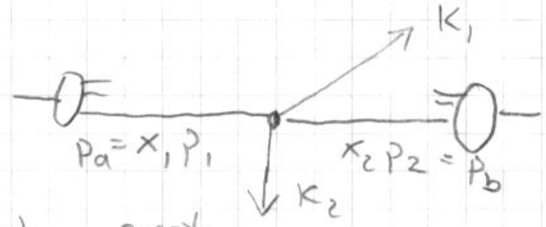
# Collider kinematics




Lab. system:

$$p_a = x_1 E(1, 0, 0, 1)$$
$$p_b = x_2 E(1, 0, 0, -1)$$

(beam along z-axis)

$$P_{cm} = p_a + p_b = E(x_1 + x_2, 0, 0, x_1 - x_2)$$

is boosted along z-axis by rapidity $y_{cm}$

$$\begin{pmatrix} E \\ P_z \end{pmatrix}_{cm} = \begin{pmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} \sqrt{s}/2 \\ 0 \end{pmatrix}$$

$$= \underbrace{\begin{pmatrix} \cosh y_{cm} & \sinh y_{cm} \\ \sinh y_{cm} & \cosh y_{cm} \end{pmatrix}}_{\exp\left\{ \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} y_{cm} \right\}} \begin{pmatrix} \sqrt{s}/2 \\ 0 \end{pmatrix}$$

boost generator rapidity

$$\left[ \frac{E + p_z}{E - p_z} = \frac{\cosh + \sinh}{\cosh - \sinh} = \frac{e^{y}}{e^{-y}} \right]$$

$$\Rightarrow \quad y_{cm} = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right)_{cm} = \frac{1}{2} \ln \frac{x_1}{x_2}$$

Product of boosts along z axis: add rapidities!

Rapidity of general 4 vector $k^\mu$

$$y = \frac{1}{2} \ln \frac{k^0 + k_z}{k^0 - k_z}$$

Boost along z axis leaves transverse momentum invariant

$k_z = 0$: $\quad k_0^\mu = (E_T, p_T \cos\varphi, p_T \sin\varphi, 0)$

$$E_T = \sqrt{m^2 + p_T^2}$$

general $k_z$:

$$k^\mu = (E_T \cosh y, p_T \cos\varphi, p_T \sin\varphi, E_T \sinh y)$$

$$= E(1, \beta \sin\theta \cos\varphi, \beta \sin\theta \sin\varphi, \beta \cos\theta)$$

pseudorapidity

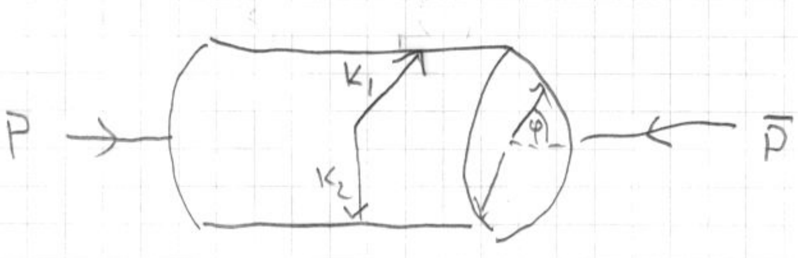
$$\eta \equiv \frac{1}{2} \ln \frac{1+\cos\theta}{1-\cos\theta}$$

massless case

$$y = \frac{1}{2} \ln \frac{k^0 + k_z}{k^0 - k_z} = \frac{1}{2} \ln \frac{E + E\cos\theta}{E - E\cos\theta} = \eta$$

- collider coordinates $E_T$ (or $p_T$), $y$ (or $\eta$), $\varphi$
- rapidity differences are boost invariant (boosts along z-axis)

Application: dijet production (massless jets)



$$p_a + p_b = k_1 + k_2$$

in lab: $\quad k_1^\mu = p_T(\cosh\eta_1, \cos\varphi, \sin\varphi_1, \sinh\eta_1)$

