

Introduction to Higgs and Electroweak Precision Physics (III)

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Cargèse 2010: Physics at TeV Colliders – From Tevatron to LHC







Indirect Constraints on the Higgs

from Electroweak Precision Data

Precision measurements allow us to probe physics at much higher energy scales than the masses of the particles directly involved in experimental reactions by exploiting contributions from quantum loops. These tests do not only require accurate and well understood experimental data but also theoretical predictions with controlled uncertainties that match the experimental precision.

The global electroweak fit

Since the Z^0 boson couples to all fermion-antifermion pairs, it is ideal for measuring and studying electroweak and strong interactions

The global electroweak fit

Most important experimental inputs to fit:

- Electroweak precision data measured at the Z⁰ resonance
- Processes studied: e⁺e[−] → fermion + anti-fermion (quarks, charged leptons, neutrinos)
- Lowest order diagrams:



- Photon exchange cross section falls with s⁻¹
- Resonance at $E_{CM} = M_Z$ (LEP-1, SLC), *W*-pair production at $E_{CM} > 2 M_W$ (LEP-2)
- Electroweak unification: relation between weak and electromagnetic couplings
- Gauge sector of SM on tree level is given by 3 free parameters, e.g., α , M_Z , G_F
- Sensitivity to heavy fermions (top) and bosons (Higgs) via radiative corrections:



And – remember – radiative corrections are important:

From electroweak unification: $M_W = M_Z \cdot \cos \theta_W$, or: $\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$

- Use, e.g., $\sin^2 \theta_W = 0.2314 \pm 0.0002$ as obtained from deep-inelastic neutrino-nucleon scattering
- ...and using world average mass measurements from LEP and Tevatron gives:

 $M_{W} = (80.399 \pm 0.023) \text{ GeV}|_{\text{LEP+Tevatron}} \stackrel{?}{=} (79.944 \pm 0.010) \text{ GeV}|_{\text{EW theory (tree level)}}$ $\sin^{2}\theta_{W} = 0.2314 \pm 0.0002 \stackrel{?}{=} 0.2226 \pm 0.0004|_{\text{EW theory (tree level)}}$

Strong disagreement (20o) !

Logarithmic Higgs dependence enters through virtual corrections, e.g. :

$$\frac{H}{m_{W}^{2}} + \frac{H}{m_{W}^{2}} \Delta \rho_{(\text{Higgs})} = \frac{11G_{F}m_{Z}^{2}\cos^{2}\theta_{W}}{24\sqrt{2}\pi^{2}}\log\left(\frac{m_{H}^{2}}{m_{W}^{2}}\right)$$
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Electroweak fits have a long history ...

Based on a huge amount of preparatory work

- Needed to understand importance of loop corrections
- Precise Standard Model (SM) predictions and measurements required

EW fits routinely performed by many groups

- D. Bardinet al. (ZFITTER), G. Passarinoet al. (TOPAZ0), LEP EW WG (M. Grünewald, K. Mönig *et al.*), J. Erler (GAPP), ...
- Important results obtained !

Global SM fits also used at lower energies

- CKMfitter (J. Charles et al.), UTfit (M. Bona et al.), ...
- Mostly concentrating on CKM matrix

Also many groups pursuing global beyond-SM fits



Experimental Inputs



Important experimental input to the fit: electroweak precision data measured at the Z⁰-resonance

Process under study: $e^+e^- \rightarrow f\bar{f}$

f = all fermions (quarks, charged leptons, neutrinos) light enough to be pair produced

Hadronic cross-section:

- s^{-1} fall-off due to virtual photon exchange
- Resonance at $\sqrt{s} = M_Z$
- For $\sqrt{s} > 2M_W$: pair-production of *W*'s kinematically allowed
- Measurements around M_z: SLC, LEP I

Combined paper LEP + SLC: Phys. Rept. 427, 257 (2006)



<u>LEP I:</u>



- Four experiments: ADLO
- 1989–1995: √s ~ M_Z
- \sqrt{s} extremely well measured (2 MeV)
- Peak $L = 2 \cdot 10^{31} \, \mathrm{cm}^{-2} \mathrm{s}^{-1}$
 - 1000 Z's per hour per experiment
 - "Z-Factory"
- In total: ~17 million Z decays (SLD: 600k)

SLC:



- Low repetition rate (120 Hz cf. LEP: 45 kHz)
- Longitudinally polarized electron beam (up to $P_e \sim 80\%$)
- Small beam dimensions (1.5×0.7 μm², LEP: 150×5 μm²) + low bunch rate allowed use of slow but high-res. CCD arrays
 - \rightarrow superior vertex reconstruction



A look at the theory – tree level relations

Vector and axial-vector couplings for $Z \rightarrow ff$ in SM:

 $g_{V,f}^{(0)} \equiv g_{L,f}^{(0)} + g_{R,f}^{(0)} = I_3^f - 2Q^f \sin^2 \theta_W \qquad \sin^2 \theta_W = 1 - \frac{M_W^2}{M_-^2}$

 $g_{A,f}^{(0)} \equiv g_{L,f}^{(0)} - g_{R,f}^{(0)} = l_3^f$ Electroweak unification: relation between weak and electromagnetic couplings:

Since uncertainty on G_F and M_Z small, relation often used to eliminate direct M_W dependence:

Gauge sector of SM on tree level is given by 3 free parameters, *e.g.*: α , *M_z*, *G_F*

$$G_{F} = \frac{\pi \alpha(0)}{\sqrt{2} (M_{W}^{(0)})^{2} \left(1 - (M_{W}^{(0)})^{2} / M_{Z}^{2}\right)}$$
$$M_{W}^{2} = \frac{M_{Z}^{2}}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi \alpha}{G_{F}M_{Z}^{2}}}\right)$$



Radiative corrections – modifying propagators and vertices

Parametrisation of radiative corrections: "electroweak form-factors": ρ , κ , Δr

• Modified ("effective") couplings at the *Z* pole:

$$g_{V,f} = \sqrt{\rho_Z^f} \left(I_3^f - 2Q^f \sin^2 \theta_{\text{eff}}^f \right)$$
$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$
$$\rho: \text{ overall scale}$$
$$\sin^2 \theta_{\text{eff}}^f = \kappa_Z^f \sin^2 \theta_W$$
$$\kappa: \text{ on-shell mixi}$$

Modified W mass:

$$M_W^2 = \frac{M_Z^2}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha \cdot (1 - \Delta r)}{G_F M_Z^2}} \right)$$



Radiative corrections – modifying propagators and vertices

Important consequences

- → All other SM parameters enter the calculations
- In particular corrections are $\sim m_{top}^2$ and $\sim \ln(M_H)$
- Loop correction of the order ~1%.
- Precision observables measured at LEP/SLC to much better precision !

