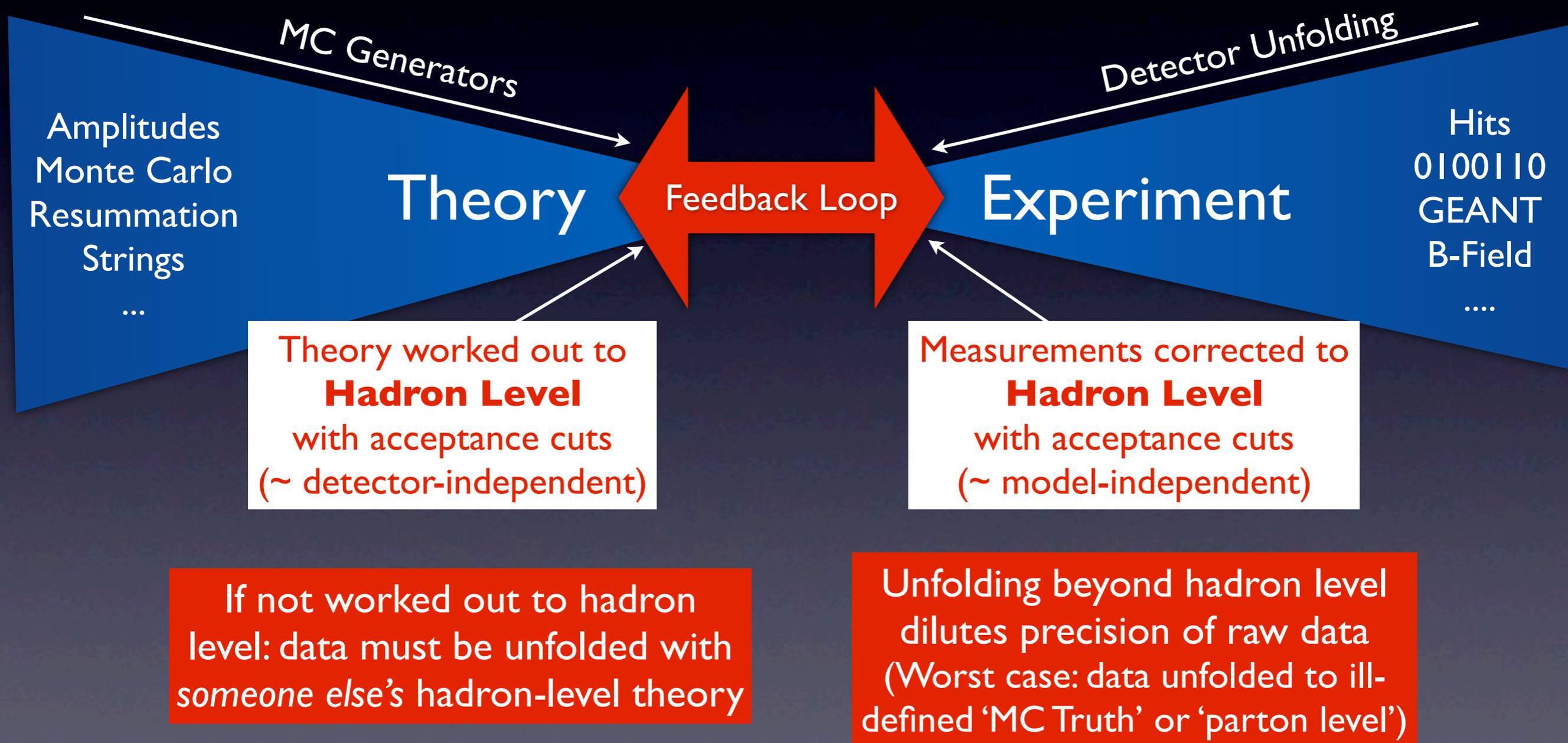


Monte Carlo Generators

P. Skands (CERN TH)

Count what is Countable
Measure what is Measurable
(and keep working on the beam) G. Galilei



From Partons ...

- Main Tool
 - Lowest-Order Matrix Elements calculated in a fixed-order perturbative expansion → parton-parton scattering cross sections



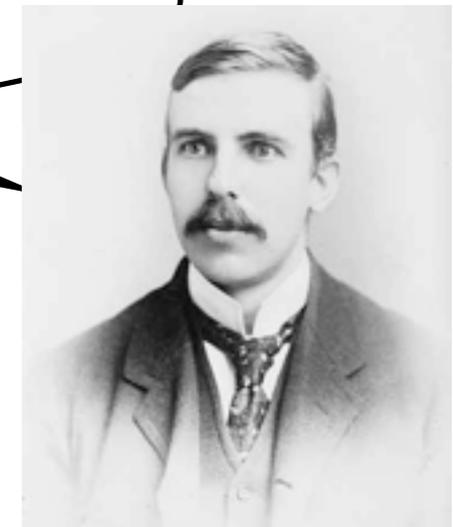
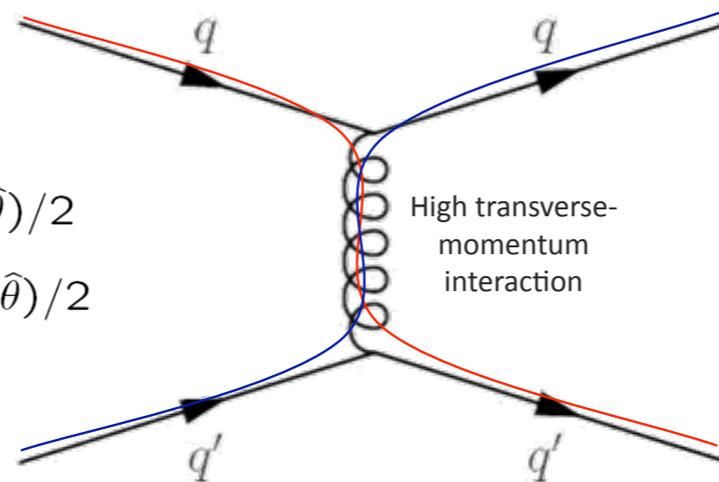
L = ...

$$qq' \rightarrow qq' : \frac{d\hat{\sigma}}{d\hat{t}} = \frac{\pi}{\hat{s}^2} \frac{4}{9} \alpha_s^2 \frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}$$

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2 = -\hat{s}(1 - \cos\hat{\theta})/2$$

$$\hat{u} = (p_1 - p_4)^2 = -\hat{s}(1 + \cos\hat{\theta})/2$$



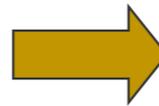
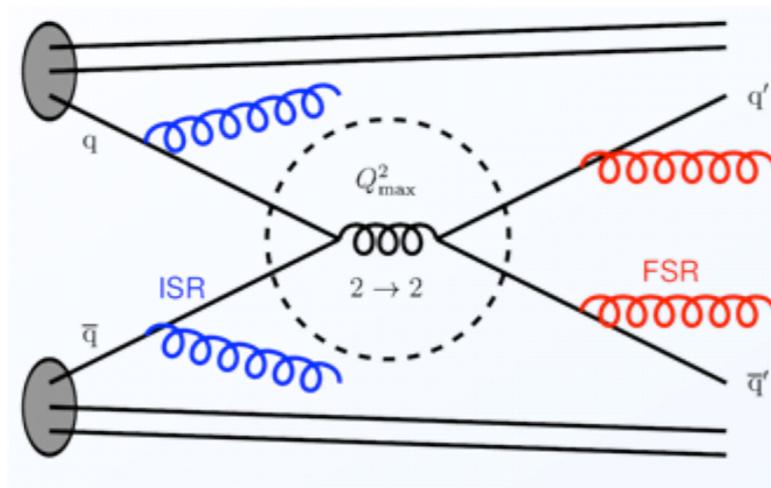
L → FeynRules/LanHEP → AlpGen/MadGraph/CalcHEP/CompHEP/... → partons

... to Pions



Reality is more complicated

Monte Carlo Generators



Calculate Everything \approx solving QCD \rightarrow requires compromise!

Improve Born-level perturbation theory, by including the 'most significant' corrections
 \rightarrow complete events \rightarrow any observable you want

1. Parton Showers

2. Matching

3. Hadronisation

4. The Underlying Event



1. Soft/Collinear Logarithms

2. Finite Terms, "K"-factors

3. Power Corrections (more if not IR safe)

4. ?

(+ many other ingredients: resonance decays, beam remnants, Bose-Einstein, ...)

Starting Point

Flashback:
factorization

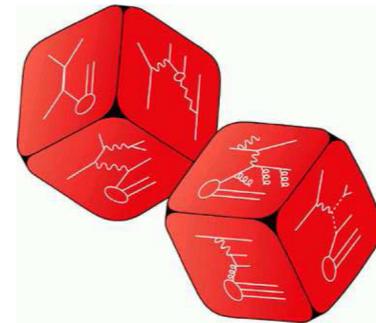
$$\frac{d\sigma}{dX} = \sum_{a,b} \sum_f \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{d\hat{\sigma}_{ab \rightarrow f}(x_a, x_b, f, Q_i^2, Q_f^2)}{d\hat{X}_f} D(\hat{X}_f \rightarrow X, Q_i^2, Q_f^2)$$

