Diffraction - 2018

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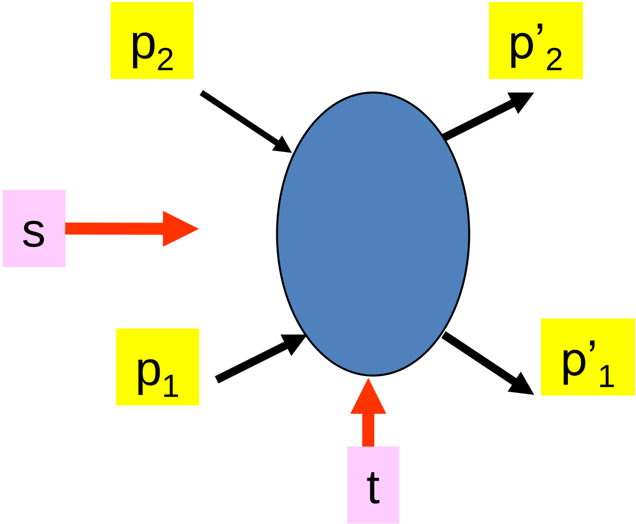
Double-Logarithmic contribution to Pomeron

talk based on results obtained in collaboration with S.I. Troyan

Classic/phenomenological Pomeron in Regge theory of 60's

Regge theory is based on applying very general concepts such as **Analyticity**, **Unitarity** and **Causality** to predict high-energy asymptotics of scattering amplitudes

Consider amplitude of 2 -> 2 scattering of hadrons at high energies



$= A\,(s, t)$

s,t are standard Mandelstam variables

c.m.f. hadron energy

$$s = (p_1 + p_2)^2 = 4\,E^2$$

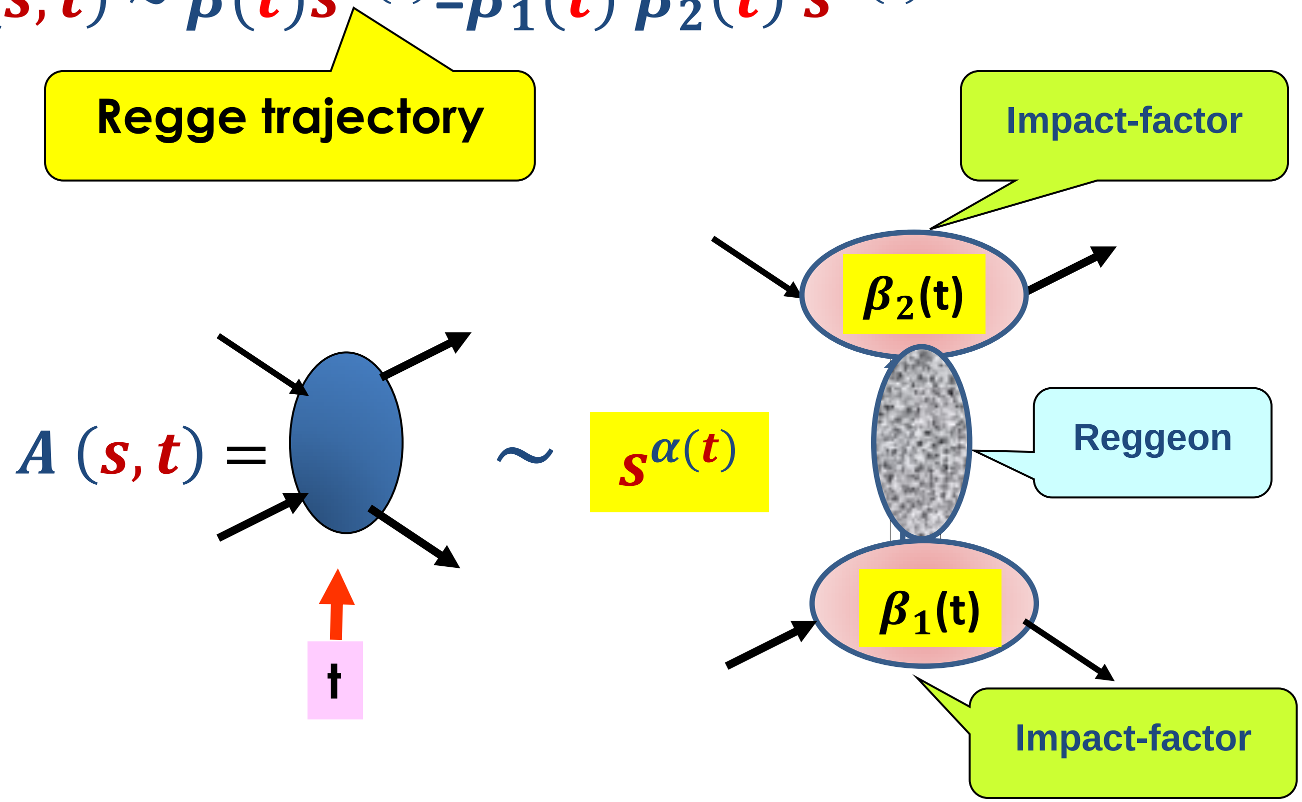
$$t = (p'_1 - p_1)^2 \approx -2E^2(1 - \cos\theta)$$

c.m.f. scattering angle

Forward kinematics $\theta \ll 1 \rightarrow s = 4\,E^2 \gg -t = E^2\theta^2$

Regge theory predicts that high-energy asymptotics of any scattering amplitude in the forward kinematics is

$$A\,(s,t) \sim \beta(t)s^{\alpha(t)} = \beta_1(t)\,\beta_2(t)\,s^{\alpha(t)}$$



The trajectories are expanded in the series in *t*. However *t* is small, so the linear approximation can be used:

$$\alpha(t) = \alpha(0) + \alpha'(0)\, t$$

intercept

slope

Optical theorem:

$$\sigma_{tot} \sim Im\ A_{el}(s,0)/s \sim (s/s_0)^{\alpha(0)-1}$$

Total cross section

Unitarity → Froissart-Martin bound

$$\sigma_{tot}(s) \leq c\, ln^2 s \rightarrow \alpha(0) \leq 1$$

I.Y. Pomeranchuk (1958) suggested existence of Reggeon with intercept = 1

Pomeron $\alpha_P(0) \equiv \alpha(0)_{max} = 1$

$$\alpha_P(0) \equiv \alpha(0)_{max} = 1$$

If so, $\sigma_{tot} \sim (s/s_0)^{\alpha_P(0)-1}$ does not have a power dependency on **s**

Phenomenological Regge theory cannot predict numerical values of all parameters it involves and cannot give an estimate for the energy scale when the asymptotic expressions can be used

There are various models of Pomeron in the literature: both pre-QCD models and QCD Pomerons (e.g. P. Landchoff, G. Preparata…).