→ Can test the SM and constraint the unknown SM Parameters



Radiative corrections – modifying propagators and vertices

Leading order terms $(M_H \ll M_W)$

• ρ_Z and κ_Z can be split into sum of universal contributions from propagator self-energies:

$$\Delta \rho_{Z} = \frac{3G_{F}M_{W}^{2}}{8\sqrt{2}\pi^{2}} \left[\frac{m_{t}^{2}}{M_{W}^{2}} - \tan^{2}\theta_{W} \left(\ln \frac{M_{H}^{2}}{M_{W}^{2}} - \frac{5}{6} \right) + \dots \right]$$
$$\Delta \kappa_{Z} = \frac{3G_{F}M_{W}^{2}}{8\sqrt{2}\pi^{2}} \left[\frac{m_{t}^{2}}{M_{W}^{2}} \cot^{2}\theta_{W} - \frac{10}{9} \left(\ln \frac{M_{H}^{2}}{M_{W}^{2}} - \frac{5}{6} \right) + \dots \right]$$

 and flavour-specific vertex corrections, which are very small, except for top quarks, due to large |V_{tb}| CKM element

$$\Delta \rho^{f} = -2\Delta \kappa^{f} = -\frac{G_{F}m_{t}^{2}}{2\sqrt{2}\pi^{2}} + \dots$$



Example – electroweak cross-section formula for unpolarised beams (LEP)

Neglects photon ISR & FSR, gluon FSR, fermion masses

The \propto (1 + cos² θ) terms contribute to total **cross-sections**

• Measure cross-sections around M_Z via corrected event counts: $\sigma = (N_{sel} - N_{bg})/\varepsilon_{sel}L$

The $\propto \cos\theta$ terms contribute only to **asymmetries**

• Measure Forward–Backward asymmetries in angular distributions final-state fermions: $A_{FB} = (N_F - N_B)/(N_F + N_B)$



Other asymmetries (not in above cross section formula)

- Dependence of Z^0 production on helicities of initial state fermions (SLC) \rightarrow Left–Right asymmetries
- Polarisation of final state fermions (can be measured in tau decays)

Total hadronic cross section – measurement and prediction

Total cross-section (from $\cos\theta$ symmetric terms) expressed in Breit-Wigner form:

$$\sigma_{f\bar{f}}^{Z} = \sigma_{f\bar{f}}^{0} \cdot \frac{s \cdot \Gamma_{Z}^{2}}{\left(s - M_{Z}^{2}\right)^{2} + s^{2} \Gamma_{Z}^{2} / M_{Z}^{2}} \cdot \frac{1}{R_{\text{QED}}} \qquad \sigma_{f\bar{f}}^{0} = \frac{12\pi}{M_{Z}^{2}} \frac{\Gamma_{ee} \Gamma_{f\bar{f}}}{\Gamma_{Z}^{2}} \qquad \text{Corrected for QED radiation}$$

Partial widths add up to full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{hadronic} + \Gamma_{invisible}$

- Measured cross sections depend on products of partial and total widths
- Highly correlated set of parameters !

Instead: use less correlated set of six measurements

- Z mass and width: M_7 , Γ_7
- Three leptonic ratios (use lepton-univ.): $R_{\ell}^{0} = R_{e}^{0} = \Gamma_{had} / \Gamma_{ee} = R_{\mu}^{0} = R_{\tau}^{0}$ Hadronic width ratios: R_{b}^{0} , R_{c}^{0}

precise √s
high statistics Include also SLD:

> higher effi./purity for heavy guarks

Taken from LEP:

Partial width – sensitive to QCD and QED corrections

Partial width are defined **inclusively**, *i.e.*, they contain final state QED and QCD vector and axial-vector corrections via "radiator functions": $R_{A,f}$, $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(\left| g_{A,f} \right|^2 R_{A,f} + \left| g_{V,f} \right|^2 R_{V,f} \right)$$

QCD corrections only affect final states with quarks

• To first order in α_{s} corrections are flavour independent and identical for A and V

$$R_{V,QCD} = R_{A,QCD} = R_{QCD} = 1 + \frac{\alpha_{S}(M_{Z}^{2})}{\pi} + \dots = 1 + 0.038 + \dots$$

• 3NLO (!) calculation available [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022]

QED corrections similar:
$$R_{V,QED} = R_{A,QED} = R_{QED} = 1 + \frac{3}{4} Q_f^2 \frac{\alpha(M_Z^2)}{\pi} + \dots$$
 What is this? (though much smaller due to $\alpha \ll \alpha_s$)

Digression: Running of $\alpha_{QED}(M_Z)$

Define: photon vacuum polarisation function $\Pi_{\gamma}(q^2)$ $i\int d^4x \ e^{iqx} \langle 0 | T J^{\mu}_{em}(x) (J^{\nu}_{em}(0))^{\dagger} | 0 \rangle = -(g^{\mu\nu}q^2 - q^{\mu}q^{\nu}) \prod_{\gamma}(q^2)$

0

Only vacuum polarisation "screens" electron charge

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta \alpha(s)} \quad \text{with:} \quad \Delta \alpha(s) = -4\pi\alpha \operatorname{Re}\left[\prod_{\gamma}(s) - \prod_{\gamma}(0)\right]$$
$$= \Delta \alpha_{\operatorname{len}}(s) + \Delta \alpha_{\operatorname{had}}(s)$$

Leptonic $\Delta \alpha_{\text{lep}}(s)$ calculable in QED (known to 3-loops). However, quark loops are modified by long-distance hadronic physics, cannot be calculated with perturbative QCD

Way out: Optical Theorem
(unitarity) ...
... and the subtracted
dispersion relation of
$$\Pi_{\gamma}(q^2)$$

(analyticity)
 $\operatorname{Born:} \sigma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$
 $12\pi \operatorname{Im}[\Pi_{\gamma}(s) = \frac{\sigma^{(0)}[e^+e^- \rightarrow \text{hadrons}]}{\sigma^{(0)}[e^+e^- \rightarrow \mu^+\mu^-]} \equiv R(s)$
 $\operatorname{Im}[\neg \gamma^{(0)}(s) = \sigma(s)(\alpha/\alpha(s))^2$

$$\prod_{\gamma}(s) - \prod_{\gamma}(0) = \frac{s}{\pi} \int_{0}^{\infty} ds' \frac{\operatorname{Im} \prod_{\gamma}(s')}{s'(s'-s) - i\varepsilon} \qquad \Delta \alpha_{\operatorname{had}}(s) = -\frac{\alpha s}{3\pi} \operatorname{Re} \int_{0}^{\infty} ds' \frac{R(s')}{s'(s'-s) - i\varepsilon}$$

Digression: Running of $\alpha_{QED}(M_Z)$

Hadronic dispersion integral solved by combination of experimental data and perturbative QCD

$$\Delta \alpha_{\text{had}}(M_Z^2) = -\frac{\alpha M_Z^2}{3\pi} P \int_0^\infty ds' \frac{R(s')}{s'(s' - M_Z^2)}$$

The task is to properly correct, average and integrate the cross section data.

Use perturbative QCD where possible ("global quark–hadron duality" allows one to extend perturbative QCD into the non-continuum regions)

Traditionally separate:

 $\Delta \alpha_{\rm had}(M_Z^2) = \Delta \alpha_{\rm had}^{(5)}(M_Z^2) + \Delta \alpha_{\rm top}(M_Z^2)$

Results [Hagiwara et al, PLB 649:, 73 (2007)]

 $\Delta \alpha (M_Z^2) = 0.03149769_{lep} + 0.02768(22)_{had (5)} - 0.000073(02)_{top}$ $\alpha^{-1} (M_Z^2) = 128.937 \pm 0.030$



Remember this slide ?

