Flavor Physics at the LHC Era*

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Abstract

This is a written version of a series of lectures aimed at graduate students in particle theory/string theory/particle experiment familiar with the basics of the Standard Model. We explain the many reasons for the interest in flavor physics. We describe flavor physics and the related CP violation within the Standard Model, and explain how the B-factories proved that the Kobayashi-Maskawa mechanism dominates the CP violation that is observed in meson decays. We explain the implications of flavor physics for new physics, with emphasis on the "new physics flavor puzzle", and present the idea of minimal flavor violation as a possible solution. We give an example of the possible impact of a signal for new physics (rather than a bound) by considering a dimuon CP asymmetry in B_s decays of order a percent. We explain how the ATLAS and CMS experiments can solve the new physics flavor puzzle and perhaps shed light on the standard model flavor puzzle.

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I. WHAT IS FLAVOR?

The term "**flavors**" is used, in the jargon of particle physics, to describe several copies of the same gauge representation, namely several fields that are assigned the same quantum charges. Within the Standard Model, when thinking of its unbroken $SU(3)_{\rm C}\times U(1)_{\rm EM}$ gauge group, there are four different types of particles, each coming in three flavors:

- Up-type quarks in the $(3)_{+2/3}$ representation: u, c, t;
- Down-type quarks in the $(3)_{-1/3}$ representation: d, s, b;
- Charged leptons in the $(1)_{-1}$ representation: e, μ, τ;
- Neutrinos in the $(1)_0$ representation: ν_1, ν_2, ν_3.

The term "**flavor physics**" refers to interactions that distinguish between flavors. By definition, gauge interactions, namely interactions that are related to unbroken symmetries and mediated therefore by massless gauge bosons, do not distinguish among the flavors and do not constitute part of flavor physics. Within the Standard Model, flavor-physics refers to the weak and Yukawa interactions.

The term "**flavor parameters**" refers to parameters that carry flavor indices. Within the Standard Model, these are the nine masses of the charged fermions and the four "mixing parameters" (three angles and one phase) that describe the interactions of the charged weak-force carriers ($W^\pm$) with quark-antiquark pairs. If one augments the Standard Model with Majorana mass terms for the neutrinos, one should add to the list three neutrino masses and six mixing parameters (three angles and three phases) for the $W^\pm$ interactions with lepton-antilepton pairs.

The term "**flavor universal**" refers to interactions with couplings (or to flavor parameters) that are proportional to the unit matrix in flavor space. Thus, the strong and electromagnetic interactions are flavor-universal.[1] An alternative term for "flavor-universal" is "**flavor-blind**".

The term "**flavor diagonal**" refers to interactions with couplings (or to flavor parameters) that are diagonal, but not necessarily universal, in the flavor space. Within the Standard Model, the Yukawa interactions of the Higgs particle are flavor diagonal in the mass basis.

The term "**flavor changing**" refers to processes where the initial and final flavor-numbers (that is, the number of particles of a certain flavor minus the number of anti-particles of the same flavor) are different. In "flavor changing charged current" processes, both up-type and down-type flavors, and/or both charged lepton and neutrino flavors are involved. Examples are (i) muon decay via $\mu \to e\bar{\nu}_i\nu_j$, and (ii) $K^- \to \mu^-\bar{\nu}_j$ (which corresponds, at the quark level, to $s\bar{u} \to \mu^-\bar{\nu}_j$). Within the Standard Model, these processes are mediated by the W-bosons and occur at tree level. In "**flavor changing neutral current**" (FCNC) processes, either up-type or down-type flavors but not both, and/or either charged lepton or neutrino flavors but not both, are involved. Example are (i) muon decay via $\mu \to e\gamma$ and (ii) $K_L \to \mu^+\mu^-$ (which corresponds, at the quark level, to $s\bar{d} \to \mu^+\mu^-$). Within the Standard Model, these processes do not occur at tree level, and are often highly suppressed.

Another useful term is "**flavor violation**". We will explain it later in these lectures.

II. WHY IS FLAVOR PHYSICS INTERESTING?

- Flavor physics can discover new physics or probe it before it is directly observed in experiments. Here are some examples from the past:
  - The smallness of $\frac{\Gamma(K_L\to\mu^+\mu^-)}{\Gamma(K^+\to\mu^+\nu)}$ led to predicting a fourth (the charm) quark;
  - The size of Δm_K led to a successful prediction of the charm mass;

[1] In the interaction basis, the weak interactions are also flavor-universal, and one can identify the source of all flavor physics in the Yukawa interactions among the gauge-interaction eigenstates.

  - The size of Δm_B led to a successful prediction of the top mass;
  - The measurement of ε_K led to predicting the third generation.
  - The measurement of neutrino flavor transitions led to the discovery of neutrino masses.

- CP violation is closely related to flavor physics. Within the Standard Model, there is a single CP violating parameter, the Kobayashi-Maskawa phase $\delta_{\rm KM}$ [1]. Baryogenesis tells us, however, that there must exist new sources of CP violation. Measurements of CP violation in flavor changing processes might provide evidence for such sources.

- The fine-tuning problem of the Higgs mass, and the puzzle of the dark matter imply that there exists new physics at, or below, the TeV scale. If such new physics had a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. The question of why this does not happen constitutes the *new physics flavor puzzle*.

- Most of the charged fermion flavor parameters are small and hierarchical. The Standard Model does not provide any explanation of these features. This is the *Standard Model flavor puzzle*. The puzzle became even deeper after neutrino masses and mixings were measured because, so far, neither smallness nor hierarchy in these parameters have been established.

III. FLAVOR IN THE STANDARD MODEL

A model of elementary particles and their interactions is defined by the following ingredients: (i) The symmetries of the Lagrangian and the pattern of spontaneous symmetry breaking; (ii) The representations of fermions and scalars. The Standard Model (SM) is defined as follows:
(i) The gauge symmetry is

$$G_{\rm SM} = SU(3)_{\rm C} \times SU(2)_{\rm L} \times U(1)_{\rm Y}. \tag{1}$$

It is spontaneously broken by the VEV of a single Higgs scalar, $\phi(1,2)_{1/2}$ ($\langle\phi^0\rangle = v/\sqrt{2}$):

$$G_{\rm SM} \to SU(3)_{\rm C} \times U(1)_{\rm EM}. \tag{2}$$

(ii) There are three fermion generations, each consisting of five representations of $G_{\rm SM}$:

$$Q_{Li}(3,2)_{+1/6}, \quad U_{Ri}(3,1)_{+2/3}, \quad D_{Ri}(3,1)_{-1/3}, \quad L_{Li}(1,2)_{-1/2}, \quad E_{Ri}(1,1)_{-1}. \tag{3}$$

A. The interactions basis

The Standard Model Lagrangian, $\mathcal{L}_{\rm SM}$, is the most general renormalizable Lagrangian that is consistent with the gauge symmetry (1), the particle content (3) and the pattern of spontaneous symmetry breaking (2). It can be divided to three parts:

$$\mathcal{L}_{\rm SM} = \mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs} + \mathcal{L}_{\rm Yukawa}. \tag{4}$$

As concerns the kinetic terms, to maintain gauge invariance, one has to replace the derivative with a covariant derivative:

$$D^\mu = \partial^\mu + ig_s G^\mu_a L_a + igW^\mu_b T_b + ig' B^\mu Y. \tag{5}$$

Here G^μ_a are the eight gluon fields, W^μ_b the three weak interaction bosons and B^μ the single hypercharge boson. The L_a's are $SU(3)_{\rm C}$ generators (the 3×3 Gell-Mann matrices $\frac{1}{2}\lambda_a$ for triplets, 0 for singlets), the T_b's are $SU(2)_{\rm L}$ generators (the 2×2 Pauli matrices $\frac{1}{2}\tau_b$ for doublets, 0 for singlets), and the Y's are the $U(1)_{\rm Y}$ charges. For example, for the quark doublets Q_L, we have

$$\mathcal{L}_{\rm kinetic}(Q_L) = i\overline{Q_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2}g_s G^\mu_a \lambda_a + \frac{i}{2} g W^\mu_b \tau_b + \frac{i}{6} g' B^\mu\right)\delta_{ij} Q_{Lj}, \tag{6}$$

while for the lepton doublets L^I_L, we have

$$\mathcal{L}_{\rm kinetic}(L_L) = i\overline{L_{Li}}\gamma_\mu \left(\partial^\mu + \frac{i}{2} g W^\mu_b \tau_b - \frac{i}{2} g' B^\mu\right)\delta_{ij} L_{Lj}. \tag{7}$$

The unit matrix in flavor space, δ_{ij}, signifies that these parts of the interaction Lagrangian are flavor-universal. In addition, they conserve CP.