Among them, the most known of the QCD Pomerons is BFKL Pomeron
V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1976); I.I. Balitsky, L.N. Lipatov (1978); V.S. Fadin, L.N. Lipatov (1998), G. Camici, M. Ciafaloni (1998)

Double-Logarithmic Approximation (DLA)

V.V. Sudakov (1956)
V.G. Gorshkov, V.N. Gribov, L.N. Lipatov, G.V. Frolov (1967)

QED scattering amplitudes at $t=0$:

$$M_{DL} = M_0 \left[1 + c_1 (\alpha \, ln^2 (s/m^2)) + c_2 \left(\alpha \, ln^2 (s/m^2)\right)^2 + \cdots\right]$$

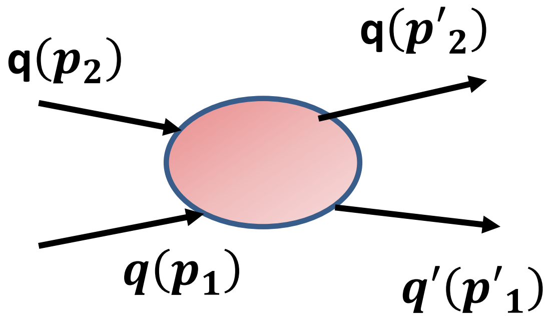
QCD scattering amplitudes:

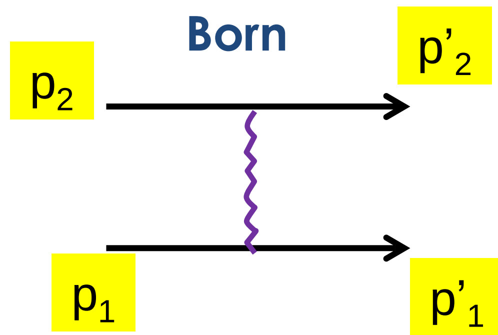
$$M_{DL} = M_0 \left[1 + c_1 (\alpha_s \, ln^2 (s/m^2)) + c_2 \left(\alpha_s \, ln^2 (s/m^2)\right)^2 + \cdots\right]$$

In DLA each power of the coupling multiplied by two logs

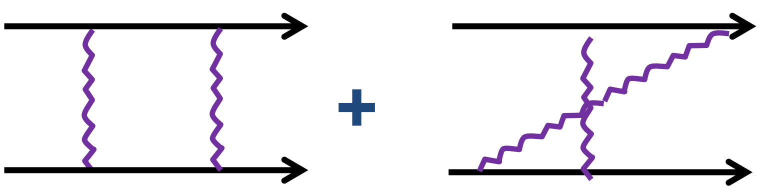
Leading Logarithmic Approximation (LLA)

quark-quark scattering in the forward kinematics $s \gg -t = (p_1 - p'_1)^2$





First loop



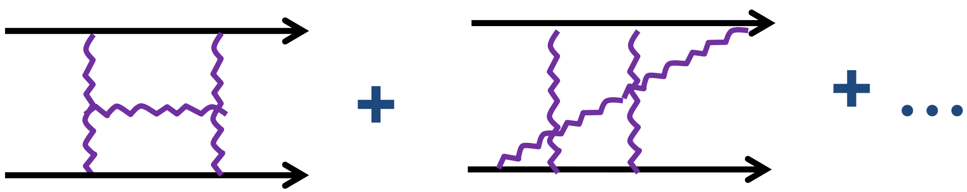
$$= A^{(1)}$$

$$A^{(1)} \sim \alpha^2{}_s s \, [ln\,(-s) \;\pm\; ln\, s] \sim$$

large factor

(-) for color singlet and (+) for octet

second-loop graphs



$$= A^{(2)}$$

H.T. Nieh, Y.P. Yao (1974)

$$A^{(2)} \sim [\alpha^2{}_s ln\, s]\, s\, (\alpha_s ln^2 s), \quad A^{(3)} \sim [\alpha^2{}_s ln\, s]\, s\, (\alpha_s ln^2 s)^2$$

Hypothesis: $A^{(n)} \sim [\alpha^2{}_s ln\, s]\, s\, (\alpha_s\, ln^2\, s)^{n-1}$

i.e. in higher orders the factor s is accompanied by DL contributions

B.M. McCoy, T.T. Wu (1975) demonstrated that that result was wrong: Actually, DL terms vanish because logs from integrations in the transverse space kill each other, so only SL remain. Accounting for SL accompanying the overall factor s is called Leading Logarithmic Approximation

$$A^{(LLA)} = s \sum c_n (\alpha_s\ ln\, s)^n$$

Resummation of Leading SL contributions accompanying the factor s was done with the BFKL equation. This equation describes the photon-photon scattering. Asymptotics of the solution of BFKL equation is addressed as BFKL Pomeron

LL Approximation

DL Approximation

$$A^{(LLA)} = s \sum c_n (\alpha_s \; \ln s)^n$$

overall factor

$$A^{(DLA)} = \sum c'_n (\alpha_s \; \ln^2 s)^n$$

overall factor **S** is absent

and because of it, DL contributions were commonly neglected compared to LL ones

However such estimate by-parameters proved to be too rough
We demonstrate that DL contribution is comparable to LL one