Want to generate events

In as much detail as Mother Nature

Get average and fluctuations right

Make random choices \approx as in nature



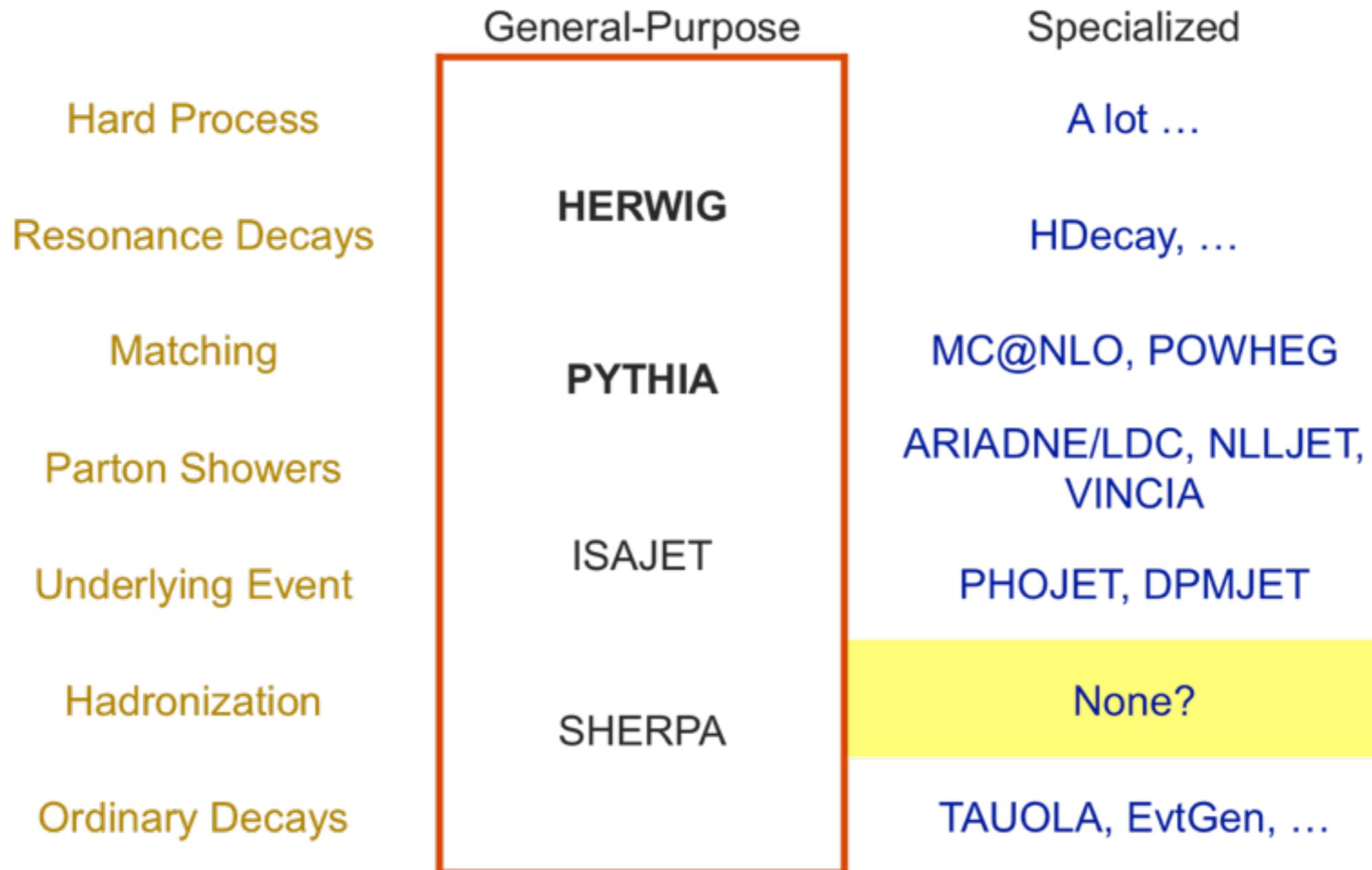
$$\sigma_{\text{final state}} = \sigma_{\text{hard process}} \mathcal{P}_{\text{tot,hard process} \rightarrow \text{final state}}$$

where $\mathcal{P}_{\text{tot}} = \mathcal{P}_{\text{res}} \mathcal{P}_{\text{ISR}} \mathcal{P}_{\text{FSR}} \mathcal{P}_{\text{MI}} \mathcal{P}_{\text{remnants}} \mathcal{P}_{\text{hadronization}} \mathcal{P}_{\text{decays}}$

with $\mathcal{P}_i = \prod_j \mathcal{P}_{ij} = \prod_j \prod_k \mathcal{P}_{ijk} = \dots$ in its turn

\implies **divide and conquer**

Generator Landscape



Main Workhorses

HERWIG, PYTHIA and SHERPA intend to offer a convenient framework for LHC physics studies, but with slightly different emphasis:



PYTHIA (successor to JETSET, begun in 1978):

- originated in hadronization studies: the Lund string
- leading in development of multiple parton interactions
- pragmatic attitude to showers & matching
- the first multipurpose generator: machines & processes

HERWIG (successor to EARWIG, begun in 1984):

- originated in coherent-shower studies (angular ordering)
- cluster hadronization & underlying event pragmatic add-on
- large process library with spin correlations in decays



SHERPA (APACIC++/AMEGIC++, begun in 2000):

- own matrix-element calculator/generator
- extensive machinery for CKKW matching to showers
- leans on PYTHIA for MPI and hadronization

Hard Processes

Wide spectrum from “general-purpose” to “one-issue”, see e.g.

<http://www.cedar.ac.uk/hepcode/>

Free for all as long as Les-Houches-compliant output.

I) General-purpose, leading-order:

- MadGraph/MadEvent (amplitude-based, ≤ 7 outgoing partons):

<http://madgraph.physics.uiuc.edu/>

- CompHEP/CalcHEP (matrix-elements-based, $\sim \leq 4$ outgoing partons)

- Comix: part of SHERPA (Behrends-Giele recursion)

- HELAC–PHEGAS (Dyson-Schwinger)

II) Special processes, leading-order:

- ALPGEN: $W/Z+ \leq 6j$, $nW + mZ + kH+ \leq 3j$, ...

- AcerMC: $t\bar{t}b\bar{b}$, ...

- VECBOS: $W/Z+ \leq 4j$

III) Special processes, next-to-leading-order:

- MCFM: NLO $W/Z+ \leq 2j$, WZ , WH , $H+ \leq 1j$

- GRACE+Bases/Spring

Note: NLO codes not yet generally interfaced to shower MCs

QCD at Fixed Order

Distribution of observable: \mathcal{O}

In production of X + anything

Fixed Order
(all orders)

$$\left. \frac{d\sigma}{d\mathcal{O}} \right|_{\text{ME}} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

↑ Cross Section differentially in \mathcal{O}
↑ Phase Space
↑ Sum over "anything" \approx legs
↑ Matrix Elements for $X+k$ at (ℓ) loops
↑ Sum over identical amplitudes, then square
↑ Momentum configuration
↑ Evaluate observable \rightarrow differential in \mathcal{O}

Truncate at $k=0, \ell=0$
 \rightarrow **Born Level = First Term**
 Lowest order at which X happens

QCD at Fixed Order

Distribution of observable: \mathcal{O}

In production of X + anything

Fixed Order
(all orders)

$$\frac{d\sigma}{d\mathcal{O}} \Big|_{\text{ME}} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

↑ Cross Section differentially in \mathcal{O}
↑ Phase Space
↑ Sum over "anything" \approx legs
↑ Matrix Elements for $X+k$ at (ℓ) loops
↑ Sum over identical amplitudes, then square
↑ Momentum configuration
↑ Evaluate observable \rightarrow differential in \mathcal{O}

Truncate at $k=n, \ell=0$
 \rightarrow **Leading Order** for $X + n$
 Lowest order at which $X + n$ happens

QCD at Fixed Order

Distribution of observable: \mathcal{O}

In production of X + anything

Fixed Order
(all orders)

$$\frac{d\sigma}{d\mathcal{O}} \Big|_{\text{ME}} = \sum_{k=0} \int d\Phi_{X+k} \left| \sum_{\ell=0} M_{X+k}^{(\ell)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+k}))$$

↑ Cross Section differentially in \mathcal{O}
↑ Phase Space
↑ Sum over "anything" \approx legs
↑ Matrix Elements for $X+k$ at (ℓ) loops
↑ Sum over identical amplitudes, then square
↑ Momentum configuration
↑ Evaluate observable \rightarrow differential in \mathcal{O}