The Higgs potential, which describes the scalar self interactions, is given by:

$$\mathcal{L}_{\rm Higgs} = \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger\phi)^2. \tag{8}$$

For the Standard Model scalar sector, where there is a single doublet, this part of the Lagrangian is also CP conserving.

The quark Yukawa interactions are given by

$$-\mathcal{L}_{\rm Y}^{q} = Y^d_{ij}\overline{Q_{Li}}\phi D_{Rj} + Y^u_{ij}\overline{Q_{Li}}\tilde{\phi} U_{Rj} + {\rm h.c.}, \tag{9}$$

(where $\tilde{\phi} = i\tau_2\phi^\dagger$) while the lepton Yukawa interactions are given by

$$-\mathcal{L}_{\rm Y}^{\ell} = Y^e_{ij}\overline{L_{Li}}\phi E_{Rj} + {\rm h.c.}. \tag{10}$$

This part of the Lagrangian is, in general, flavor-dependent (that is, $Y^f \not\propto \mathbf{1}$) and CP violating.

B. Global symmetries

In the absence of the Yukawa matrices Y^d, Y^u and Y^e, the SM has a large $U(3)^5$ global symmetry:

$$G_{\rm global}(Y^{u,d,e} = 0) = SU(3)^3_q \times SU(3)^2_\ell \times U(1)^5, \tag{11}$$

where

$$\begin{aligned} SU(3)^3_q &= SU(3)_Q \times SU(3)_U \times SU(3)_D, \\ SU(3)^2_\ell &= SU(3)_L \times SU(3)_E, \\ U(1)^5 &= U(1)_B \times U(1)_L \times U(1)_Y \times U(1)_{\rm PQ} \times U(1)_E. \end{aligned} \tag{12}$$

Out of the five $U(1)$ charges, three can be identified with baryon number (B), lepton number (L) and hypercharge (Y), which are respected by the Yukawa interactions. The two remaining $U(1)$ groups can be identified with the PQ symmetry whereby the Higgs and D_R, E_R fields have opposite charges, and with a global rotation of E_R only.

The point that is important for our purposes is that $\mathcal{L}_{\rm kinetic} + \mathcal{L}_{\rm Higgs}$ respect the non-Abelian flavor symmetry $S(3)^3_q \times SU(3)^2_\ell$, under which

$$Q_L \to V_Q Q_L, \quad U_R \to V_U U_R, \quad D_R \to V_D D_R, \quad L_L \to V_L L_L, \quad E_R \to V_E E_R, \tag{13}$$

where the V_i are unitary matrices. The Yukawa interactions (9) and (10) break the global symmetry,

$$G_{\rm global}(Y^{u,d,e} \neq 0) = U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau. \tag{14}$$

(Of course, the gauged $U(1)_Y$ also remains a good symmetry.) Thus, the transformations of Eq. (13) are not a symmetry of $\mathcal{L}_{\rm SM}$. Instead, they correspond to a change of the interaction basis. These observations also offer an alternative way of defining flavor physics: it refers to interactions that break the $SU(3)^5$ symmetry (13). Thus, the term "**flavor violation**" is often used to describe processes or parameters that break the symmetry.

One can think of the quark Yukawa couplings as spurions that break the global $SU(3)_q^3$ symmetry (but are neutral under $U(1)_B$),

$$Y^u \sim (3, \bar{3}, 1)_{SU(3)_q^3}, \quad Y^d \sim (3, 1, \bar{3})_{SU(3)_q^3}, \tag{15}$$

and of the lepton Yukawa couplings as spurions that break the global $SU(3)_\ell^2$ symmetry (but are neutral under $U(1)_e \times U(1)_\mu \times U(1)_\tau$),

$$Y^e \sim (3, \bar{3})_{SU(3)_\ell^2}. \tag{16}$$

The spurion formalism is convenient for several purposes: parameter counting (see below), identification of flavor suppression factors (see Section V), and the idea of minimal flavor violation (see Section VIII).

C. Counting parameters

How many independent parameters are there in $\mathcal{L}_Y^q$? The two Yukawa matrices, Y^u and Y^d, are 3×3 and complex. Consequently, there are 18 real and 18 imaginary parameters in these matrices. Not all of them are, however, physical. The pattern of $G_{\rm global}$ breaking means that there is freedom to remove 9 real and 17 imaginary parameters (the number of parameters in three 3×3 unitary matrices minus the phase related to $U(1)_B$). For example, we can use the unitay transformations $Q_L \to V_Q Q_L$, $U_R \to V_U U_R$ and $D_R \to V_D D_R$, to lead to the following interaction basis:

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u, \tag{17}$$

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d = {\rm diag}(y_d, y_s, y_b), \quad \lambda_u = {\rm diag}(y_u, y_c, y_t), \tag{18}$$

while V is a unitary matrix that depends on three real angles and one complex phase. We conclude that there are 10 quark flavor parameters: 9 real ones and a single phase. In the mass basis, we will identify the nine real parameters as six quark masses and three mixing angles, while the single phase is $\delta_{\rm KM}$.

How many independent parameters are there in $\mathcal{L}_Y^\ell$? The Yukawa matrix Y^e is 3×3 and complex. Consequently, there are 9 real and 9 imaginary parameters in this matrix. There is, however, freedom to remove 6 real and 9 imaginary parameters (the number of parameters in two 3×3 unitary matrices minus the phases related to $U(1)^3$). For example, we can use the unitay transformations $L_L \to V_L L_L$ and $E_R \to V_E E_R$, to lead to the following interaction basis:

$$Y^e = \lambda_e = \text{diag}(y_e, y_\mu, y_\tau). \tag{19}$$

We conclude that there are 3 real lepton flavor parameters. In the mass basis, we will identify these parameters as the three charged lepton masses. We must, however, modify the model when we take into account the evidence for neutrino masses.

D. The mass basis

Upon the replacement $\mathcal{R}e(\phi^0) \to \frac{v+H^0}{\sqrt{2}}$, the Yukawa interactions (9) give rise to the mass matricess

$$M_q = \frac{v}{\sqrt{2}} Y^q. \tag{20}$$

The mass basis corresponds, by definition, to diagonal mass matrices. We can always find unitary matrices V_{qL} and V_{qR} such that

$$V_{qL} M_q V_{qR}^\dagger = M_q^{\text{diag}} \equiv \frac{v}{\sqrt{2}} \lambda_q. \tag{21}$$

The four matrices V_{dL}, V_{dR}, V_{uL} and V_{uR} are then the ones required to transform to the mass basis. For example, if we start from the special basis (17), we have $V_{dL} = V_{dR} = V_{uR} = \mathbf{1}$ and $V_{uL} = V$. The combination $V_{uL} V_{dL}^\dagger$ is independent of the interaction basis from which we start this procedure.

We denote the left-handed quark mass eigenstates as U_L and D_L. The charged current interactions for quarks [that is the interactions of the charged $SU(2)_\text{L}$ gauge bosons $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp i W_\mu^2)$], which in the interaction basis are described by (6), have a complicated form in the mass basis:

$$-\mathcal{L}_{W^\pm}^q = \frac{g}{\sqrt{2}} \overline{U_{Li}} \gamma^\mu V_{ij} D_{Lj} W_\mu^+ + \text{h.c.}. \tag{22}$$

where V is the 3×3 unitary matrix ($VV^\dagger = V^\dagger V = \mathbf{1}$) that appeared in Eq. (17). For a general interaction basis,

$$V = V_{uL} V_{dL}^\dagger. \tag{23}$$

V is the Cabibbo-Kobayashi-Maskawa (CKM) *mixing matrix* for quarks [1, 2]. As a result of the fact that V is not diagonal, the $W^{\pm}$ gauge bosons couple to quark mass eigenstates of different generations. Within the Standard Model, this is the only source of *flavor changing* quark interactions.