To this end, trying to be close to BFKL,

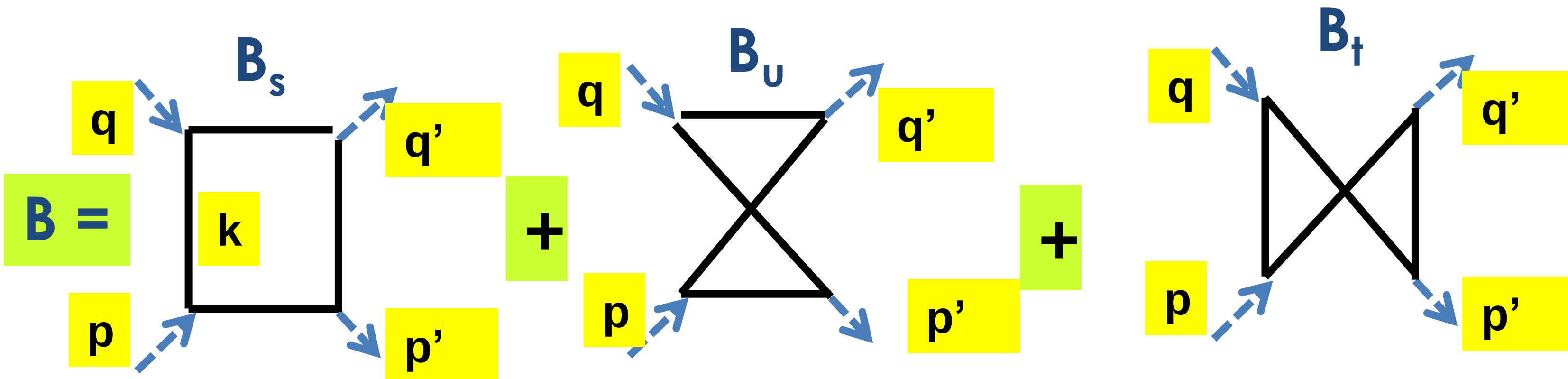
We consider amplitudes of the elastic scattering of virtual photons

$$\gamma^*(p)\gamma^*(\mathbf{q}) \rightarrow \gamma^*(p')\gamma^*(\mathbf{q'})$$

in the forward kinematics. We presume that all the photons are non-polarized

This process, apart of its experimental importance, is interesting from the theoretical point of view because, in contrast to hadronic reactions, it is free of non-perturbative contributions, so it can be regarded as a test-field for various theoretical approaches

Born (lowest order) approximation:



B_T can be neglected in the forward kinematics because it does not contain logs of s and u

We use the standard notations:

$$s = (p+q)^2 \approx -u = (p - q')^2 \gg -t = (p - q')^2 ,$$

$$|p^2| \equiv Q_1{}^2 \approx |p'^2|, |q^2| \approx |q'^2| \equiv Q_2{}^2 \gg \mu^2$$

IR cut-off

There are two kinematic regions in the Born approximation :

Moderately virtual photons: $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$B_s = -\left(e^4/16\pi^2\right)\left[ln^2\left(-s/\mu^2\right) - ln^2\left({Q_2}^2/\mu^2\right) - ln^2\left({Q_1}^2/\mu^2\right)\right]$$

