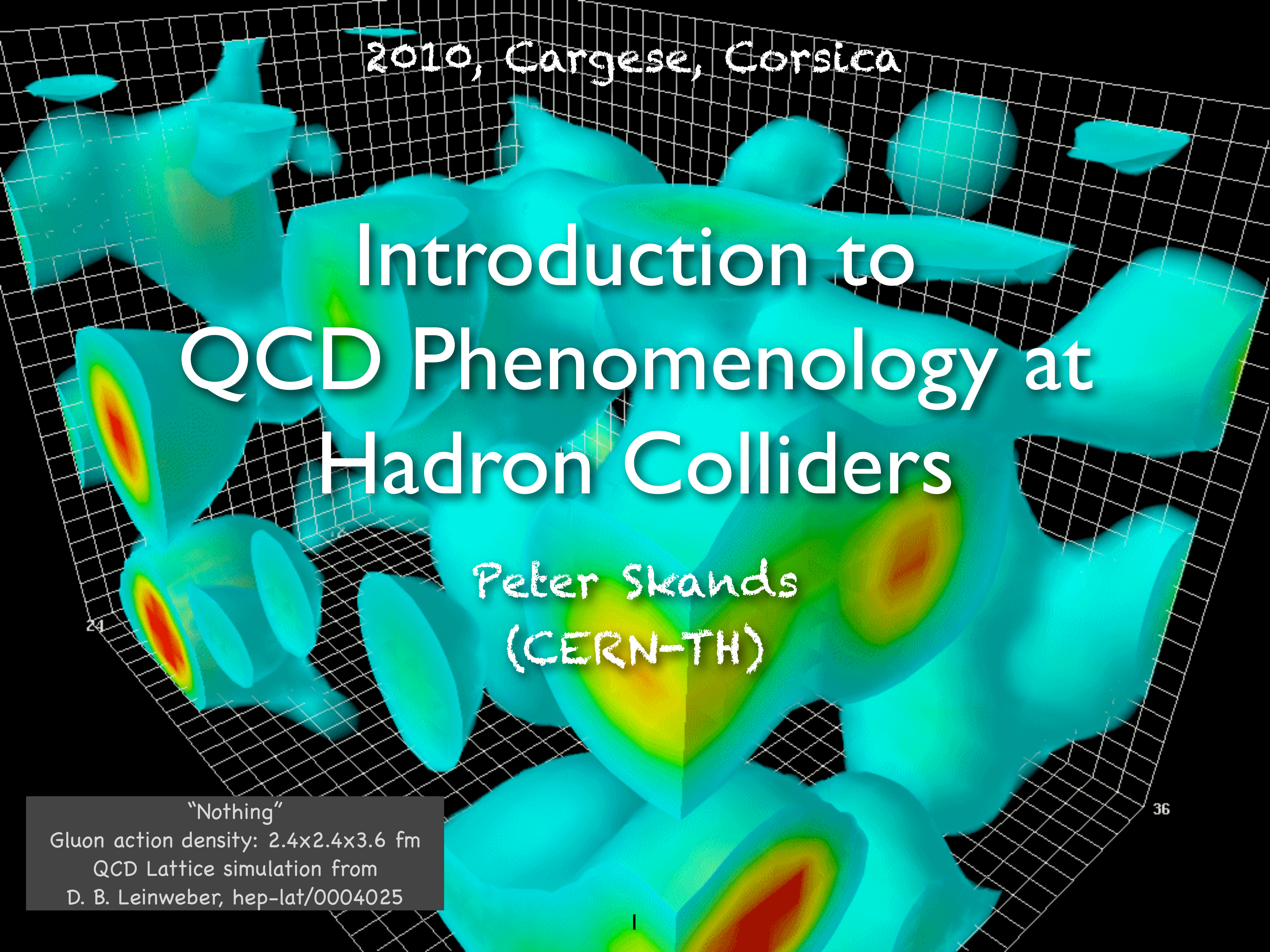
2010, Cargese, Corsica

Introduction to QCD Phenomenology at Hadron Colliders

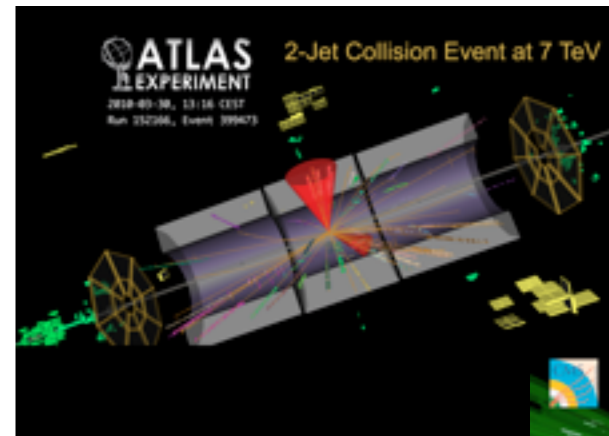
Peter Skands
(CERN-TH)

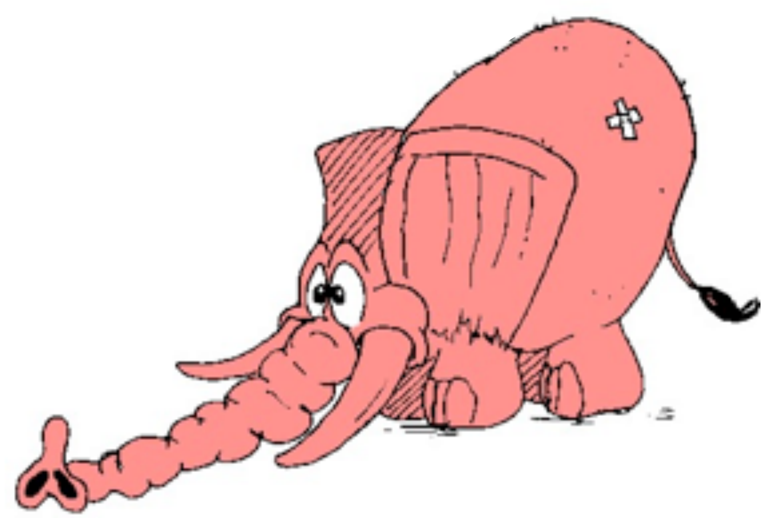


"Nothing"
Gluon action density: 2.4x2.4x3.6 fm
QCD Lattice simulation from
D. B. Leinweber, hep-lat/0004025

Collider Physics

Comparisons
to Collider
observables

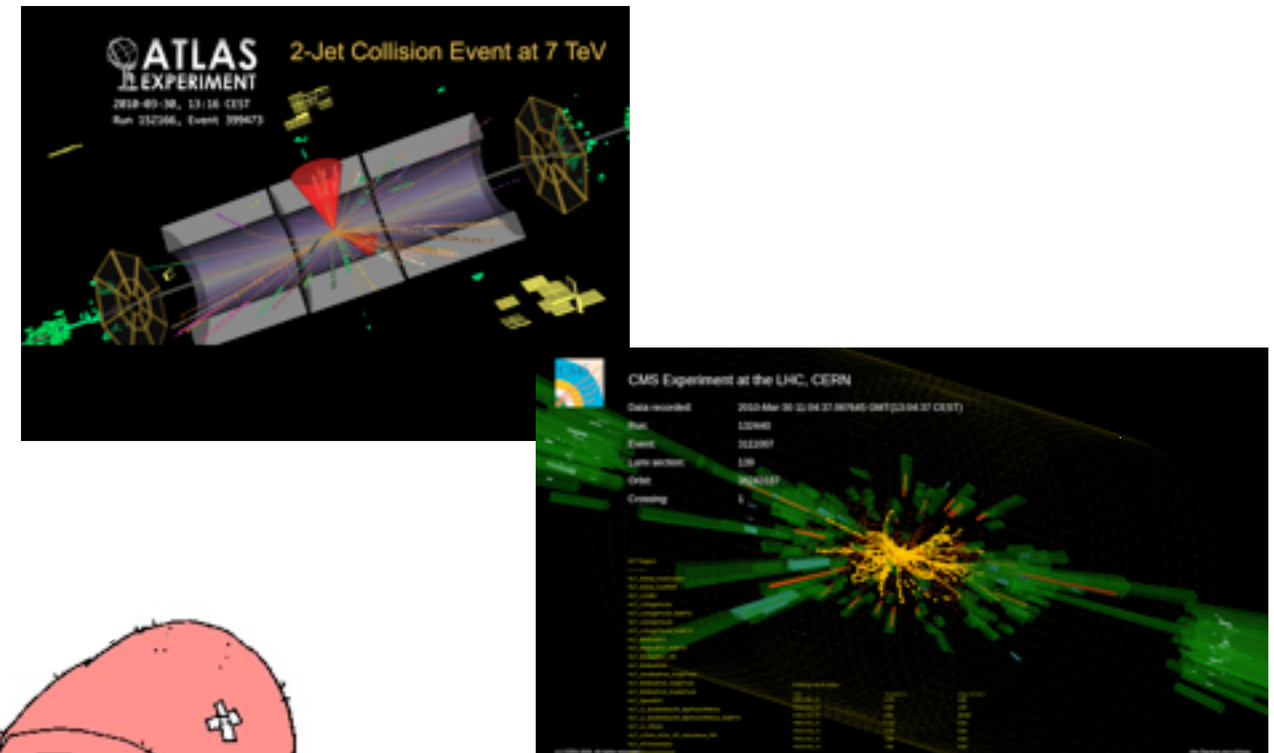


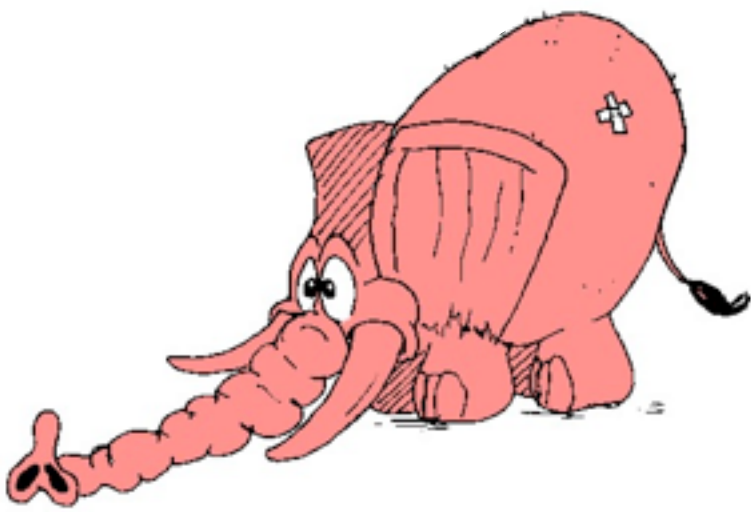


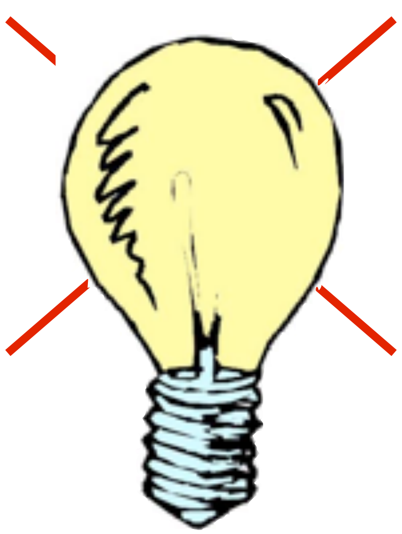
L=...