Exercise 1: *Prove that, in the absence of neutrino masses, there is no mixing in the lepton sector.*

Exercise 2: *Prove that there is no mixing in the Z couplings. (In the physics jargon, there are no flavor changing neutral currents at tree level.)*

The detailed structure of the CKM matrix, its parametrization, and the constraints on its elements are described in Appendix A.

IV. TESTING CKM

Measurements of rates, mixing, and CP asymmetries in B decays in the two B factories, BaBar abd Belle, and in the two Tevatron detectors, CDF and D0, signified a new era in our understanding of CP violation. The progress is both qualitative and quantitative. Various basic questions concerning CP and flavor violation have received, for the first time, answers based on experimental information. These questions include, for example,

- Is the Kobayashi-Maskawa mechanism at work (namely, is $\delta_{\rm KM} \neq 0$)?
- Does the KM phase dominate the observed CP violation?

As a first step, one may assume the SM and test the overall consistency of the various measurements. However, the richness of data from the B factories allow us to go a step further and answer these questions model independently, namely allowing new physics to contribute to the relevant processes. We here explain the way in which this analysis proceeds.

A. $S_{\psi K_S}$

The CP asymmetry in $B \to \psi K_S$ decays plays a major role in testing the KM mechanism. Before we explain the test itself, we should understand why the theoretical interpretation of the asymmetry is exceptionally clean, and what are the theoretical parameters on which it depends, within and beyond the Standard Model.

The CP asymmetry in neutral meson decays into final CP eigenstates f_{CP} is defined as follows:

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] - d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}] + d\Gamma/dt[B^0{}_{\rm phys}(t) \to f_{CP}]} \,. \qquad (24)$$

A detailed evaluation of this asymmetry is given in Appendix B. It leads to the following form:

$$\begin{aligned} \mathcal{A}_{f_{CP}}(t) &= S_{f_{CP}} \sin(\Delta m t) - C_{f_{CP}} \cos(\Delta m t), \\ S_{f_{CP}} &\equiv \frac{2\,\mathcal{I}m(\lambda_{f_{CP}})}{1+|\lambda_{f_{CP}}|^2}, \quad C_{f_{CP}} \equiv \frac{1-|\lambda_{f_{CP}}|^2}{1+|\lambda_{f_{CP}}|^2} \,, \end{aligned} \qquad (25)$$

where

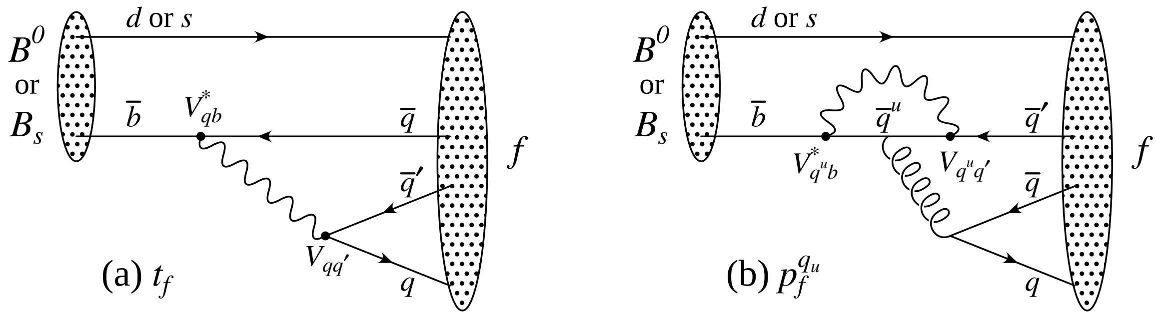
$$\lambda_{f_{CP}} = e^{-i\phi_B}(\overline{A}_{f_{CP}}/A_{f_{CP}}) \,. \qquad (26)$$

Here ϕ_B refers to the phase of M_{12} [see Eq. (B23)]. Within the Standard Model, the corresponding phase factor is given by

$$e^{-i\phi_B} = (V^*_{tb}V_{td})/(V_{tb}V^*_{td}) \,. \qquad (27)$$

The decay amplitudes A_f and $\overline{A}_f$ are defined in Eq. (B1).

FIG. 1: Feynman diagrams for (a) tree and (b) penguin amplitudes contributing to $B^0 \to f$ or $B_s \to f$ via a $\bar{b} \to \bar{q}q\bar{q}'$ quark-level process.



The $B^0 \to J/\psi K^0$ decay [3, 4] proceeds via the quark transition $\bar{b} \to \bar{c}c\bar{s}$. There are contributions from both tree (t) and penguin (p^{q_u}, where $q_u = u, c, t$ is the quark in the loop) diagrams (see Fig. 1) which carry different weak phases:

$$A_f = (V^*_{cb}V_{cs})\, t_f + \sum_{q_u=u,c,t} \left(V^*_{q_u b}V_{q_u s}\right) p_f^{q_u} \,. \qquad (28)$$

(The distinction between tree and penguin contributions is a heuristic one, the separation by the operator that enters is more precise. For a detailed discussion of the more complete operator product approach, which also includes higher order QCD corrections, see, for example, ref. [5].) Using CKM unitarity, these decay amplitudes can always be written in terms of just two CKM combinations:

$$A_{\psi K} = (V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P_{\psi K}^u, \tag{29}$$

where $T_{\psi K} = t_{\psi K} + p^c_{\psi K} - p^t_{\psi K}$ and $P^u_{\psi K} = p^u_{\psi K} - p^t_{\psi K}$. A subtlety arises in this decay that is related to the fact that $B^0 \to J/\psi K^0$ and $\overline{B}^0 \to J/\psi \overline{K}^0$. A common final state, e.g. $J/\psi K_S$, can be reached via $K^0 - \overline{K}^0$ mixing. Consequently, the phase factor corresponding to neutral K mixing, $e^{-i\phi_K} = (V_{cd}^* V_{cs})/(V_{cd} V_{cs}^*)$, plays a role:

$$\frac{\overline{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb} V_{cs}^*)\, T_{\psi K} + (V_{ub} V_{us}^*)\, P^u_{\psi K}}{(V_{cb}^* V_{cs})\, T_{\psi K} + (V_{ub}^* V_{us})\, P^u_{\psi K}} \times \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*}. \tag{30}$$

The crucial point is that, for $B \to J/\psi K_S$ and other $\bar{b} \to \bar{c} c \bar{s}$ processes, we can neglect the P^u contribution to $A_{\psi K}$, in the SM, to an approximation that is better than one percent:

$$|P^u_{\psi K}/T_{\psi K}| \times |V_{ub}/V_{cb}| \times |V_{us}/V_{cs}| \sim (\text{loop factor}) \times 0.1 \times 0.23 \lesssim 0.005. \tag{31}$$

Thus, to an accuracy of better than one percent,

$$\lambda_{\psi K_S} = \left(\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right)\left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}\right) = -e^{-2i\beta}, \tag{32}$$

where β is defined in Eq. (A9), and consequently

$$S_{\psi K_S} = \sin 2\beta, \quad C_{\psi K_S} = 0\,. \tag{33}$$

(Below the percent level, several effects modify this equation [6–9].)

Exercise 3: *Show that, if the $B \to \pi\pi$ decays were dominated by tree diagrams, then $S_{\pi\pi} = \sin 2\alpha$.*

Exercise 4: *Estimate the accuracy of the predictions $S_{\phi K_S} = \sin 2\beta$ and $C_{\phi K_S} = 0$.*

When we consider extensions of the SM, we still do not expect any significant new contribution to the tree level decay, $b \to c\bar{c}s$, beyond the SM W-mediated diagram. Thus, the expression $\bar{A}_{\psi K_S}/A_{\psi K_S} = (V_{cb} V_{cd}^*)/(V_{cb}^* V_{cd})$ remains valid, though the approximation of neglecting sub-dominant phases can be somewhat less accurate than Eq. (31). On the other hand, M_{12}, the $B^0-\overline{B}^0$ mixing amplitude, can in principle get large and even dominant contributions from new physics. We can parametrize the modification to the SM in terms of two parameters, r_d^2 signifying the change in magnitude, and $2\theta_d$ signifying the change in phase:

$$M_{12} = r_d^2\, e^{2i\theta_d}\, M_{12}^{\rm SM}(\rho,\eta). \tag{34}$$

This leads to the following generalization of Eq. (33):

$$S_{\psi K_S} = \sin(2\beta + 2\theta_d), \quad C_{\psi K_S} = 0\ . \tag{35}$$

The experimental measurements give the following ranges [10]:

$$S_{\psi K_S} = +0.671 \pm 0.024, \quad C_{\psi K_S} = +0.005 \pm 0.019\ . \tag{36}$$

B. Self-consistency of the CKM assumption

The three generation standard model has room for CP violation, through the KM phase in the quark mixing matrix. Yet, one would like to make sure that indeed CP is violated by the SM interactions, namely that $\sin\delta_{\rm KM} \neq 0$. If we establish that this is the case, we would further like to know whether the SM contributions to CP violating observables are dominant. More quantitatively, we would like to put an upper bound on the ratio between the new physics and the SM contriubtions.