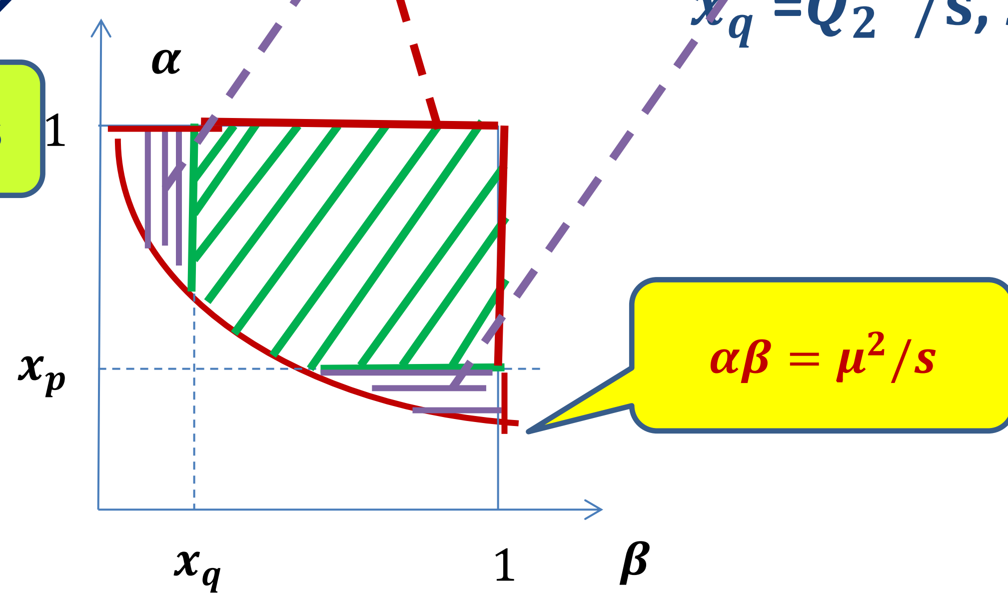
Sudakov parametrization

$$k = -\,\alpha\, q' + \,\beta\, p' + \,k_\perp$$

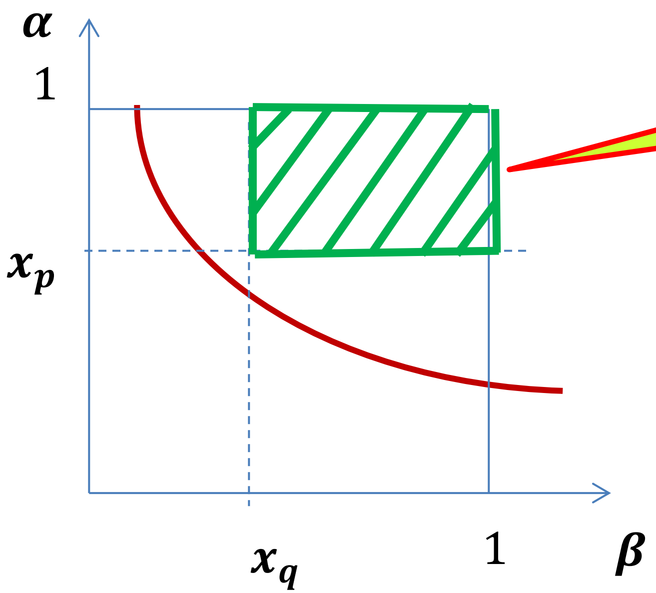
light-cone vectors

$$q' = q - x_q\, p,\;\; p' = p - x_p\, q$$

$$x_q = {Q_2}^2/s,\; x_p = {Q_1}^2/s$$



Deeply virtual photons $s\,\mu^2 \ll Q_1{}^2 Q_2{}^2$



Integration region does not involve μ^2

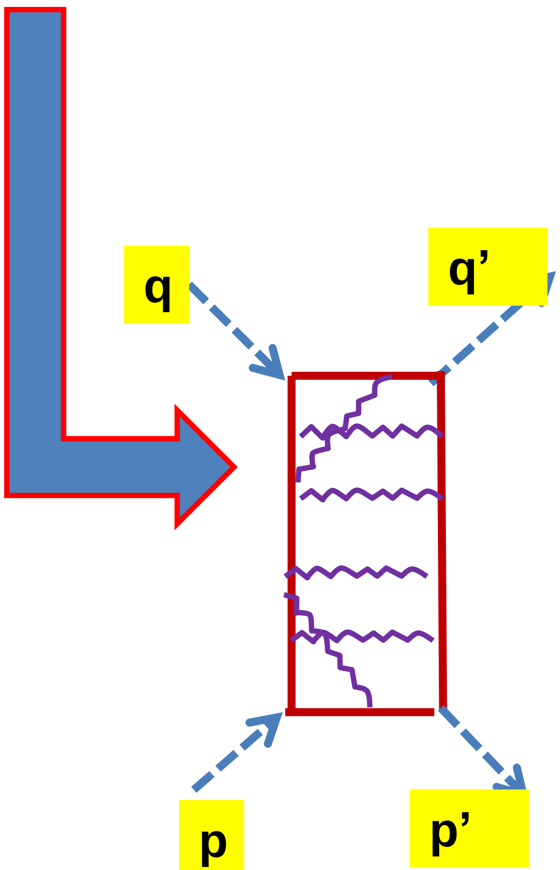
$$B_s = -\left(e^4/16\pi^2\right)\left[\,2\,ln\left(-s/Q_1{}^2\right)ln\left(-s/Q_2{}^2\right)\right]$$

does not depend on μ^2

$$B_u\,(\mathrm{u}) \approx B_s\,(\mathrm{s})$$

Beyond the Born approximation

Step 1 Amplitudes of photon- photon scattering via overall quark loop in DLA



Bartels-Lublinsky 2003

Explicit DLA expressions for such amplitudes at $t = 0$ (collinear scattering) obtained with several different approaches

Ermolaev-Ivanov-Troyan 2017

Generalization to $s \gg -t \neq 0$ and accounting for the running coupling

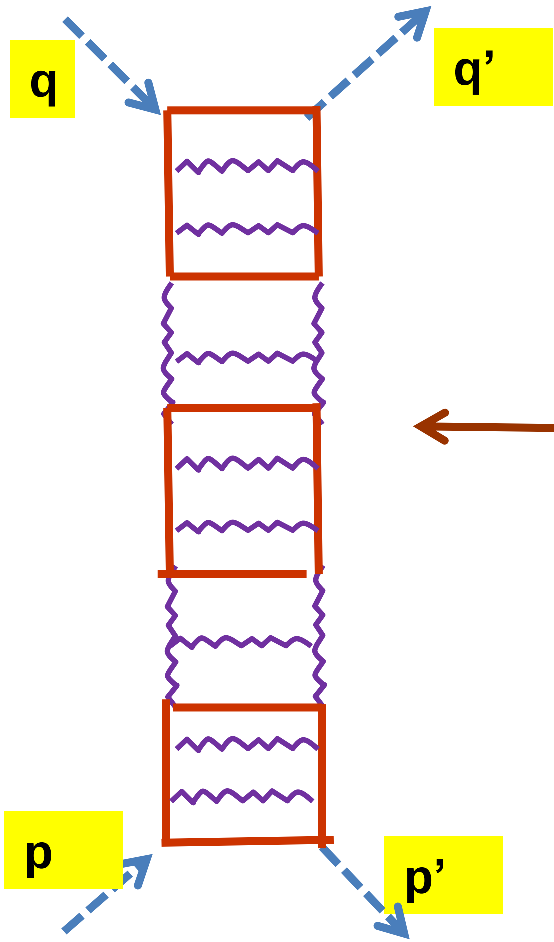
We used InfraRed Evolution Equations (IREE)
This method was invented by L.N. Lipatov and applied to both Gravity (Lipatov, 1982) and quark-quark elastic scattering (Kirschner-Lipatov, 1982)

After that IREE approach has been applied to many problems of QED, QCD and EW interactions