Muon magnetic moment: comparison between measurement (blue band) and predictions (dots)



The magnetic moment of the muon is predicted to a precision of several 10^{-6} , ...

E

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... but some tension exists ...

Remember this slide ?

Standard Model prediction:

$$a_{\mu}^{SM} = a_{\mu}^{QED} + a_{\mu}^{EW} + a_{\mu}^{Had-LO} + a_{\mu}^{Had-NLO}$$

= (116 591 834 ± 2 ± 41 ± 26) × 10⁻¹¹
where: $a_{\mu}^{QED} = (116 584 718.09 \pm 0.15) \times 10^{-11}$
 $a_{\mu}^{EW} = (154 \pm 1_{\text{theo}} \pm 2_{M_{H} = 100...500 \text{ GeV}}) \times 10^{-11}$
 $a_{\mu}^{Had-LO} = (6 955 \pm 40_{\text{exp}} \pm 7_{pQCD}) \times 10^{-11}$
 $a_{\mu}^{Had-NLO} = (7 \pm 1_{\text{NLO disp}} \pm 26_{\text{LPLS}}) \times 10^{-11}$

Experimental result (BNL): $a_{\mu}^{exp} = (116\ 592\ 089 \pm 54_{stat} \pm 33_{syst}) \times 10^{-11}$ Deviation exp - SM: $(255 \pm 63 \pm 49) \times 10^{-11}$ ("3.2 σ ")



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$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(\left| g_{A,f} \right|^2 R_{A,f} + \left| g_{V,f} \right|^2 R_{V,f} \right)$$

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QED corrections similar:
$$R_{V,QED} = R_{A,QED} = R_{QED} = 1 + \frac{3}{4} Q_f^2 \frac{\alpha(M_Z^2)}{\pi} + \dots$$
 We know what this is ! We know what this is !

Asymmetry and polarisation – quantify parity violation

Distinguish vector and axial-vector couplings of the Z (*i.e.*, $sin^2 \theta^{f}_{eff}$) Convenient to use "asymmetry parameters":

$$A_{f} = \frac{g_{L,f}^{2} - g_{R,f}^{2}}{g_{L,f}^{2} + g_{R,f}^{2}} = 2 \frac{g_{V,f} / g_{A,f}}{1 + (g_{V,f} / g_{A,f})^{2}} \quad \text{dependent on } \sin^{2}\theta_{\text{eff}}^{f} \colon \frac{\text{Re}(g_{V,f})}{\text{Re}(g_{A,f})} = 1 - 4 |Q_{f}| \sin^{2}\theta_{\text{eff}}^{f}$$

Via final state (FS) angular distribution in unpolarised scattering (LEP)

- Forward-backward asymmetries: $A_{FB}^{f} = \frac{N_{F} N_{B}}{N_{F} + N_{B}}, A_{FB}^{0,f} = \frac{3}{4}A_{e}A_{f}$
- LEP measurements: $A_{FB}^{0,l}$, $A_{FB}^{0,c}$, $A_{FB}^{0,b}$

$$Via \ IS \ polarisation \ (SLC): \ A_{LR} = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\left\langle \left| P \right|_e \right\rangle}, \ A_{LRFB} = \frac{\left(N_F - N_B\right)_L - \left(N_F - N_B\right)_R}{\left(N_F + N_B\right)_L + \left(N_F + N_B\right)_R} \frac{1}{\left\langle \left| P_e \right| \right\rangle}$$

• Left-right, and left-right forward-backward asymmetries: $A_{LR}^0 = A_e^0$, $A_{LRFB}^{0,f} = \frac{3}{4}A_f^0$



Via final state (FS) angular distribution in unpolarised scattering (LEP)

- Forward-backward asymmetries: $A_{FB}^{f} = \frac{N_{F} N_{B}}{N_{-} + N_{-}}, A_{FB}^{0,f} = \frac{3}{4}A_{e}A_{f}$
- LEP measurements: $A_{FB}^{0,l}$, $A_{FB}^{0,c}$, $A_{FB}^{0,b}$

$$Via \ IS \ polarisation \ (SLC): \ A_{LR} = \frac{N_L - N_R}{N_L + N_R} \frac{1}{\left\langle \left| P \right|_e \right\rangle}, \ A_{LRFB} = \frac{\left(N_F - N_B\right)_L - \left(N_F - N_B\right)_R}{\left(N_F + N_B\right)_L + \left(N_F + N_B\right)_R} \frac{1}{\left\langle \left| P_e \right| \right\rangle}$$

• Left-right, and left-right forward-backward asymmetries: $A_{LR}^0 = A_e^0$, $A_{LRFB}^{0,f} = \frac{3}{4}A_f^0$

Asymmetry and polarisation – quantify parity violation

Distinguish vector and axial-vector couplings of the Z (*i.e.*, $sin^2 \theta_{eff}^f$) Convenient to use "asymmetry parameters":

$$A_{f} = \frac{g_{L,f}^{2} - g_{R,f}^{2}}{g_{L,f}^{2} + g_{R,f}^{2}} = 2\frac{g_{V,f}/g_{A,f}}{1 + (g_{V,f}/g_{A,f})^{2}}$$

Via final state polarisation (LEP):

• Tau polarisation:

$$P_{\tau}(\cos\theta) = -\frac{A_{\tau}(1+\cos^2\theta)+2A_{e}\cos\theta}{1+\cos^2\theta+2A_{\tau}A_{e}\cos\theta}$$

- Measure τ spin versus from energy and angular correlations in τ decays
- Fit at LEP determines: A_τ, A_e

dependent on
$$\sin^2 \theta_{\text{eff}}^f$$
: $\frac{\text{Re}(g_{V,f})}{\text{Re}(g_{A,f})} = 1 - 4 |Q_f| \sin^2 \theta_{\text{eff}}^f$



Asymmetry and polarisation – quantify parity violation

Distinguish vector and axial-vector couplings of the Z (*i.e.*, $sin^2 \theta^{f}_{eff}$) Convenient to use "asymmetry parameters":

$$A_{f} = \frac{g_{L,f}^{2} - g_{R,f}^{2}}{g_{L,f}^{2} + g_{R,f}^{2}} = 2 \frac{g_{V,f} / g_{A,f}}{1 + \left(g_{V,f} / g_{A,f}\right)^{2}} \quad \text{dependent on } \sin^{2}\theta_{\text{eff}}^{f} \colon \frac{\text{Re}(g_{V,f})}{\text{Re}(g_{A,f})} = 1 - 4 \left|Q_{f}\right| \sin^{2}\theta_{\text{eff}}^{f}$$

Via final state polarisation (LEP):

- The measured asymmetries are corrected for QED radiation, γ –Z interference to give "pole" quantities ⁽⁰⁾.
- In case of e⁺e⁻ final state, must also take into account t-channel scattering.

Initial and final state QED radiation

Measured cross-section and asymmetries are modified by initial and final state QED radiation

• Effects are corrected for by the collaborations (using the programs TOPAZ0 and ZFITTER)

 $\sigma(s) = \int_{4m_f^2/s}^{1} dz \cdot H_{\text{QED}}^{\text{tot}}(z,s) \cdot \sigma(zs) \quad \begin{array}{c} \text{Convolution of kernel cross} \\ \text{section by QED radiator function} \end{array}$

- Very large corrections applied in some cases!
- Measured observables become "pseudo-observables"
- *E.g.*, hadronic pole-cross section σ^{0}_{had}

In the electroweak fit the published "pseudo-observables" are used

Important: these QED corrections are independent of the electroweak corrections discussed before!