As a first step, one can assume that flavor changing processes are fully described by the SM, and check the consistency of the various measurements with this assumption. There are four relevant mixing parameters, which can be taken to be the Wolfenstein parameters λ, A, ρ and η defined in Eq. (A4). The values of λ and A are known rather accurately [11] from, respectively, $K \to \pi\ell\nu$ and $b \to c\ell\nu$ decays:

$$\lambda = 0.2257 \pm 0.0010, \quad A = 0.814 \pm 0.022. \tag{37}$$

Then, one can express all the relevant observables as a function of the two remaining parameters, ρ and η, and check whether there is a range in the $\rho - \eta$ plane that is consistent with all measurements. The list of observables includes the following:

- The rates of inclusive and exclusive charmless semileptonic B decays depend on $|V_{ub}|^2 \propto \rho^2 + \eta^2$;

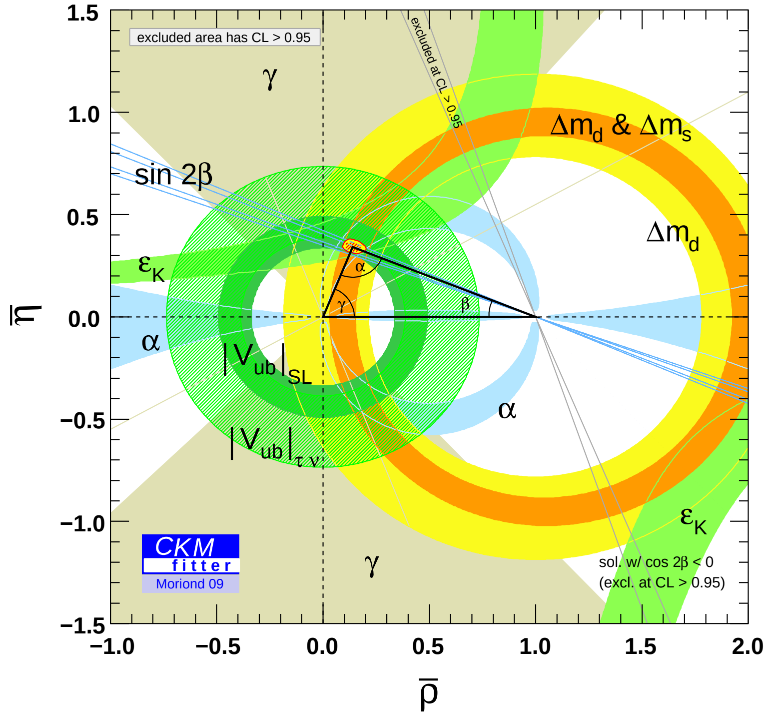


FIG. 2: Allowed region in the ρ, η plane. Superimposed are the individual constraints from charmless semileptonic B decays ($|V_{ub}/V_{cb}|$), mass differences in the B^0 (Δm_d) and B_s (Δm_s) neutral meson systems, and CP violation in $K \to \pi\pi$ (ε_K), $B \to \psi K$ ($\sin 2\beta$), $B \to \pi\pi, \rho\pi, \rho\rho$ (α), and $B \to DK$ (γ). Taken from [12].

- The CP asymmetry in $B \to \psi K_S$, $S_{\psi K_S} = \sin 2\beta = \frac{2\eta(1-\rho)}{(1-\rho)^2+\eta^2}$;
- The rates of various $B \to DK$ decays depend on the phase γ, where $e^{i\gamma} = \frac{\rho+i\eta}{\sqrt{\rho^2+\eta^2}}$;
- The rates of various $B \to \pi\pi, \rho\pi, \rho\rho$ decays depend on the phase $\alpha = \pi - \beta - \gamma$;
- The ratio between the mass splittings in the neutral B and B_s systems is sensitive to $|V_{td}/V_{ts}|^2 = \lambda^2[(1-\rho)^2 + \eta^2]$;
- The CP violation in $K \to \pi\pi$ decays, ϵ_K, depends in a complicated way on ρ and η.

The resulting constraints are shown in Fig. 2.

The consistency of the various constraints is impressive. In particular, the following ranges for ρ and η can account for all the measurements [11]:

$$\rho = +0.135^{+0.031}_{-0.016}, \quad \eta = +0.349 \pm 0.017. \tag{38}$$

One can make then the following statement [13]:

Very likely, CP violation in flavor changing processes is dominated by the Kobayashi-Maskawa phase.

In the next two subsections, we explain how we can remove the phrase "very likely" from this statement, and how we can quantify the KM-dominance.

C. Is the KM mechanism at work?

In proving that the KM mechanism is at work, we assume that charged-current tree-level processes are dominated by the W-mediated SM diagrams (see, for example, [14]). This is a very plausible assumption. I am not aware of any viable well-motivated model where this assumption is not valid. Thus we can use all tree level processes and fit them to ρ and η, as we did before. The list of such processes includes the following:

1. Charmless semileptonic B-decays, $b \to u\ell\nu$, measure R_u [see Eq. (A8)].

2. $B \to DK$ decays, which go through the quark transitions $b \to c\bar{u}s$ and $b \to u\bar{c}s$, measure the angle γ [see Eq. (A9)].

3. $B \to \rho\rho$ decays (and, similarly, $B \to \pi\pi$ and $B \to \rho\pi$ decays) go through the quark transition $b \to u\bar{u}d$. With an isospin analysis, one can determine the relative phase between the tree decay amplitude and the mixing amplitude. By incorporating the measurement of $S_{\psi K_S}$, one can subtract the phase from the mixing amplitude, finally providing a measurement of the angle γ [see Eq. (A9)].

In addition, we can use loop processes, but then we must allow for new physics contributions, in addition to the (ρ, η)-dependent SM contributions. Of course, if each such measurement adds a separate mode-dependent parameter, then we do not gain anything by using this information. However, there is a number of observables where the only relevant loop process is $B^0 - \overline{B}^0$ mixing. The list includes $S_{\psi K_S}$, Δm_B and the CP asymmetry in semileptonic B decays:

$$\begin{aligned} S_{\psi K_S} &= \sin(2\beta + 2\theta_d), \\ \Delta m_B &= r_d^2 (\Delta m_B)^{\rm SM}, \\ \mathcal{A}_{\rm SL} &= -\mathcal{R}e\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\sin 2\theta_d}{r_d^2} + \mathcal{I}m\left(\frac{\Gamma_{12}}{M_{12}}\right)^{\rm SM} \frac{\cos 2\theta_d}{r_d^2}. \end{aligned} \tag{39}$$

As explained above, such processes involve two new parameters [see Eq. (34)]. Since there are three relevant observables, we can further tighten the constraints in the (ρ, η)-plane. Similarly, one can use measurements related to $B_s - \overline{B}_s$ mixing. One gains three new observables at the cost of two new parameters (see, for example, [15]).

The results of such fit, projected on the $\rho - \eta$ plane, can be seen in Fig. 3. It gives [12]

$$\eta = 0.44^{+0.05}_{-0.23} \ \ (3\sigma). \tag{40}$$

[A similar analysis in Ref. [16] obtains the 3σ range $(0.31 - 0.46)$.] It is clear that $\eta \neq 0$ is well established:

The Kobayashi-Maskawa mechanism of CP violation is at work.