STEP 2: photon scattering via intermittence of quark and gluon loops

Ermolaev-Troyan 2017

IREE approach was used



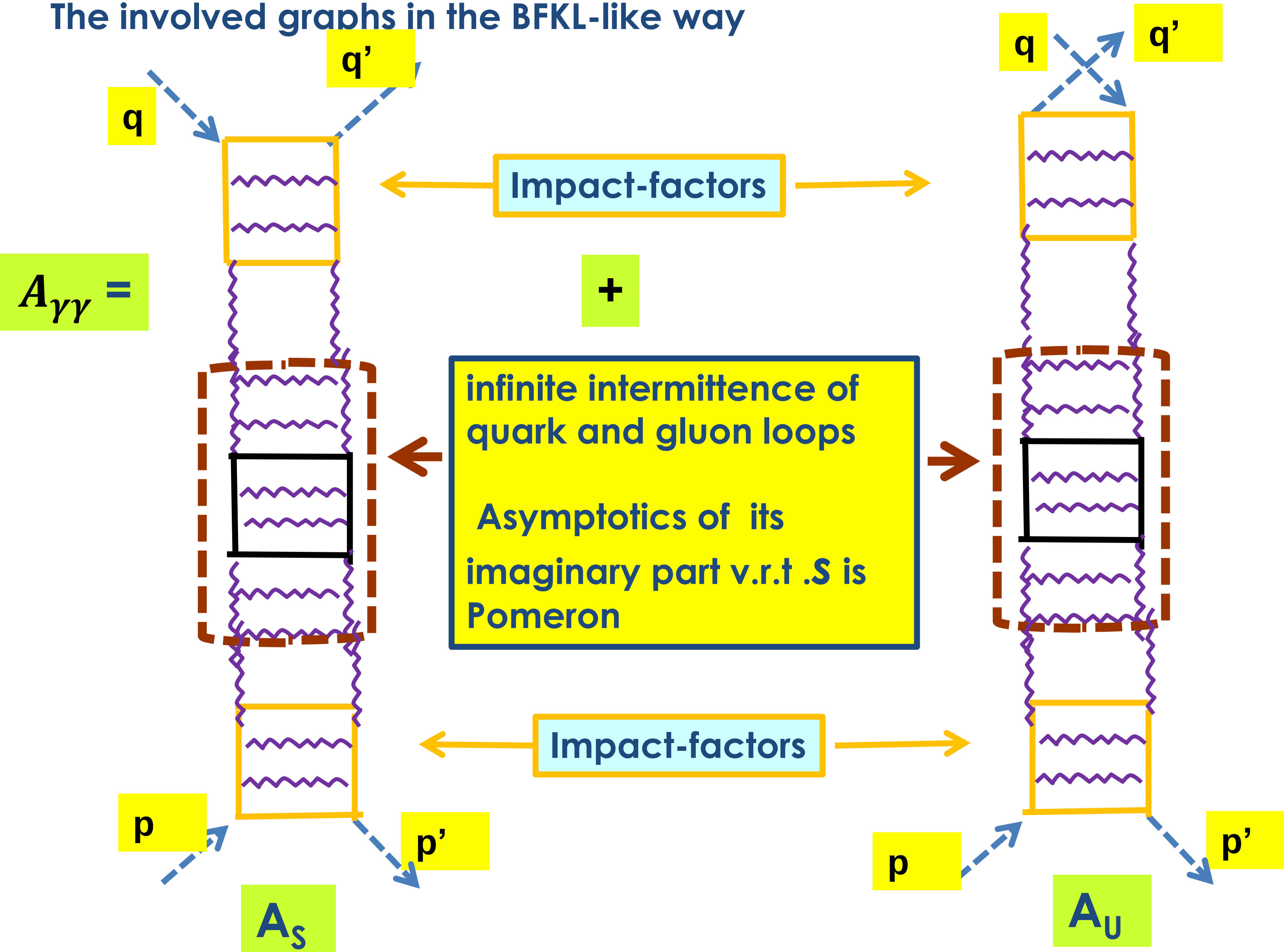
Non-ladder gluons are not depicted though implied

Such graphs yield both LL and DL contributions

BFKL accounts for LL terms

In contrast, we account for DL terms

The involved graphs in the BFKL-like way



Impact-factors are calculated independently of BFKL with Pert QCD means

It is convenient to use the Mellin transform in both DLA and LLA calculations

$$A^{(\pm)}(s/\mu^2) = \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\mu^2}\right)^{\omega} \xi^{(\pm)}(\omega)\, f^{(\pm)}{}_0(\omega)$$

signature factor

$$\xi^{(\pm)} = -\left(1 \pm e^{-i\pi\omega}\right)/2$$

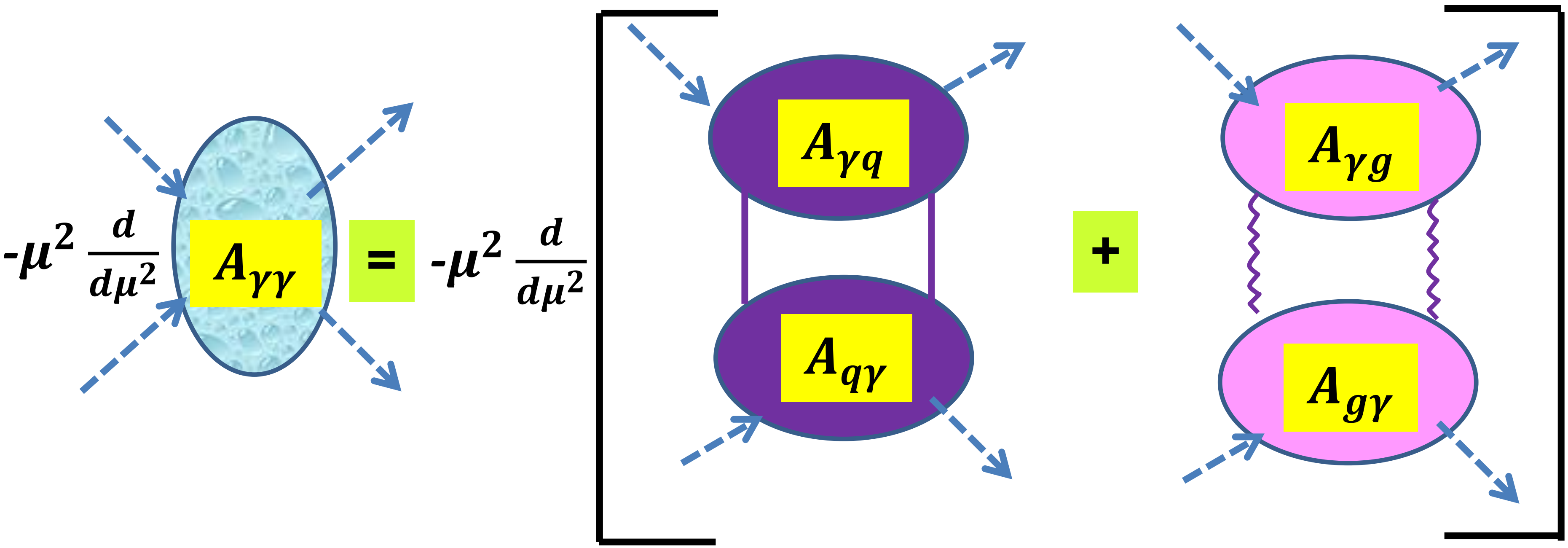
logarithmic variables:

$$\rho = ln\left(s/\mu^2\right) \; y_1 = ln\left(Q_1{}^2/\mu^2\right) \; y_2 = ln\left(Q_2{}^2/\mu^2\right)$$

$$A(s, Q_1{}^2, Q_2{}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \quad F(\omega, y_1, y_2)$$

IREEs for $A_{\gamma\gamma}(s, Q_1{}^2, Q_2{}^2)$ involve auxiliary amplitudes

$A_{\gamma q}(s, Q_1{}^2)$, $A_{q\gamma}(s, Q_2{}^2)$, $A_{\gamma g}(s, Q_1{}^2)$, $A_{g\gamma}(s, Q_2{}^2)$