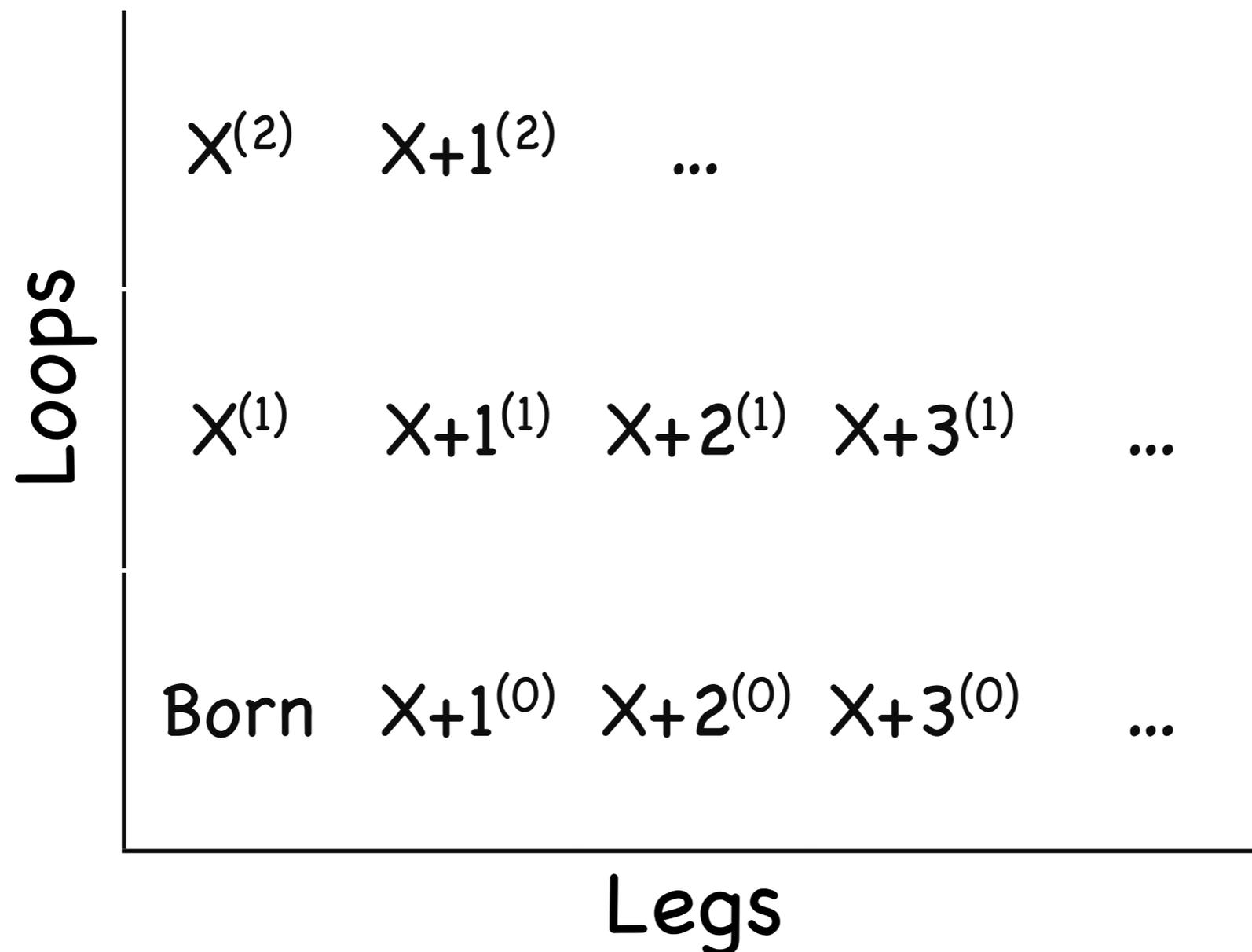
Truncate at $k+l \leq n$

\rightarrow $N^n\text{LO}$ for X

Includes $N^{n-1}\text{LO}$ for $X+1$, $N^{n-2}\text{LO}$ for $X+2$, ...