Collider Physics

Comparisons to Collider observables





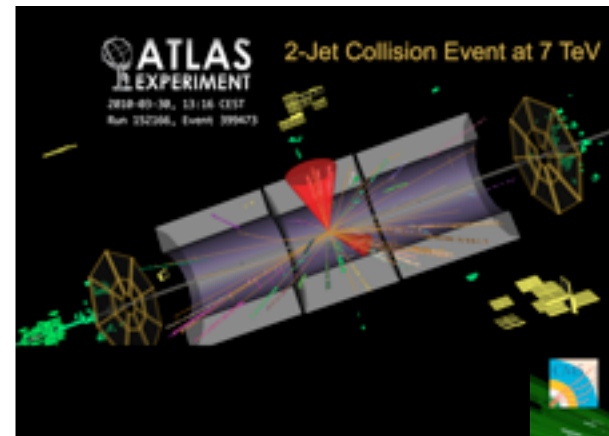


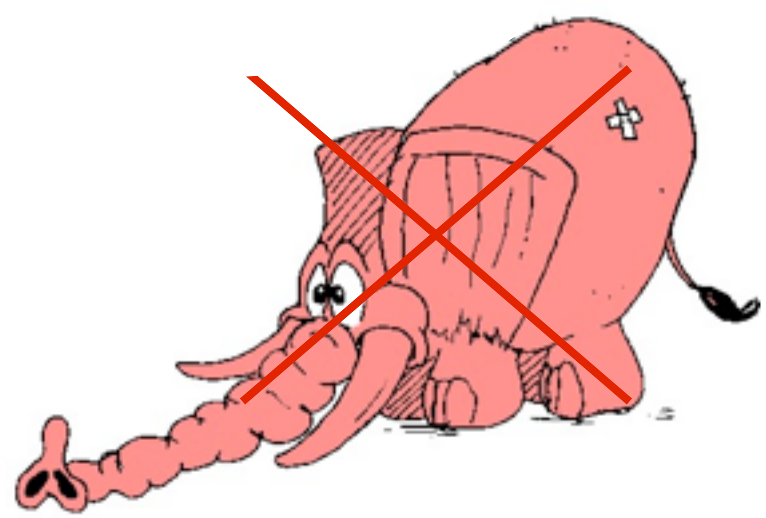
L=...

A) Theoretical Idea is wrong

Collider Physics

Comparisons to Collider observables





L=...

A) Theoretical Idea

B) SM Physics Model is wrong

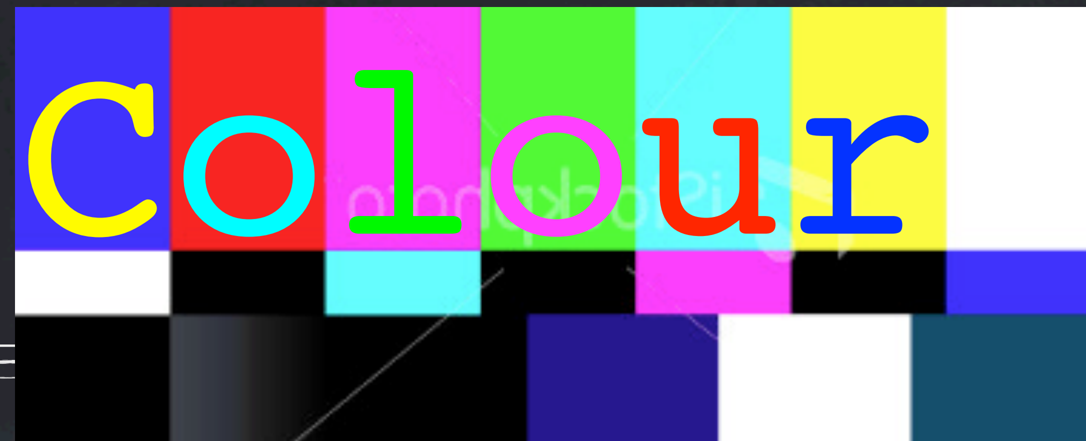
Disclaimer

Focus on (perturbative) QCD for collider physics

- QCD, Factorization, Hard Processes
- Monte Carlo Event Generators
- Matching & Tuning

Still, some topics not touched, or only briefly

- Not much time for Underlying Event, Hadronization, Min-Bias, ... ← This is my hobby / specialty, so please feel free to ask me offline
- Heavy flavor physics (e.g., B mesons, J/Psi, ...)
- Physics of hadrons, Lattice QCD
- Heavy ion physics
- DIS
- New Physics
- Prompt photon production, polarized beams, forward physics, diffraction, BFKL, ...



Gauge Group (= local internal space)

Special Unitary group in 3 (complex) dimensions, SU(3)

(Group of 3x3 unitary complex matrices with det=1)

Gluons

One gauge boson for each linearly independent such matrix

$3^2-1 = 8$: gluons are octets

Quarks

One quark color for each degree of SU(3)

3 : quarks are triplets (e.g., vectors on which matrices operate)

$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

(Antonio used $\mathbf{G}_\mu$ instead of TA_μ and G_μ instead of A_μ)

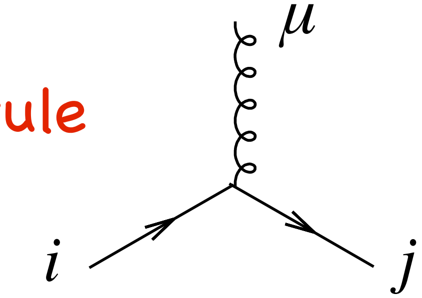
Quark fields

$$\psi_q^j = \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}$$

Covariant Derivative

$$D_{\mu ij} = \delta_{ij}\partial_\mu - i g_s T_{ij}^a A_\mu^a$$

$\Rightarrow$ Feynman rule



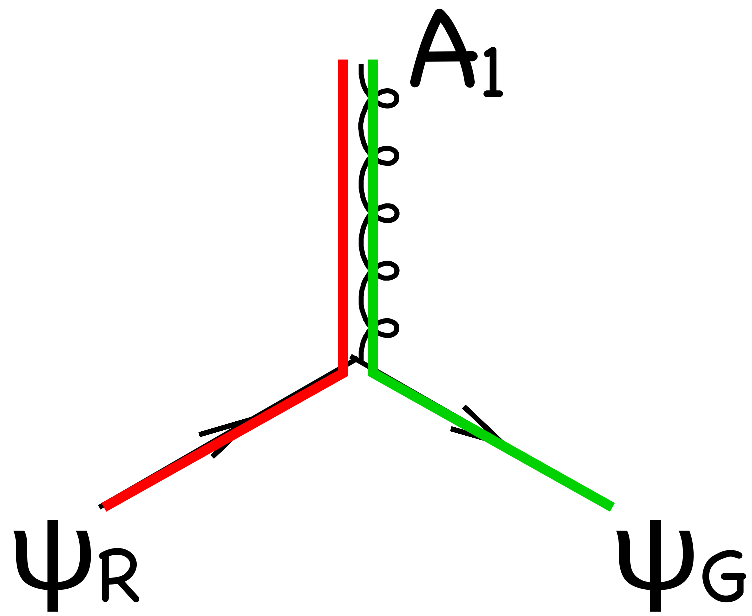
Gell-Mann Matrices $(T^a = \lambda^a/2)$

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \lambda^8 = \begin{pmatrix} \frac{1}{\sqrt{3}} & 0 & 0 \\ 0 & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & \frac{-2}{\sqrt{3}} \end{pmatrix}$$

Interactions in Colour Space

Quark-Gluon interactions



$$\underbrace{\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}}_{A_1} \underbrace{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}}_{\psi_R} = \underbrace{\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}}_{\psi_G}$$

Interactions in Colour Space

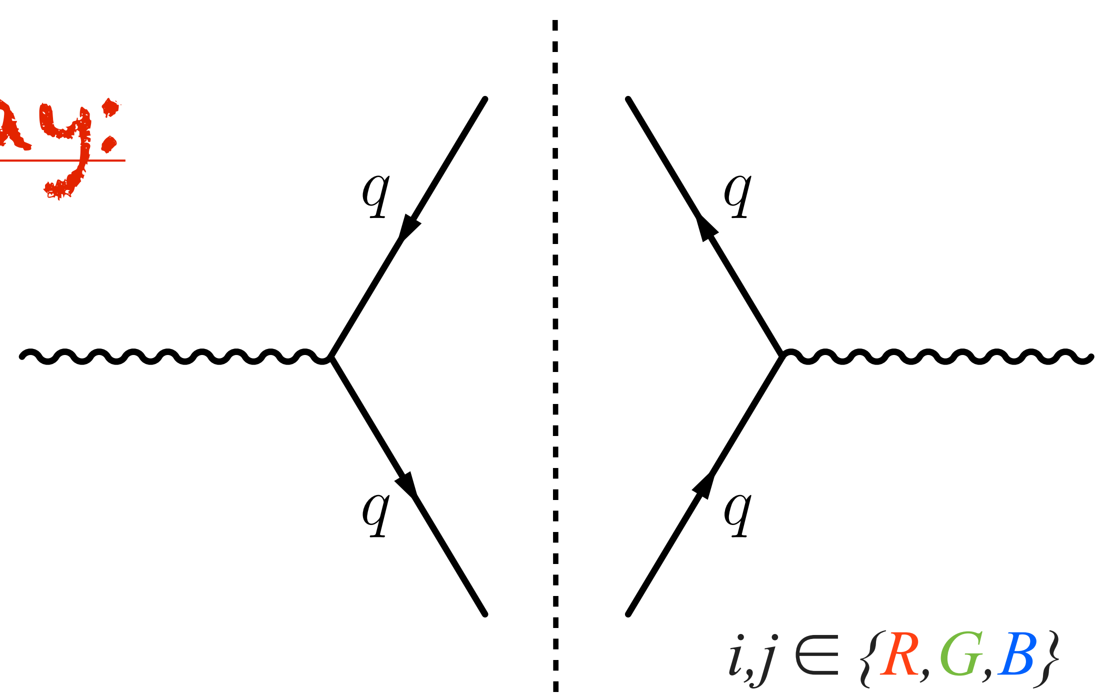
Colour Factors

We already saw pion decay and the "R" ratio depended on how many "color paths" we could take

All QCD processes have a "colour factor". It counts the enhancement from the sum over colours.

Z Decay:

$$\sum_{\text{colours}} |M|^2 =$$



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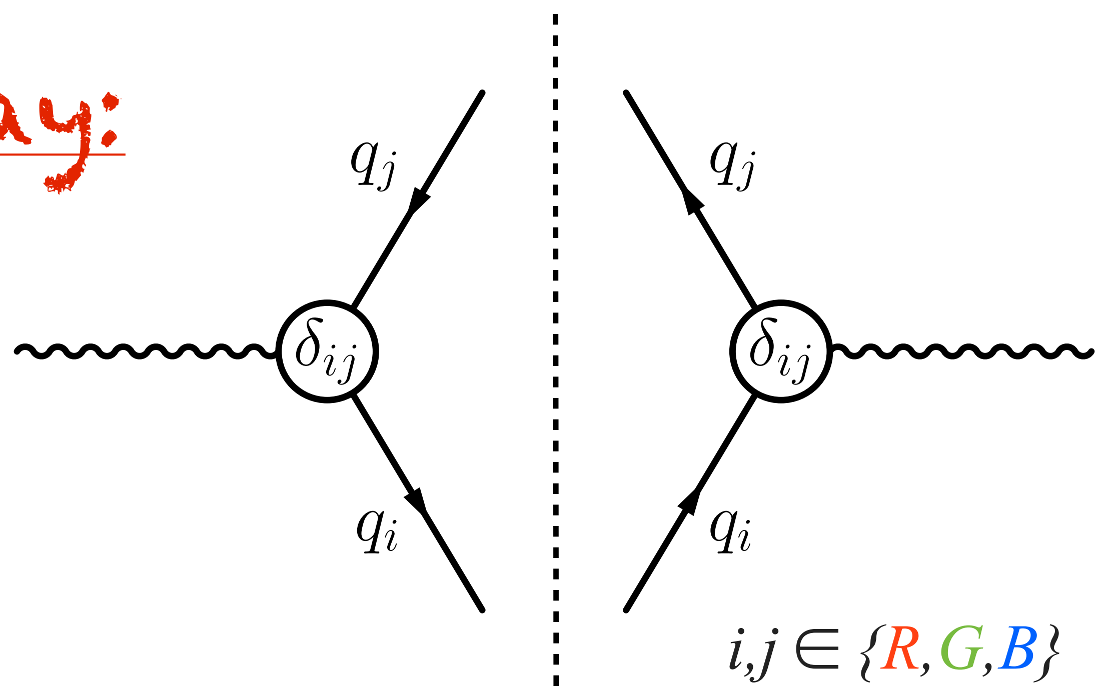