Applying the standard Feynman rules , arrive at IREEs in analytic form
They look simpler in the Melin space

In analytic form the l.h.s of the IREE is

$$-\mu^2 \frac{d}{d\mu^2} A(s, {Q_1}^2, {Q_2}^2) = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} \left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] F(\omega, y_1, y_2)$$

The l.h.s. is the same for both Moderately and Deeply Virtual photons but r.h.s. are differerent:

Moderately virtual photons $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] {F^{(M)}}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y_1) F_{q\gamma}(\omega, y_2) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y_1) F_{g\gamma}(\omega, y_2)$$

Deeply virtual photons $s\,\mu^2 \ll {Q_1}^2 {Q_2}^2$

$$\left[\omega + \frac{\partial}{\partial y_1} + \frac{\partial}{\partial y_2}\right] {F^{(D)}}_{\gamma\gamma}(\omega, y_1, y_2) = 0$$

Solution for deeply virtual photons $s\,\mu^2 \gg {Q_1}^2 {Q_2}^2$

$$A^{(M)}{}_{\gamma\gamma}(\omega, y_1, y_2) = \frac{1}{8\,\pi^2} \int_{-i\infty}^{i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{\sqrt{{Q_1}^2 {Q_1}^2}}\right)^{\omega} \sum_{r=q,g} \left[{}^{1}\!/_{\omega}\, f_{\gamma r}(\omega) f_{r\gamma}(\omega) + f_{r\gamma}(\omega) \int_o^{\eta} dz F_{\gamma r}(\omega, z) + {}^{1}\!/_{2} \int_{\eta}^{2\rho\,-\xi} dz\; e^{\omega z/2} F_{\gamma r}(\omega, z) F_{r\gamma}(\omega, z) \right]$$

where the symmetrical variables are used

$\xi = y_1 + y_2$,

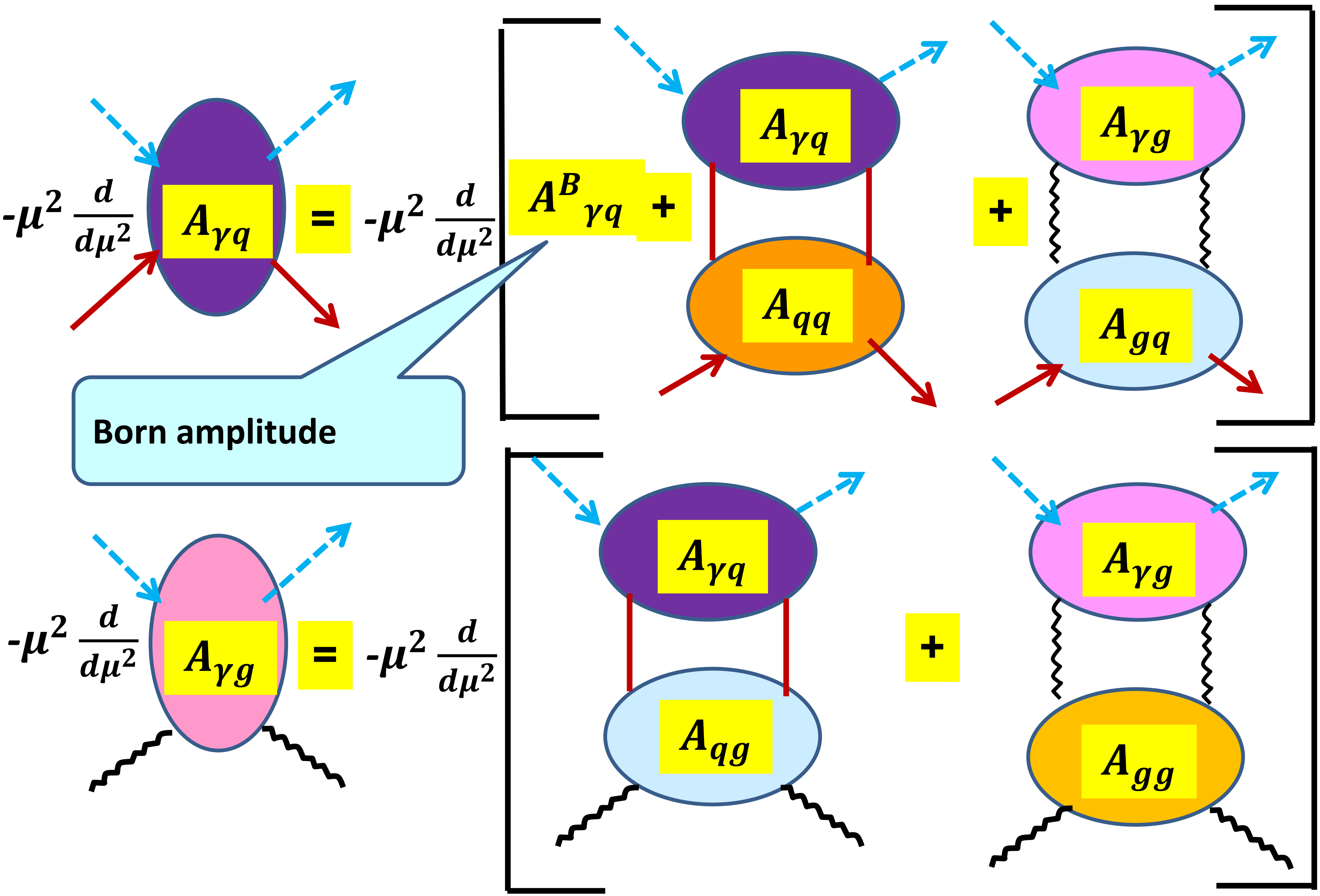
$\eta == |y_1 - y_2|$

Auxiliary amplitudes $F_{\gamma r}(\omega, z)$, $f_{\gamma r}(\omega)$ describe photon-parton scattering

$Q^2 \gg \mu^2$

$Q^2 \sim \mu^2$

IREE for auxiliary amplitudes



Applying the standard Feynman rules , arrive at IREEs in analytic form

Born contribution

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma q}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qq}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gq}(\omega)$$

$$\left[\omega + \frac{\partial}{\partial y}\right] F_{\gamma g}(\omega, y) = \frac{1}{8\pi^2} F_{\gamma q}(\omega, y) f_{qg}(\omega) + \frac{1}{8\pi^2} F_{\gamma g}(\omega, y) f_{gg}(\omega)$$

parton-parton amplitudes

It is easy to find general solutions so as to obtain the auxiliary amplitudes in terms of parton-parton amplitudes.

