Multi particle production in proton-nucleus collisions at high-energy

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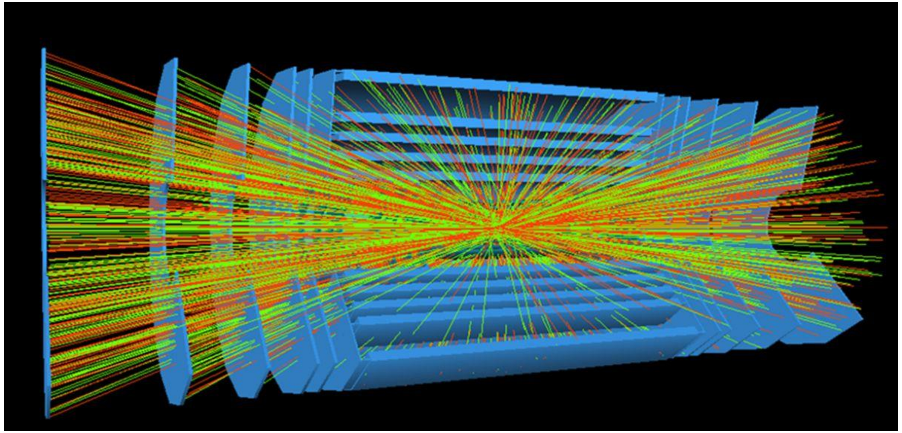
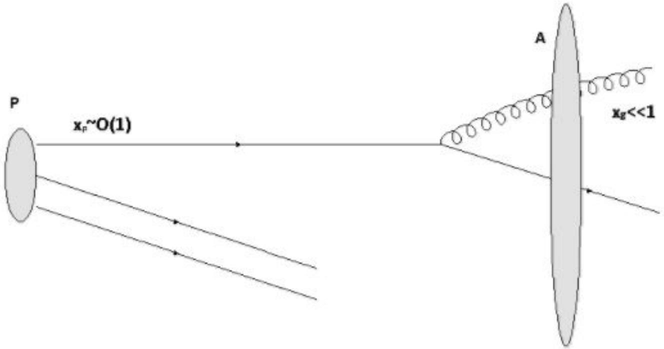
Together with E. Iancu, To appear soon



Leading Order Forward Dijet Cross Section

By using the formalism of the light-cone wave function in perturbative QCD, together with the hybrid factorization, the first derivation of the forward LO dijet cross-section appeared in hep-ph/0708.0231 (C. Marquet).

The basic setup: a large-x parton from the proton scatters off the small-x gluon distribution in the target nucleus. Large-x parton is most likely a quark.



Quark fragmentation in the presence of a shockwave.

The time evolution of the initial (bare) quark state is given by:

$$\left|q^{\alpha}_{\lambda}(q^{+},\boldsymbol{q})\right\rangle_{\mathrm{in}} \equiv U(0,-\infty)\left|q^{\alpha}_{\lambda}(q^{+},\boldsymbol{q})\right\rangle$$

Where U denotes a unitary operator:

$$U(t,t_0) = \mathrm{T}\exp\left\{-i\int_{t_0}^{t} dt_1\, H_I(t_1)\right\}$$

The information both on the time evolution and interaction of the bare quark with the target nucleus is given by the “outgoing state”:

$$\left|q^{\alpha}_{\lambda}(q^{+},\boldsymbol{w})\right\rangle^{(g)}_{out} \equiv U(\infty,0)\,\hat{S}\,U(0,-\infty)\left|q^{\alpha}_{\lambda}(q^{+},\boldsymbol{w})\right\rangle$$

This state will be shown to generate all the possible insertions of the shockwave. More importantly, the outgoing state is directly related to expectation values:

$$\left\langle \hat{\mathcal{O}}\right\rangle = \left\langle \langle q|\, U^{\dagger}\,\hat{S}\,U\,\hat{\mathcal{O}}\,U^{\dagger}\,\hat{S}\,U\,|q\rangle\right\rangle_{cgc}$$

The LO Outgoing State

The production state at leading order is given by

$$\left|q^{\alpha}_{\lambda}(q^{+},\,\boldsymbol{w})\right\rangle^{(g)}_{out} \equiv U(\infty,\,0)\,\hat{S}\,U(0,\,-\infty)\,\left|q^{\alpha}_{\lambda}(q^{+},\,\boldsymbol{w})\right\rangle$$

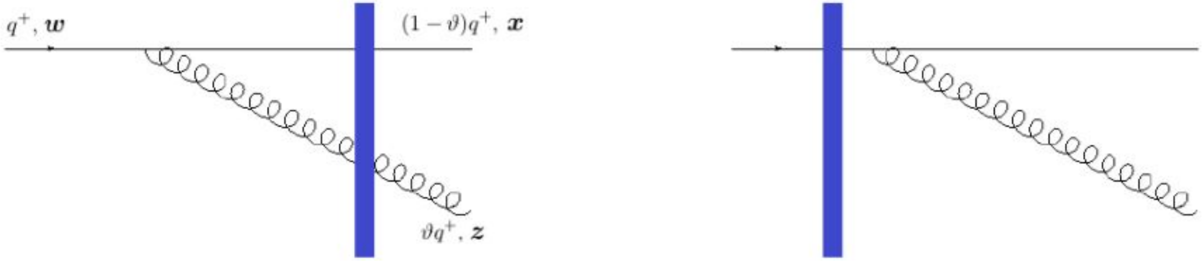
$$= \left|q^{\gamma}g^{b}\right\rangle\left(-\left\langle q^{\gamma}g^{b}\right|\hat{S}\left|q^{\beta}g^{a}\right\rangle\frac{\left\langle q^{\beta}g^{a}\right|H_{q\to qg}\left|q^{\alpha}\right\rangle}{E_{qg}-E_{q}}+\frac{\left\langle q^{\gamma}g^{b}\right|H_{q\to qg}\left|q^{\beta}\right\rangle}{E_{qg}-E_{q}}\left\langle q^{\beta}\right|\hat{S}\left|q^{\alpha}\right\rangle\right)+\left|q^{\alpha}\right\rangle$$