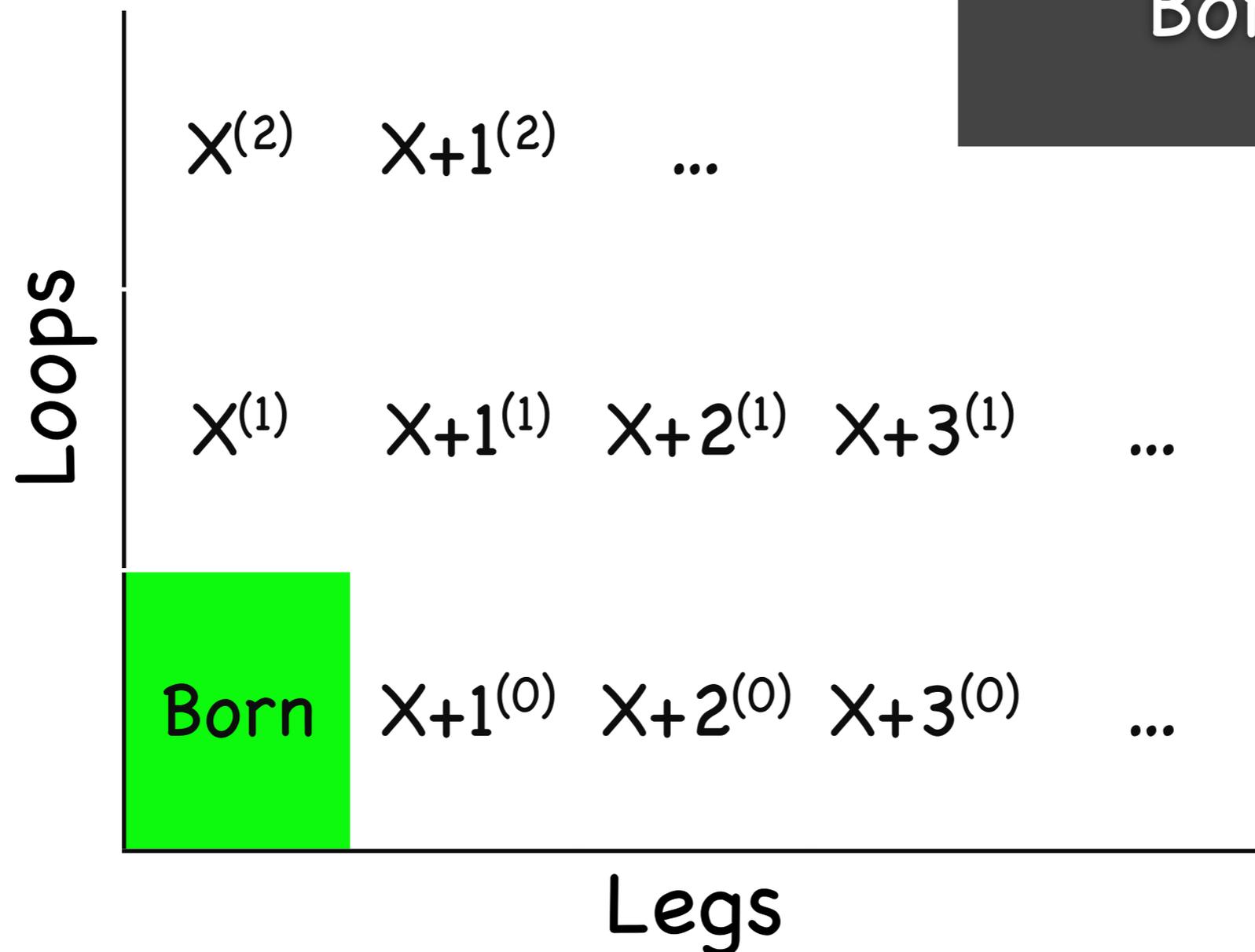
Loops and Legs

Another representation



Loops and Legs

Another representation



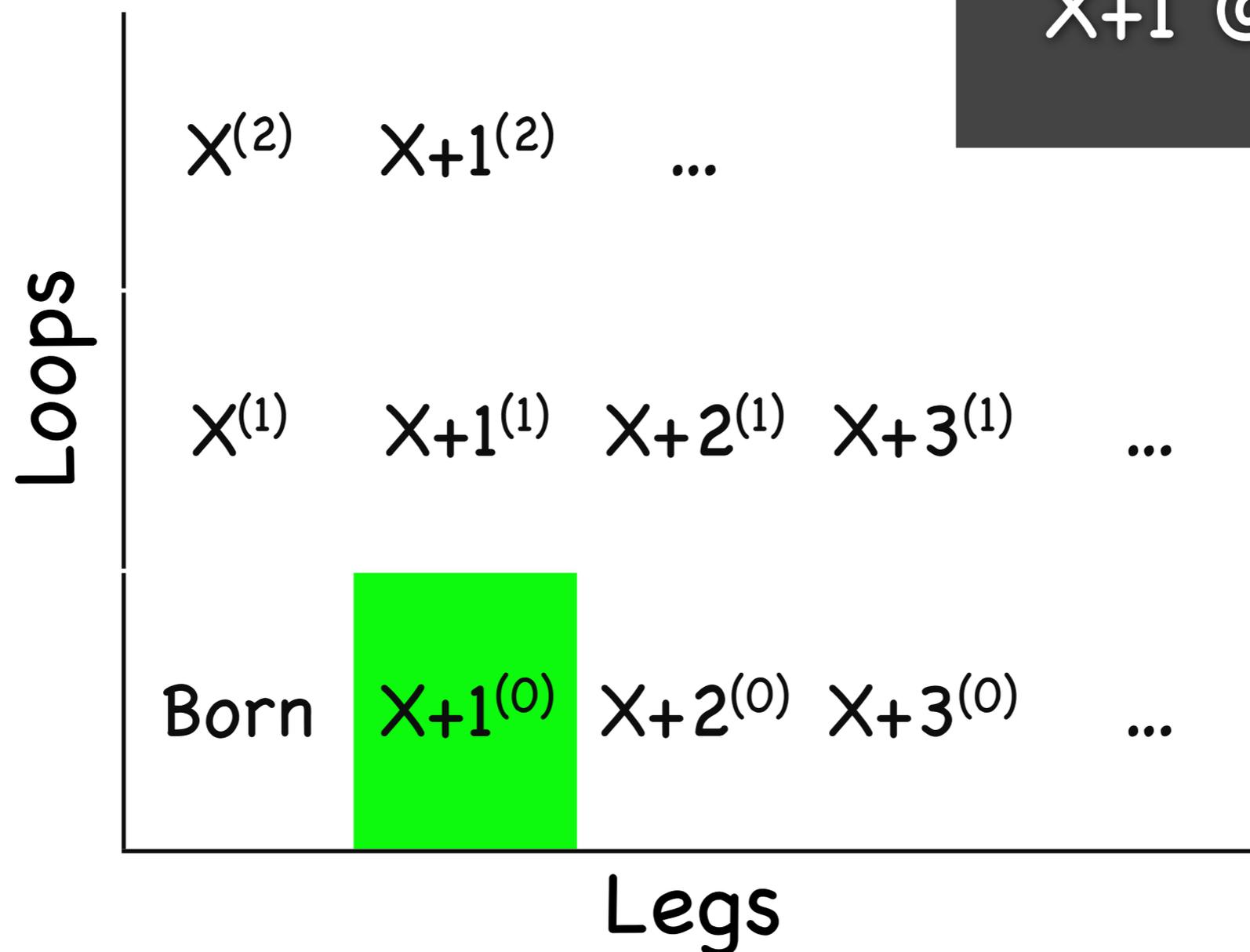
Born



M. Born
(1882-1970)
Nobel 1954

Loops and Legs

Another representation

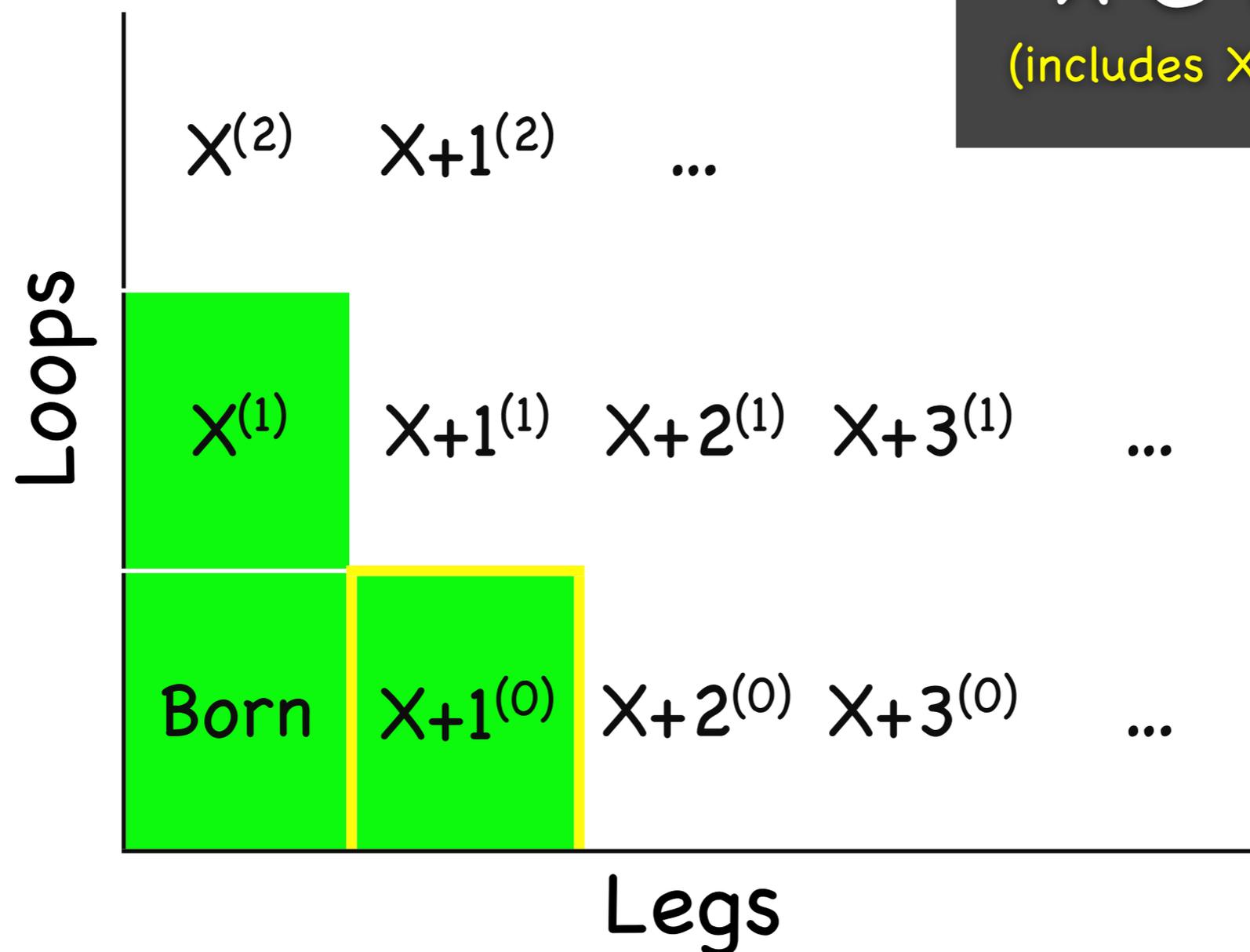


$X_{+1} @ LO$

Note: $\sigma \rightarrow \infty$
if jet not
resolved

Loops and Legs

Another representation



$X @ NLO$
(includes $X+1 @ LO$)

Note: $X+1$ jet
observables
only correct
at LO

Fixed-Order QCD

What kind of observables can we evaluate this way?

Perturbation theory valid $\rightarrow \alpha_s$ must be small
 \rightarrow All $Q_i \gg \Lambda_{\text{QCD}}$

Multi-scale: absence of enhancements from soft/collinear singular (conformal) dynamics
 \rightarrow All $Q_i/Q_j \approx 1$

All resolved scales $\gg \Lambda_{\text{QCD}}$ AND no large hierarchies*

*At "leading twist" (not counting underlying event)

Fixed-Order QCD

All resolved scales $\gg \Lambda_{\text{QCD}}$ AND no large hierarchies*

*At "leading twist" (not counting underlying event)

Trivially untrue for QCD

We're colliding, and observing, hadrons \rightarrow small scales

We want to consider high-scale processes \rightarrow large scale differences

\rightarrow A Priori, no perturbatively calculable observables in hadron-hadron collisions

Resummed QCD

All resolved scales $\gg \Lambda_{\text{QCD}}$ AND no large hierarchies*

*At "leading twist" (not counting underlying event)

Trivially untrue for QCD

We're colliding, and observing, hadrons \rightarrow small scales

We want to consider high-scale processes \rightarrow large scale differences

$$\frac{d\sigma}{dX} = \sum_{a,b} \sum_f \int_{\hat{X}_f} f_a(x_a, Q_i^2) f_b(x_b, Q_i^2) \frac{d\hat{\sigma}_{ab \rightarrow f}(x_a, x_b, f, Q_i^2, Q_f^2)}{d\hat{X}_f} D(\hat{X}_f \rightarrow X, Q_i^2, Q_f^2)$$

PDFs: needed to compute inclusive cross sections

FFs: needed to compute (semi-) exclusive cross sections

All resolved scales $\gg \Lambda_{\text{QCD}}$ AND X Infrared Safe

*At "leading twist" (not counting underlying event)

Parton Showers

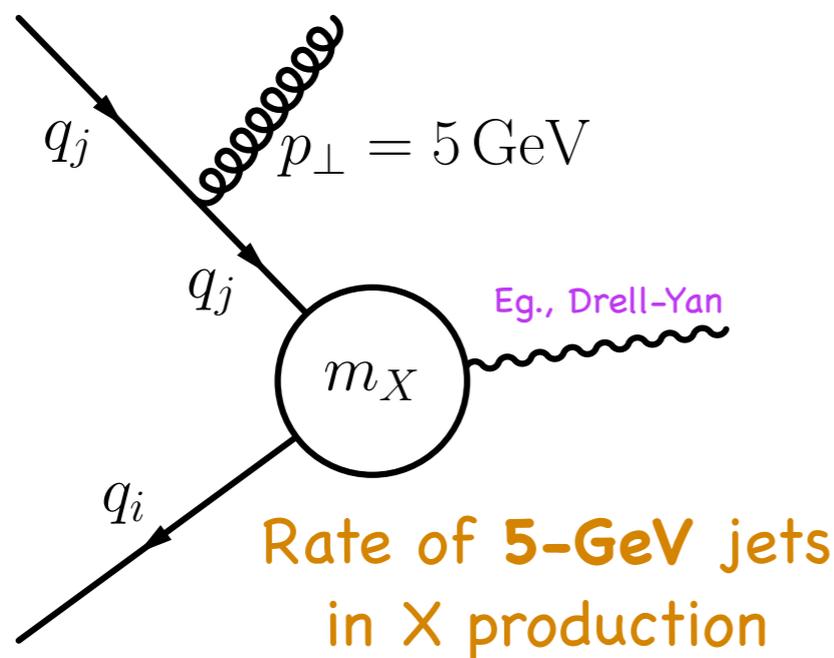
≈ Exclusive Resummation

Conformal QCD

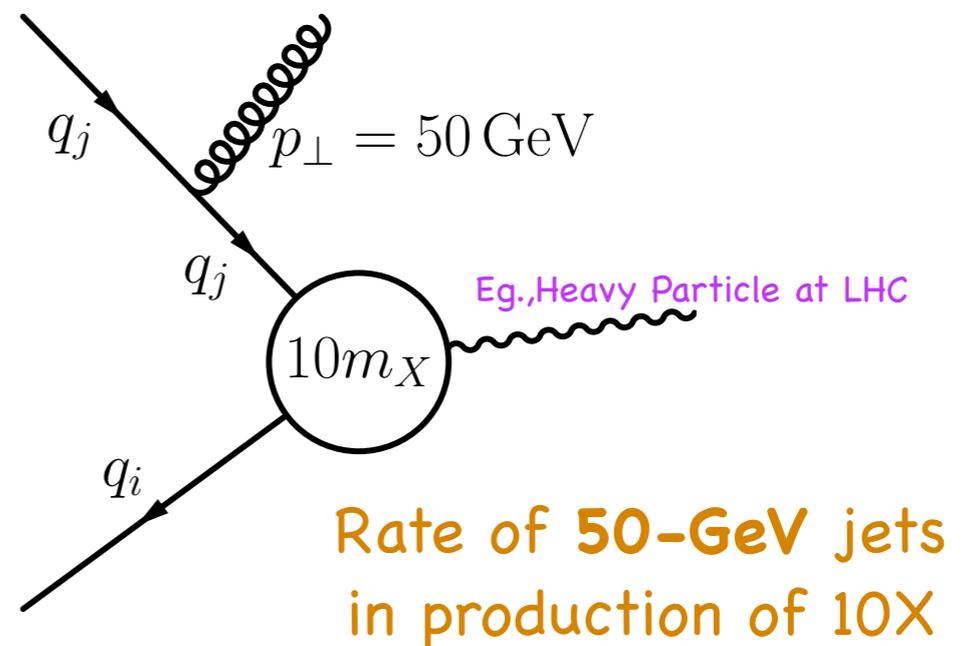
Flashback:
Conformal QCD

Bremsstrahlung

Rate of bremsstrahlung jets mainly depends on the **RATIO** of the jet p_T to the "hard scale"



\approx



See, e.g.,

Plehn, Rainwater, PS: PLB645(2007)217

Plehn, Tait: 0810.2919 [hep-ph]

Alwall, de Visscher, Maltoni:

JHEP 0902(2009)017

Bremsstrahlung



$$d\sigma_X = \dots$$

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}} \frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}} \frac{ds_{3b}}{s_{3b}}$$