$$\propto \delta_{ij}\delta^*_{ji} = \mathrm{Tr}[\delta_{ij}] = N_C$$

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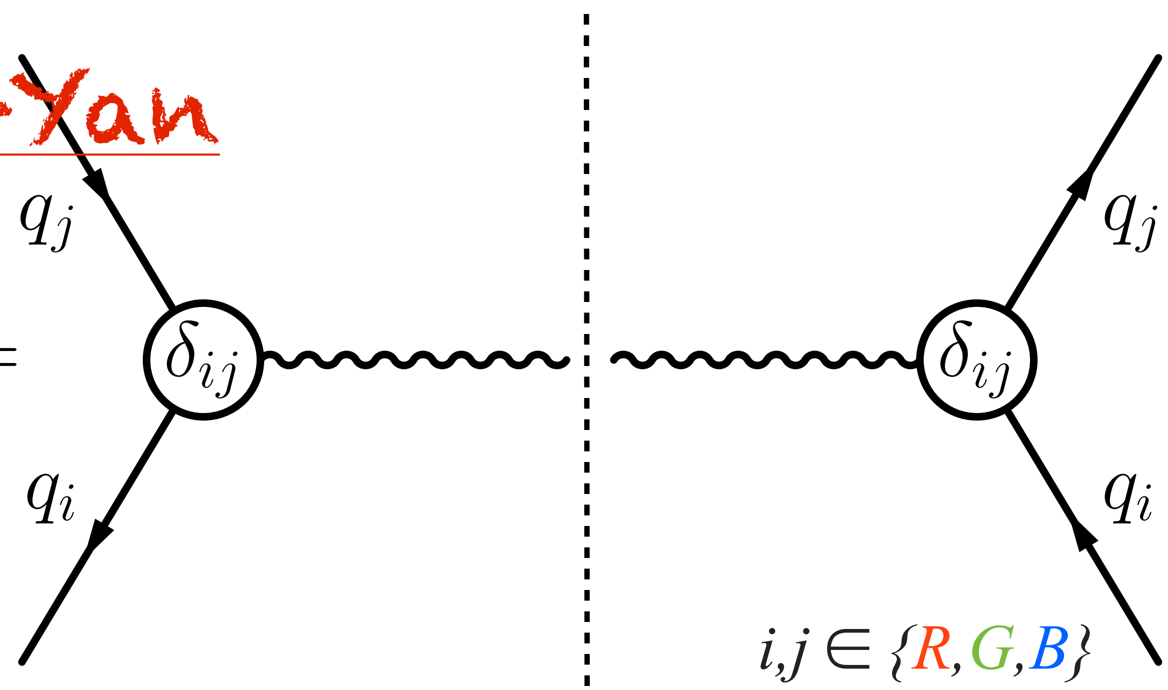
Drell-Yan

$$\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij}\delta^*_{ji}$$
$$= \mathrm{Tr}[\delta_{ij}]$$
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Interactions in Colour Space

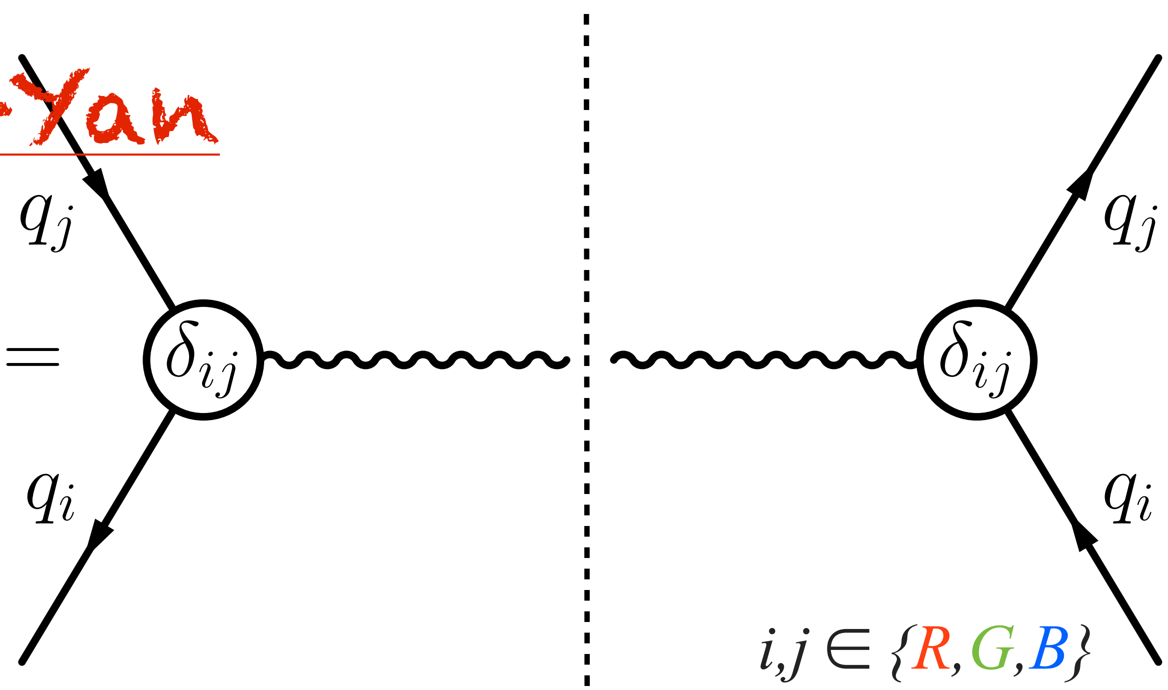
Colour Factors

We already saw pion decay and the “R” ratio depended on how many “color paths” we could take

All QCD processes have a “colour factor”. It counts the enhancement from the sum over colours.

Drell-Yan

$$\frac{1}{9}\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij}\delta^*_{ji}$$
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Interactions in Colour Space

Colour Factors

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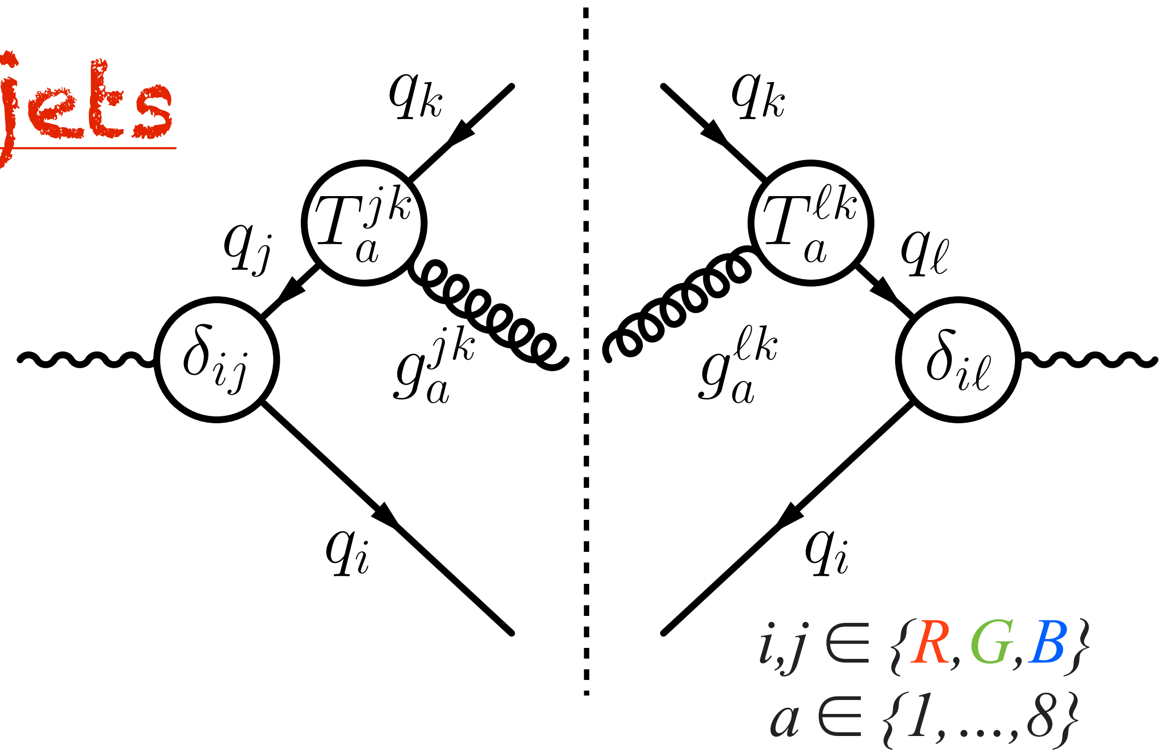
Z→3 jets

$$\sum_{\text{colours}} |M|^2 =$$



$$\propto \delta_{ij} T_a^{jk} (T_a^{\ell k} \delta_{i\ell})^*$$

$$= \mathrm{Tr}[T_a T_a]$$

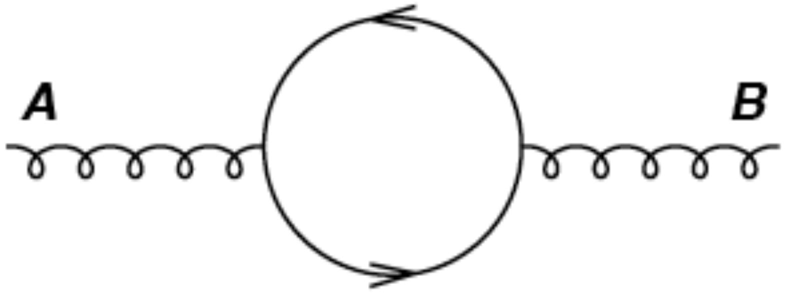
$$= \frac{1}{2} \mathrm{Tr}\,\delta_{ab}$$

$$= 4$$

Quick Guide to Colour Algebra

Colour factors squared produce traces

$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}, \quad T_R = \frac{1}{2}$$



(from lectures by G. Salam)

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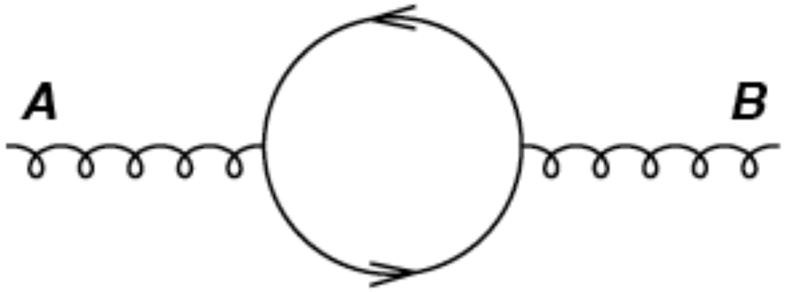
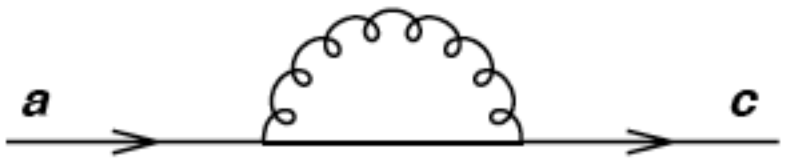


$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



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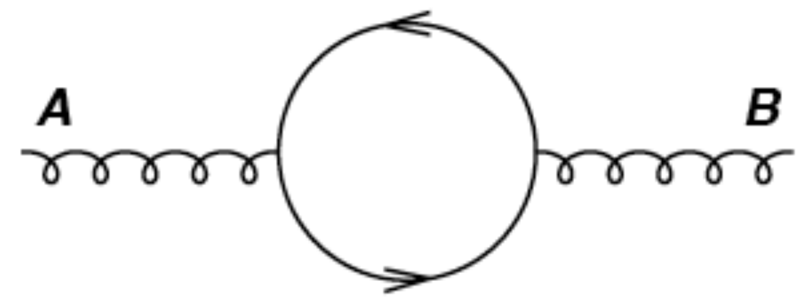
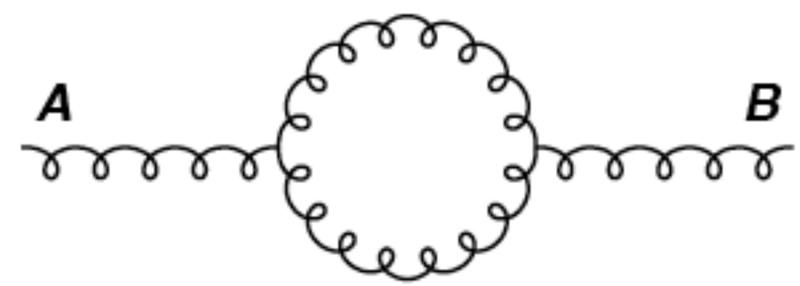
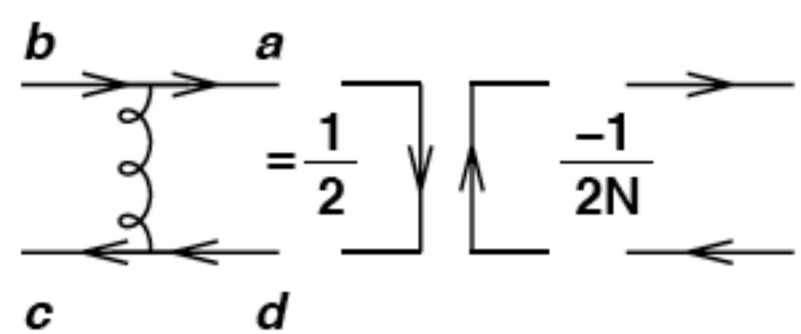