Where only terms of order g were kept. The following result is obtained for the |qg> contribution:

$$\left|\psi^{\alpha}_{\lambda}(q^{+},\,\boldsymbol{w})\right\rangle_{qg} = \int_{\boldsymbol{x},\,\boldsymbol{z}}\int_{0}^{1}d\vartheta\,\frac{ig\phi^{ij}_{\lambda_{1}\lambda}(\vartheta)\sqrt{q^{+}}\,\boldsymbol{X}^{j}}{4\pi^{3/2}\sqrt{\vartheta}\,\boldsymbol{X}^{2}}\,\delta^{(2)}(\boldsymbol{w}-(1-\vartheta)\boldsymbol{x}-\vartheta\boldsymbol{z})$$

$$\times\left[V^{\gamma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{z})\,t^{a}_{\beta\alpha}-t^{b}_{\gamma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]\left|q^{\gamma}_{\lambda_{1}}((1-\vartheta)q^{+},\,\boldsymbol{x})\,g^{b}_{i}(\vartheta q^{+},\,\boldsymbol{z})\right\rangle$$

Diagrammatically:



One gluon production at leading order with shockwave before and after the emission.

The leading-order dijet cross-section

From the production state we can pass easily to the quark-gluon dijet cross section:

$$\frac{d\sigma_{\rm LO}^{qA\to qg+X}}{d^3k\,d^3p} \equiv \frac{1}{2N_c\,L}\,{}^{(g)}_{out}\langle q_\lambda^\alpha(q^+,\boldsymbol{q})|\,\hat{\mathcal{N}}_q(p)\,\hat{\mathcal{N}}_g(k)\,|q_\lambda^\alpha(q^+,\boldsymbol{q})\rangle_{out}^{(g)}$$
$$= \frac{1}{2N_c\,L}\int_{\boldsymbol{w},\overline{\boldsymbol{w}}} e^{i(\boldsymbol{w}-\overline{\boldsymbol{w}})\cdot\boldsymbol{q}}\,{}_{qg}\langle\psi_\lambda^\alpha(q^+,\overline{\boldsymbol{w}})|\,\hat{\mathcal{N}}_q(p)\,\hat{\mathcal{N}}_g(k)\,|\psi_\lambda^\alpha(q^+,\boldsymbol{w})\rangle_{qg}$$

The following number density operators were introduced:

$$\hat{\mathcal{N}}_q(p) \equiv \frac{1}{(2\pi)^3}\,b_\lambda^{\alpha\dagger}(p)\,b_\lambda^\alpha(p) \qquad\qquad \hat{\mathcal{N}}_g(k) \equiv \frac{1}{(2\pi)^3}\,a_i^{a\dagger}(k)\,a_i^a(k)$$

Then the result for the leading-order dijet cross section is given by:

$$\frac{d\sigma_{\rm LO}^{q\to qgX}}{dk^+\,d^2\boldsymbol{k}\,dp^+\,d^2\boldsymbol{p}} = \frac{\alpha_s\,C_F\,N_c\,\left(1+(1-\vartheta)^2\right)}{4\pi^3\vartheta(q^+)^3}\,\delta(q^+-k^+-p^+)$$
$$\times\int_{\boldsymbol{x},\overline{\boldsymbol{x}},\boldsymbol{z},\overline{\boldsymbol{z}}}\frac{\boldsymbol{X}\cdot\overline{\boldsymbol{X}}}{\boldsymbol{X}^2\,\overline{\boldsymbol{X}}^2}\,e^{-i\boldsymbol{p}\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{k}\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})}$$
$$\times\left[S^{(1)}_{q\bar{q}gg}(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\boldsymbol{x},\,\boldsymbol{z}) - S_{q\bar{q}g}(\overline{\boldsymbol{w}},\,\boldsymbol{x},\,\boldsymbol{z}) - S_{q\bar{q}g}(\overline{\boldsymbol{x}},\,\boldsymbol{w},\,\overline{\boldsymbol{z}}) + \mathcal{S}(\overline{\boldsymbol{w}},\,\boldsymbol{w})\right]$$

with $\boldsymbol{X}\equiv\boldsymbol{x}-\boldsymbol{z}$, $\overline{\boldsymbol{X}}\equiv\overline{\boldsymbol{x}}-\overline{\boldsymbol{z}}$, $\boldsymbol{w}=(1-\vartheta)\boldsymbol{x}+\vartheta\boldsymbol{z}$ and $\overline{\boldsymbol{w}}=(1-\vartheta)\overline{\boldsymbol{x}}+\vartheta\overline{\boldsymbol{z}}$.

Where the following combinations of Wilson lines were introduced (in the large Nc limit these combinations represent the quadropole-dipole and dipole-dipole interactions):

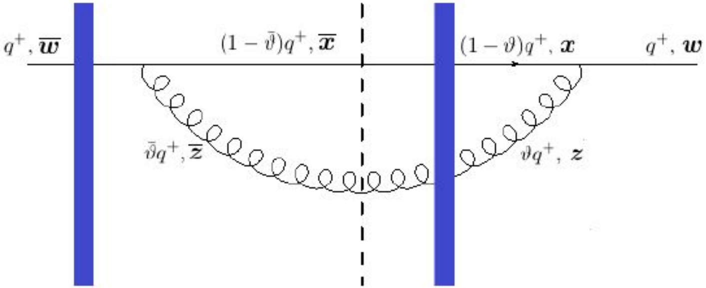
$$S^{(1)}_{q\bar{q}gg}\left(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \boldsymbol{x},\, \boldsymbol{z}\right) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^{\dagger}(\overline{\boldsymbol{x}})\, V(\boldsymbol{x})\, t^a\, t^c\right)\left[U^{\dagger}(\overline{\boldsymbol{z}})\, U(\boldsymbol{z})\right]^{ca}$$