Interpretation: the structure evolves

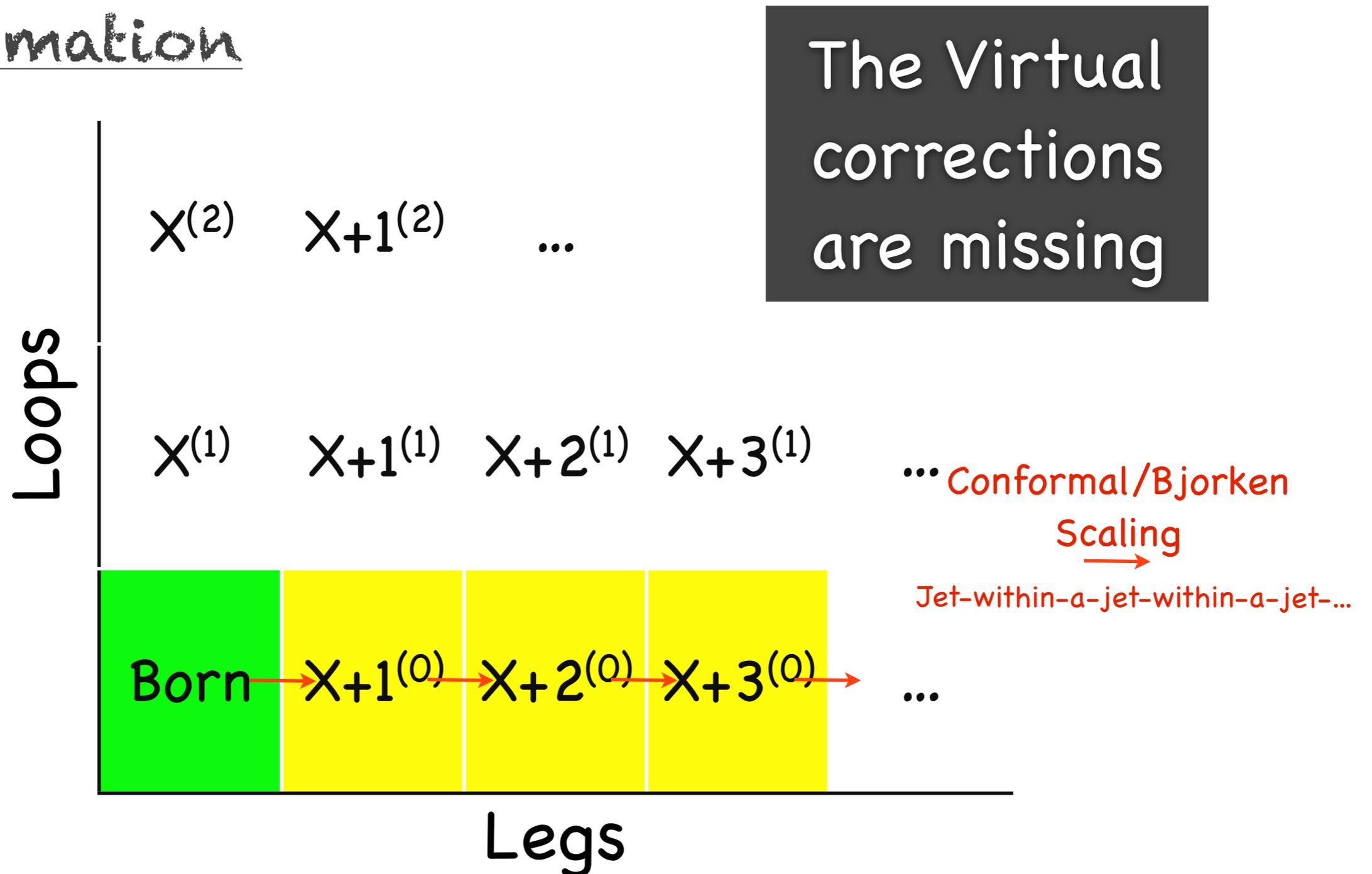
But something's not right...

This is an approximation to infinite-order tree-level cross sections

Total cross section would be infinite ...

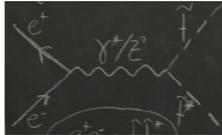
Loops and Legs

Summation



Resummation



$$d\sigma_X = \dots$$


$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}} \frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}} \frac{ds_{3b}}{s_{3b}}$$

- ▶ Interpretation: the structure evolves! (example: $X = 2$ -jets)
 - Take a jet algorithm, with resolution measure “Q”, apply it to your events
 - At a very crude resolution, you find that everything is 2-jets

Resummation



$$d\sigma_X = \dots$$

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

$$d\sigma_{X+2} \sim 2g^2 d\sigma_{X+1} \frac{ds_{a2}}{s_{a2}} \frac{ds_{2b}}{s_{2b}}$$

$$d\sigma_{X+3} \sim 2g^2 d\sigma_{X+2} \frac{ds_{a3}}{s_{a3}} \frac{ds_{3b}}{s_{3b}}$$

Interpretation: the structure evolves

$$\sigma_{X+1}(Q) = \sigma_{X;\text{incl}} - \sigma_{X;\text{excl}}(Q)$$

This includes both real and virtual corrections

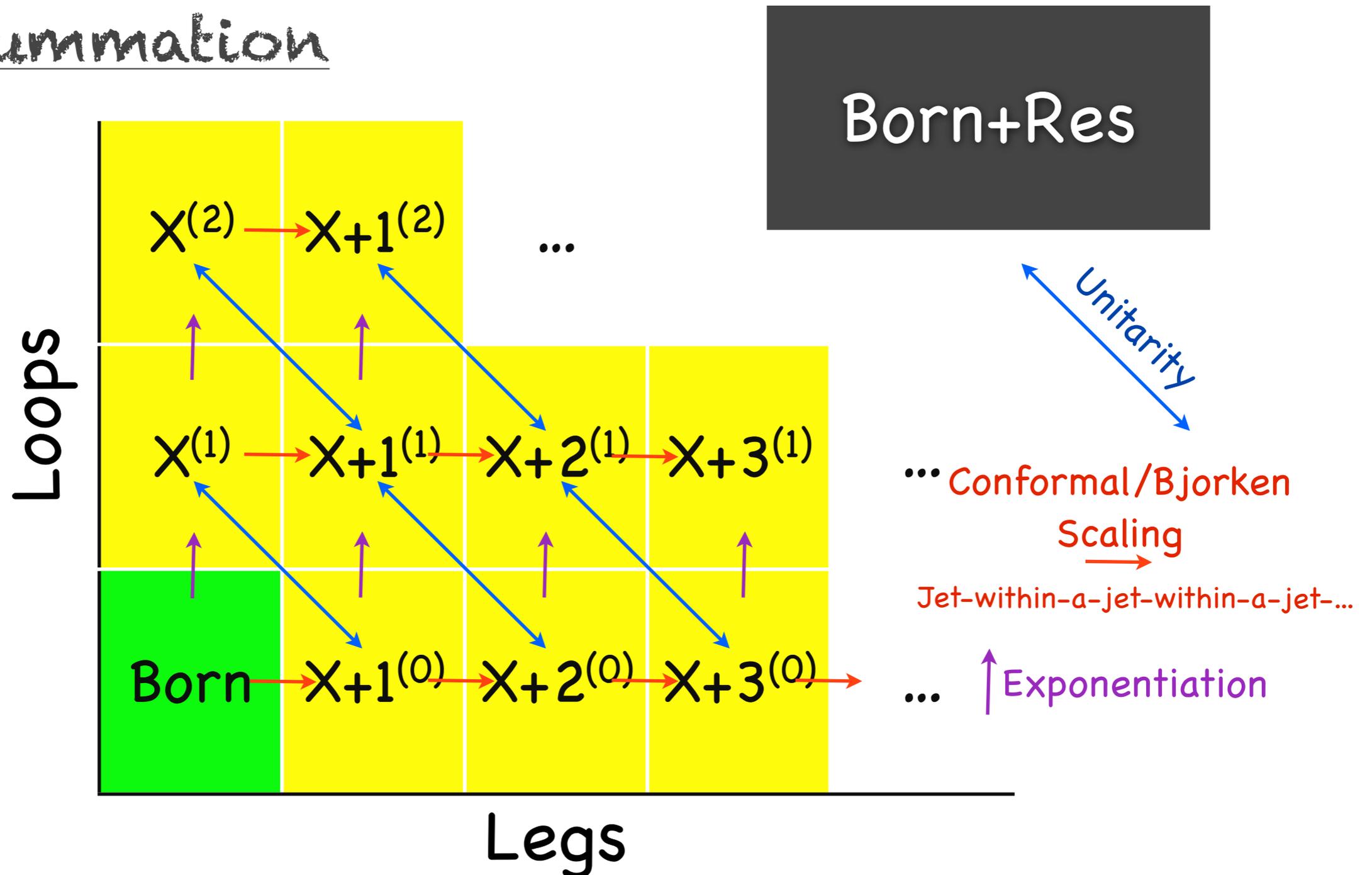
+ UNITARITY:

Virt = - Int(Tree) + F
(or: given a jet definition, an event has either 0, 1, 2, or n jets)

$$\begin{aligned} \sigma_{X;\text{excl}} &= \sigma_X - \sigma_{X+1} \\ &= \sigma_X - \sigma_{X+1;\text{excl}} - \sigma_{X+2;\text{excl}} - \dots \end{aligned}$$

Loops and Legs

Resummation



Born to Shower

Born $\frac{d\sigma}{d\mathcal{O}} \Big|_{\text{Born}} = \int d\Phi_X w_X^{(0)} \delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$ $\{p\} : \text{partons}$
 $w_X^{(0)} \propto \text{PDFs} \times |M_X^{(0)}|^2$

But instead of evaluating \mathcal{O} directly on the Born final state,
first insert a showering operator

Born + shower $\frac{d\sigma}{d\mathcal{O}} \Big|_{\text{PS}} = \int d\Phi_X w_X^{(0)} S(\{p\}_X, \mathcal{O})$ $\{p\} : \text{partons}$
 $S : \text{showering operator}$

To first order, S does nothing

$$S(\{p\}_X, \mathcal{O}) = \delta(\mathcal{O} - \mathcal{O}(\{p\}_X)) + \mathcal{O}(\alpha_s)$$

The Shower Operator



To Lowest Order

$$S(\{p\}_X, \mathcal{O}) = \delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

To First Order

(unitarity)

$$S(\{p\}_X, \mathcal{O}) = \left(1 - \int_{t_{\text{start}}}^{t_{\text{had}}} dt \frac{d\mathcal{P}}{dt} \right) \delta(\mathcal{O} - \mathcal{O}(\{p\}_X)) \\ + \int_{t_{\text{start}}}^{t_{\text{had}}} dt_{X+1} \frac{d\mathcal{P}}{dt_{X+1}} \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+1}))$$