Summary – experimental results on the *Z* pole

Parameter	Input value	
M_Z [GeV]	91.1875 ± 0.0021	
Γ_Z [GeV]	2.4952 ± 0.0023	
$\sigma_{ m had}^0$ [nb]	41.540 ± 0.037	Ц
R^0_ℓ	20.767 ± 0.025	
$A_{ m FB}^{0,\ell}$	0.0171 ± 0.0010	
$A_\ell \ ^{(\star)}$	0.1499 ± 0.0018	
A_c	0.670 ± 0.027	SLO I
A_b	0.923 ± 0.020	
$A_{ m FB}^{0,c}$	0.0707 ± 0.0035	6
$A_{ m FB}^{0,b}$	0.0992 ± 0.0016	Ш
R_c^0	0.1721 ± 0.0030	၂ပ
R_b^0	0.21629 ± 0.00066	ທ
$\sin^2\!\theta_{ m eff}^\ell(Q_{ m FB})$	0.2324 ± 0.0012	

Total cross-sections around M_Z

- Measured by: M_Z , Γ_Z , σ^0_{had} , R_I^0 , R_c^0 , R_b^0
- Sensitive to the total coupling strength of the *Z* to fermions

Asymmetries on the *Z* pole

- Measured by: $A_{FB}^{0,I}$, $A_{FB}^{0,b}$, $A_{FB}^{0,c}$, A_{I} , A_{c} , A_{b} , $\sin^{2}\theta'_{eff}(Q_{FB})$
- Sensitive to the ratio of the Z⁰ vector to axial-vector couplings (*i.e.* sin²θ_{eff})

And additional observables in fit

- M_W is subject to rad. corr. via: $\Delta r = \Delta \alpha (M_Z) - \Delta \rho + \dots$ (higher orders)
- m_t due to its large loop corrections

Experimental Input

All Observables Entering the Fit

Experimental results:

- Z-pole observables: LEP/SLD results (corrected for ISR/FSR QED effects) [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
- M_W and Γ_W : LEP + Tevatron (incl. Moriond-09 result from D0)

[ADLO, hep-ex/0612034] [D0 Conference Note 5893-CONF] [CDF, Phys Rev. D77, 112001 (2008)] [CDF, Phys. Rev. Lett. 100, 071801 (2008)] [CDF+D0, Phys. Rev. D 70, 092008 (2004)]

- *m_t*: latest Tevatron average [arXiv:0903.2503]
- *m_c*, *m_b*: world averages [PDG, J. Phys. G33,1 (2006)]
- $\Delta \alpha_{had}(M_Z)$: [K. Hagiwara et al., Phys. Lett. B649, 173 (2007)] + rescaling mechanism to account for α_s dependency
- Direct Higgs searches at LEP and Tevatron (incl. Moriond-09 Tevatron average) [ADLO: Phys. Lett. B565, 61 (2003)] [CDF+D0: arXiv:0903.4001]

Not considered: results on $\sin^2\theta_{eff}$ from

- NuTeV: unclear theoretical uncertainties from QCD effects (NLO corrections, nuclear effects of the bound nucleon PDFs)
- APV, fixed target polarised Möller scattering: present experimental accuracy too low

Parameter	Input value			Parameter	Input value	
M_Z [GeV]	91.1875 ± 0.0021		(M_H [GeV] ^(\circ)	Likelihood ratios	
Γ_Z [GeV]	2.4952 ± 0.0023	ЦЩ —	ſ	M_W [GeV]	80.399 ± 0.023	
$\sigma_{ m had}^0$ [nb]	41.540 ± 0.037	-	L L	Γ_W [GeV]	2.098 ± 0.048	
R^0_ℓ	20.767 ± 0.025					
$A_{ m FB}^{0,\ell}$	0.0171 ± 0.0010			\overline{m}_c [GeV]	1.25 ± 0.09	
$A_{\ell}^{(\star)}$	0.1499 ± 0.0018			\overline{m}_b [GeV]	4.20 ± 0.07	
A_{c}	0.670 ± 0.027	SL()	(m_t [GeV]	173.1 ± 1.3	
A_b	0.923 ± 0.020			$\Delta \alpha_{\rm had}^{(5)}(M_Z^2) \ ^{(\dagger \triangle)}$	2769 ± 22	
$A_{ m FB}^{0,c}$	0.0707 ± 0.0035	<u>م</u>		$\alpha_s(M_Z^2)$	_	
$A_{ m FB}^{0,b}$	0.0992 ± 0.0016	12		$\delta_{ m th} M_W$ [MeV]	$[-4, 4]_{\rm theo}$	
R_c^0	0.1721 ± 0.0030	ုပ		$\delta_{\rm th} \sin^2 \theta_{\rm eff}^{\ell} $ ^(†)	$[-4.7, 4.7]_{\rm theo}$	
R_b^0	0.21629 ± 0.00066	SI		$\delta_{ m th} ho_{Z}^{f} (\dagger)$	$[-2, 2]_{\text{theo}}$	
$\sin^2 \theta_{\rm eff}^{\ell}(Q_{\rm FB})$	0.2324 ± 0.0012	Ι.		$\delta_{ m th}\kappa_Z^f$ (†)	$[-2,2]_{\mathrm{theo}}$	
$\frac{\sin^2 \theta_{\rm eff}^\ell(Q_{\rm FB})}{2}$	0.2324 ± 0.0012			$\frac{\delta_{\rm th}\rho_Z^f}{\delta_{\rm th}\kappa_Z^f} \stackrel{(\dagger)}{}$	$[-2, 2]_{\text{theo}}$ $[-2, 2]_{\text{theo}}$	

12 million

Correlations for observables from Z lineshape fit					
	M_Z	Γ_Z	$\sigma_{ m had}^0$	R^0_ℓ	$A^{0,\ell}_{ ext{FB}}$
M_Z	1	-0.02	-0.05	0.03	0.06
Γ_Z		1	-0.30	0.00	0.00
$\sigma_{ m had}^0$			1	0.18	0.01
R_ℓ^0				1	-0.06
$A_{ ext{FB}}^{0,\ell}$					1

Correlations for heavy-flavour observables at Z pole						
	$A^{0,c}_{ ext{FB}}$	$A^{0,b}_{\scriptscriptstyle\mathrm{FB}}$	A_c	A_b	R_c^0	R_b^0
$A^{0,c}_{\scriptscriptstyle \mathrm{FB}}$	1	0.15	0.04	-0.02	-0.06	0.07
$A^{0,b}_{\scriptscriptstyle\mathrm{FB}}$		1	0.01	0.06	0.04	-0.10
A_c			1	0.11	-0.06	0.04
A_b				1	0.04	-0.08
R_c^0					1	-0.18

LEP & Tevatron

Tevatron

Precision Measurement of the W mass

Results from LEP-2:

- 10 pb⁻¹ per experiment recorded close to the WW threshold
 - M_W from σ_{WW} measurements
 - Much less precise result than kinematic W reconstruction (200 MeV statistical error)
- 700 pb⁻¹ per experiment above the threshold
 - *M_W* directly reconstructed from invariant mass of observed leptons (dominant) and jets
 - Large "colour reconnection" systematics in hadronic channel (35 MeV)
 - Combination: $M_W = (80.376 \pm 0.025 \pm 0.022)$ GeV

Results from Tevatron:

- Using leptonic W decays
 - M_W from template fits to the transverse mass or transverse momentum of lepton
 - Extremely challenging, systematics dominated measurement (energy calibration)
 - Combination (2009): M_W = (80.420 ± 0.031) GeV

 $4 \times 700 \text{ pb}^{-1}$ taken for $\sqrt{s} = 161-209 \text{ GeV}$ between 1996 and 2000 at LEP-2



Precision Measurement of the W mass

Recent D0 measurement of M_W in $W \rightarrow e_V$

- Analysis relies on energy calibration with $Z \rightarrow ee$
- Result: M_W = (80.401 ± 0.021 ± 0.038) GeV
- Greatly deserves the label "precision measurement"





The (a) m_T , (b) p_T^e , and (c) $E_{T,\text{miss}}$ distributions for data and fastmc simulation with backgrounds. The χ values are shown below each distribution where $\chi_i = [N_i - (\text{fastmc}_i)]/\sigma_i$ for each point in the distribution, N_i is the data yield in bin *i* and only the statistical uncertainty is used. The fit ranges are indicated by the double-ended horizontal arrows.

Measurement of the top mass

Top quark mass is measured in di-lepton (4%), lepton-jets (30%), and jets-jets (46%) modes

- Analysis relies strongly on identification of *b*-jets for background suppression and reduction of jet combinatorics
- Use multivariate methods to suppress backgrounds
- "In situ" jet energy scale (JES) calibration in modes with jets

Fit method: parameterise templates depending on top mass and JES for sensitive variables (e.g., $M_{\text{jet-jet}}, M_{\text{letp-jet}}, \dots$), construct and maximise overall likelihood function



The lepton-jets channel provides most precise m_t measurment



Measurement of the top mass

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Can also extract m_t from top cross section measurement

- Complementary method [PRD 80, 071102 (2009)]
- Unambiguous definition of running top mass, but limited by precision on luminosity



The lepton-jets channel provides most precise m_t measurment



2009 Γ_W , M_W (left) and m_{top} (right) world averages



[CDF + D0, 3.6 fb⁻¹, arXiv:0903.2503]


2009 Γ_W , M_W (left) and m_{top} (right) world averages





[CDF + D0, 3.6 fb⁻¹, arXiv:0903.2503]

Constraints from Direct Higgs Searches

LEP-2: Higgs production via "Higgs-Strahlung"

 ee → ZH (H → bb, ττ) [ADLO: Phys. Lett. B565, 61 (2003)]

Tevatron: see yesterday's lecture

Statistical interpretation in global fit: two-sided CL_{S+B}

- Experiments measure test statistics LLR = $-2\ln Q$, where $Q = L_{S+B} / L_B$
- LLR is transformed by experiments into CL_{S+B} using pseudo-MC experiments
- We transform one-sided CL_{S+B} into a two-sided CL_{S+B} (measure *deviation* from SM !)
- Contribution to χ^2 estimator obtained via inverse error function: $\Delta \chi^2 = \text{Erf}^{-1} (1 \text{CL}_{S+B}^{2-\text{sided}})$
- Alternative treatments:
 - Use one-sided CL_{S+B}: however, different interpretation want SM Higgs (not any Higgs)
 - − Directly use $\Delta \chi^2 \approx$ LLR: Bayesian interpretation, lacks pseudo-MC information

Direct Higgs Searches





Note: "hypothesis-only" test

(like the -2lnQ curves delivered by the collaborations)

- Procedure tests only the M_H under consideration
- It neglects that a given SM signal hypothesis entails background hypotheses
 → if SM Higgs is found at a certain M_H other values of M_H are excluded
- Effect expected to be small today, but relevant once the Higgs is discovered.

The Global Electroweak Fit

Fit parameters – unknown SM parameters are floating in fit

Naïve set of free parameters relevant for the electroweak analysis:

- Coupling constants: electromagnetic (α), weak (G_F), strong (α_S)
- Boson masses: M_{γ} , M_Z , M_W , M_H
- Fermion masses: m_f with $f = e, \mu, \tau, v_e, v_{\mu}, v_{\tau}, u, c, t, d, s, b$

Simplification: massless neutrinos : $m_{\nu e} = m_{\nu \mu} = m_{\nu \tau} = 0$

Simplification from electroweak unification:

- Massless photon: $M_{\gamma} = 0$
- Can express M_W as a function of M_Z and the couplings α and G_F

Further simplification by fixing parameters with insignificant uncertainties compared to sensitivity of the fit

• G_{F} , and leptonic contribution to running of α

The Global Electroweak Fit

Fit parameters – unknown SM parameters are floating in fit

Masses of leptons and light quarks are small and/or sufficiently well known \rightarrow uncertainties are negligible in the fit

- Masses are fixed to world-averages from PDG
- Except for the running charm, bottom and top masses $\rightarrow m_t$ strongest impact on fit !

List of remaining parameters in the SM fit: $\Delta \alpha_{had}^{(5)}(M_Z), \ \alpha_S(M_Z), \ M_Z, \ M_H, \ \overline{m}_c, \ \overline{m}_b, \ m_t$

State-of-the art calculations, in particular:

- *M_W* and sin²θ^f_{eff}: full two-loop + leading beyond-two-loop form factor corrections
 [M. Awramik et al., Phys. Rev D69, 053006 (2004) and ref.] [M. Awramik et al., JHEP 11, 048 (2006) and refs.]
- Radiator functions: 3NLO prediction of the massless QCD cross section [P.A. Baikov et al., Phys. Rev. Lett. 101 (2008) 012022]

The Global Electroweak Fit

Fit parameters – theoretical uncertainties are also floating parameters

Fit minimises test statistics: $\chi^2 = -2 \ln L$

The likelihood function "*L*" is a product of contributions that

- Measure the agreement between $x_{\text{theo}}(y_{\text{mod}})$ and x_{exp}
- Expresses prior knowledge of some of the y_{mod} parameters (if available)

The fit shown in the following treats errors as follows:

- Statistical and systematic experimental errors obey Gaussian likelihood functions
- Theoretical errors have bound uniform likelihoods \rightarrow allowed ranges

Most important theoretical uncertainties included in the SM fit:

- Uncertainty on M_W from missing higher order corrections: $\delta M_W = 4 6$ MeV
- Uncertainty on $\sin^2\theta'_{eff}$ from missing higher order corrections: $\delta \sin^2\theta'_{eff} = 4.7 \cdot 10^{-5}$

Fit Results^(*)

Distinguish two fit types:

Standard Fit: all data except for direct Higgs searches Complete Fit: all data including direct Higgs searches