$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z},\, \overline{\boldsymbol{z}})\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z}) - \mathcal{S}(\overline{\boldsymbol{x}},\, \boldsymbol{x})\right) \simeq \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z},\, \overline{\boldsymbol{z}})\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z})$$

$$S_{q\bar{q}g}\left(\overline{\boldsymbol{w}},\, \boldsymbol{x},\, \boldsymbol{z}\right) \equiv \frac{1}{C_F N_c}\,\mathrm{tr}\left(V^{\dagger}(\overline{\boldsymbol{w}})\, t^b\, V(\boldsymbol{x})\, t^a\right) U^{ba}(\boldsymbol{z})$$

$$= \frac{1}{2C_F N_c}\left(N_c^2\, \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z},\, \boldsymbol{x}) - \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{x})\right) \simeq \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z},\, \boldsymbol{x})$$

In total there are four different insertions of Wilson lines (each diagram corresponds to a different term in the rectangular brackets). For example, below is the relevant diagram which corresponds to $S_{q\bar{q}g}\left(\overline{\boldsymbol{w}},\, \boldsymbol{x},\, \boldsymbol{z}\right)$



$$\mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^{\dagger}(\overline{\boldsymbol{w}})\, V(\boldsymbol{w})\right]$$

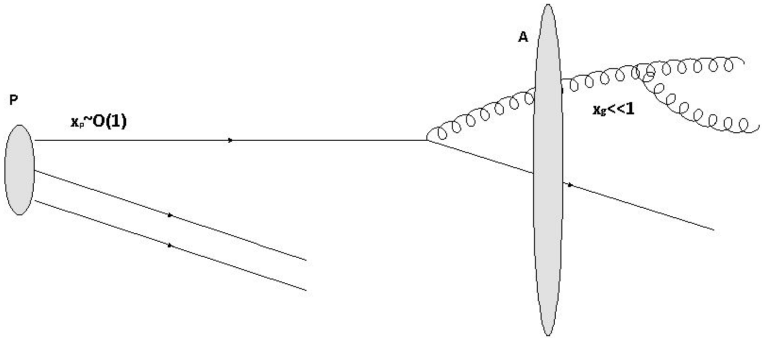
$$\mathcal{Q}\left(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z},\, \overline{\boldsymbol{z}}\right) \equiv \frac{1}{N_c}\,\mathrm{tr}\left[V^{\dagger}(\overline{\boldsymbol{x}})\, V(\boldsymbol{x})\, V^{\dagger}(\boldsymbol{z})\, V(\overline{\boldsymbol{z}})\right]$$

The Trijet Setup

In the new setup, we have to produce three particles in the final state. There are two configurations of particles:

a) Quark, quark and anti-quark
b) Quark together with two gluons.

Due to the fact that we are using the light-cone gauge, the production of these configurations can happen both instantaneously (via one emission), or in the regular way, via two successive emissions or emission followed by splitting process.



An example for a contribution with three particles in the final state

The Trijet Outgoing State

The perturbative expression for the outgoing state is:

$$|out\rangle = |in\rangle + |out\rangle^{(1)} + |out\rangle^{(2)} + \cdots$$

with:

$$|out\rangle^{(1)} = -\sum_{f,j} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\text{int}}|in\rangle}{E_j - E_{in}} + \sum_{f,j} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle}{E_f - E_j} \langle j|S|in\rangle$$

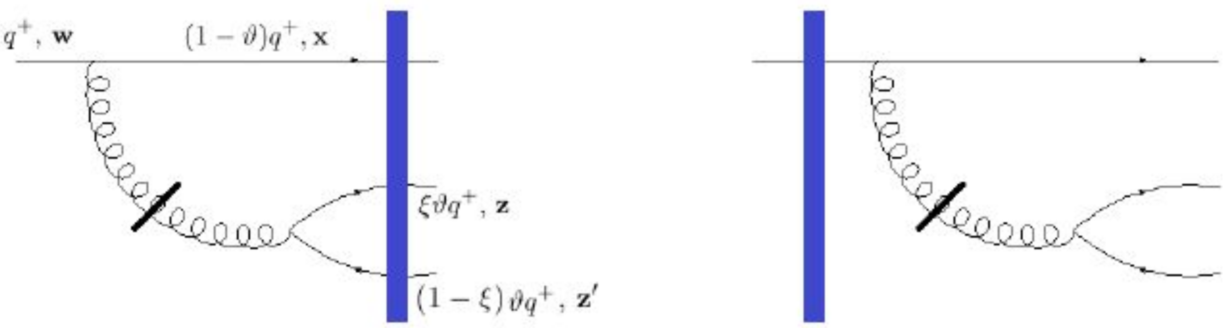
$$|out\rangle^{(2)} = \sum_{f,j,i} |f\rangle\langle f|S|j\rangle \frac{\langle j|H_{\text{int}}|i\rangle\,\langle i|H_{\text{int}}|in\rangle}{(E_j - E_{in})(E_i - E_{in})} + \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle\,\langle j|H_{\text{int}}|i\rangle}{(E_f - E_j)(E_f - E_i)} \langle i|S|in\rangle$$
$$- \sum_{f,j,i} |f\rangle \frac{\langle f|H_{\text{int}}|j\rangle}{E_f - E_j} \langle j|S|i\rangle \frac{\langle i|H_{\text{int}}|in\rangle}{E_i - E_{in}}$$