Particular solution are found through Matching:

$$F_{\gamma q}(\omega, y = 0) = f_{\gamma q}(\omega)$$

amplitudes of photon-parton scattering when y =0.
They must be calculated independently

It is convenient to introduce amplitudes $h_{\gamma q,\gamma g}(\omega) = f_{\gamma q,\gamma g}(\omega)/(8\,\pi^2)$

IREEs for them are algebraic:

$$\omega h_{\gamma q}(\omega) = a_{\gamma q} + h_{\gamma q}(\omega) h_{qq}(\omega) + h_{\gamma g}(\omega) h_{gq}(\omega)$$

$$\omega h_{\gamma g}(\omega) = a_{\gamma g} + h_{\gamma q}(\omega) h_{qg}(\omega) + h_{\gamma g}(\omega) h_{gg}(\omega)$$

with $a_{\gamma q} = \alpha/2\,\pi, a_{\gamma q} = 0$

Solution to these equations allows us to express the auxiliary amplitudes in terms of the parton-parton amplitudes

IREE for the parton-parton amplitudes $h_{ik} = (1/8\,\pi^2) f_{ik}$

$$\omega h_{qq} = b_{qq} + h_{qq} h_{qq} + h_{qg} h_{gq}$$
$$\omega h_{qg} = b_{qg} + h_{qq} h_{qg} + h_{qg} h_{gg}$$
$$\omega h_{gq} = b_{gq} + h_{gq} h_{qq} + h_{gg} h_{gq}$$
$$\omega h_{gg} = b_{gg} + h_{gq} h_{qg} + h_{gg} h_{gg}$$

$$\mathrm{b}_{\mathrm{ik}}(\omega) = a_{ik}(\omega) + V_{ik}(\omega)$$

Born contributions. They are independent of ω when QCD coupling is fixed but depend on it when the coupling is running

Contributions of the color octet (non-ladder graphs)

$a_{ik}(\omega)$ coincide with analogous factors for g_1 singlet, see e.g. Bartels-Ermolaev-Ryskin (1996) and Blumlein (1997)

However such coincidence does not take place for $V_{ik}(\omega)$

Fixed QCD coupling

$$a_{qq} = (\alpha_s/2\pi)\ C_F\ , \qquad a_{qg} = (\alpha_s/\pi) C_F,$$
$$a_{gq} = -(\alpha_s/2\pi)\ n_f\ , \qquad a_{gg} = (\alpha_s/\pi)\ 2N$$

Running QCD coupling

$$a_{qq} = (A(\omega)/2\pi)\ C_F\ , \qquad a_{qg} = (A'(\omega)/\pi) C_F,$$
$$a_{gq} = -(A'(\omega)/2\pi)\ n_f\ , \qquad a_{gg} = (A(\omega)/\pi)\ 2N$$

where

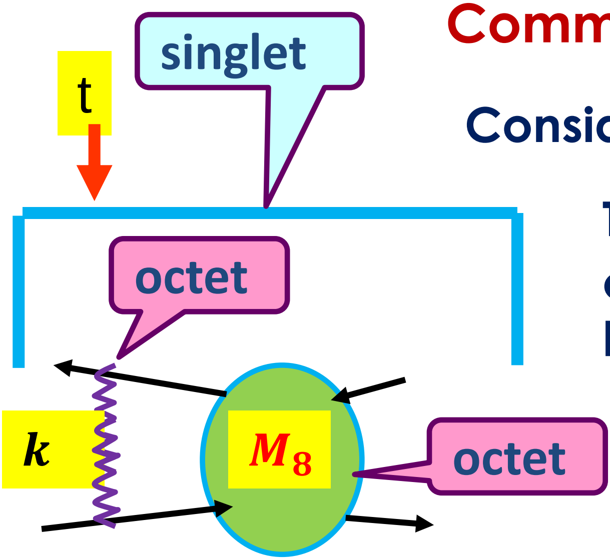
$$A(\omega) = (1/b)\left[\frac{\zeta}{\zeta^2+\pi^2} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\zeta)^2+\pi^2}\right]$$

$$A'(\omega) = (1/b)\left[\frac{1}{\zeta} - \int_0^\infty dz \frac{e^{-z\omega}}{(z+\zeta)^2}\right]$$

Ermolaev-Greco-Troyan

$$\zeta = ln(\mu^2/\Lambda_{QCD}{}^2), \qquad b = (11N - 2\,n_f)/(12\,\pi)$$

Comment on the color octets $V_{ik}(\omega)$



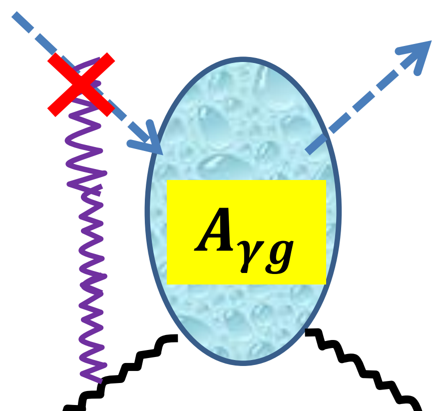
Consider a term of IREE with the factorized gluon.