Another way to establish that CP is violated by the CKM matrix is to find, within the same procedure, the allowed range for $\sin 2\beta$ [16]:

$$\sin 2\beta^{\rm tree} = 0.80 \pm 0.03. \tag{41}$$

Thus, $\beta \neq 0$ is well established.

The consistency of the experimental results (36) with the SM predictions (33,41) means that the KM mechanism of CP violation dominates the observed CP violation. In the next subsection, we make this statement more quantitative.

D. How much can new physics contribute to $B^0 - \overline{B}^0$ mixing?

All that we need to do in order to establish whether the SM dominates the observed CP violation, and to put an upper bound on the new physics contribution to $B^0 - \overline{B}^0$ mixing, is to project the results of the fit performed in the previous subsection on the $r_d^2 - 2\theta_d$ plane. If we find that $\theta_d \ll \beta$, then the SM dominance in the observed CP violation will be established. The constraints are shown in Fig. 4(a). Indeed, $\theta_d \ll \beta$.

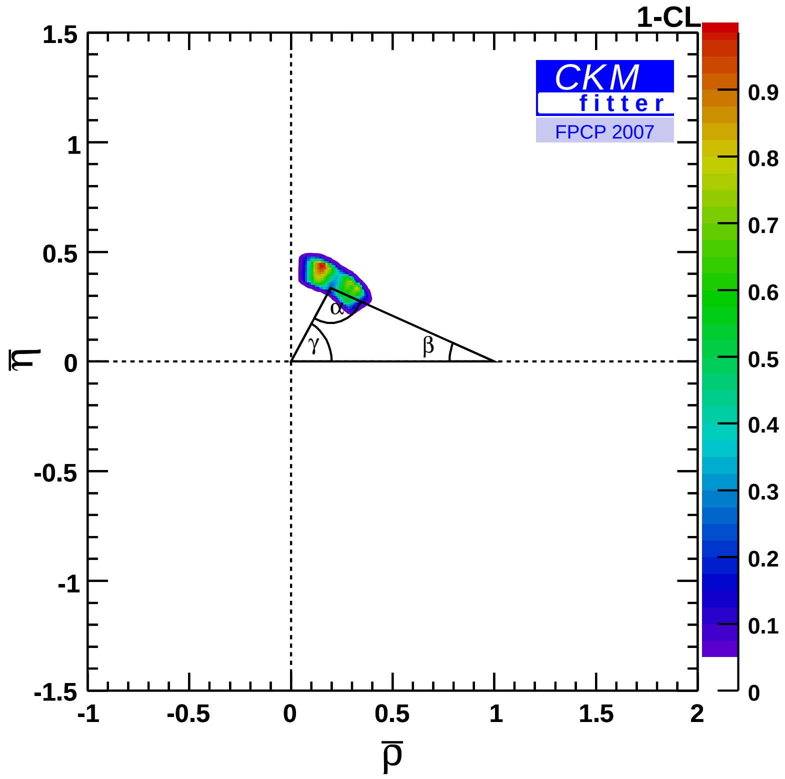


FIG. 3: The allowed region in the $\rho - \eta$ plane, assuming that tree diagrams are dominated by the Standard Model [12].

An alternative way to present the data is to use the h_d, σ_d parametrization,

$$r_d^2 e^{2i\theta_d} = 1 + h_d e^{2i\sigma_d}. \tag{42}$$

While the r_d, θ_d parameters give the relation between the full mixing amplide and the SM one, and are convenient to apply to the measurements, the h_d, σ_d parameters give the relation between the new physics and SM contributions, and are more convenient in testing theoretical models:

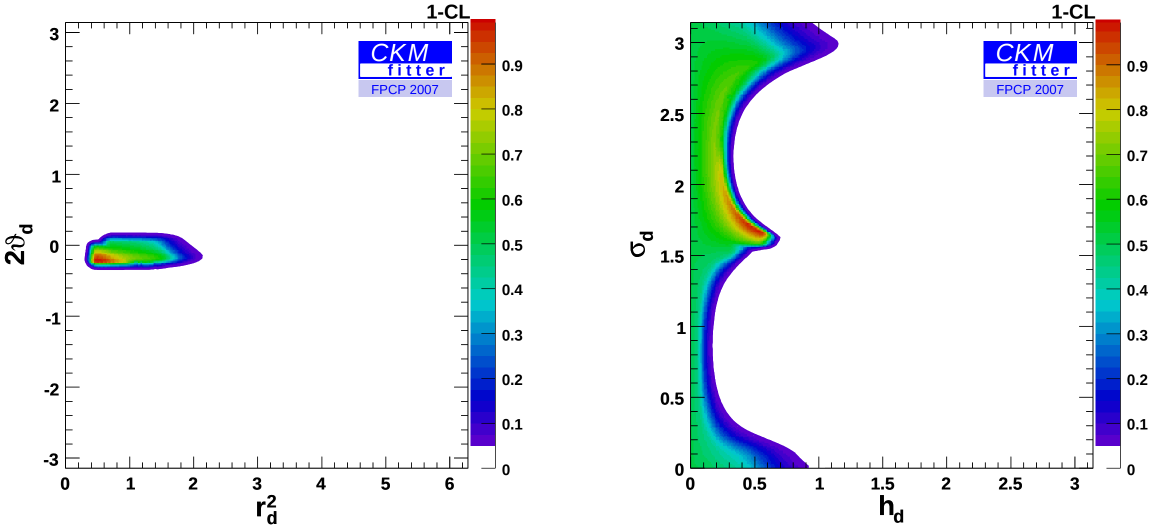
$$h_d e^{2i\sigma_d} = \frac{M_{12}^{\rm NP}}{M_{12}^{\rm SM}}. \tag{43}$$

The constraints in the $h_d - \sigma_d$ plane are shown in Fig. 4(b). We can make the following two statements:

1. A new physics contribution to $B^0 - \overline{B}^0$ mixing amplitude that carries a phase that is significantly different from the KM phase is constrained to lie below the 20-30% level.

2. A new physics contribution to the $B^0 - \overline{B}^0$ mixing amplitude which is aligned with the KM phase is constrained to be at most comparable to the CKM contribution.

FIG. 4: Constraints in the (a) $r_d^2 - 2\theta_d$ plane, and (b) $h_d - \sigma_d$ plane, assuming that NP contributions to tree level processes are negligible [12].



One can reformulate these statements as follows:

1. The KM mechanism dominates CP violation in $B^0 - \overline{B}^0$ mixing.

2. The CKM mechanism is a major player in $B^0 - \overline{B}^0$ mixing.

V. THE NEW PHYSICS FLAVOR PUZZLE

A. A model independent discussion

It is clear that the Standard Model is not a complete theory of Nature:

1. It does not include gravity, and therefore it cannot be valid at energy scales above $m_{\rm Planck} \sim 10^{19}$ GeV:

2. It does not allow for neutrino masses, and therefore it cannot be valid at energy scales above $m_{\rm seesaw} \sim 10^{15}$ GeV;

3. The fine-tuning problem of the Higgs mass and the puzzle of the dark matter suggest that the scale where the SM is replaced with a more fundamental theory is actually much lower, $\Lambda_{\rm NP} \lesssim 1$ TeV.

Given that the SM is only an effective low energy theory, non-renormalizable terms must be added to $\mathcal{L}_{\rm SM}$ of Eq. (4). These are terms of dimension higher than four in the fields which, therefore, have couplings that are inversely proportional to the scale of new physics $\Lambda_{\rm NP}$. For example, the lowest dimension non-renormalizable terms are dimension five:

$$-\mathcal{L}_{\rm Yukawa}^{\rm dim-5} = \frac{Z^\nu_{ij}}{\Lambda_{\rm NP}} L^I_{Li} L^I_{Lj} \phi\phi + {\rm h.c.}. \tag{44}$$

These are the seesaw terms, leading to neutrino masses.