Where i, j and k runs over the relevant bare states, and Hint represent the interaction part of the QCD Hamiltonian.In the following we will focus only on the contribution to the outgoing states from the quark, quark, anti-quark configuration:

$$|\psi^\alpha\rangle^{inst}_{qq\bar{q}} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma |\, \mathsf{H}_{q\to qq\bar{q}}\, |q^\beta\rangle\,\langle q^\beta |\, \hat{S}\, |q^\alpha\rangle}{E_{qq\bar{q}} - E_q} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma |\, \hat{S}\, |\bar{q}^\epsilon q^\delta q^\beta\rangle\,\langle \bar{q}^\epsilon q^\delta q^\beta |\, \mathsf{H}_{q\to qq\bar{q}}\, |q^\alpha\rangle}{E_{qq\bar{q}} - E_q} \right)$$

$$|\psi^\alpha\rangle^{reg}_{qq\bar{q}} \equiv |\bar{q}^\rho q^\varrho q^\sigma\rangle \left(\frac{\langle \bar{q}^\rho q^\varrho q^\sigma |\, \hat{S}\, |\bar{q}^\delta q^\epsilon q^\kappa\rangle\,\langle \bar{q}^\delta q^\epsilon q^\kappa |\, \mathsf{H}_{g\to q\bar{q}}\, |q^\beta g^i\rangle\,\langle q^\beta g^i |\, \mathsf{H}_{q\to qg}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_q)(E_{qg} - E_q)} \right.$$
$$\left. + \frac{\langle \bar{q}^\rho q^\varrho q^\sigma |\, \mathsf{H}_{g\to q\bar{q}}\, |q^\gamma g^i\rangle\,\langle q^\gamma g^i |\, \mathsf{H}_{q\to qg}\, |q^\beta\rangle\,\langle q^\beta |\, \hat{S}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qq\bar{q}} - E_q)} - \frac{\langle \bar{q}^\rho q^\varrho q^\sigma |\, \mathsf{H}_{g\to q\bar{q}}\, |q^\gamma g^j\rangle\,\langle q^\gamma g^j |\, \hat{S}\, |q^\beta g^i\rangle\,\langle q^\beta g^i |\, \mathsf{H}_{q\to qg}\, |q^\alpha\rangle}{(E_{qq\bar{q}} - E_{qg})(E_{qg} - E_q)} \right)$$

The Results for the Quark Anti-quark Outgoing State (Instantaneous Emission)



(a)

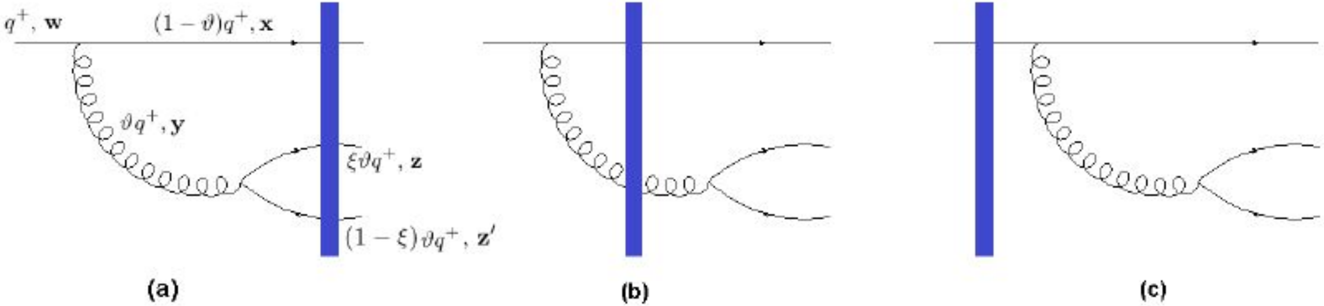
(b)

$$\left|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle^{inst}_{q q \bar{q}} = -\int_{\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'} \int_0^1 d\vartheta\, d\xi\, \frac{g^2\,(1-\vartheta)\xi(1-\xi)q^+}{4\pi^3\,\left(\xi(1-\xi)\boldsymbol{Z}^2 \,+\, (1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2\right)}$$
$$\times\,\left[V^{\varrho\delta}(\boldsymbol{z}')\,t^a_{\delta\epsilon}\,V^{\dagger\epsilon\rho}(\boldsymbol{z})\,V^{\sigma\beta}(\boldsymbol{x})\,t^a_{\beta\alpha} - t^a_{\varrho\rho}\,t^a_{\sigma\beta}\,V^{\beta\alpha}(\boldsymbol{w})\right]$$
$$\times\,\delta^{(2)}\,(\boldsymbol{w}-\boldsymbol{C})\left|\bar{q}^{\rho}_{\lambda_1}((1-\xi)\vartheta q^+,\,\boldsymbol{z})\,q^{\varrho}_{\lambda_1}(\xi\vartheta q^+,\,\boldsymbol{z}')\,q^{\sigma}_{\lambda}((1-\vartheta)q^+,\boldsymbol{x})\right\rangle$$

C denotes the c.o.m for of the three produced particles:

$$\boldsymbol{C} \,\equiv\, (1-\vartheta)\boldsymbol{x} \,+\, \xi\vartheta\boldsymbol{z} \,+\, (1-\xi)\vartheta\boldsymbol{z}'.$$

Regular Emission



$$\left|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\right\rangle^{reg}_{qq\bar q} = -\int_{\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'}\int_0^1 d\vartheta\, d\xi\, \frac{g^2\,\varphi^{il}_{\lambda_2\lambda_3}(\xi)\,\phi^{ij}_{\lambda_1\lambda}(\vartheta)\,\boldsymbol{Z}^l\,\left(\boldsymbol{X}^j+\xi\boldsymbol{Z}^j\right)\,q^+}{8\pi^3\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2\,\boldsymbol{Z}^2}$$
$$\times\Big[\Theta_1(\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}')\,V^{\varrho\delta}(\boldsymbol{z}')\,t^a_{\delta\epsilon}\,V^{\dagger\epsilon\rho}(\boldsymbol{z})\,V^{\sigma\beta}(\boldsymbol{x})\,t^a_{\beta\alpha} + \Theta_2(\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}')\,t^a_{\varrho\rho}\,t^a_{\sigma\beta}\,V^{\beta\alpha}(\boldsymbol{w})$$
$$-\,t^b_{\varrho\rho}\,V^{\sigma\beta}(\boldsymbol{x})\,U^{ba}(\boldsymbol{y})\,t^a_{\beta\alpha}\Big]\,\delta^{(2)}\left(\boldsymbol{w}-\boldsymbol{C}\right)\Big|\bar q^{\rho}_{\lambda_3}((1-\xi)\vartheta q^+,\,\boldsymbol{z})\,q^{\varrho}_{\lambda_2}(\xi\vartheta q^+,\,\boldsymbol{z}')\,q^{\sigma}_{\lambda_1}((1-\vartheta)q^+,\boldsymbol{x})\Big\rangle$$