The gluon belongs to the vector representation of the color group SU(3). Hence, the amplitude M_8 belongs to the octet representation too

$$8 \otimes 8 = 1 \oplus 8_s \oplus 8_A \oplus \ \ldots$$

We have to compose IREEs for the parton-parton octet amplitudes.
It is done absolutely in the same way

Fortunately, we need not octet components for photon-parton amplitudes:
Photon-gluon vertices are absent in the QCD Lagrangian



Explicit expressions for amplitudes h_{ik}

$$h_{qq} = \frac{1}{2}\left[\omega - Z - \frac{b_{gg} - b_{qq}}{Z}\right] \qquad h_{qg} = \frac{b_{qg}}{Z}$$

$$h_{gq} = \frac{b_{gq}}{Z} \qquad h_{gg} = \frac{1}{2}\left[\omega - Z + \frac{b_{gg} - b_{qq}}{Z}\right]$$

$$Z = \frac{1}{\sqrt{2}}\left[U + W\right]$$

$$U = \omega^2 - 2\left(b_{qq} + b_{gg}\right)$$

$$W = \left[\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq}\right]^{1/2}$$

Substituting them in expressions for auxiliary amplitudes $F_{\gamma q}$ $F_{\gamma q}$ and then in expressions for photon-photon amplitudes, we arrive at explicit expressions for $A^{(M)}{}_{\gamma\gamma}$, $A^{(D)}{}_{\gamma\gamma}$

Asymptotics of light-by-light amplitudes at $s \to \infty$ can be found with Saddle-Point method:

$$A(s, Q_1{}^2, Q_2{}^2) = -\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho} F(\omega, y_1, y_2) = -\int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} e^{\omega\rho + \Psi(\omega, y_1, y_2)}$$

We remind that $\rho = ln(s/\mu^2), y_1 = ln(Q_1{}^2/\mu^2), y_2 = ln(Q_2{}^2/\mu^2)$

Then the asymptotics is: At $\rho \to \infty$, $A \sim \frac{e^{\Psi(\omega_0, y_{1,2})}}{\sqrt{2\pi\Psi''(\omega_0, y_{1,2})}} \left(\frac{s}{\mu^2}\right)^{\omega_0}$

$\rho + \Psi'(\omega_0) = 0$

Stationary point equation

The leading singularity is the rightmost branching point and is given by largest root of

$$\left(\omega^2 - 2b_{gg} - 2b_{qq}\right)^2 - 4\left(b_{gg} - b_{qq}\right)^2 - 16 b_{qg} b_{gq} = 0$$

Asymptotics of light-by-light amplitudes is of the Regge form:

depends on impact-factors

Reggeon

$$A_{\gamma\gamma} \sim \frac{\lambda}{ln^{3/2}(s/\mu^2)} \left(\frac{s}{\sqrt{Q_1{}^2 Q_1{}^2}}\right)^{\omega_0}$$

This Reggeon has vacuum quantum numbers, so it is a new, DL contribution to Pomeron. It has nothing in common with BFKL Pomeron where $A_{\gamma\gamma} \sim s^{1+\Delta}$

We define $x = \left(\sqrt{Q_1{}^2 Q_1{}^2}\right)/s$

BFKL Pomeron is asymptotics of the SL series:

$$\left(\frac{1}{x}\right)\left[1 + c_1(\alpha_s ln(1/x)) + c_2(\alpha_s ln(1/x))^2 + c_3(\alpha_s ln(1/x))^3 + \cdots\right]$$

Asymptotics $x^{-\omega_0}$

$\omega_0 = 1 + \Delta$

comes from resummation

comes from the overall factor 1/x

DL Pomeron is asymptotics of the DL series:

$$1 + c'_1\left(\alpha_s ln^2(1/x)\right) + c'_2\left(\alpha_s ln^2(1/x)\right)^2 + c'_3\left(\alpha_s ln^2(1/x)\right)^3 + \cdots$$

The factor 1/x is absent, so the whole ω_0 comes entirely from calculations

For comparison with BFKL, it is convenient to introduce $\Delta \equiv \omega_0 - 1$

when α_s is fixed

$$\omega_{0_{fix}} = (\alpha_s/\pi)^{1/2}\left[4N + C_F + \sqrt{(4N - C_F)^2 - 8n_f C_F}\right]^{1/2}$$

α_s=0.24

on basis of PMS
Ermolaev-Greco-Troyan

A. quark contributions neglected, i.e. purely gluonic Pomeron

Δ_{fix}= 0.35

Close to LO BFKL intercept

B. both gluon and quark contributions accounted for

Δ_{fix}= 0.29

Accounting for the running α_s effects

C. Purely gluonic Pomeron $\quad \boldsymbol{\Delta = 0.25}$

D. Both gluon and quark contributions are taken into account

$$\boldsymbol{\Delta = 0.066}$$ ← Close to NLO BFKL intercept

We think that there is no a physical reason whatsoever for DL intercepts be close to BFKL ones and consider it as coincidence

OBSERVATION: The higher accuracy, the smaller the Pomeron intercept

Δ_{fix}= 0.35 — Fixed coupling, gluons only

Δ_{fix}= 0.29 — Fixed coupling, gluons and quarks

$\Delta = 0.25$ — Coupling runs, gluons only

$\Delta = 0.07$ — Coupling runs, gluons and quarks

SUGGESTION: this tendency suggests that eventually the intercept will go down to zero

Applicability region of the high-energy asymptotics

We introduce $R_{as} = Asympt\ A_{\gamma\gamma}/A_{\gamma\gamma}$

and plot it against *s*. The plot demonstrates that $R_{as} > 0.9$ when

$$s > s_{min} = 10^6 \sqrt{{Q_1}^2 {Q_2}^2}$$

CONCLUSIONS

We have obtained explicit expressions for light-by-light scattering amplitudes $A_{\gamma\gamma}$ in DLA, with fixed and running QCD coupling

Applying Saddle-Point method to these expressions, we arrive at the Regge asymptotics, with the Reggeon bearing the vacuum quantum numbers. So, it a new, DL contribution to Pomeron.

Altough intercepts of DL Pomeron are not far from the ones of BFKL and both of them are supercritical, DL Pomeron has nothing in common with BFKL Pomeron which is asymptotics of total resummation of single-logarithmic contributions while we deal with double logarithms.

Value of the DL Pomeron intercept monotonically decreases with increase of accuracy of calculations. It tempts us to suggest that the further increase of accuracy should make the intercept be= 1, which would agree with the Froissart-Martin bound i.e. with Unitarity

Explicit expressions for scattering amplitudes are quite complicated. In contrast, their Regge asymptotics are represent by simple exponential expressions so they are often used though their applicability regions are unknown. Comparing $A_{\gamma\gamma}$ to their asymptotics , we fixed the applicability region of the high-energy asymptotics of $A_{\gamma\gamma}$

We think that Interference of BFKL and DL Pomerons contributions to different reactions should be examined in detail

This work was initiated by discussions during Diffraction-2016.
In this regard we are grateful to D.A. Ross, W. Schafer, A. Szczurek and especially to D.Yu. Ivanov