Splitting Operator

$$\mathcal{P} = \int \frac{d\Phi_{X+1}}{d\Phi_X} \frac{w_{X+1}}{w_X} \Big|_{\text{PS}}$$

= Shower approximation
of $X \rightarrow X+1$

The Shower Operator



To ALL Orders

(Markov Chain)

$$S(\{p\}_X, \mathcal{O}) = \Delta(t_{\text{start}}, t_{\text{had}}) \delta(\mathcal{O} - \mathcal{O}(\{p\}_X))$$

"Nothing Happens" → "Evaluate Observable"

$$- \int_{t_{\text{start}}}^{t_{\text{had}}} dt \frac{d\Delta(t_{\text{start}}, t)}{dt} S(\{p\}_{X+1}, \mathcal{O})$$

"Something Happens" → "Continue Shower"

All-orders Probability that nothing happens

$$\Delta(t_1, t_2) = \exp \left(- \int_{t_1}^{t_2} dt \frac{d\mathcal{P}}{dt} \right)$$

(Exponentiation)

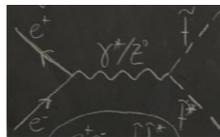
Analogous to nuclear decay

$$N(t) \approx N(0) \exp(-ct)$$

Splitting Functions



“DLA” $\propto \frac{s_{ab}}{s_{ai}s_{ib}}$

$d\sigma_X = \dots$ 

$$d\sigma_{X+1} \sim 2g^2 d\sigma_X \frac{ds_{a1}}{s_{a1}} \frac{ds_{1b}}{s_{1b}}$$

Splitting Operator

Examples

$$\mathcal{P} = \int \frac{d\Phi_{X+1}}{d\Phi_X} \frac{w_{X+1}}{w_X} \Big|_{\text{PS}}$$

$$\mathcal{P}_{\text{DGLAP}} = \sum_i \int \frac{dQ^2}{Q^2} dz P_i(z)$$

$$\mathcal{P}_{\text{Antenna}} = \int \frac{ds_{ij} ds_{jk}}{16\pi^2 s} \frac{|M_3(s_{ij}, s_{jk}, s)|^2}{|M_2(s)|^2}$$

Splitting Functions

DGLAP

(E.g., HERWIG, PYTHIA)

$$d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_{abc}}{2\pi} P_{a \rightarrow bc}(z) dt dz .$$

$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z} ,$$

$$P_{g \rightarrow gg}(z) = N_C \frac{(1-z(1-z))^2}{z(1-z)} ,$$

$$P_{g \rightarrow q\bar{q}}(z) = T_R (z^2 + (1-z)^2) ,$$

$$P_{q \rightarrow q\gamma}(z) = e_q^2 \frac{1+z^2}{1-z} ,$$

$$P_{\ell \rightarrow \ell\gamma}(z) = e_\ell^2 \frac{1+z^2}{1-z} ,$$

Dipole-Antennae

(E.g., ARIADNE, VINCIA)

$$d\mathcal{P}_{IK \rightarrow ijk} = \frac{ds_{ij} ds_{jk}}{16\pi^2 s} a(s_{ij}, s_{jk})$$

$$a_{q\bar{q} \rightarrow qg\bar{q}} = \frac{2C_F}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2)$$

$$a_{qg \rightarrow qgg} = \frac{C_A}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3)$$

$$a_{gg \rightarrow ggg} = \frac{C_A}{s_{ij}s_{jk}} (2s_{ik}s + s_{ij}^2 + s_{jk}^2 - s_{ij}^3 - s_{jk}^3)$$

$$a_{qg \rightarrow q\bar{q}'q'} = \frac{T_R}{s_{jk}} (s - 2s_{ij} + 2s_{ij}^2)$$

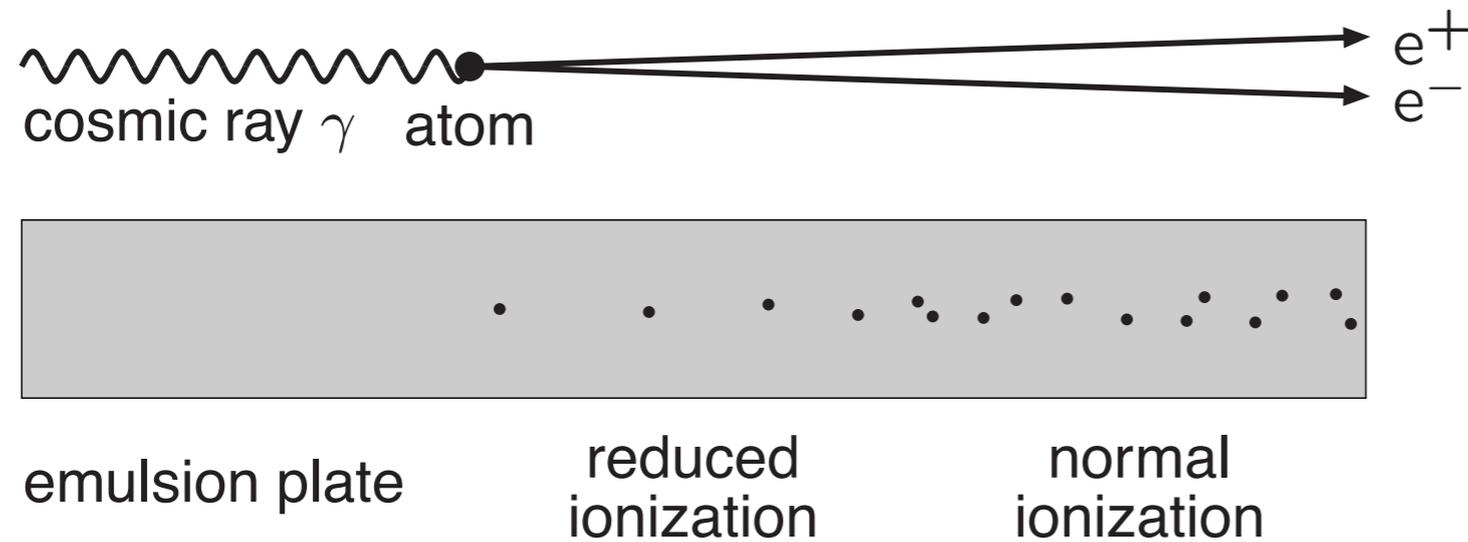
$$a_{gg \rightarrow g\bar{q}'q'} = a_{qg \rightarrow q\bar{q}'q'}$$

... + non-singular terms

NB: Also others, e.g., Catani-Seymour (SHERPA), Sector Antennae,

Coherence

QED: Chudakov effect (mid-fifties)



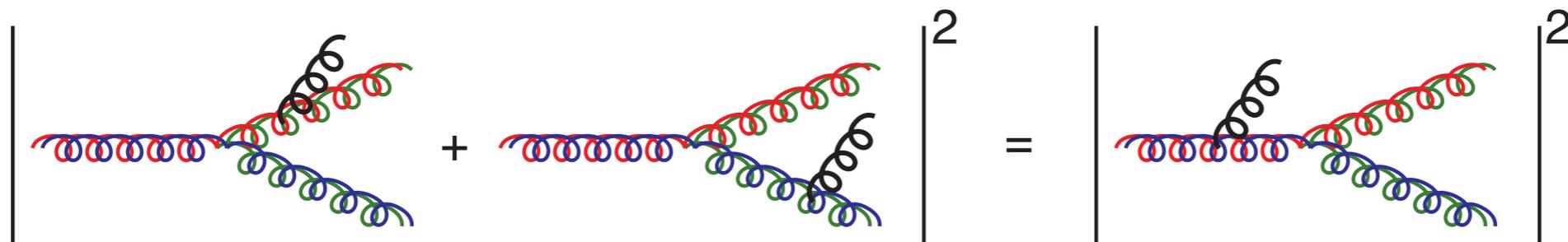
Approximations to Coherence:

Angular Ordering (HERWIG)

Angular Vetos (PYTHIA)

Coherent Dipoles/Antennae (ARIADNE, CS, VINCIA)

QCD: colour coherence for **soft** gluon emission



The Initial State

Parton Densities and Initial-State Showers

Parton Densities for MC

LO

Consistent with LO matrix elements in LO generators
Effectively 'tuned' to absorb missing NLO contributions
But they give quite bad fits compared to NLO ...

NLO

Formally consistent with NLO matrix elements
Effectively 'tuned' with NLO theory
→ badly tuned for LO matrix elements (not enough low-x glue)?
Suggest to only use for NLO generators?

LO*,

MC

pdfs,

...

Best of both worlds?

PDF has always had an impact on generator tuning

But now we are going the other way: tune the PDF!