Parameter	Input value	Free in fit	Results from global EW fits:Standard fitComplete fit		<i>Complete fit w/o exp. input in line</i>
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1874 ± 0.0021	91.1877 ± 0.0021	$91.1974^{+0.0146}_{-0.0159}$
Γ_Z [GeV]	2.4952 ± 0.0023	_	2.4959 ± 0.0015	$2.4954_{-0.0013}^{+0.0016}$	$2.4954^{+0.0008}_{-0.0012}$
$\sigma_{ m had}^0$ [nb]	41.540 ± 0.037	_	41.478 ± 0.014	41.472 ± 0.001	41.469 ± 0.015
R^0_ℓ	20.767 ± 0.025	_	20.742 ± 0.018	20.741 ± 0.018	20.718 ± 0.027
$A_{ m FB}^{0,\ell}$	0.0171 ± 0.0010	_	0.01638 ± 0.0002	0.01624 ± 0.0002	0.01618 ± 0.0002
$A_\ell \ ^{(\star)}$	0.1499 ± 0.0018	_	0.1478 ± 0.0010	0.1472 ± 0.0009	_
A_c	0.670 ± 0.027	_	$0.6682^{+0.00045}_{-0.00044}$	$0.6679^{+0.00043}_{-0.00037}$	$0.6679^{+0.00044}_{-0.00034}$
A_b	0.923 ± 0.020	_	0.93469 ± 0.00009	$0.93464^{+0.00006}_{-0.00007}$	$0.93463^{+0.00006}_{-0.00007}$
$A_{ m FB}^{0,c}$	0.0707 ± 0.0035	_	$0.0741^{+0.0006}_{-0.0005}$	0.0737 ± 0.0005	0.0738 ± 0.0005
$A_{ m FB}^{0,b}$	0.0992 ± 0.0016	_	0.1036 ± 0.0007	0.1032 ± 0.0006	$0.1037^{+0.0004}_{-0.0005}$
R_c^0	0.1721 ± 0.0030	_	0.17225 ± 0.00006	0.17226 ± 0.00006	0.17225 ± 0.00006
R_b^0	0.21629 ± 0.00066	_	$0.21579^{+0.00004}_{-0.00006}$	0.21577 ± 0.00005	0.21577 ± 0.00005
$\sin^2 \theta_{\rm eff}^{\ell}(Q_{\rm FB})$	0.2324 ± 0.0012	-	$0.23145^{+0.00011}_{-0.00016}$	0.23151 ± 0.00011	$0.23148^{+0.00013}_{-0.00010}$
M_H [GeV] ^(\circ)	Likelihood ratios	yes	$82.8^{+30.2[+75.2]}_{-23.2[-41.5]}$	$119.1_{-4.0[-4.9]}^{+13.4[+37.9]}$	$82.8^{+30.2[+75.2]}_{-23.2[-41.5]}$
M_W [GeV]	80.399 ± 0.023	-	$80.384^{+0.014}_{-0.015}$	$80.370^{+0.008}_{-0.010}$	$80.365^{+0.009}_{-0.026}$
Γ_W [GeV]	2.098 ± 0.048	_	2.092 ± 0.001	2.091 ± 0.001	2.092 ± 0.001
\overline{m}_c [GeV]	1.25 ± 0.09	yes	1.25 ± 0.09	1.25 ± 0.09	_
\overline{m}_b [GeV]	4.20 ± 0.07	yes	4.20 ± 0.07	4.20 ± 0.07	_
$m_t \; [\text{GeV}]$	173.1 ± 1.3	yes	173.2 ± 1.2	173.6 ± 1.2	$177.9^{+11.2}_{-3.5}$
$\Delta \alpha_{\rm had}^{(5)}(M_Z^2) ^{(\dagger \triangle)}$	2769 ± 22	yes	2772 ± 22	2764 ± 22	2733^{+57}_{-46}
$\alpha_s(M_Z^2)$	-	yes	$0.1192^{+0.0028}_{-0.0027}$	0.1193 ± 0.0028	0.1193 ± 0.0028
$\delta_{ m th} M_W$ [MeV]	$[-4,4]_{\mathrm{theo}}$	yes	4	4	_
$\delta_{ m th} \sin^2 \! \theta_{ m eff}^{\ell} ^{(\dagger)}$	$[-4.7, 4.7]_{\rm theo}$	yes	4.7	0.8	_
$\delta_{ m th} ho_Z^f$ (†)	$[-2,2]_{\mathrm{theo}}$	yes	2	2	_
$\delta_{ m th}\kappa^f_Z$ (†)	$[-2,2]_{\mathrm{theo}}$	yes	2	2	_

Parameter	$\ln M_H$	$\Delta \alpha^{(5)}_{\rm had}(M_Z^2)$	M_Z	$\alpha_s(M_Z^2)$	m_t	\overline{m}_c	\overline{m}_b
$\ln M_H$	1	-0.395	0.113	0.041	0.309	-0.001	-0.006
$\Delta \alpha_{\rm had}^{(5)}(M_Z^2)$		1	-0.006	0.101	-0.007	0.001	0.003
M_Z			1	-0.019	-0.015	-0.000	0.000
$\alpha_s(M_Z^2)$				1	0.021	0.011	0.043
m_t					1	0.000	-0.003
\overline{m}_c						1	0.000

Correlation coefficients of free fit parameters

Goodness-of-Fit

Goodness-of-fit:

- Standard fit: $\chi^2_{min} = 16.4 \rightarrow \text{Prob}(\chi^2_{min}, 13) = 0.23$
- Complete fit: $\chi^2_{min} = 17.9 \rightarrow \text{Prob}(\chi^2_{min}, 14) = 0.21$

Pull values for complete fit (right figure \rightarrow)

- No individual pull exceeds 3σ
- FB(*b*) asymmetry largest contributor to χ^2_{min}
- Small contributions from M_Z, Δα^{had}(M_Z), m_c, m_b indicate that their input accuracies exceed fit requirements → parameters could have been fixed in fit
- Can describe data with only two floating parameters (α_{s}, M_{H})



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Goodness-of-Fit

Toy analysis: p-value for wrongly rejecting the SM = $0.20 \pm 0.01 - 0.02_{theo}$



M_H from Standard fit:

- Central value $\pm 1\sigma$: $M_{H} = 83^{+30}_{-23}$ GeV
- 2σ interval: [42, 158] GeV

Green band due to *R*fit treatment of theory errors, fixed errors lead to larger χ^2_{min}

M_H from Complete fit:

- Central value $\pm 1\sigma$: $M_{H} = 119^{+13}_{-4.0}$ GeV
- 2o interval: [114, 157] GeV



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M_H from Complete fit:

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Verify Gaussian Prob($\Delta\chi^2$,1) approximation

- Fix M_H , perform two fits and calculate $Prob(\Delta \chi^2(M_H) = \chi^2_{min}(M_H) - \chi^2_{min}, 1)$
- Generate pseudo experiments ("toy MC") using fitted values for M_H with experimental errors
- For each toy experiment perform two fits and compute $\Delta \chi^2_{toy}(M_H)$ exactly as in real data
- Compute 1–CL at M_H by integrating normalised $\Delta \chi^2_{toy}(M_H)$ distribution from $\Delta \chi^2(M_H)$ to infinity
- Assumes that $\Delta \chi^2_{toy}(M_H)$ distribution is independent of nuisance parameters !