With the following definitions:

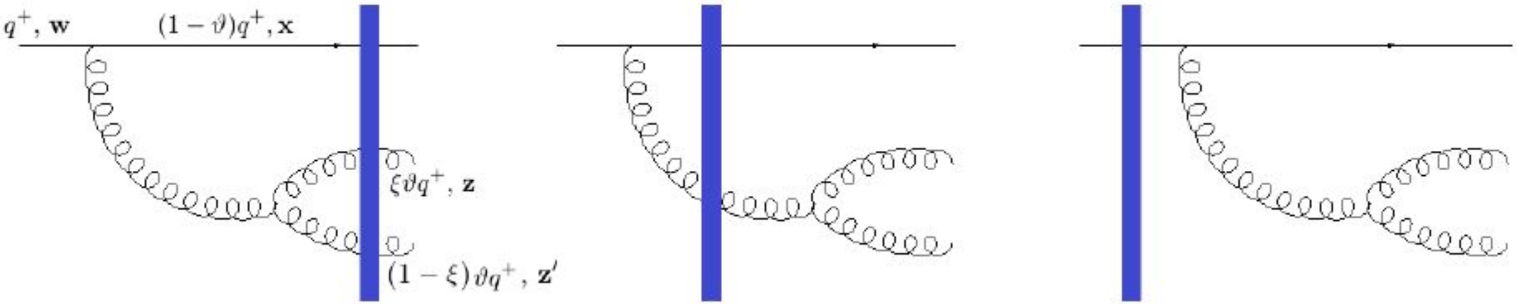
$$\boldsymbol{y}\equiv\xi\boldsymbol{z}'+(1-\xi)\boldsymbol{z}$$

$$\Theta_1(\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}')\equiv\frac{(1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2}{(1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2+\xi(1-\xi)\boldsymbol{Z}^2}$$

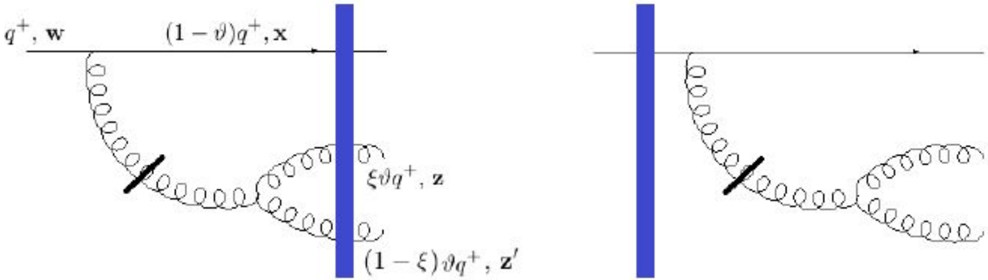
$$\Theta_2(\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}')\equiv\frac{\xi(1-\xi)\boldsymbol{Z}^2}{(1-\vartheta)\,(\boldsymbol{X}+\xi\boldsymbol{Z})^2+\xi(1-\xi)\boldsymbol{Z}^2}$$

Note that both the result above and in the previous slide vanishes under the limit S->1. This property of the results has to be expected since the new particles are produced by the shockwave.

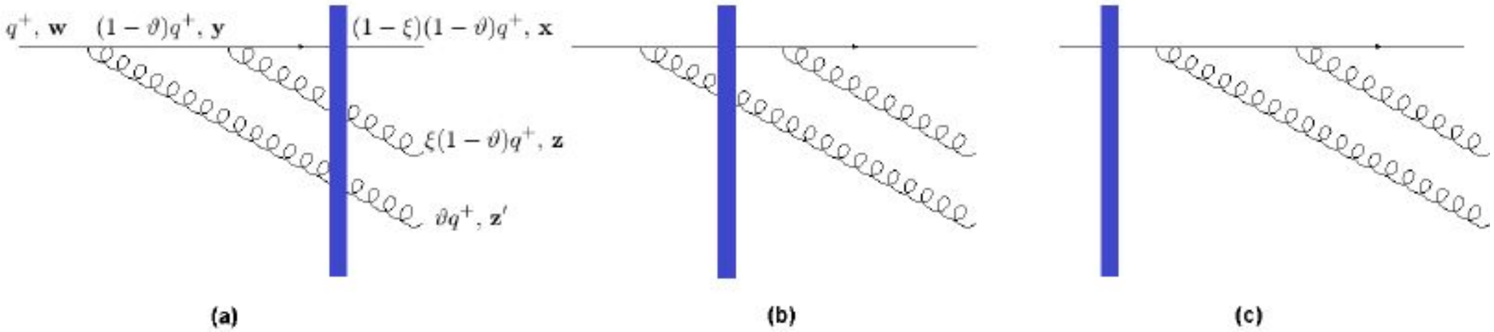
The Diagrams for the Quark and Two Gluons Outgoing States