$\qquad\quad k_2^\mu = p_T(\cosh\eta_2, -\cos\varphi, -\sin\varphi, \sinh\eta_2)$

cm frame $\quad \eta_i \to \bar{\eta}_i \quad \eta_1 = y_{cm} + \bar{\eta}_1$

$\qquad\qquad\qquad\qquad\qquad \eta_2 = y_{cm} - \bar{\eta}_1$

$$x_{1/2} = \frac{2p_T}{\sqrt{s}} \cosh \frac{\eta_1 - \eta_2}{2} \, e^{\pm \frac{1}{2}(\eta_1 + \eta_2)}$$

$$= \frac{p_T}{\sqrt{s}} \left( e^{\frac{\eta_1 - \eta_2}{2}} + e^{\frac{-\eta_1 + \eta_2}{2}} \right) e^{\pm \frac{\eta_1 + \eta_2}{2}}$$

$$\Rightarrow x_1 = \frac{p_T}{\sqrt{s}} \left( e^{\eta_1} + e^{\eta_2} \right)$$

$$x_2 = \frac{p_T}{\sqrt{s}} \left( e^{-\eta_2} + e^{-\eta_1} \right)$$

$$\Rightarrow \quad Y_{cm} = \frac{1}{2}(\eta_1 + \eta_2)$$

$$\hat{\eta}_1 = \frac{1}{2}(\eta_1 - \eta_2) = \frac{1}{2} \ln \frac{1+\cos\theta_{CM}}{1-\cos\theta_{CM}}$$

$\rightarrow$ c.m scattering angle

in c.m frame : $\hat{k}_1^0 = \frac{\sqrt{\hat{s}}}{2} = p_T \cosh \hat{\eta}_1$

$$\Rightarrow \quad \hat{s} = 4 p_T^2 \cosh^2 \frac{\eta_1 - \eta_2}{2} = \underbrace{x_1 x_2}_{\tau} s$$

$$Y_{cm} = \frac{1}{2} \ln \frac{x_1}{x_2}$$

$$\Rightarrow \quad \boxed{\begin{aligned} x_1 &= \sqrt{\tau}\, e^{Y_{cm}} \\ x_2 &= \sqrt{\tau}\, e^{-Y_{cm}} \end{aligned}} \quad \begin{aligned} &= \frac{p_T}{\sqrt{s}} (e^{\eta_1} + e^{\eta_2}) \\ &= \frac{p_T}{\sqrt{s}} (e^{-\eta_1} + e^{-\eta_2}) \end{aligned}$$

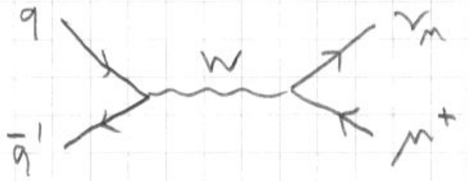
Note : in cross section formula integrate over $x_1, x_2$

$$dx_1\, dx_2 = \underbrace{\begin{vmatrix} \frac{\partial x_1}{\partial \tau} & \frac{\partial x_2}{\partial \tau} \\ \frac{\partial x_1}{\partial Y_{cm}} & \frac{\partial x_2}{\partial Y_{cm}} \end{vmatrix}}_{\begin{vmatrix} \frac{1}{2\sqrt{\tau}} e^{Y} & \frac{1}{2\sqrt{\tau}} e^{-Y} \\ \sqrt{\tau}\, e^{Y} & -\sqrt{\tau}\, e^{-Y} \end{vmatrix} = 1} d\tau\, dY_{cm} = d\tau\, dY_{cm}$$

- determine $x_1, x_2$ from final state
- determine $f_{a/p}(x_1) f_{b/p}(x_2)$ from measured rate

## W production

Consider $W^+ \to \mu^+ \nu_\mu$ signal




$q\bar{q}' = u\bar{d},\ u\bar{s},\ c\bar{d},\ c\bar{s}$

$$\frac{d\hat{\sigma}}{d\cos\theta} \sim \overline{\sum} |\mathcal{M}|^2 \sim |V_{qq'}|^2 \frac{1}{(\hat{s}-m_W^2)^2 + (m_W\Gamma_W)^2} (1+\cos\theta)^2$$

$V_{qq'}$ : CKM matrix

$$V_{qq'} \approx \begin{array}{cc} \begin{array}{ccc} d & s & b \end{array} & \\ \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix} & \begin{array}{c} u \\ c \\ t \end{array} \end{array} \qquad \begin{array}{c} \sin^2\theta_c = |V_{us}|^2 \\ \approx 0.05 \end{array}$$

$u\bar{b} \to W$ or $c\bar{b} \to W$ negligible because

$$|V_{ub}|^2 \ll |V_{cb}|^2 \approx 0.0017$$

and small b pdf

$\Rightarrow$ CKM mixing effect small compared to NLO QCD corrections

Corrections are proportional to

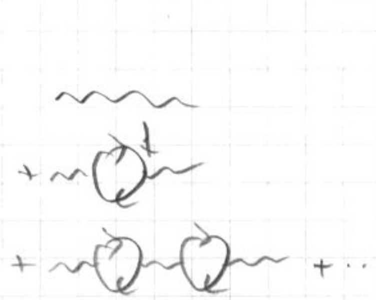
$$\sin^2\theta_c \underbrace{(u(x_1)-c(x_1))(\bar{d}(x_2)-\bar{s}(x_2))}_{\text{suppressed at small } x \text{, ie. LHC}}$$