$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$

$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}, \quad C_A = N_c = 3$$



$$t^A_{ab} t^A_{cd} = \frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{2N_c}\delta_{ab}\delta_{cd} \quad \text{(Fierz)}$$



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Homework

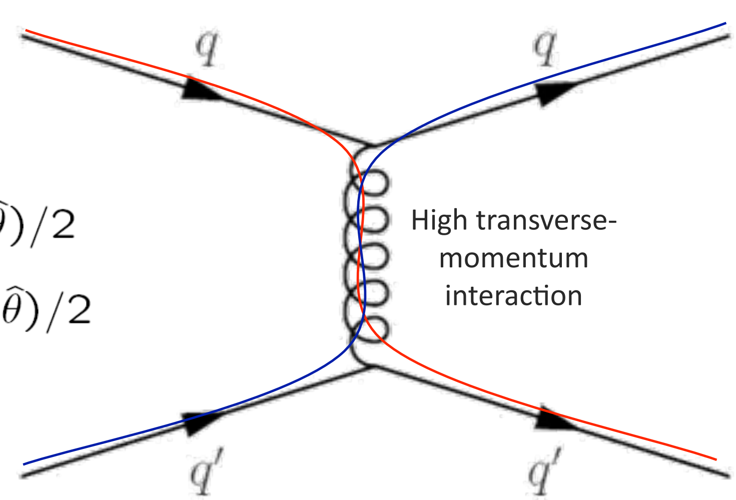
- The dominant process at Hadron Colliders is QCD 2→2 scattering (Rutherford Scattering)

$$qq' \to qq' \quad : \quad \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2}\frac{4}{9}\alpha_s^2\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}$$

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$



Question: what is the colour factor?

(hint: important to keep track of who has 3 indices and who has 8)

The Strong Coupling

Bjorken scaling

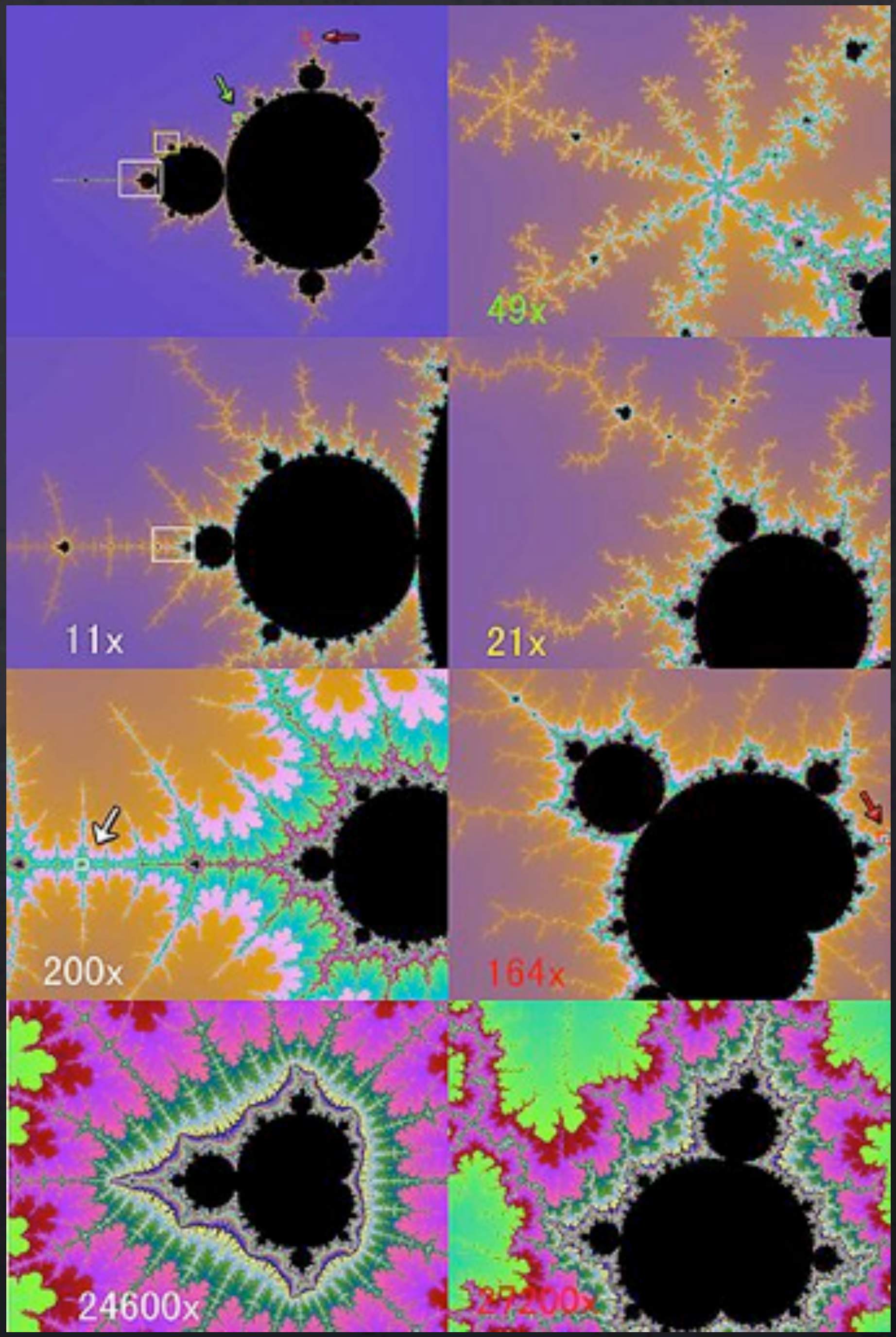
To first approximation, QCD is SCALE INVARIANT

(a.k.a. conformal)

A jet inside a jet inside a jet inside a jet ...

If the strong coupling did not run, this would be absolutely true

(e.g., N=4 SYM)

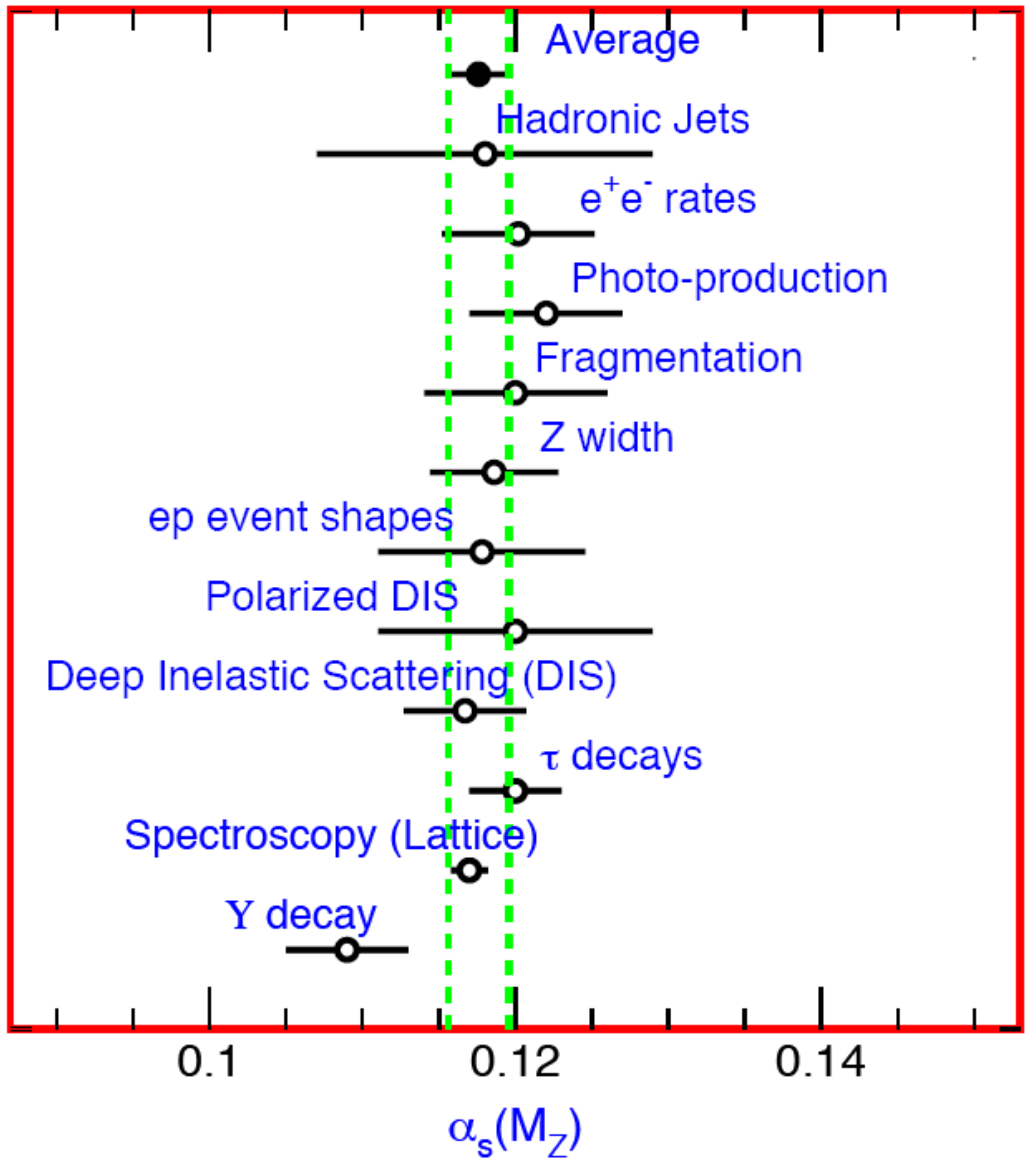


Conformal QCD

No running

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = 0$$

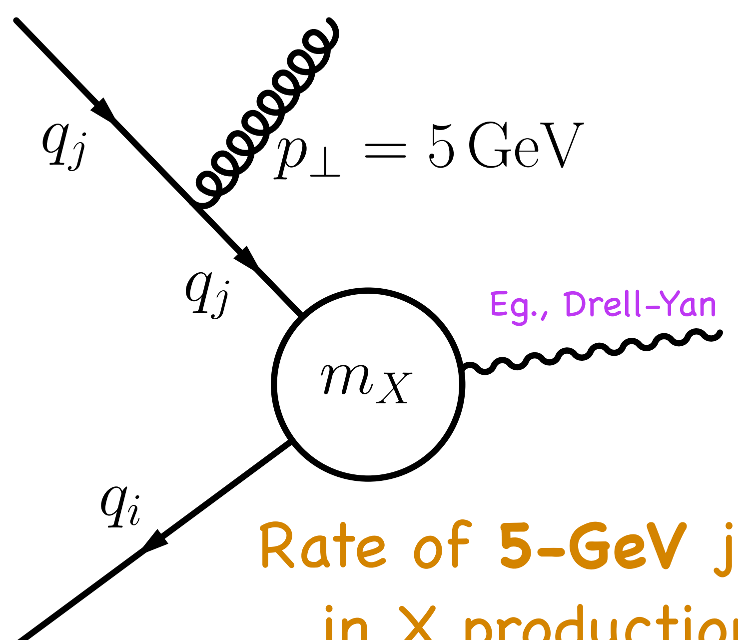
This simplification (QCD at fixed coupling) already captures some of the important properties of QCD