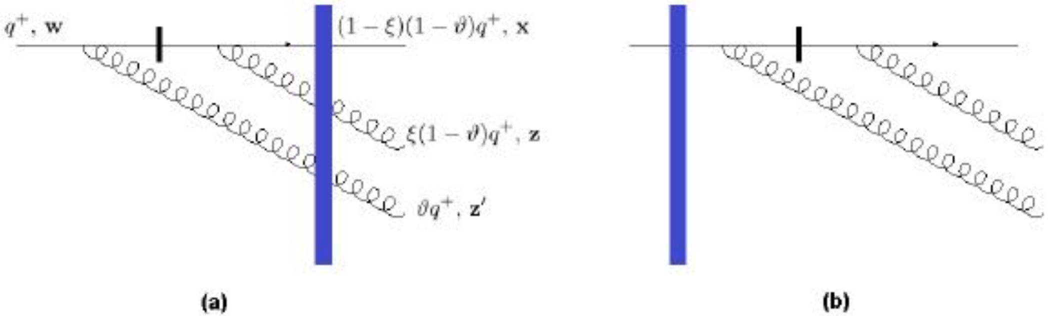


(a) (b) (c)



(a) (b)


q^+, $\mathbf{w}$
$(1-\vartheta)q^+$, $\mathbf{y}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$
(a)
(b)
(c)


q^+, $\mathbf{w}$
$(1-\xi)(1-\vartheta)q^+$, $\mathbf{x}$
$\xi(1-\vartheta)q^+$, $\mathbf{z}$
ϑq^+, $\mathbf{z}'$
(a)
(b)

The results for the forward trijet cross section

The expression for the forward trijet cross section is composed by two contributions:

$$\frac{d\sigma^{pA\to 3jet+X}}{d^3q_1\,d^3q_2\,d^3q_3} = \int dx_p\, q(x_p,\mu^2)\left(\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\,d^3q_2\,d^3q_3} + \frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3}\right)$$

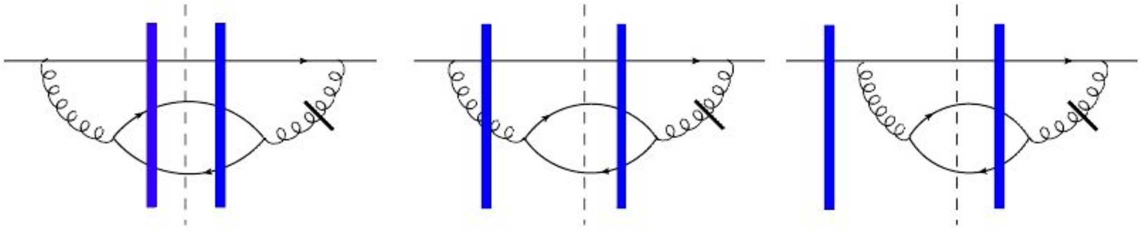
The two contributions to the two final partonic state:

$$\begin{aligned}\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\,d^3q_2\,d^3q_3} &\equiv \frac{1}{2N_c\,L}\,{}^{(g^2)}_{out}\langle q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{q}=0_\perp)|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_q(q_2)\,\hat{\mathcal{N}}_{\bar{q}}(q_3)\,|q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{q}=0_\perp)\rangle^{(g^2)}_{out}\\ &= \frac{1}{2N_c\,L}\int_{\boldsymbol{w},\,\overline{\boldsymbol{w}}}{}_{qq\bar{q}}\langle\psi^{\alpha}_{\lambda}(q^+,\,\overline{\boldsymbol{w}})|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_q(q_2)\,\hat{\mathcal{N}}_{\bar{q}}(q_3)\,|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\rangle_{qq\bar{q}}\end{aligned}$$

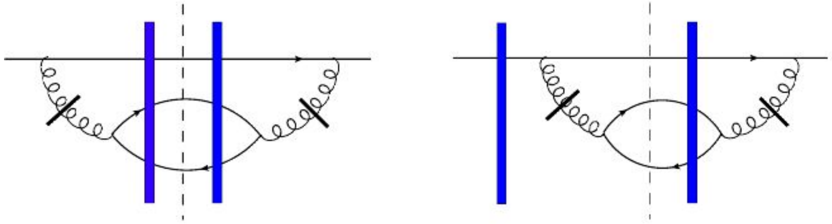
$$\begin{aligned}\frac{d\sigma^{qA\to qgg+X}}{d^3q_1\,d^3q_2\,d^3q_3} &\equiv \frac{1}{2N_c\,L}\,{}^{(g^2)}_{out}\langle q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{q}=0_\perp)|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_g(q_2)\,\hat{\mathcal{N}}_g(q_3)\,|q^{\alpha}_{\lambda}(q^+,\,\boldsymbol{q}=0_\perp)\rangle^{(g^2)}_{out}\\ &= \frac{1}{2N_c\,L}\int_{\boldsymbol{w},\,\overline{\boldsymbol{w}}}{}_{qgg}\langle\psi^{\alpha}_{\lambda}(q^+,\,\overline{\boldsymbol{w}})|\,\hat{\mathcal{N}}_q(q_1)\,\hat{\mathcal{N}}_g(q_2)\,\hat{\mathcal{N}}_g(q_3)\,|\psi^{\alpha}_{\lambda}(q^+,\,\boldsymbol{w})\rangle_{qgg}\end{aligned}$$

The results for the cross section (quark contribution)

The contribution of the quarks to the cross section is given in terms of four blocks which represent two direct (regular / instantaneous gluon both in the amplitude and its conjugate) and two interference (regular - inst.) contributions.