Exercise 5: *How does the global symmetry breaking pattern (14) change when (44) is taken into account?*

Exercise 6: *What is the number of physical lepton flavor parameters in this case? Identify these parameters in the mass basis.*

As concerns quark flavor physics, consider, for example, the following dimension-six, four-fermion, flavor changing operators:

$$\mathcal{L}_{\Delta F=2} = \frac{z_{sd}}{\Lambda^2_{\rm NP}}(\overline{d_L}\gamma_\mu s_L)^2 + \frac{z_{cu}}{\Lambda^2_{\rm NP}}(\overline{c_L}\gamma_\mu u_L)^2 + \frac{z_{bd}}{\Lambda^2_{\rm NP}}(\overline{d_L}\gamma_\mu b_L)^2 + \frac{z_{bs}}{\Lambda^2_{\rm NP}}(\overline{s_L}\gamma_\mu b_L)^2. \tag{45}$$

Each of these terms contributes to the mass splitting between the corresponding two neutral mesons. For example, the term $\mathcal{L}_{\Delta B=2} \propto (\overline{d_L}\gamma_\mu b_L)^2$ contributes to Δm_B, the mass difference between the two neutral B-mesons. We use $M_{12}^B = \frac{1}{2m_B}\langle B^0|\mathcal{L}_{\Delta F=2}|\overline{B}^0\rangle$ and

$$\langle B^0|(\overline{d_{La}}\gamma^\mu b_{La})(\overline{d_{Lb}}\gamma_\mu b_{Lb})|\overline{B}^0\rangle = -\frac{1}{3} m_B^2 f_B^2 B_B. \tag{46}$$

Analogous expressions hold for the other neutral mesons.[2] This leads to $\Delta m_B/m_B = 2|M_{12}^B|/m_B \sim (|z_{bd}|/3)(f_B/\Lambda_{\rm NP})^2$. Experiments give, for CP conserving observables (the experimental evidence for Δm_D is at the 3σ level):

$$\begin{aligned}
\Delta m_K/m_K &\sim 7.0\times 10^{-15},\\
\Delta m_D/m_D &\sim 8.7\times 10^{-15},\\
\Delta m_B/m_B &\sim 6.3\times 10^{-14},\\
\Delta m_{B_s}/m_{B_s} &\sim 2.1\times 10^{-12},
\end{aligned} \tag{47}$$

[2] The PDG [11] quotes the following values, extracted from leptonic charged meson decays: $f_K \approx 0.16\ GeV$, $f_D \approx 0.23\ GeV$, $f_B \approx 0.18\ GeV$. We further use $f_{B_s} \approx 0.20\ GeV$.

and for CP violating ones

$$\begin{aligned}
\epsilon_K &\sim 2.3 \times 10^{-3}, \\
A_\Gamma / y_{\rm CP} &\lesssim 0.2, \\
S_{\psi K_S} &= 0.67 \pm 0.02, \\
S_{\psi\phi} &\lesssim 1.
\end{aligned} \tag{48}$$

These measurements give then the following constraints:

$$\Lambda_{\rm NP} \gtrsim \begin{cases} \sqrt{z_{sd}}\ 1 \times 10^3\ TeV & \Delta m_K \\ \sqrt{z_{cu}}\ 1 \times 10^3\ TeV & \Delta m_D \\ \sqrt{z_{bd}}\ 4 \times 10^2\ TeV & \Delta m_B \\ \sqrt{z_{bs}}\ 7 \times 10^1\ TeV & \Delta m_{B_s} \end{cases} \tag{49}$$

and, for maximal phases,

$$\Lambda_{\rm NP} \gtrsim \begin{cases} \sqrt{z_{sd}}\ 2 \times 10^4\ TeV & \epsilon_K \\ \sqrt{z_{cu}}\ 3 \times 10^3\ TeV & A_\Gamma \\ \sqrt{z_{bd}}\ 8 \times 10^2\ TeV & S_{\psi K} \\ \sqrt{z_{bs}}\ 7 \times 10^1\ TeV & S_{\psi\phi} \end{cases} \tag{50}$$

If the new physics has a generic flavor structure, that is $z_{ij} = \mathcal{O}(1)$, then its scale must be above $10^3 - 10^4$ TeV (or, if the leading contributions involve electroweak loops, above $10^2 - 10^3$ TeV).[3]

If indeed $\Lambda_{\rm NP} \gg TeV$, it means that we have misinterpreted the hints from the fine-tuning problem and the dark matter puzzle. There is, however, another way to look at these constraints:

$$\begin{aligned}
z_{sd} &\lesssim 8 \times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2, \\
z_{cu} &\lesssim 5 \times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2, \\
z_{bd} &\lesssim 5 \times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2, \\
z_{bs} &\lesssim 2 \times 10^{-4}\ (\Lambda_{\rm NP}/TeV)^2,
\end{aligned} \tag{51}$$

$$z^I_{sd} \lesssim 6 \times 10^{-9}\ (\Lambda_{\rm NP}/TeV)^2,$$

[3] The bounds from the corresponding four-fermi terms with LR structure, instead of the LL structure of Eq. (45), are even stronger.

$$
\begin{aligned}
z_{cu}^{I} &\lesssim 1\times 10^{-7}\ (\Lambda_{\rm NP}/TeV)^2,\\
z_{bd}^{I} &\lesssim 1\times 10^{-6}\ (\Lambda_{\rm NP}/TeV)^2,\\
z_{bs}^{I} &\lesssim 2\times 10^{-4}\ (\Lambda_{\rm NP}/TeV)^2.
\end{aligned} \tag{52}
$$

It could be that the scale of new physics is of order TeV, but its flavor structure is far from generic.

One can use that language of effective operators also for the SM, integrating out all particles significantly heavier than the neutral mesons (that is, the top, the Higgs and the weak gauge bosons). Thus, the scale is $\Lambda_{\rm SM}\sim m_W$. Since the leading contributions to neutral meson mixings come from box diagrams, the z_{ij} coefficients are suppressed by α_2^2. To identify the relevant flavor suppression factor, one can employ the spurion formalism. For example, the flavor transition that is relevant to $B^0-\overline{B}^0$ mixing involves $\overline{d_L}b_L$ which transforms as $(8,1,1)_{SU(3)_q^3}$. The leading contribution must then be proportional to $(Y^uY^{u\dagger})_{13}\propto y_t^2V_{tb}V_{td}^*$. Indeed, an explicit calculation (using VIA for the matrix element and neglecting QCD corrections) gives[4]

$$
\frac{2M_{12}^B}{m_B}\approx-\frac{\alpha_2^2}{12}\frac{f_B^2}{m_W^2}S_0(x_t)(V_{tb}V_{td}^*)^2, \tag{53}
$$

where $x_i=m_i^2/m_W^2$ and

$$
S_0(x)=\frac{x}{(1-x)^2}\left[1-\frac{11x}{4}+\frac{x^2}{4}-\frac{3x^2\ln x}{2(1-x)}\right]. \tag{54}
$$

Similar spurion analyses, or explicit calculations, allow us to extract the weak and flavor suppression factors that apply in the SM:

$$
\begin{aligned}
\mathcal{I}m(z_{sd}^{\rm SM}) &\sim \alpha_2^2y_t^2|V_{td}V_{ts}|^2\sim 1\times 10^{-10},\\
z_{sd}^{\rm SM} &\sim \alpha_2^2y_c^2|V_{cd}V_{cs}|^2\sim 5\times 10^{-9},\\
z_{bd}^{\rm SM} &\sim \alpha_2^2y_t^2|V_{td}V_{tb}|^2\sim 7\times 10^{-8},\\
z_{bs}^{\rm SM} &\sim \alpha_2^2y_t^2|V_{ts}V_{tb}|^2\sim 2\times 10^{-6}.
\end{aligned} \tag{55}
$$

(We did not include $z_{cu}^{\rm SM}$ in the list because it requires a more detailed consideration. The naively leading short distance contribution is $\propto\alpha_2^2(y_s^4/y_c^2)|V_{cs}V_{us}|^2\sim 5\times 10^{-13}$. However, higher dimension terms can replace a y_s^2 factor with $(\Lambda/m_D)^2$ [18]. Moreover, long distance contributions are expected to dominate. In particular, peculiar phase space effects [19, 20]

[4] A detailed derivation can be found in Appendix B of [17].

have been identified which are expected to enhance Δm_D to within an order of magnitude of the its measured value.)

It is clear then that contributions from new physics at $\Lambda_{\rm NP} \sim 1\, TeV$ should be suppressed by factors that are comparable or smaller than the SM ones. Why does that happen? This is the new physics flavor puzzle.