Still gaining experience. Proceed with caution & sanity checks

PDF Uncertainties

Much debate recently on PDF errors

Attempt to propagate experimental errors properly \rightarrow 68% CL

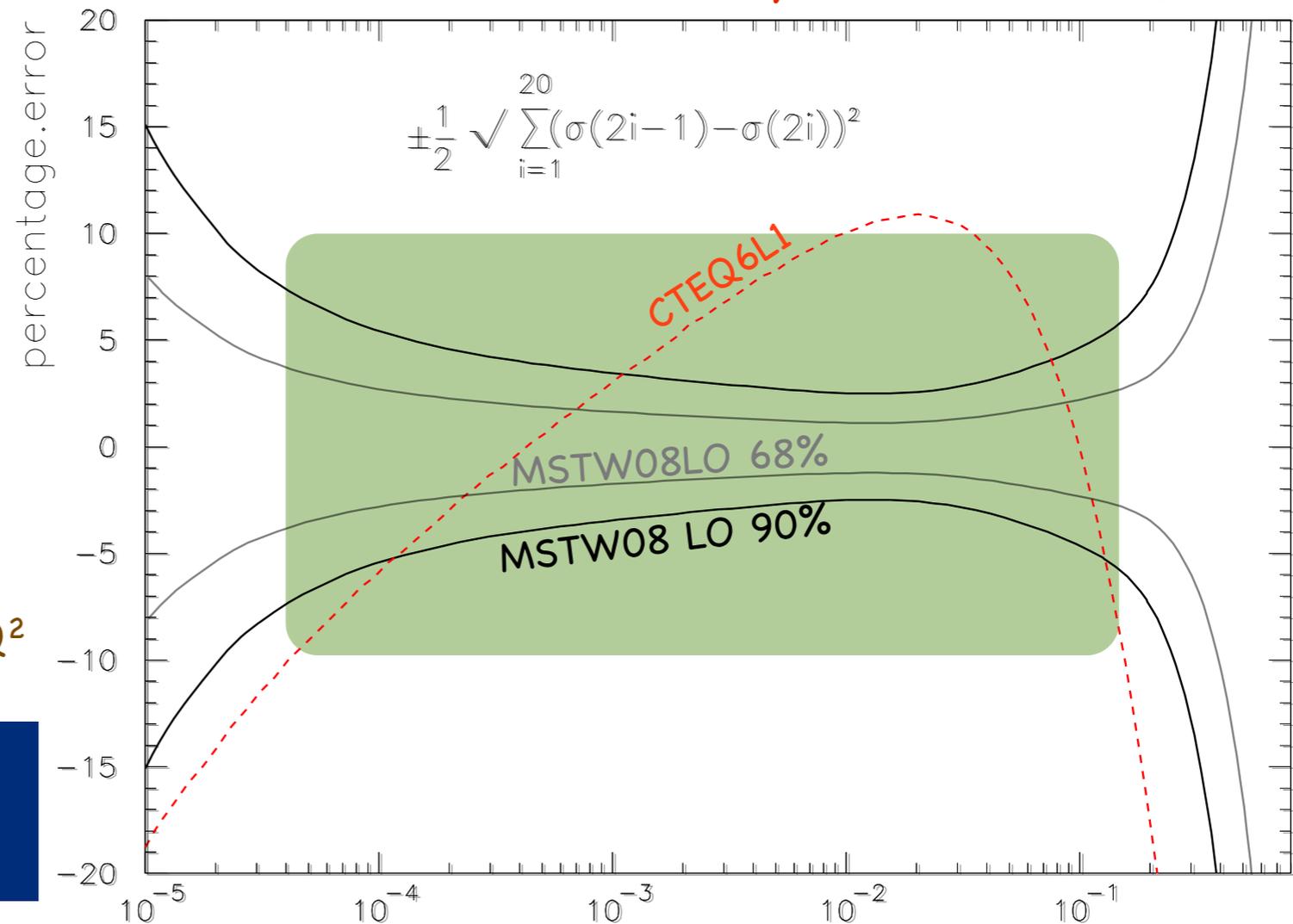
But "tensions" between different badly compatible data sets \rightarrow ... ?

\rightarrow 90%, something else?

+ unknown uncertainty from starting parametrization at low Q^2

Still, good to $\approx 10\%$ even for LO gluon in $10^{-4} < x < 10^{-1}$
(bigger errors at lower Q^2)

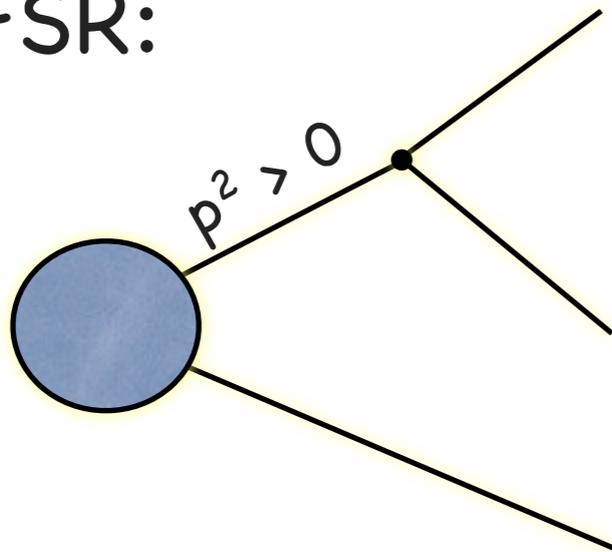
Gluon PDF uncertainty, $Q^2 = (10 \text{ GeV})^2$



Initial-State Evolution

= Spacelike (backwards) Evolution

FSR:



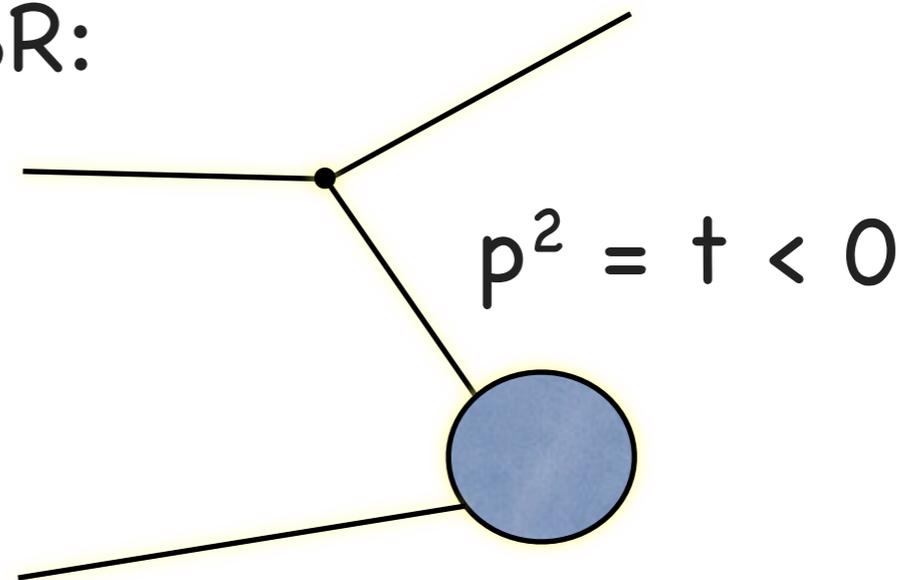
Virtualities are

Timelike: $p^2 > 0$

Start at $Q^2 = Q_f^2$

Unconstrained forwards evolution

ISR:



Virtualities are

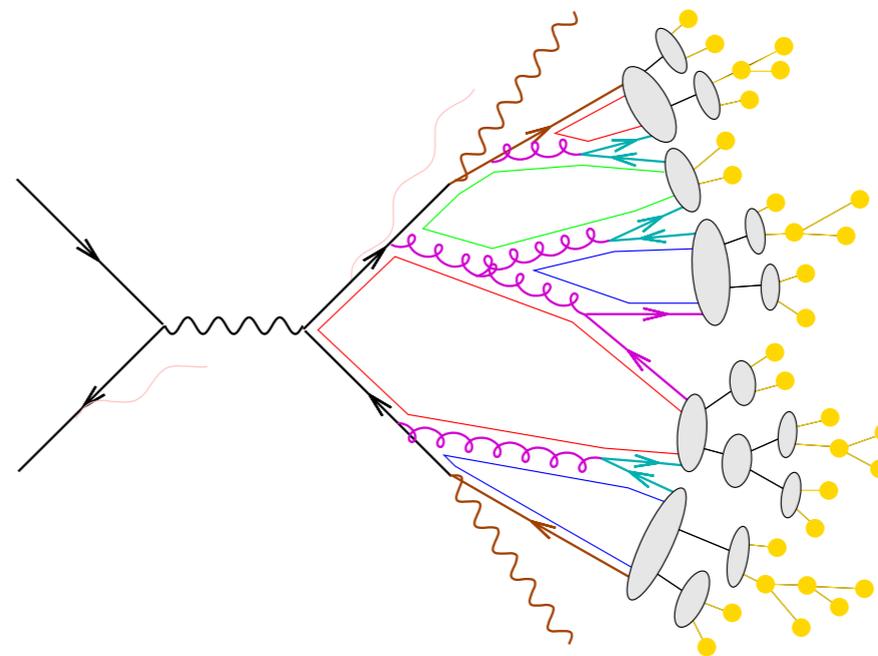
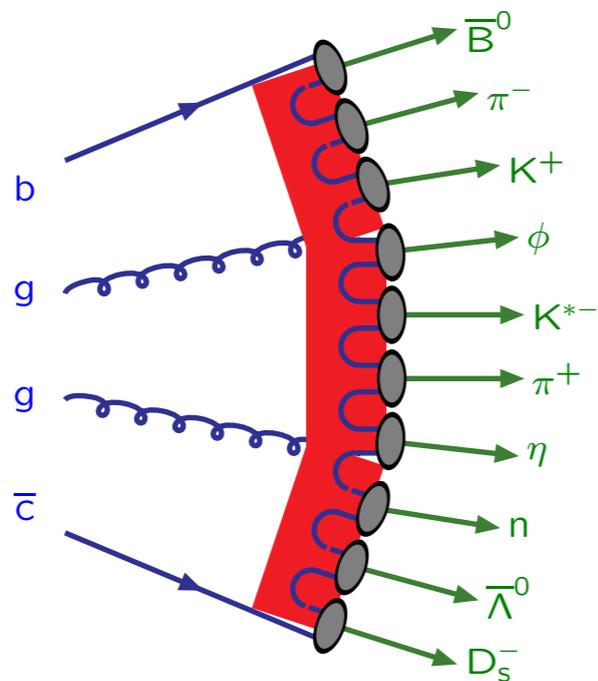
Spacelike: $p^2 < 0$

Start at $Q^2 = Q_i^2$

Constrained backwards evolution towards boundary condition = proton

+ Look Out! (Especially Tricky): ISR-FSR interference! FSR off ISR!