## Breit-Wigner propagator

$$\mathcal{M} \sim \frac{1}{\hat{s}-m_W^2 + i m_W \Gamma_W}$$

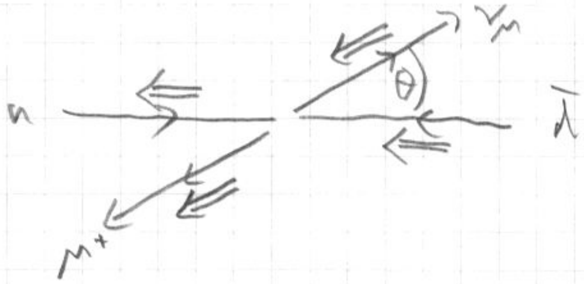
: resum

$i\,\mathrm{Im}\,\Pi_{WW}(\hat{s})$



→ fixed order perturbation theory does not work : need to resum unstable particle propagators

$\underline{A_{FB}}$ : W couples to <u>left</u> handed fermions




angular momentum not conserved at $\theta = \pi$

$\Rightarrow \mathcal{M} \sim 1 + \cos\theta$

<u>Problem</u> $\nu_\mu$ not observed $\Rightarrow \cos\theta$ cannot be measured : only $p_T$ and $\eta_{\mu^+}$ available

but

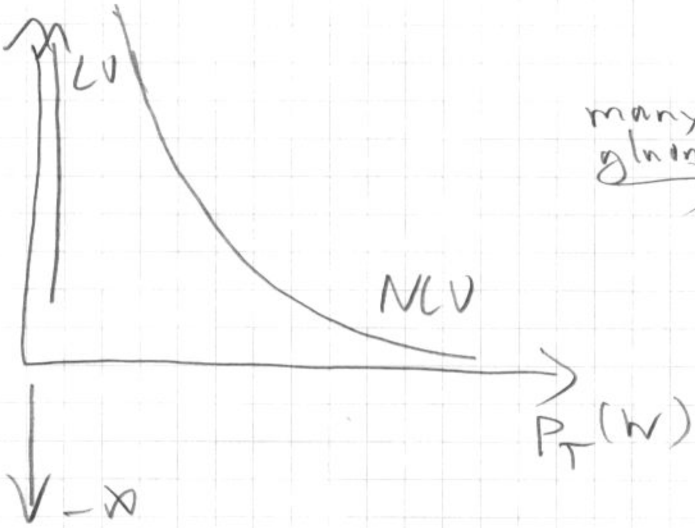
$$\frac{d\sigma}{dp_T} = \frac{d\sigma}{d\cos\theta}\left|\frac{d\cos\theta}{dp_T}\right| = \frac{d\sigma}{d\cos\theta}\,\frac{4p_T}{\hat{s}}\,\frac{1}{\sqrt{1-4p_T^2/\hat{s}}}$$

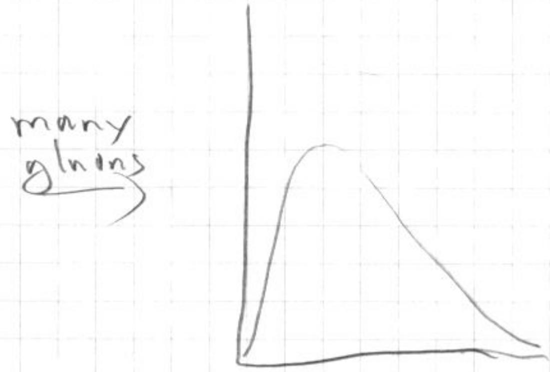
with $p_T = \frac{\sqrt{\hat{s}}}{2}\sin\theta \Rightarrow \cos\theta = \sqrt{1-\frac{4p_T^2}{\hat{s}}}$

Jacobian factor, diverges at

$p_{T\,max} = \frac{\sqrt{\hat{s}}}{2} = \frac{m_W}{2}$ in narrow width approx

W produced at finite $p_T$ at NLO


Absorb $p_T$ kick by using transverse mass instead

$$m_T^2 \equiv (\overbrace{E_{Te}}^{|\vec{p}_{Te}|} + E_{T\nu})^2 - (\vec{p}_{Te} + \vec{p}_{T\nu})^2$$

$$= 2(E_{Te}E_{T\nu} - \vec{p}_{Te}\cdot\vec{p}_{T\nu}) \leq m_W^2$$