Conformal QCD

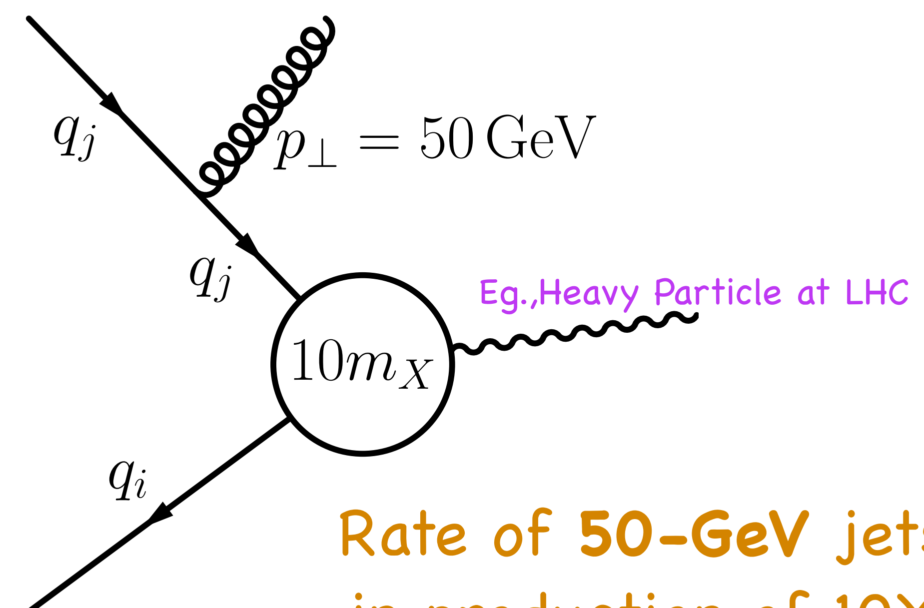
Bremsstrahlung

Rate of bremsstrahlung jets mainly depends on the **RATIO** of the jet p_T to the "hard scale"



Rate of **5-GeV** jets
in X production

$\approx$



Rate of **50-GeV** jets
in production of 10X

See, e.g.,

Plehn, Rainwater, PS: PLB645(2007)217
Plehn, Tait: 0810.2919 [hep-ph]
Alwall, de Visscher, Maltoni: JHEP 0902(2009)017

Conformal QCD

Naively, brems suppressed by $\alpha_s \approx 0.1$

Truncate at fixed order = LO, NLO, …

But beware the jet-within-a-jet-within-a-jet …

Know your signal
Especially if looking for decay
jets of similar $p_\perp$

Example:

100 GeV can be "soft" at the LHC

SUSY pair production at 14 TeV, with $M_{SUSY} \approx 600$ GeV

LHC - sps1a - m~600 GeV

Plehn, Rainwater, PS PLB645(2007)217

FIXED ORDER pQCD	σ_{tot}[pb]	$\tilde{g}\tilde{g}$	$\tilde{u}_L\tilde{g}$	$\tilde{u}_L\tilde{u}_L^*$	$\tilde{u}_L\tilde{u}_L$	TT
$p_{T,j} >$ 100 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
inclusive **X + 1 "jet"** →	σ_{1j}	2.89	2.74	0.136	0.145	0.73
inclusive **X + 2 "jets"** →	σ_{2j}	1.09	0.85	0.049	0.039	0.26
$p_{T,j} >$ 50 GeV	σ_{0j}	4.83	5.65	0.286	0.502	1.30
	σ_{1j}	5.90	5.37	0.283	0.285	1.50
	σ_{2j}	4.17	3.18	0.179	0.117	1.21

(Computed with SUSY-MadGraph)

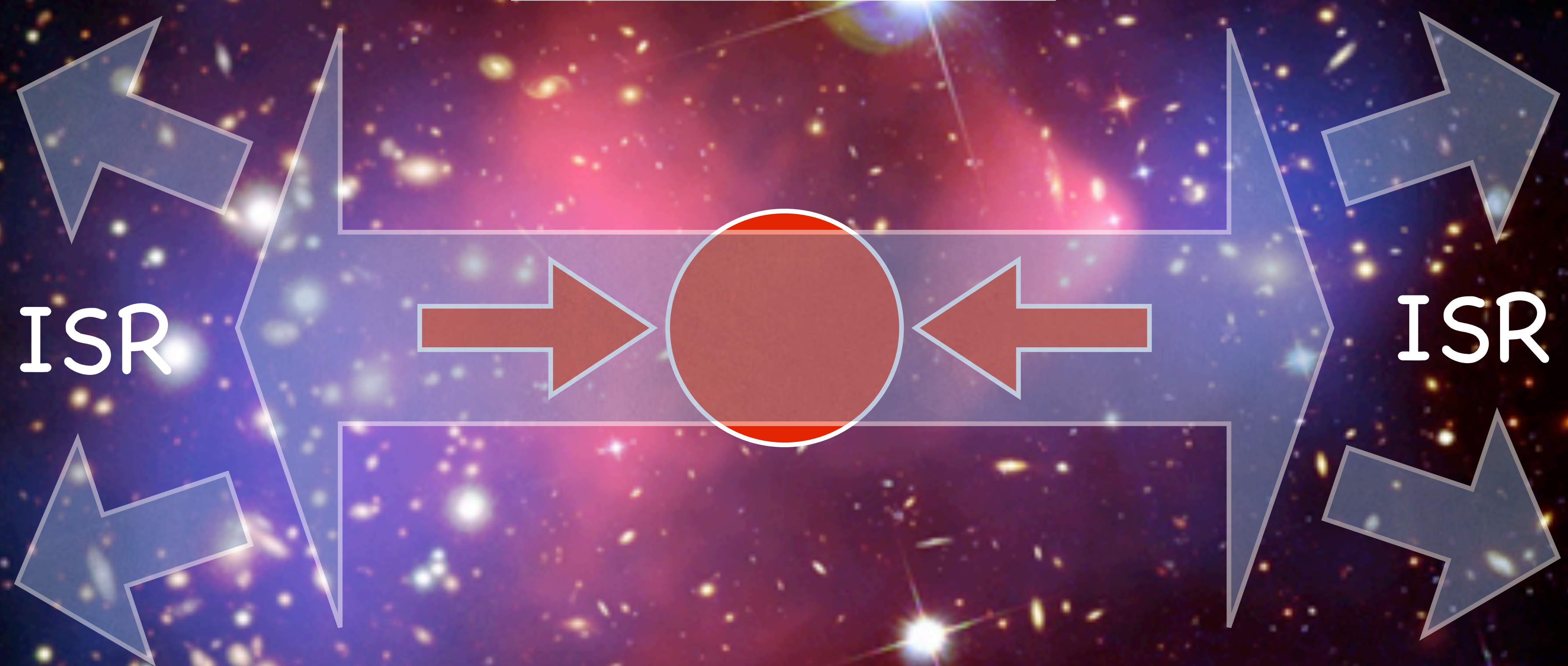
σ for X + jets much larger than naive estimate

σ for 50 GeV jets ≈ larger than total cross section → not under control

Caused by the conformal nature of quantum fluctuations inside fluctuations inside fluctuations …

Brems

Charges Stopped



The harder they stop, the harder the fluctations that continue to become strahlung

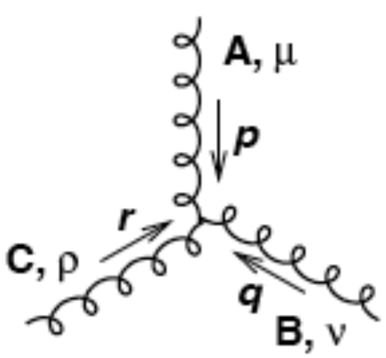
Gluons ≠ Photons

Gluon-Gluon Interactions

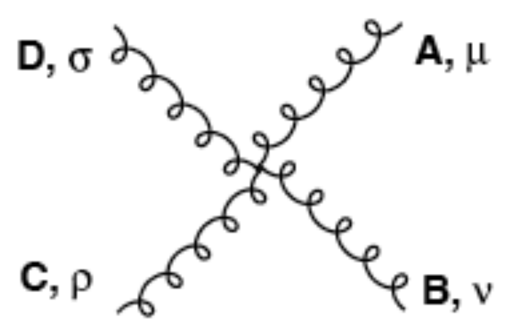
$$\mathcal{L} = \bar{\psi}_q^i (i\gamma^\mu)(D_\mu)_{ij}\psi_q^j - m_q \bar{\psi}_q^i \psi_{qi} - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$$

Gluon field strength tensor:

$$F^a_{\mu\nu} \;=\; \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f^{abc} A^b_\mu A^c_\nu$$



$-g_s f^{ABC}[(p-q)^\rho g^{\mu\nu}$
$+(q-r)^\mu g^{\nu\rho}$
$+(r-p)^\nu g^{\rho\mu}]$



$-ig_s^2 f^{XAC} f^{XBD}[g^{\mu\nu} g^{\rho\sigma} -$
$g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow$
$(D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$

Structure constants of SU(3):

$$f_{123} = 1$$

$$f_{147} = f_{246} = f_{257} = f_{345} = \frac{1}{2}$$

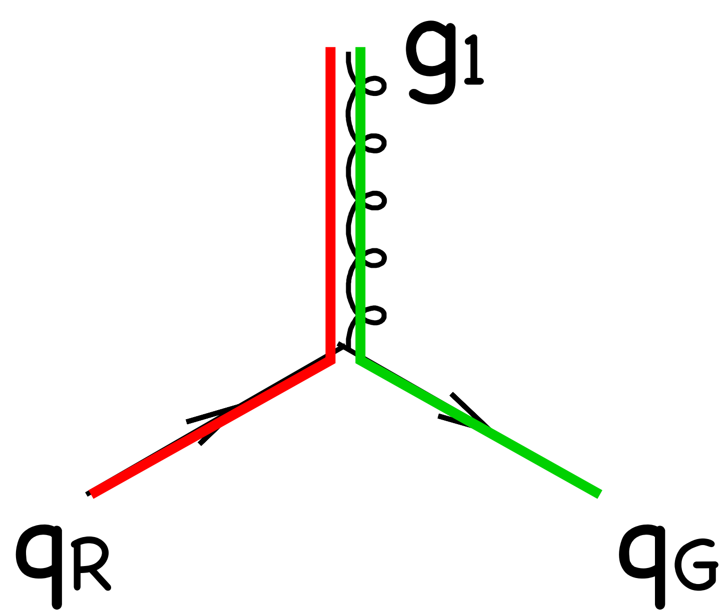
$$f_{156} = f_{367} = -\frac{1}{2}$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}$$

Antisymmetric in all indices

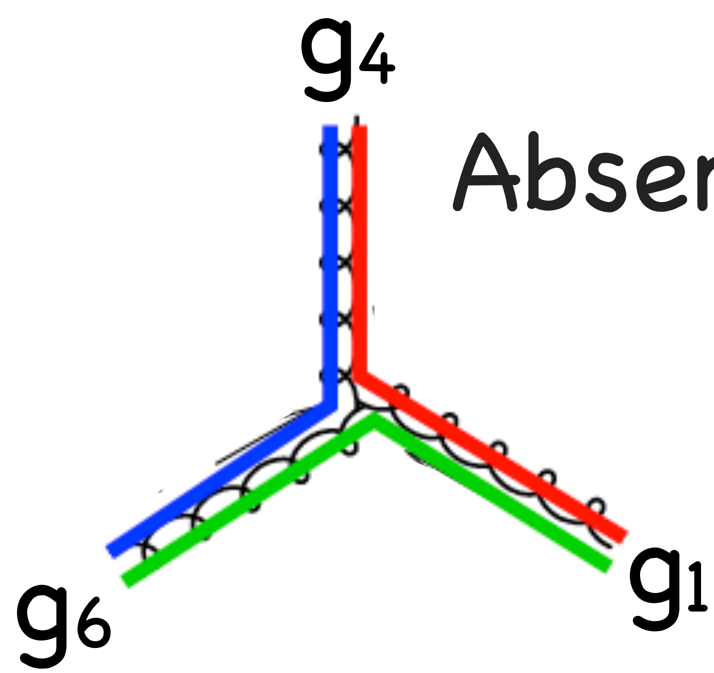
All other $f_{ijk} = 0$

Gluon self-interaction



$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

g_1 q_R q_G



Absent in QED

Twice as large as quark

Scaling Violation

In real QCD

The "beta function" of QCD

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s), \qquad \beta(\alpha_s) = -\alpha_s^2 (b_0 + b_1 \alpha_s + b_2 \alpha_s^2 + \ldots),$$