Hadronization



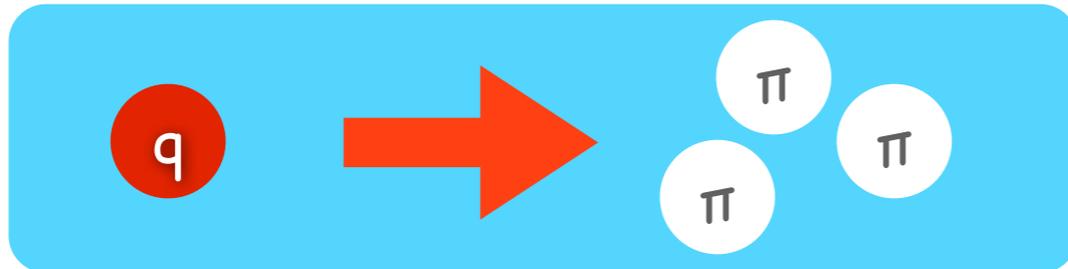
program	PYTHIA	HERWIG (&SHERPA)
model	string	cluster
energy-momentum picture	powerful	simple
parameters	predictive	unpredictive
flavour composition	few	many
parameters	messy	simple
	unpredictive	in-between
	many	few

Small strings → clusters. Large clusters → strings

Independent Fragmentation?

≈ Local Parton-Hadron Duality (LPHD)

Universal fragmentation of a parton into hadrons



This is awfully wrong!

The point of confinement is that partons are colored
Hadronization = the process of color neutralization

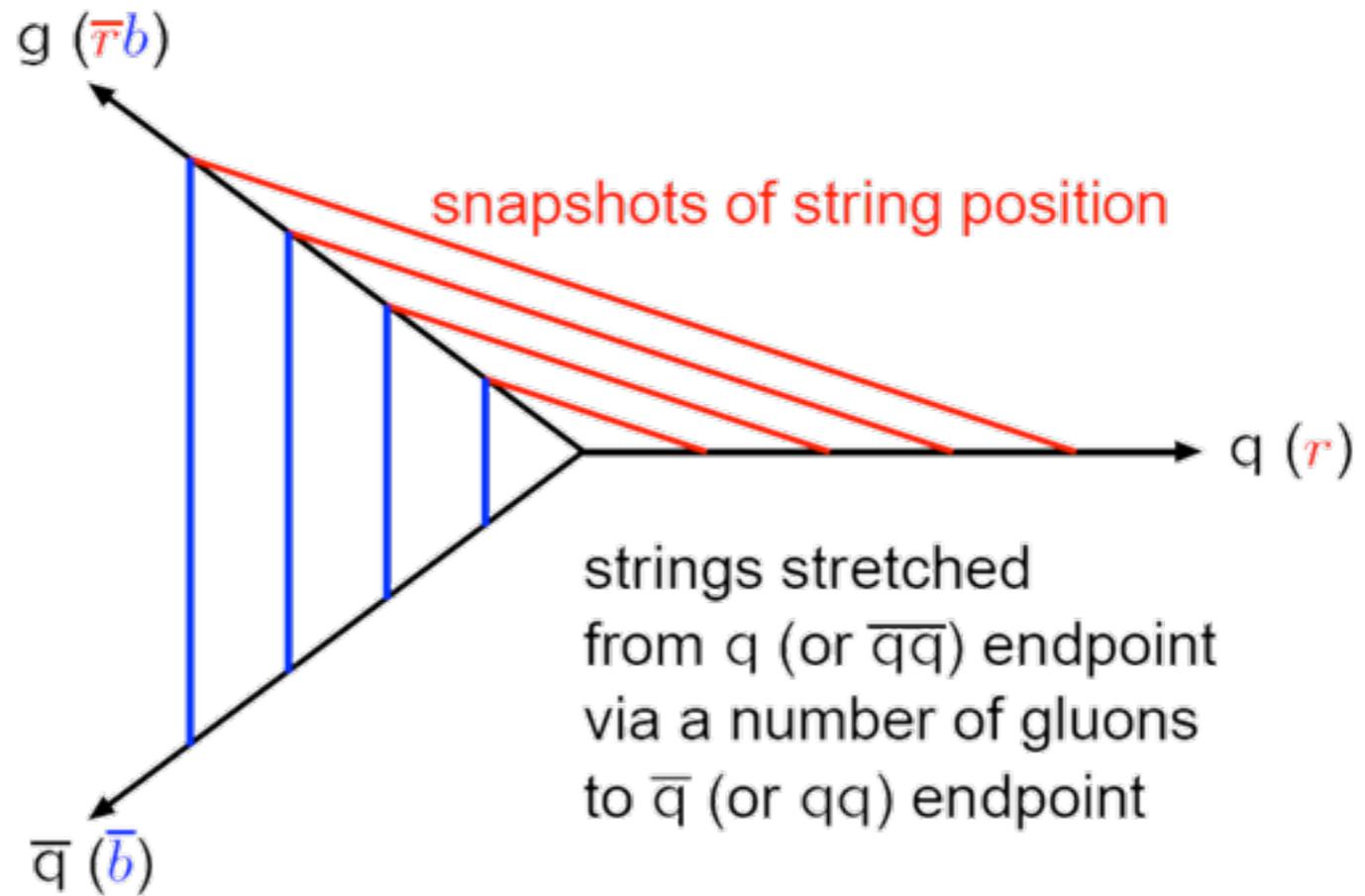
I.e, the one question NOT addressed by LPHD or I.F.

My opinion: *despite some success at describing inclusive quantities, it is fundamentally misguided to think about independent fragmentation of individual partons*

The (Lund) String Model

Map:

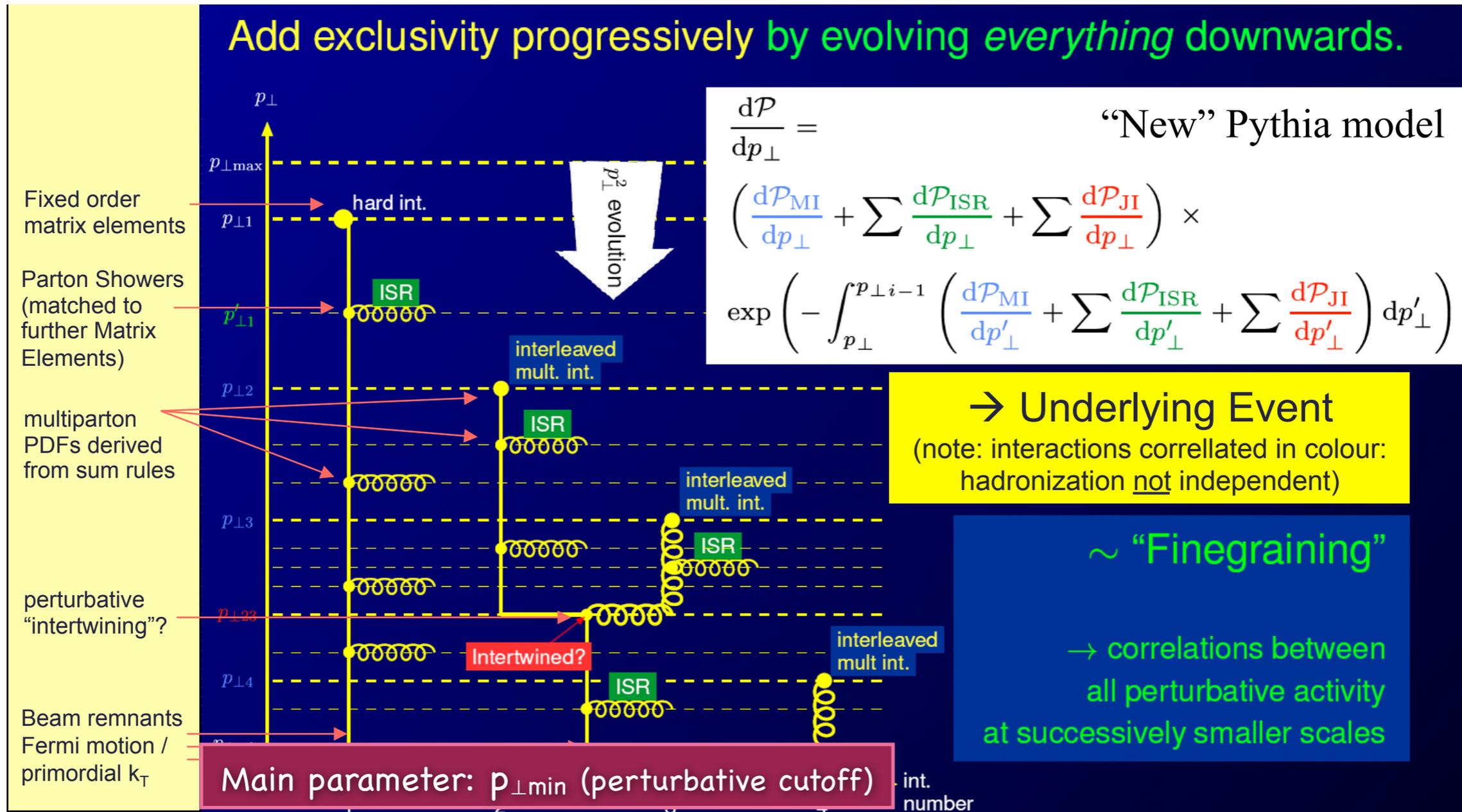
- **Quarks** → String Endpoints
- **Gluons** → Transverse Excitations (kinks)
- Physics then in terms of string worldsheet evolving in spacetime
- Probability of string break constant per unit area → **AREA LAW**



Gluon = kink on string, carrying energy and momentum

Simple space-time picture + no separate params for g jets
Details of string breaks more complicated ...

Underlying Event: Multiple Parton-Parton Interactions



Generators - Summary

- Allow to connect theory \leftrightarrow experiment
 - On PHYSICAL OBSERVABLES
 - Precision is a function of Model & Constraints
- Random Numbers to Simulate Quantum Behaviour
 - Fixed-Order pQCD supplemented with showers, hadronization, decays, underlying event, matching, ...
- No single program does it all
 - + Variations needed for uncertainty estimates!
 - Rapid evolution of theory/models/constraints/tunes/...
 - Emphasis on interfaces, interoperability

(Some) Possible Discussion Topics

- What's the difference (relation?) between zero bias, minimum-bias, and underlying event?
 - + What's (the role of) diffraction?
- How does resummation get around the problem of infinities at fixed order? Where do the infinities go?
- Where does the motivation for the string model come from? How much can we “know” about non-perturbative physics?
 - + how do strings break?
- Multiple interactions: perturbative or a non-perturbative component? Beam remnants and PDFs? Is it a theory or a model?
- Factorization