M_H from Standard fit:

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M_H from Complete fit:

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Curve gives the probability of wrongly rejecting SM hypothesis assuming a certain value for M_H



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Known tension between $A^{0,b}_{FB}$ and $A_{Iep}(SLD)$ and M_W :

- Pseudo-MC analysis to evaluate
 "Probability to observe a Δχ² = 8.0 when removing the least compatible input "
 → accounts for "look-elsewhere effect"
- Find: 1.4% (2.5 σ)



Top Mass

Quadratic sensitivity to m_{top}

- Standard fit: $m_{top} = 177.2_{-7.8}^{+10.5} \text{ GeV}$
- Complete fit: $m_{top} = 177.9_{-3.5}^{+11.2} \text{ GeV}$

Tevatron average: (173.1 ± 1.3) GeV

→ What is this mass? Pole mass? "MC" mass?

Note: profile of the *standard fit* exhibits an asymmetry opposite to the naïve expectation from $\sim m_t^2$ dependence of loop corrections

Reasons: floating Higgs mass and its positive correlation with m_t



Top Mass

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For Standard fit with free m_{top} find: $m_{H} = 116^{+184}_{-61}$ GeV

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For Standard fit with free m_{top} find: $m_{H} = 116^{+184}_{-61}$ GeV

Fit (*i.e.* excluding the Higgs searches and the respective measurements)

Fit + Higgs searches

Fit + Higgs searches + direct measurements → best knowledge of SM



$\Delta \alpha_{had}(M_z)$

Strong sensitivity to $\Delta \alpha_{had}(M_z)$

• Complete fit: $\Delta \alpha_{had}^{(5)}(M_Z^2) = (273.3_{-4.6}^{+5.7}) \cdot 10^{-4}$

Phenomenological value: $(277.2 \pm 2.2) 10^{-4}$

Fit (*i.e.* excluding the Higgs searches and the respective measurements)

Fit + Higgs searches

Fit + Higgs searches + direct measurements → best knowledge of SM



- The structures reflect presence of local minima in $(\Delta \chi^2 \text{ vs. } M_H)$ -plot
- Today's precision in m_t and $\Delta \alpha_{had}(M_Z)$ sufficient for the EW fit

3NLO Determination of $\alpha_{\rm s}$



From Complete Fit:

 $\alpha_{\rm s}(M_Z) = 0.1193 \pm 0.0028 \pm 0.0001$

• First error experimental

```
• Second error theoretical (!)
```

[incl. variation of renorm. scale from $M_Z/2$ to $2M_Z$ and massless terms of order/beyond $a_S^5(M_Z)$ and massive terms of order/beyond $a_S^4(M_Z)$]

Excellent agreement with N³LO result from hadronic τ decays
 [M. Davier et al., arXiv:0803.0979]

 Best current test of asymptotic freedom property of QCD !

The Fate of the Standard Model



Driving the SM to *M*_{Planck}

Remember, the behaviour of the quartic Higgs couplings as function of the cut-off scale Λ puts bounds on M_H

- For too large M_H , the couplings become non-perturbative ("triviality" or "blow-up" scenario)
- For too small M_{H} , the vacuum becomes unstable

 \rightarrow obtain three lower bounds on M_H from different requirement: absolute stability, finite-T and zero-T metastability



Convolve Bounds with *M*_{*H*} **Constraints**

Can we obtain likelihoods on vacuum stability (or, likewise, the cut-off = new physics scale Λ) from constraint on M_H ?

- Non-perturbativity excluded at 95.7% CL → raise to 99.1% with Tevatron Higgs searches !
- Cannot distinguish between vacuum stability, metastability or collapse scenarios
 - \rightarrow requires M_H > 122 GeV to exclude collapse scenario at 95% CL



Convolve Bounds with *M_H* **Constraints**

Can we obtain likelihoods on vacuum stability (or, likewise, the cut-off = new physics scale Λ) from constraint on M_H ?

- Requiring absolute vacuum stability (at all times), one can obtain upper bound Λ
 - Left plot: current situation → no significant information
 - Right plot: case for precise M_H measurement of **120 GeV**



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Convolve Bounds with *M*_{*H*} **Constraints**

Can we obtain likelihoods on vacuum stability (or, likewise, the cut-off = new physics scale Λ) from constraint on M_H ?

- Requiring absolute vacuum stability (at all times), one can obtain upper bound Λ
 - Left plot: current situation → no significant information
 - Right plot: case for precise M_H measurement of **115 GeV**



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Prospects for the Standard Model Fit



Prospects for LHC, ILC and ILC with Giga-Z

New colliders (LHC/ILC) will increase precision in electroweak observables

- Improvement of the predictive power of the fit
- Higgs discovery \rightarrow testing goodness-of-fit \rightarrow sensitivity to new physics

Expected improvement from LHC (10 fb⁻¹):

- δM_W : 25 MeV \rightarrow 15 MeV (at least)
- δm_t : 1.2 GeV \rightarrow 1.0 GeV

Expected improvement from ILC:

• From threshold scan $\delta m_t = 50$ MeV, translates to 100–200 MeV on the running mass

Expected improvement from GigaZ:

- From *WW* threshold scan: $\delta M_W = 6$ MeV
- From A_{LR} : $\delta \sin^2 \theta'_{eff}$: $17 \cdot 10^{-5} \rightarrow 1.3 \cdot 10^{-5}$
- δR_l^0 : 2.5.10⁻² \rightarrow 0.4.10⁻²

Improved determination of $\Delta \alpha_{had}^{(5)}(M_Z)$ will become necessary

- Needs improvement in hadronic cross section data around *cc* res.
- Expected uncertainty of 7·10⁻⁵ (today 22·10⁻⁵) if relative crosssection precision below J/Ψ at 1% [Jegerlehner, hep-ph/0105283]
- Experiments with better acceptances and control of systematics needed
- Promising: ISR analyses at B and Φ factories; new data from BES-III

Prospects for LHC, ILC and ILC with Giga-Z

Assumed experimental improvements for prospective study:

- LHC: *M_W*, *m*_{top}
- ILC: *M_W*, *m*_{top}
- Giga-Z: M_W , m_{top} , $\sin^2\theta'_{eff}$, R_{lep}
- ISR-based (BABAR) and BESIII cross-section measurements should improve $\Delta \alpha^{had}(M_Z)$

	Expected uncertainty				
Quantity	Present	LHC	ILC	GigaZ (ILC)	
$\overline{M_W [MeV]}$	23	15	15	6	
$m_t \; [\; { m GeV}]$	1.3	1.0	0.2	0.1	
$\sin^2 \theta_{ m eff}^{\ell} \ [10^{-5}]$	17	17	17	1.3	
$R^0_\ell \; [10^{-2}]$	2.5	2.5	2.5	0.4	
$\Delta lpha_{ m had}^{(5)}(M_Z^2) \ [10^{-5}]$	22 (7)	22 (7)	22 (7)	22 (7)	
$M_H (= 120 \; { m GeV}) \; [\; { m GeV}] \ lpha_{\scriptscriptstyle S}(M_Z^2) \; [10^{-4}]$	$^{+54}_{-40} \begin{pmatrix} +51\\ -38 \end{pmatrix} \begin{bmatrix} +38\\ -30 \end{bmatrix}$ 28	$^{+45}_{-35} \begin{pmatrix} +42\\ -33 \end{pmatrix} \begin{bmatrix} +30\\ -25 \end{bmatrix}$ 28	$^{+42}_{-33} \begin{pmatrix} +39\\ -31 \end{pmatrix} \begin{bmatrix} +28\\ -23 \end{bmatrix}$ 28	$^{+26}_{-23} \begin{pmatrix} +20\\ -18 \end{pmatrix} \begin{bmatrix} +8\\ -8 \end{bmatrix}$ 6	

