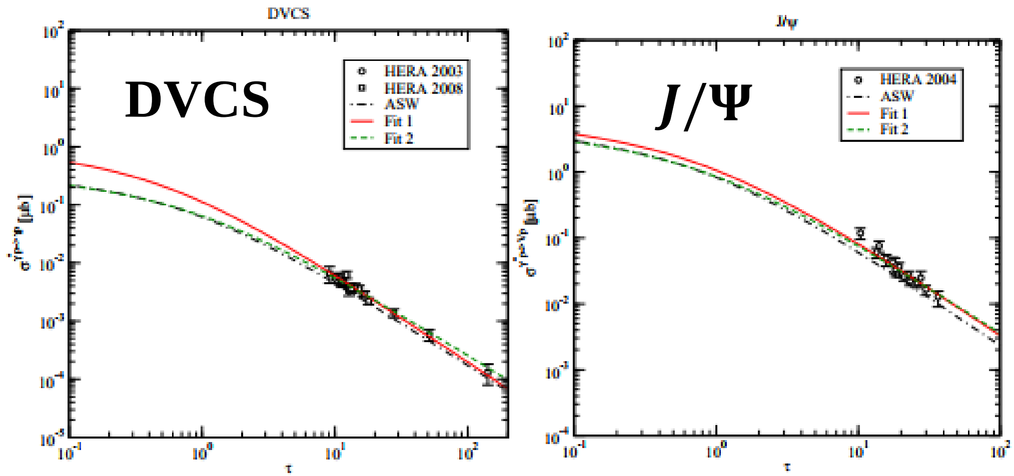

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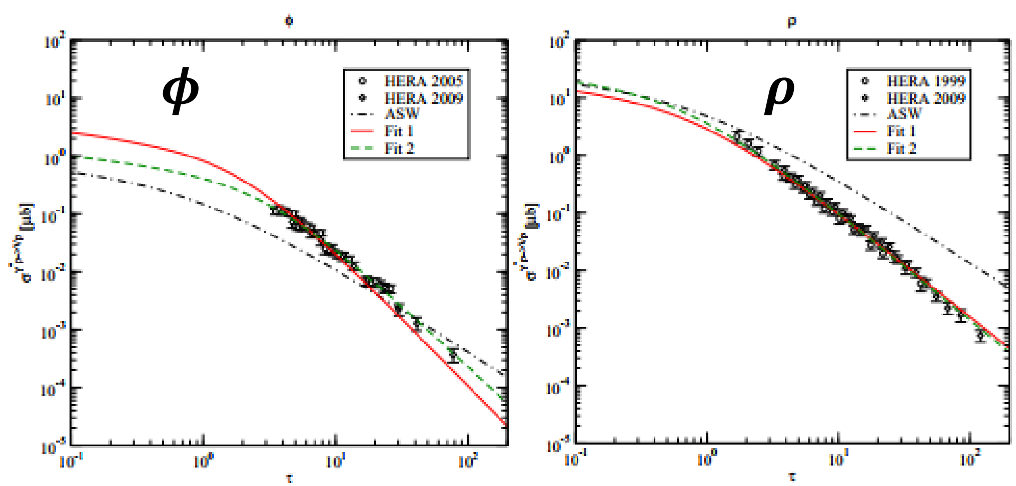

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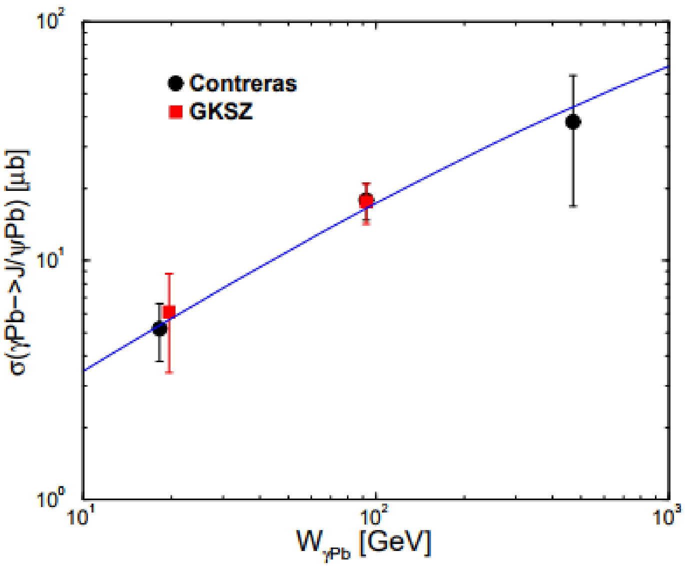
Nuclear targets

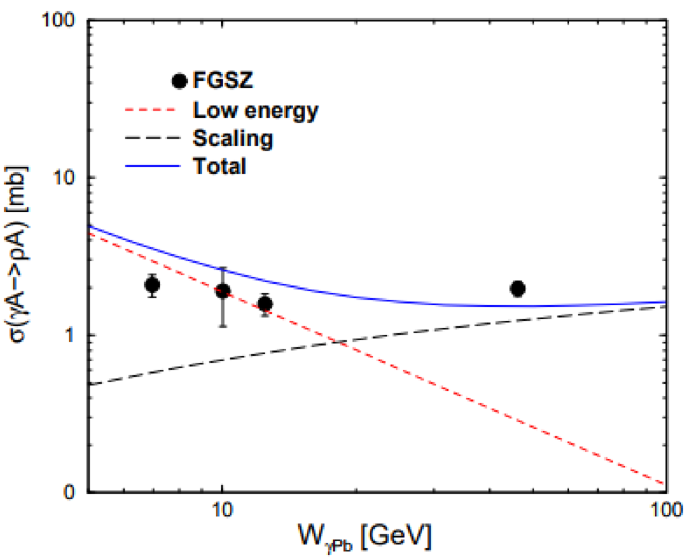
Geometrical scaling translates to nuclear targets

$$\sigma^{\gamma^* A \to EA}(\tau_A) = \frac{\pi R_A^2}{\pi R_P^2} \sigma^{\gamma^* p \to Ep}(\tau = \tau_A)$$

Where $\tau_A = \tau_P \left[\frac{\pi R_A^2}{A \pi R_P^2}\right]^\Delta$

Parameters from Fit 2 for γp data





Summary

- Geometrical scaling and simple considerations on the scope of the eikonal model
- Scaling curve derived for the first time for the exclusive case, generalizing the curve found for the inclusive case
- Able to describe available data for vector meson production and DVCS with a universal scaling function without any further parameter
- Parametrization can be extrapolated to nuclear targets to be tested in future

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Thank you!

Felipe G. Ben
felipeben@gmail.com

Post credit F.A.Q. backup!

What about diffraction?

$\sigma_{DIFF}^{\gamma^* p \to Xp}$ data also suggests scaling behaviour $\sigma_D(x_P, Q^2) = \sigma_D(\tau_d)$

Ask me about it on the next conference!