The fact that the flavor structure of new physics at the TeV scale must be non-generic means that flavor measurements are a good probe of the new physics. Perhaps the best-studied example is that of supersymmetry. Here, the spectrum of the superpartners and the structure of their couplings to the SM fermions will allow us to probe the mechanism of dynamical supersymmetry breaking.

B. Lessons from CP violation in $B_s - \overline{B_s}$ mixing

An interesting experimental result concerning CP violation in $B_s - \overline{B_s}$ mixing have been recently presented by the D0 experiment [21]:

$$(a^b_{\rm SL})^{D0} = (-9.6 \pm 2.5 \pm 1.5) \times 10^{-3}, \tag{56}$$

to be compared with the Standard Model (SM) prediction [22]:

$$(a^b_{\rm SL})^{\rm SM} = (-0.23^{+0.05}_{-0.06}) \times 10^{-3}. \tag{57}$$

The measured asymmetry is a combination of the asymmetries in B^0_d and B^0_s decays [21]:

$$a^b_{\rm SL} = (0.51 \pm 0.04) a^d_{\rm SL} + (0.49 \pm 0.04) a^s_{\rm SL}. \tag{58}$$

To explain the difference between the experimental result (56) and the SM prediction (57), a new physics contribution to $B_s - \overline{B_s}$ and/or $B_d - \overline{B_d}$ mixing is required that is comparable in size to the SM contribution and carries a new phase of order one.

The like-sign dimuon charge asymmetry in semileptonic b decys is experimentally defined as

$$a^b_{\rm SL} \equiv \frac{N_b^{++} - N_b^{--}}{N_b^{++} + N_b^{--}}, \tag{59}$$

where N_b^{++} is the number of $b\bar{b} \to \mu^+\mu^+ X$ events, and similarly for N_b^{--}. Another way to write the asymmetry is (we take the asymmetry in B_s decays as an example)

$$a^s_{\rm SL} = \frac{d\Gamma/dt[\overline{B_s}(t) \to \mu^+ X] - d\Gamma/dt[B_s(t) \to \mu^- X]}{d\Gamma/dt[\overline{B_s}(t) \to \mu^+ X] + d\Gamma/dt[B_s(t) \to \mu^- X]}, \tag{60}$$

where $B_s(t)$ is defined in Eq. (B14). It is straightforward to show that

$$a_{\rm SL}^s = \frac{1-|q/p|^4}{1+|q/p|^4}. \tag{61}$$

Thus, $a_{\rm SL}^s \neq 0$ signals that the B_s mass eigenstates are not CP eigenstates.

In the approximation that $|\Gamma_{12}/M_{12}| \ll 1$, which is valid for both B_d and B_s, the CP asymmetry in semileptonic decays is given by

$$a_{\rm SL}^s = \mathcal{I}m(\Gamma_{12}^s/M_{12}^s). \tag{62}$$

Exercise 7: *Using Eqs. (B16,B17), and taking into account that* $|A_{\ell^+X}| = |\overline{A}_{\ell^-X}|$ *and* $A_{\ell^-X} = \overline{A}_{\ell^+X} = 0$*, prove Eq. (61).*

Exercise 8: *Using Eq. (B8), prove Eq. (62).*

The SM prediction for $a_{\rm SL}^s$ is tiny, because it involves three flavor suppression factors:

1. While $(M_{12}^s)^{\rm SM}$ is given by box diagrams with intermediate top quarks, $(\Gamma_{12}^s)^{\rm SM}$ is given by box diagrams (with a cut) with intermediate up and/or charm quarks. Thus, $(\Gamma_{12}^s/M_{12}^s)^{\rm SM} \sim 4\pi m_b^2/m_t^2 = \mathcal{O}(10^{-2})$.

2. Neglecting the difference in masses between the up and charm quarks, $(\Gamma_{12}^s)^{\rm SM} \propto (V_{ub}V_{us}^* + V_{cb}V_{cs}^*)^2 = (V_{tb}V_{ts}^*)^2$, and thus carries the same phase as M_{12}^s. Consequently, a phase difference appears only at the cost of an $m_c^2/m_b^2 \sim 0.1$ factor.

3. The phase difference is given by $\mathcal{I}m\,[(V_{cb}V_{cs}^*)/(V_{tb}V_{ts}^*)] = -\sin\beta_s \sim 0.02$.

The end result is (see *e.g.* [23])

$$(a_{\rm SL}^s)^{\rm SM} = -4\pi\frac{m_c^2}{m_W^2}\frac{K_1+K_2}{\eta_B S_0(m_t^2/m_W^2)}\mathcal{I}m\left(\frac{V_{cb}V_{cs}^*}{V_{tb}V_{ts}^*}\right) = \mathcal{O}(10^{-5}), \tag{63}$$

where $K_{1,2}$ are Wilson coefficients, η_B is a QCD coorrection factor, and S_0 is the Inami-Lim function for the box diagram.

Since Γ_{12}^s is dominated by tree level $b \to c\bar{c}s$ decays, it is unlikely to be significantly affected by new physics. It is much more plausible that the new physics effect comes via a contribution to M_{12}^s, which can be parametrized similarly to Eqs. (34) and (43):

$$\begin{aligned} M_{12}^s &= r_s^2 e^{2i\theta_s}(M_{12}^s)^{\rm SM}, \\ (M_{12}^s)^{\rm NP} &= h_s e^{2i\sigma_s}(M_{12}^s)^{\rm SM}. \end{aligned} \tag{64}$$

This leads to the following modifications of the SM predictions:

$$\begin{aligned}
\Delta m_s &= r_s^2 \Delta m_s^{\rm SM}, \\
\Delta\Gamma_s &= \Delta\Gamma_s^{\rm SM} \cos 2\theta_s, \\
a_{\rm SL}^s &= \mathcal{I}m \left[\Gamma_{12}^s / \left(M_{12}^{s,{\rm SM}} r_s^2 e^{2i\theta_s}\right)\right] \approx -\left(\Delta\Gamma_s/\Delta m_s\right) \tan 2\theta_s, \\
S_{\psi\phi} &= \sin\left(2\beta_s - 2\theta_s\right). \qquad (65)
\end{aligned}$$

Ref. [24] performed a fit to all relevant data, and obtained the following results:

$$(h_s, \sigma_s) \sim (0.6 \pm 0.3, 2.2 \pm 0.2) \text{ or } (1.8 \pm 0.2, 1.9 \pm 0.1). \qquad (66)$$

The effects of new physics at a high energy scale ($\Lambda \gg m_W$) on $B_q - \overline{B_q}$ mixing can be studied in an effective operator language. A complete set of four quark operators relevant to $B_s - \overline{B_s}$ transitions is given by

$$\begin{aligned}
Q_1^{sb} &= \bar{b}_L^\alpha \gamma_\mu s_L^\alpha \bar{b}_L^\beta \gamma_\mu s_L^\beta, \quad \widetilde{Q}_1^{sb} = \bar{b}_R^\alpha \gamma_\mu s_R^\alpha \bar{b}_R^\beta \gamma_\mu s_R^\beta, \\
Q_2^{sb} &= \bar{b}_R^\alpha s_L^\alpha \bar{b}_R^\beta s_L^\beta, \quad \widetilde{Q}_2^{sb} = \bar{b}_L^\alpha s_R^\alpha \bar{b}_L^\beta s_R^\beta, \\
Q_3^{sb} &= \bar{b}_R^\alpha s_L^\beta \bar{b}_R^\beta s_L^\alpha, \quad \widetilde{Q}_3^{sb} = \bar{b}_L^\alpha s_R^\beta \bar{b}_L^\beta s_R^\alpha, \\
Q_4^{sb} &= \bar{b}_R^\alpha s_L^\alpha \bar{b}_L^\beta s_R^\beta, \quad Q_5^{sb} = \bar{b}_R^\alpha s_L^\beta \bar{b}_L^\beta s_R^\alpha. \qquad (67)
\end{aligned}$$