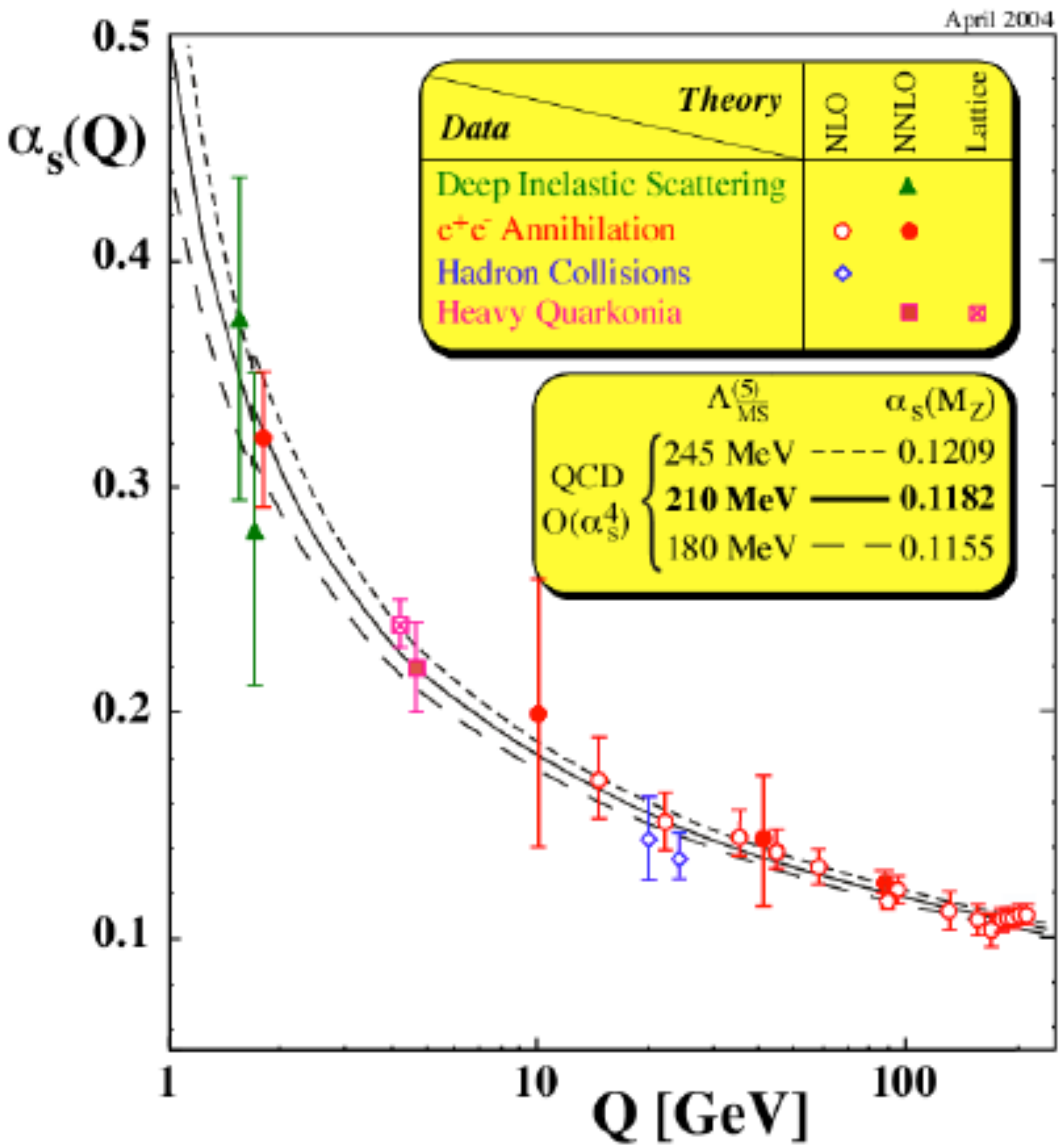
$$b_0 = \frac{11 C_A - 2 n_f}{12\pi}, \qquad b_1 = \frac{17 C_A^2 - 5 C_A n_f - 3 C_F n_f}{24\pi^2} = \frac{153 - 19 n_f}{24\pi^2}$$

The coupling runs logarithmically with the energy

Asymptotic freedom in the ultraviolet

Infrared slavery (confinement) in the IR

UV and IR



At current scales

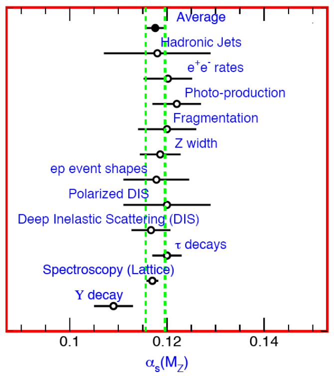
Coupling actually runs rather fast

Explodes at a scale somewhere below

≈ 1 GeV

So we usually give its value at a unique reference scale that everyone agrees on

The Fundamental Parameter(s)



QCD has one fundamental parameter

$\alpha_s(m_Z)^{\overline{\mathrm{MS}}}$

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\ \alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

+ n_f & quark masses

… and its sibling

$\Lambda_{\mathrm{QCD}}^{(n_f)\overline{\mathrm{MS}}}$

$$\alpha_s(Q^2) = \frac{1}{b_0\ \ln\frac{Q^2}{\Lambda^2}} \longleftarrow \Lambda \sim 200\,\mathrm{MeV}$$

… And all their cousins

$\alpha_s(m_Z)_{LO}$ $\alpha_s(m_Z)_{N^nLO}$ $\alpha_s(m_Z)_{N^nLO+N^nLL}$ $\alpha_s(m_Z)^{DIS}$ $\alpha_s(m_Z)^{DR}$, …

$\Lambda^{(3)}$ $\Lambda^{(4)}$ $\Lambda^{(5)}$ Λ_{CMW} Λ_{FSR} Λ_{ISR} Λ_{MPI} , …

Other parameters

> The emergent is unlike its components insofar as … it cannot be reduced to their sum or their difference."
>
> *G. Lewes (1875)*

Emergent phenomena

Cannot guess non-perturbative phenomena from perturbative QCD → "Emerge" due to confinement

Hadron masses,
Decay constants,
Fragmentation functions
Parton distribution functions,…

Difficult/Impossible to compute given only knowledge of perturbative QCD

→ Lattice QCD (only for "small" systems)
→ Experimental fits (for reference)
→ Phenomenological models (for everything else)

Image Credits: Yeimaya

The Way of the Chicken

- Who needs QCD? I'll use leptons
  - Sum inclusively over all QCD
    - Leptons almost IR safe by definition
    - WIMP-type DM, Z', EWSB → may get some leptons

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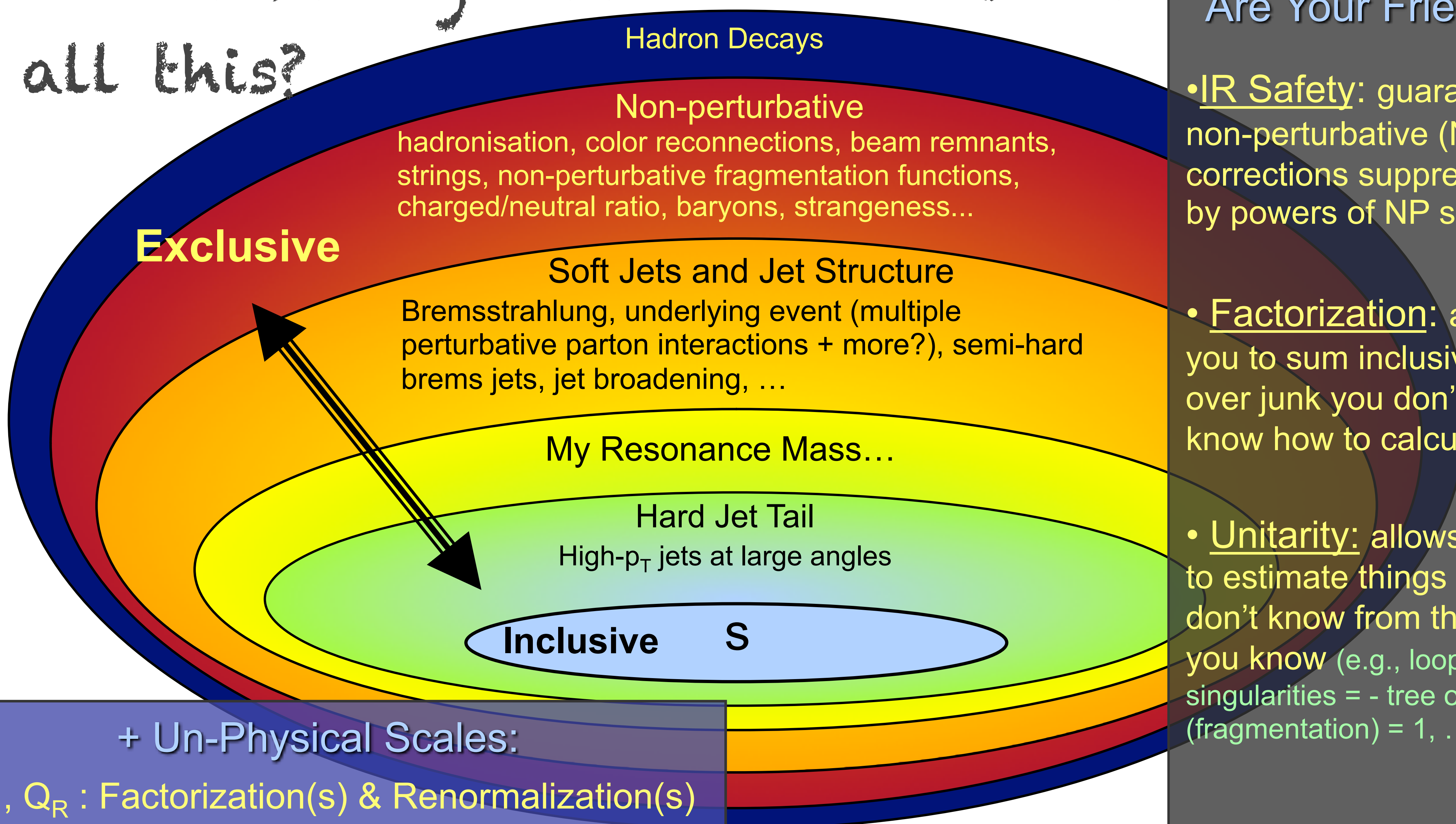
- Who needs QCD? I'll use leptons
  - Sum inclusively over all QCD
    - Leptons almost IR safe by definition
    - WIMP-type DM, Z', EWSB → may get some leptons
  - Beams = hadrons for next decade (RHIC / Tevatron / LHC)
    - At least need well-understood PDFs
    - High precision = higher orders → enter QCD (and more QED)
  - Isolation → indirect sensitivity to QCD
  - Fakes → indirect sensitivity to QCD
  - Not everything gives leptons
    - Need to be a lucky chicken …
- The unlucky chicken
  - Put all its eggs in one basket and didn't solve QCD

Collider Energy Scales

Do we really need to calculate all this?