(a) (b) (c)



(a) (b)

At the large Nc limit:

$$
\begin{aligned}
&\frac{d\sigma^{qA\to qq\bar{q}+X}}{d^3q_1\, d^3q_2\, d^3q_3} \equiv \frac{\alpha_s^2\, N_c\, N_f}{64\pi^5 (q^+)^2}\, \delta(q^+ - q_1^+ - q_2^+ - q_3^+) \int_{\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}'} e^{-i\boldsymbol{q}_1\cdot(\boldsymbol{x}-\overline{\boldsymbol{x}})-i\boldsymbol{q}_2\cdot(\boldsymbol{z}-\overline{\boldsymbol{z}})-i\boldsymbol{q}_3\cdot(\boldsymbol{z}'-\overline{\boldsymbol{z}}')}\\
&\times \Big\{K^1_{qq\bar{q}}\left(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}'\right) \Big[\overline{\Theta}_1\, \Theta_1\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z}) - \overline{\Theta}_1\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{y},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{y})\\
&- \Theta_1\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{y}})\, \mathcal{S}(\overline{\boldsymbol{y}},\, \boldsymbol{z}) + \overline{\Theta}_2\, \Theta_1\, \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z}',\, \boldsymbol{x}) + \overline{\Theta}_1\, \Theta_2\, \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{w})\\
&+ \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{y},\, \overline{\boldsymbol{y}})\, \mathcal{S}(\overline{\boldsymbol{y}},\, \boldsymbol{y}) - \overline{\Theta}_2\, \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{x})\, \mathcal{S}(\boldsymbol{x},\, \boldsymbol{y}) - \Theta_2\, \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{y}})\, \mathcal{S}(\overline{\boldsymbol{y}},\, \boldsymbol{w}) + \overline{\Theta}_2\, \Theta_2 \mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right)\Big]\\
&+ K^2_{qq\bar{q}}\left(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}'\right) \Big[\Theta_1\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z}) - \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{y},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{y})\\
&- \Theta_1\, \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z}',\, \boldsymbol{x}) + \Theta_2\, \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{w}) + \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{x})\, \mathcal{S}(\boldsymbol{x},\, \boldsymbol{y}) - \Theta_2\, \mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right)\Big]\\
&+ K^2_{qq\bar{q}}\left(\boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}',\, \overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}'\right) \Big[\overline{\Theta}_1\, \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z}) - \mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{y}})\, \mathcal{S}(\overline{\boldsymbol{y}},\, \boldsymbol{z})\\
&+ \overline{\Theta}_2\, \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z}',\, \boldsymbol{x}) - \overline{\Theta}_1\, \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{w}) + \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{y}})\, \mathcal{S}(\overline{\boldsymbol{y}},\, \boldsymbol{w}) - \overline{\Theta}_2\, \mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right)\Big]\\
&+ K^3_{qq\bar{q}}\left(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}},\, \overline{\boldsymbol{z}}',\, \boldsymbol{x},\, \boldsymbol{z},\, \boldsymbol{z}'\right) \Big[\mathcal{Q}(\overline{\boldsymbol{x}},\, \boldsymbol{x},\, \boldsymbol{z}',\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{z}) - \mathcal{S}(\overline{\boldsymbol{w}},\, \boldsymbol{z})\, \mathcal{S}(\boldsymbol{z}',\, \boldsymbol{x})\\
&- \mathcal{S}(\overline{\boldsymbol{x}},\, \overline{\boldsymbol{z}}')\, \mathcal{S}(\overline{\boldsymbol{z}},\, \boldsymbol{w}) + \mathcal{S}\left(\overline{\boldsymbol{w}},\, \boldsymbol{w}\right)\Big]\Big\} + \left(q_1^+ \leftrightarrow q_2^+,\, \boldsymbol{q}_1 \leftrightarrow \boldsymbol{q}_2\right)
\end{aligned}
$$

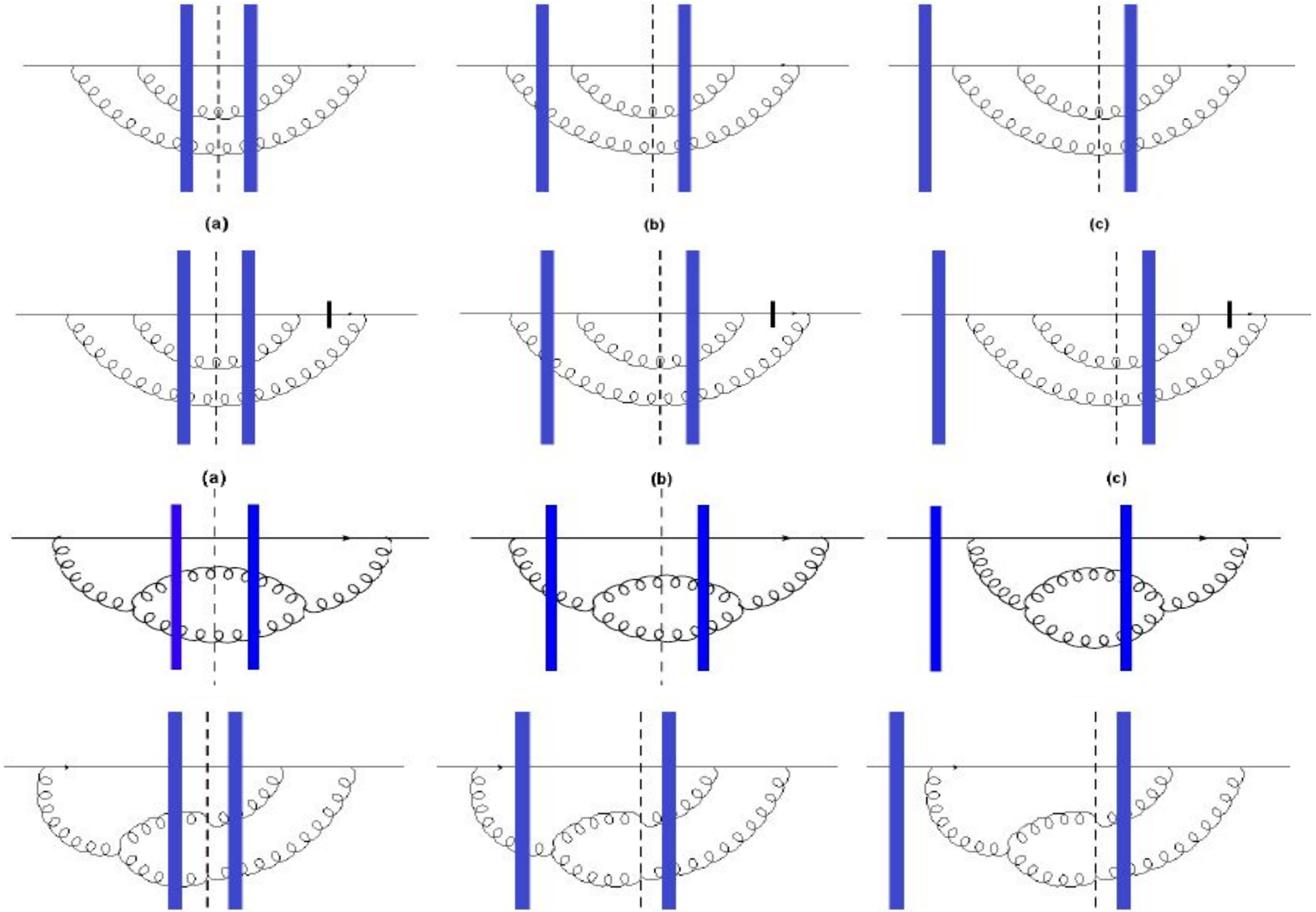
Note that the result vanishes (each block of terms in the rectangular brackets separately) when taking the limit S->1.