Input from: [ATLAS, Physics TDR (1999)] [CMS, Physics TDR (2006)] [A. Djouadi et al., arXiv:0709.1893][I. Borjanovic, EPJ C39S2, 63 (2005)] [S. Haywood et al., hep-ph/0003275] [R. Hawkings, K. Mönig, EPJ direct C1, 8 (1999)] [A. H. Hoang et al., EPJ direct C2, 1 (2000)] [M. Winter, LC-PHSM-2001-016]

Prospects for LHC, ILC and ILC with Giga-Z

Results on M_{H} , including (solid) and excluding (dotted) theoretical errors



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Probing New Physics with the EW Fit





The Two-Higgs-Doublet Model (2HDM)



Two-Higgs-Doublet Model

Extend SM by adding another scalar Higgs doublet (2HDM)

- *Type-II* 2HDM: one doublet couples to up-type and the other one to down-type fermions only
- 6 free parameters: $\underline{M_h}$, M_{A0} , M_{H0} , $M_{H\pm}$, $\tan\beta = v_2 / v_1$, α (governing $h-H^0$ mixing)
- Resembles Higgs sector of MSSM

Look, e.g., at processes sensitive to charged Higgs: $M_{H\pm}$

$$L_{H^{\pm}ff} = \frac{g}{2\sqrt{2}M_{W}} \Big\{ H^{+}\overline{U} \Big[M_{U}V_{CKM} (1-\gamma_{5}) \cot\beta + V_{CKM}M_{D} (1+\gamma_{5}) \tan\beta \Big] D + \text{h.c.} \Big\}$$

- Interaction has similar structure as *W* boson
- Left-handed coupling: $1/\tan\beta$, right-handed coupling: $\tan\beta$
- Sensitive parameters are $M_{H\pm}$ and tan β
- LEP limit: $M_{H\pm}$ > 78.6 GeV (95% CL), for any value of tan β

Sensitive observables mostly from *B*-physics sector, but also *c* and *s*

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Two-Higgs-Doublet Model

Observables used to constrain charged Higgs in 2HDM

Observable	Input value	Exp. Ref.	Calculation
R_b^{0}	0.21629 ± 0.00066	[ADLO, Phys. Rept. 427, 257 (2006)	[H. E. Haber and H. E. Logan, Phys. Rev. D62, 015011 (2000)]
$BR\;(B\to X_s\gamma)$	(3.52 ± 0.23 ± 0.09)·10 ⁻⁴	[HFAG, latest update]	[M. Misiak et al., Phys. Rev. Lett. 98, 022002 (2007)]
BR ($B \rightarrow \tau \nu$)	(1.73 ± 0.33)·10 ⁻⁴	[P.Chang, Talk at ICHEP 2008]	[W. S. Hou, Phys. Rev. D48, 2342 (1993)]
BR ($B \rightarrow \mu \nu$)	$(-5.7\pm6.8\pm7.1)\cdot10^{-4}$	[E. Baracchini, Talk at ICHEP 2008]	[W. S. Hou, Phys Rev. D48, 2342 (1993)]
BR ($K \rightarrow \mu \nu$) / BR($\pi \rightarrow \mu \nu$)	1.004 ± 0.007	[FlaviaNet,, arXiv: 0801.1817]	[FlaviaNet, arXiv: 0801.1817]
$BR(B \to D\tau v) / BR(B \to Dev)$	0.416 ± 0.117 ± 0.052	[Babar, Phys. Rev. Lett 100, 021801 (2008)]	[J. F. Kamenik and F. Mescia, arXiv: 0802.3790]

 R_b^0 and $B \rightarrow X_s \gamma$

Z-vertex correction $\propto \cot^2\beta$



Penguin dipole-moment of $B \rightarrow X_{s\gamma}$ allows combination of left- and righthanded Higgs couplings.

Wilson coefficient:

$$C_{7}^{H} \approx -\frac{m_{t}^{2}}{2M_{H}^{2}} \left(\frac{7}{36}\cot^{2}\beta + \frac{2}{3}\ln\frac{M_{H}^{2}}{m_{t}^{2}} - \frac{1}{2}\right)$$





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$B \rightarrow \tau \nu$

Weak annihilation process. BR proportional to $|V_{ub}|^2$ and *B* decay constant-squared f_B^2

$$\Gamma(B \rightarrow \tau \nu) = \frac{G_F}{8\pi} \cdot m_{B^+} m_{\tau}^2 \left(1 - \frac{m_{\tau}^2}{m_{B^+}^2}\right)^2 \cdot f_B^2 \left|V_{ub}\right|^2 \cdot \left(1 - \frac{m_{B^+}^2}{M_{H^+}^2} \tan^2 \beta\right)^2$$

b \bar{u} H^+ \bar{v}

Quadratic solution Strength of effect $\propto \tan\beta$

Conflict (2.6 σ) between direct BR measurement and SM prediction governed by CKM angle β



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Compare BR predictions based on direct measurements of $|V_{ub}|$ (left) with CKM fit (right)



Other measurements with tree level contributions

- Weak upper limit on $BR(B \rightarrow \mu \nu)$
- Favored solution of $BR(B \rightarrow \tau v)$ excluded by combination of:
 - $\triangleright \quad \mathsf{BR}(B \to X_{s}\gamma)$
 - $\triangleright \quad \mathsf{BR}(B \to D\tau v) \ / \ \mathsf{BR}(B \to Dev)$
 - $\triangleright \quad \mathsf{BR}(K \to \mu \nu) \,/\, \mathsf{BR}(\pi \to \mu \nu)$



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2HDM – Combined Fit

Fit minimum: χ^2 = 3.9 for $M_{H\pm}$ = 858 GeV and tan β = 6.8 Excluded at 95% CL

- Small $tan\beta$
- $M_{\rm H}$ < 240 GeV for all tan β
- $M_{\rm H} < 780 \text{ GeV}$ for tan $\beta = 70$ (mostly from $B \rightarrow \tau \nu$)



Cargèse 2010

Oblique Corrections

Oblique = vacuum polarisation (VP) corrections – Universal: occur in any gauge boson propagator Assume that new (heavy) physics contributes to VP only

- Direct corrections (vertex, box, bremsstrahlung) generally suppressed by m_f / Λ
- Eminent exception: $\Gamma(Z \rightarrow bb)$, which has CKM-enhanced top vertex corrections

Electroweak fit is sensitive to the new physics via these oblique corrections parametrised with "STU" parameters (and more)

$$O_{\text{measured}} = O_{\text{SM, ref}}(M_H, m_t) + c_s S + c_t T + c_u U, \quad (S = T = U = 0 \text{ in SM})$$

STU measure deviations from EW radiative correction expected in O_{SM, ref}

- S: new physics contribution to neutral current processes
- **U** (& S): new physics contribution to **charged current processes**
 - U only sensitive to M_W and Γ_W usually very small in new physics models (often: U=0)
- T: difference between neutral and charged current processes (sensitive to weak isospin violation)

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We've shaken enough!

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Let's open the box finally!

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