These Things Are Your Friends

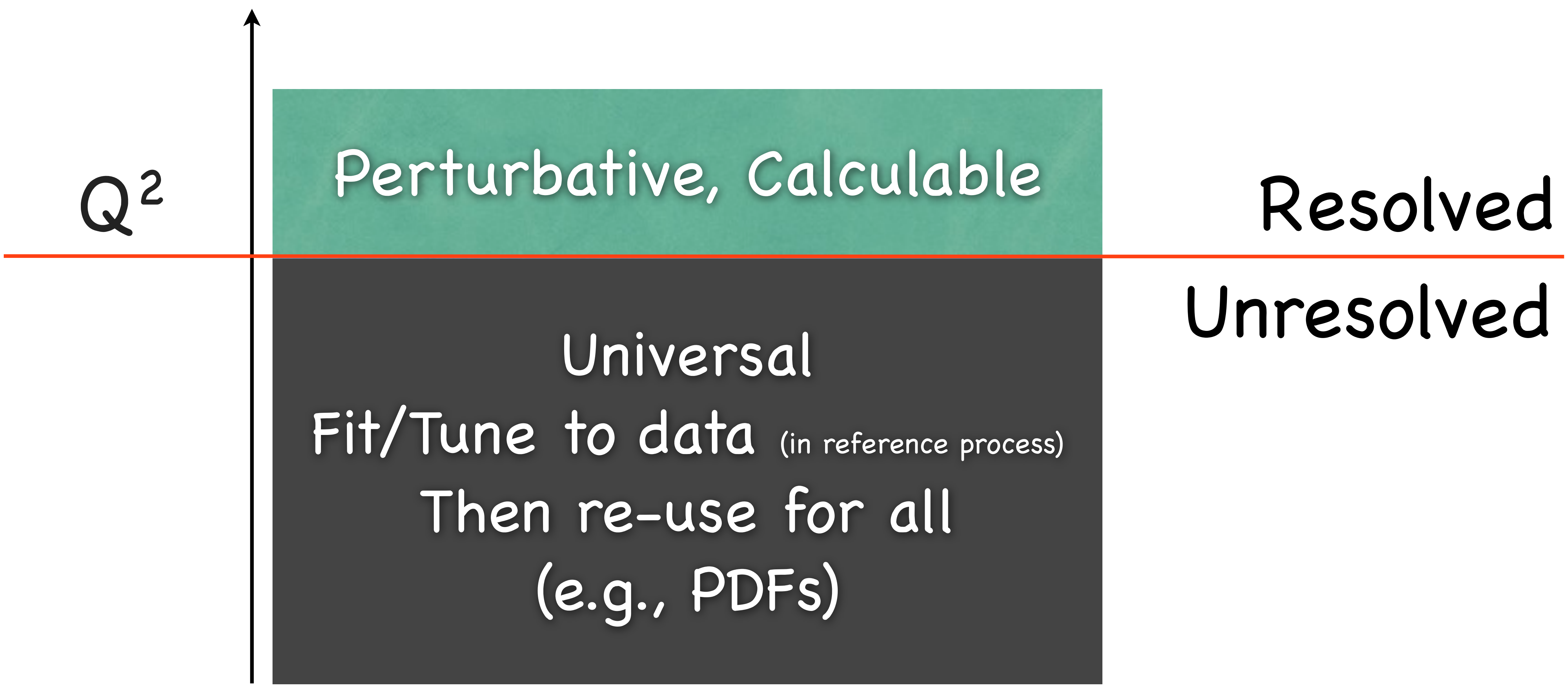
- IR Safety: guarantees non-perturbative (NP) corrections suppressed by powers of NP scale
- Factorization: allows you to sum inclusively over junk you don't know how to calculate
- Unitarity: allows you to estimate things you don't know from things you know (e.g., loop singularities = - tree ones; P (fragmentation) = 1, …)

+ Un-Physical Scales:

- Q_F , Q_R : Factorization(s) & Renormalization(s)
- Q_E : Evolution(s)

Factorization

Subdivide a calculation

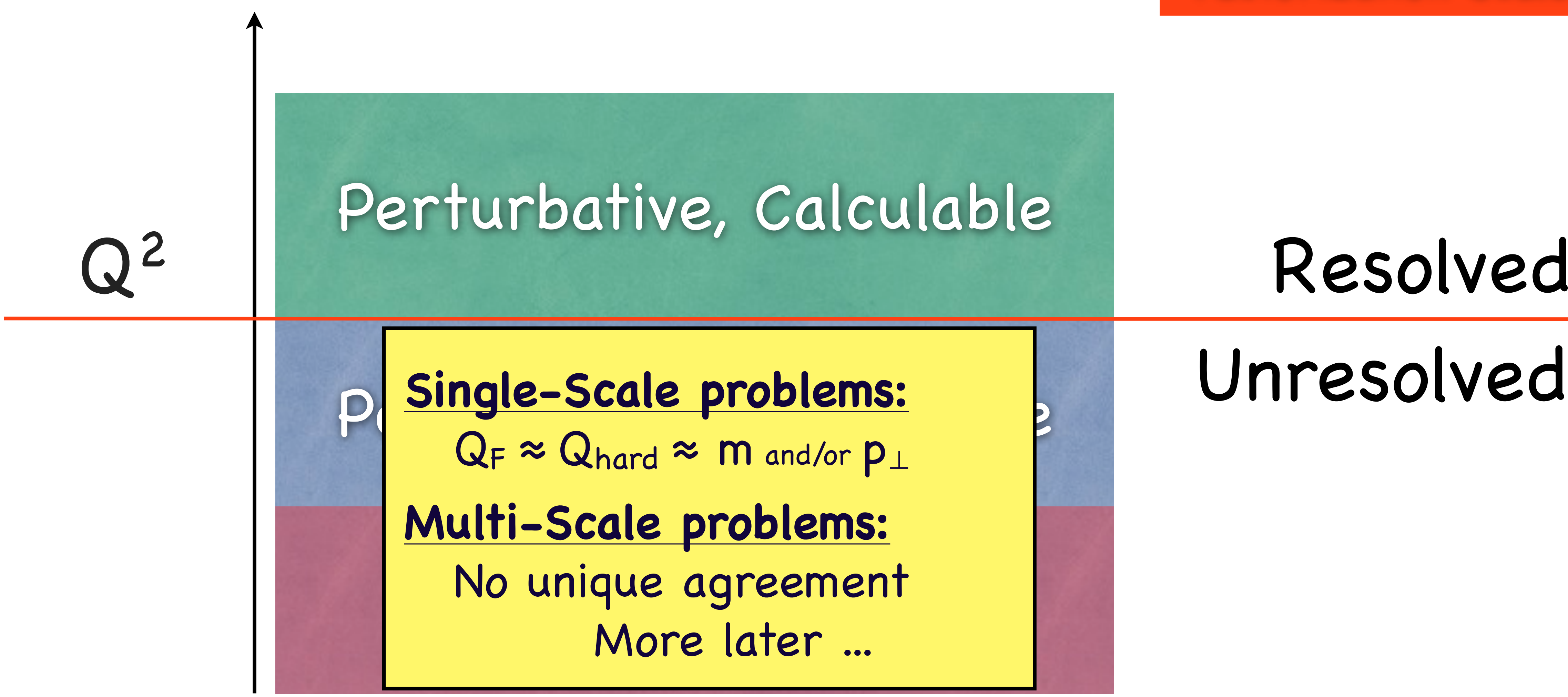


Factorization

Subdivide a calculation

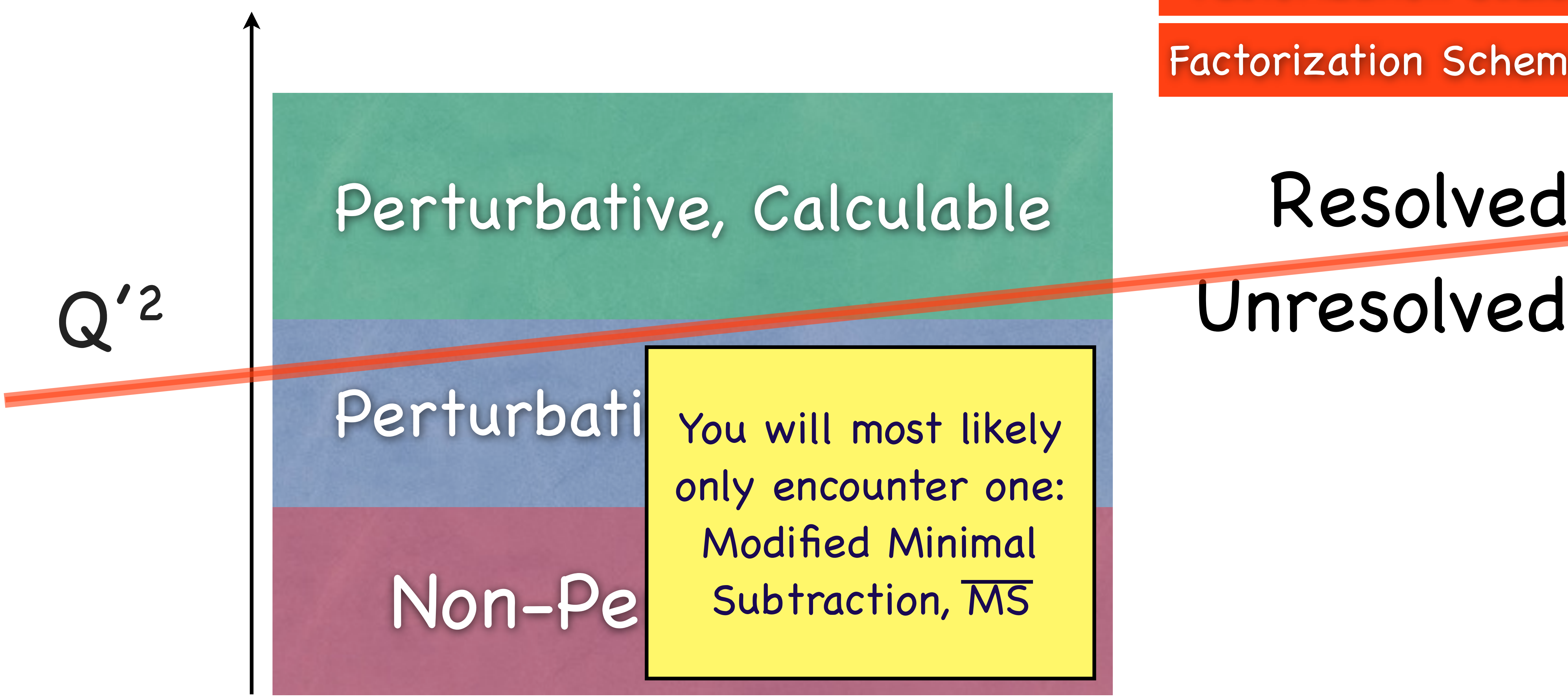
Dependence on

Factorization Scale



Factorization

Subdivide a calculation



Dependence on

Factorization Scale

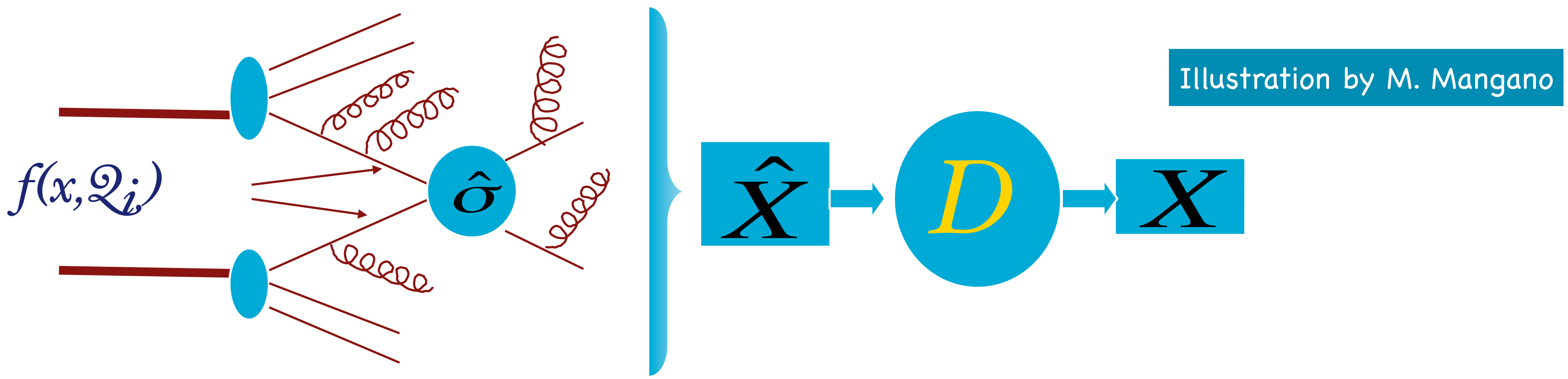
Factorization Scheme

Factorization Theorem

(See also Dieter Zeppendfeld's 1st lecture)

Factorization: expresses the independence of long-wavelength (soft) emission on the nature of the hard (short-distance) process.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}X} = \sum_{a,b} \sum_{f} \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to f}(x_a, x_b, f, Q_i^2, Q_f^2)}{\mathrm{d}\hat{X}_f} D(\hat{X}_f \to X, Q_i^2, Q_f^2)$$



$\vec{p}_j = x \vec{P}_{proton}$

$f_a(x_a, Q_i^2)$ Parton distribution functions (PDF)

- sum over long-wavelength histories leading to a with x_a at the scale Q_i^2 (ISR)

$D(\hat{X}_f \to X, Q_i^2, Q_f^2)$ Fragmentation Function (FF)

- Sum over long-wavelength histories from $\hat{X}_f$ at Q_f^2 to X (FSR and Hadronization)

+ (At H.O. each of these defined in a specific scheme, usually $\overline{\mathrm{MS}}$)

Uncalculated Orders

Naively $O(\alpha_s)$ - True in e^+e^- !

$$\sigma_1(e^+e^- \to q\bar{q}(g)) = \sigma_0(e^+e^- \to q\bar{q}) \left(1 + \frac{\alpha_s(E_{CM})}{\pi} + O(\alpha_s^2)\right)$$

Generally larger in hadron collisions

Typical "K" factor in pp (= σ_{NLO}/σ_{LO}) ≈ 1.5 ± 0.5

Why is this? Many pseudoscientific explanations

- Explosion of # of diagrams ($n_{Diagrams} \approx n!$)
- New initial states contributing at higher orders (E.g., $gq \to Zq$)
- Inclusion of low-x (non-DGLAP) enhancements
- Bad (high) scale choices at Lower Orders, ...