$$K^1_{q\bar{q}\bar{q}}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv \frac{2\,\overline{\boldsymbol{Z}}^n\,\left(\overline{\boldsymbol{X}}^p+\xi\overline{\boldsymbol{Z}}^p\right)\,\boldsymbol{Z}^l\,\left(\boldsymbol{X}^j+\xi\boldsymbol{Z}^j\right)}{\overline{\boldsymbol{Z}}^2\,\left(\overline{\boldsymbol{X}}+\xi\overline{\boldsymbol{Z}}\right)^2\,\boldsymbol{Z}^2\,\left(\boldsymbol{X}+\xi\boldsymbol{Z}\right)^2}$$
$$\times\left(\left((2\xi-1)^2(\vartheta-2)^2+\vartheta^2\right)\delta^{np}\delta^{lj}+\left((\vartheta-2)^2+2\vartheta^2\right)\delta^{pj}\delta^{nl}-\left((2\xi-1)^2\vartheta^2+(\vartheta-2)^2\right)\delta^{lp}\delta^{nj}\right)$$

$$K^2_{q\bar{q}\bar{q}}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv -\frac{4(2-\vartheta)(1-\vartheta)(2-\xi)\xi(1-\xi)\,\left(\boldsymbol{X}\cdot\boldsymbol{Z}+\xi\boldsymbol{Z}^2\right)}{\left(\xi(1-\xi)\overline{\boldsymbol{Z}}^2\,+\,(1-\vartheta)\left(\overline{\boldsymbol{X}}+\xi\overline{\boldsymbol{Z}}\right)^2\right)\,\boldsymbol{Z}^2\,\left(\boldsymbol{X}+\xi\boldsymbol{Z}\right)^2}$$

$$K^3_{q\bar{q}\bar{q}}\left(\overline{\boldsymbol{x}},\,\overline{\boldsymbol{z}},\,\overline{\boldsymbol{z}}',\,\boldsymbol{x},\,\boldsymbol{z},\,\boldsymbol{z}'\right) \equiv \frac{(1-\vartheta)^2\xi^2(1-\xi)^2}{\left(\xi(1-\xi)\overline{\boldsymbol{Z}}^2\,+\,(1-\vartheta)\left(\overline{\boldsymbol{X}}+\xi\overline{\boldsymbol{Z}}\right)^2\right)\,\left(\xi(1-\xi)\boldsymbol{Z}^2\,+\,(1-\vartheta)\left(\boldsymbol{X}+\xi\boldsymbol{Z}\right)^2\right)}$$

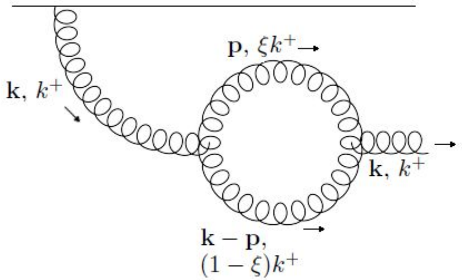
Contribution from the Gluons

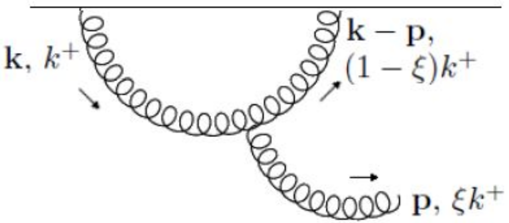


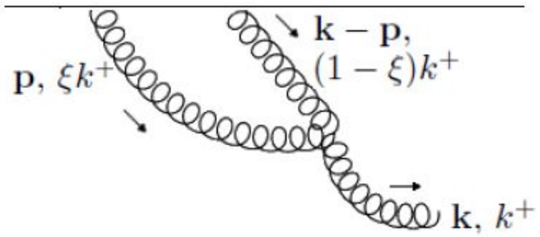
The forward NLO dijet cross section

In order to allow phenomenology to be reliable, higher order corrections as dictated by pQCD must be included in the result of hep-ph/0708.0231.

The missing part of the new outgoing state (with respect to the trijet calculation) is the part which involves the production of a quark and a gluon together with a loop / virtual correction.







Summary

1) A generalization of the method shown in hep-ph/0708.0231 to all orders is possible by adopting the outgoing state approach.

2) We computed the one loop light-cone wave function of the incoming quark and its corresponding outgoing state.

3) From the above result we managed to deduce the expression for the forward trijet cross section at the large Nc limit.

4) The NLO dijet production cross section calculation is experimentally more important, but its calculation is more tricky since it involves many more contributions, with some of them diverge.