Their's not to reason why // Their's but to do and die

The Charge of the Light Brigade, by Alfred, Lord Tennyson

1. Changing the scale(s)

Why scale variation ≈ uncertainty?

Scale dependence of calculated orders must be canceled by contribution from uncalculated ones (+ non-pert)

$$\alpha_s(Q^2) = \alpha_s(m_Z^2)\frac{1}{1 + b_0\ \alpha_s(m_Z)\ln\frac{Q^2}{m_Z^2} + \mathcal{O}(\alpha_s^2)}$$

$$b_0 = \frac{11N_C - 2n_f}{12\pi}$$

→ $\alpha_s(Q'^2)\ |M|^2 - \alpha_s(Q^2)\ |M|^2 \approx \alpha_s^2(Q^2)\ |M|^2 + \ldots$

→ Generates terms of higher order, but proportional to what you already have → a first naive* way to estimate uncertainty

*warning: some theorists believe it is the only way … be agnostic!

Dangers

$p_{\perp 1}$= 50 GeV
$p_{\perp 2}$= 50 GeV
$p_{\perp 3}$= 50 GeV

Complicated final states

Intrinsically Multi-Scale problems
with Many powers of α_s

Hardest imaginable scale

E.g., W + 3 jets in pp

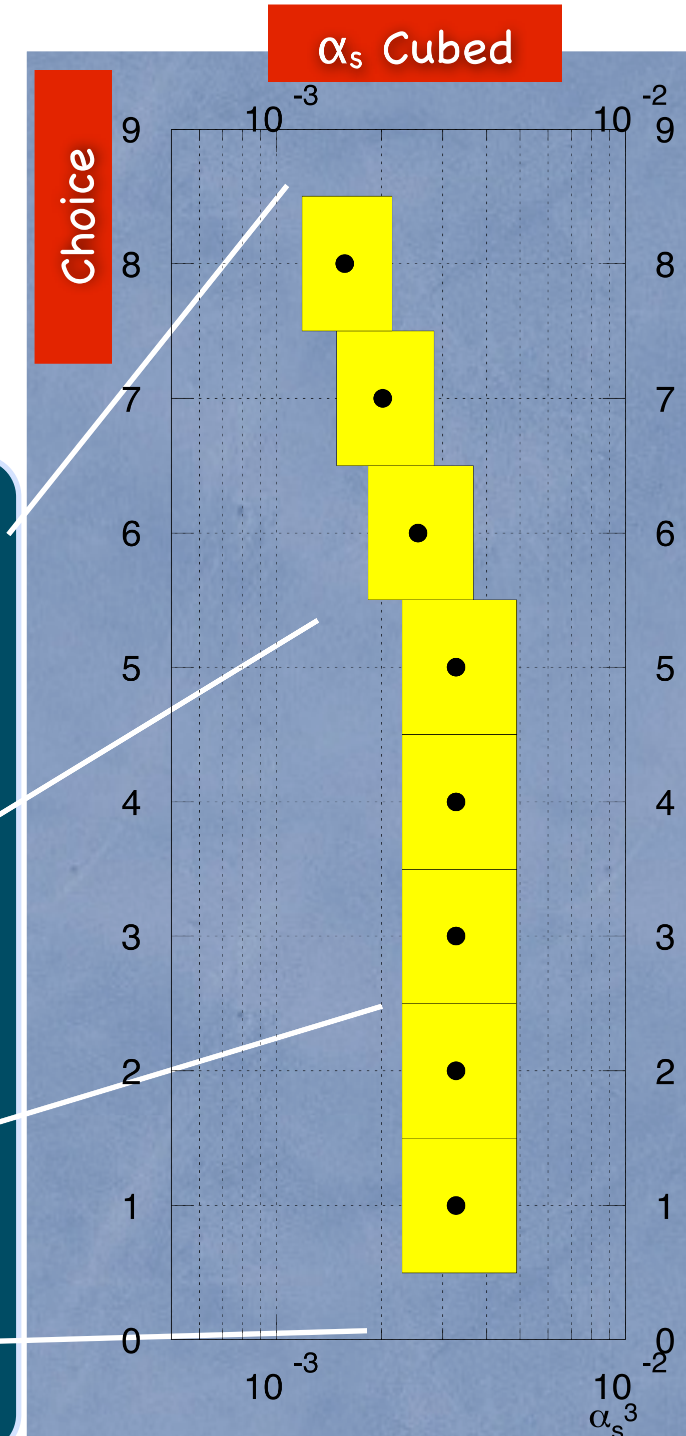
$$\overset{6}{} \alpha_s^3(m_W^2) < \alpha_s^3\left(m_W^2 + \langle p_\perp^2\rangle\right)_{7} < \overset{8}{} \alpha_s^3\left(m_W^2 + \sum_i p_{\perp i}^2\right)$$

Global Scaling: jets don't care about m_W

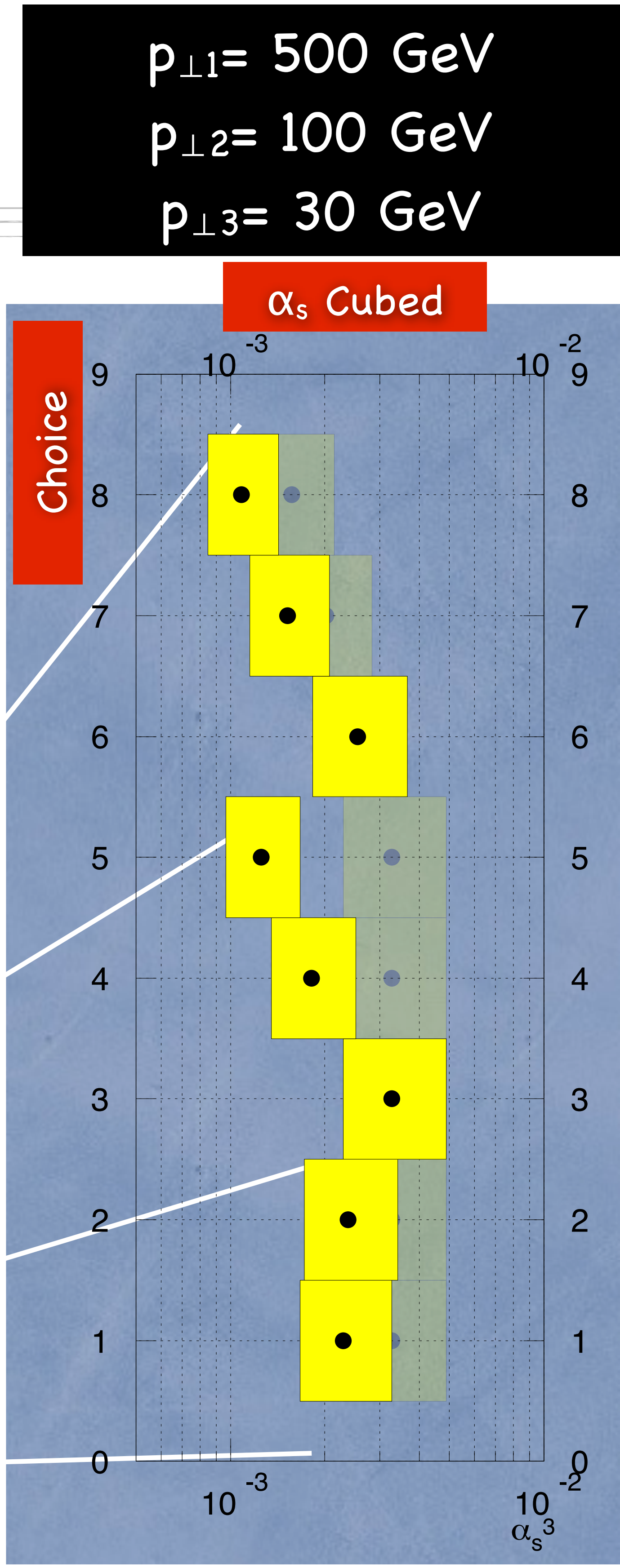
$$\overset{3}{} \alpha_s^3(\min[p_\perp^2]) < \alpha_s^3(\langle p_\perp^2\rangle)^{4} < \alpha_s^3(\max[p_\perp^2])_{5}$$

MC parton showers: "Local scaling"

$$\alpha_s(p_{\perp 1})\alpha_s(p_{\perp 2})_{2}\alpha_s(p_{\perp 3}) \sim \alpha_s^3\left(\langle p_\perp^2\rangle_{\rm geom}\right)_{1}$$



Dangers



Complicated final states

Intrinsically Multi-Scale problems

with Many powers of α_s

Whatever they might tell you

If you have multiple QCD scales

→ variation of μ_R by factor 2 in each direction not good enough! (nor is × 3, nor × 4)

Need to vary also functional dependence on each scale!

Main Points

Quarks live in 3D

Gluons live in 8D (which is ≈ 9 ≈ color + anticolor)

Bjorken Scaling: fixed coupling → scale invariance

Characteristic feature: self-similar jet-within-a-jet-within-a-jet-...

RATIOS of scales (hierarchies) : soft/collinear bremsstrahlung enhancements ⟵ (more in next lecture)

Real-World QCD is UV free ...

But take heed: Multiscale problems → large scale uncertainties

and IR confined

Factorization → meaningful perturbative calculations

Homework

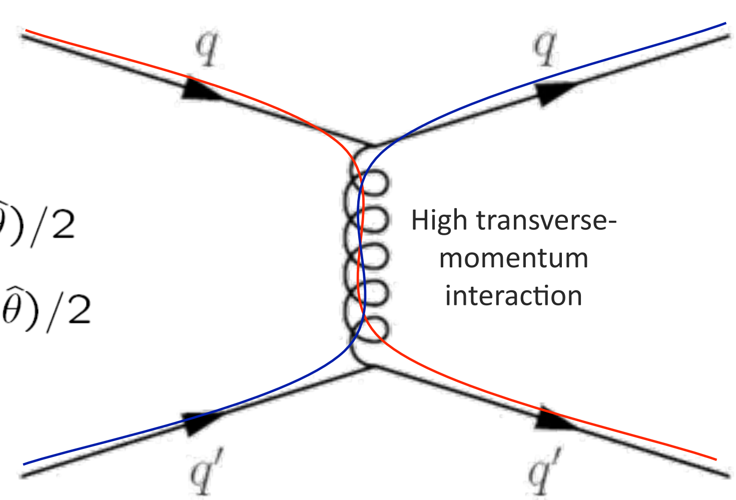
- The dominant process at Hadron Colliders is QCD 2→2 scattering (Rutherford Scattering)

$$qq' \to qq' \quad : \quad \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2}\frac{4}{9}\alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$



Question: what is the colour factor?

(hint: important to keep track of who has 3 indices and who has 8)