Here $d_L(d_R)$ represent $SU(2)$-doublets (singlets), and α, β are color-indices. The effective Hamiltonian is given by

$$\mathcal{H}_{\rm eff}^{\Delta B=\Delta S=2} = \frac{1}{\Lambda^2}\left(\sum_{i=1}^{5} z_i Q_i + \sum_{i=1}^{3} \tilde{z}_i \widetilde{Q}_i\right). \qquad (68)$$

For the new physics to give a contribution to the mixing amplitude that is > 0.3 of the SM one, we need that at least one of the following conditions will be satisfied (see *e.g.* [25]):

$$\begin{aligned}
|z_1^{bs}| &\gtrsim 1.6 \times 10^{-5} \left(\frac{\Lambda}{\rm TeV}\right)^2, \\
|z_2^{bs}| &\gtrsim 7.5 \times 10^{-6} \left(\frac{\Lambda}{\rm TeV}\right)^2, \\
|z_3^{bs}| &\gtrsim 2.7 \times 10^{-5} \left(\frac{\Lambda}{\rm TeV}\right)^2, \\
|z_4^{bs}| &\gtrsim 2.7 \times 10^{-6} \left(\frac{\Lambda}{\rm TeV}\right)^2, \\
|z_5^{bs}| &\gtrsim 7.2 \times 10^{-6} \left(\frac{\Lambda}{\rm TeV}\right)^2, \qquad (69)
\end{aligned}$$

(or a value of $|\tilde{z}_i^{bs}|$ similar to the one given for the corresponding $|z_i^{bs}|$.) We thus learn that (56) gives an upper bound on the scale of the relevant new physics:

$$\Lambda \lesssim 600 \text{ TeV}. \tag{70}$$

VI. LESSONS FOR SUPERSYMMETRY FROM NEUTRAL MESON MIXING

We consider, as an example, the contributions from the box diagrams involving the squark doublets of the second and third generations, $\tilde{Q}_{L2,3}$, to the $B_s - \overline{B_s}$ mixing amplitude. The contributions are proportional to $K_{3i}^{d*} K_{2i}^{d} K_{3j}^{d*} K_{2j}^{d}$, where K^d is the mixing matrix of the gluino couplings to a left-handed down quark and their supersymmetric squark partners ($\propto [(\delta_{LL}^d)_{23}]^2$ in the mass insertion approximation, described in Appendix C 1). We work in the mass basis for both quarks and squarks. A detailed derivation [26] is given in Appendix C 2. It gives:

$$M_{12}^s = \frac{\alpha_s^2 m_{B_s} f_{B_s}^2 B_{B_s} \eta_{\rm QCD}}{108 m_{\tilde{d}}^2} [11 \tilde{f}_6(x) + 4x f_6(x)] \frac{(\Delta \tilde{m}_{\tilde{d}}^2)^2}{\tilde{m}_{\tilde{d}}^4} (K_{32}^{d*} K_{22}^{d})^2. \tag{71}$$

Here $m_{\tilde{d}}$ is the average mass of the two squark generations, $\Delta m_{\tilde{d}}^2$ is the mass-squared difference, and $x = m_{\tilde{g}}^2 / m_{\tilde{d}}^2$.

Eq. (71) can be translated into our generic language:

$$\Lambda_{\rm NP} = m_{\tilde{q}}, \tag{72}$$

$$z_1^{bs} = \frac{11 \tilde{f}_6(x) + 4x f_6(x)}{18} \alpha_s^2 \left(\frac{\Delta \tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2} \right)^2 (K_{32}^{d*} K_{22}^{d})^2 \approx 10^{-4} (\delta_{23}^{LL})^2,$$

where, for the last approximation, we took the example of $x = 1$ $[11 \tilde{f}_6(1) + 4 f_6(1) = 1/6]$, and defined

$$\delta_{23}^{LL} = \left(\frac{\Delta \tilde{m}_{\tilde{d}}^2}{m_{\tilde{d}}^2} \right) (K_{32}^{d*} K_{22}^{d}). \tag{73}$$

Similar ezpressions can be derived for the dependence of $K^0 - \overline{K^0}$ on $(\delta_{MN}^d)_{12}$, $B^0 - \overline{B^0}$ on $(\delta_{MN}^d)_{13}$, and $D^0 - \overline{D^0}$ on $(\delta_{MN}^u)_{12}$. Then we can use the constraints of Eqs. (51,52) to put upper bounds on $(\delta_{MN}^q)_{ij}$. Some examples are given in Table I (see Ref. [27] for details and list of references).

We learn that, in most cases, we need $\delta_{ij}^q / m_{\tilde{q}} \ll 1/\text{TeV}$. One can immediately identify three generic ways in which supersymmetric contributions to neutral meson mixing can be suppressed:

TABLE I: The phenomenological upper bounds on $(\delta^q_{LL})_{ij}$ and $\langle\delta^q_{ij}\rangle = \sqrt{(\delta^q_{LL})_{ij}(\delta^q_{RR})_{ij}}$. Here $q = u, d$ and $M = L, R$. The constraints are given for $m_{\tilde{q}} = 1$ TeV and $x = m^2_{\tilde{g}}/m^2_{\tilde{q}} = 1$. We assume that the phases could suppress the imaginary part by a factor of ~ 0.3. Taken from Ref. [27].

q ij	$(\delta^q_{LL})_{ij}$	$\langle\delta^q_{ij}\rangle$
d 12	0.03	0.002
d 13	0.2	0.07
d 23	0.6	0.2
u 12	0.1	0.008

1. Heaviness: $m_{\tilde{q}} \gg 1\ TeV$;
2. Degeneracy: $\Delta m^2_{\tilde{q}} \ll m^2_{\tilde{q}}$;
3. Alignment: $K^q_{ij} \ll 1$.

When heaviness is the only suppression mechanism, as in split supersymmetry [28], the squarks are very heavy and supersymmetry no longer solves the fine tuning problem.[5] If we want to maintain supersymmetry as a solution to the fine tuning problem, either degeneracy or alignment or a combination of both is needed. This means that the flavor structure of supersymmetry is not generic, as argued in the previous section.

Take, for example, $(\delta^d_{LL})_{12} \leq 0.03$. Naively, one might expect the alignment to be of order $(V_{cd}V^*_{cs}) \sim 0.2$, which is far from sufficient by itself. Barring a very fine-tuned alignment and accidental cancellations, we are led tp conclude that the first two squark generations must be quasi-degenerate. Actually, by combining the constraints from $K^0 - \overline{K^0}$ mixing and $D^0 - \overline{D^0}$ mixing, one can show that this is the case independently of assumptions about the alignment [34, 35]. Analogous conclusions can be drawn for many TeV-scale new physics scenarios: a strong level of degeneracy is required (for definitions and detailed analysis, see [31]).

[5] When the first two squark generations are mildly heavy and the third generation is light, as in effective supersymmetry [29], the fine tuning problem is still solved, but additional suppression mechanisms are needed.

Exercise 9: *Does $K^d_{31} \sim |V_{ub}|$ suffice to satisfy the Δm_B constraint with neither degeneracy nor heaviness? (Use the two generation approximation and ignore the second generation.)*

Is there a natural way to make the squarks degenerate? Degeneracy requires that the 3×3 matrix of soft supersymmetry breaking mass-squared terms $\tilde m^2_{Q_L} \simeq \tilde m^2_{\tilde q}\mathbf{1}$. We have mentioned already that flavor universality is a generic feature of gauge interactions. Thus, the requirement of degeneracy is perhaps a hint that supersymmetry breaking is *gauge mediated* to the MSSM fields.

Let us now return to the interpretation of the D0 measurement of $a^b_{\rm SL}$ in the supersymmetric framework. From Eqs. (69) and (72) we learn that, to explain $a^s_{\rm SL} \sim 0.01$ from the contribution of left-handed squarks, we need

$$\delta^{LL}_{23} \geq 0.4\ (m_{\tilde d}/1\ \text{TeV}). \tag{74}$$

While this is allowed by present constraints, it is still puzzling that $|\delta^{LL}_{23}| \gg |V_{tb}V_{ts}|$. Indeed, it is very difficult to obtain such a large value in models where the supersymmetric flavor structure has a natural explanation. Moreover, $|\delta^{LR}_{23}|$ is strongly constrained by the rates of $b \to s\gamma$ and $b \to s\ell^+\ell^-$ and cannot account for a large $a^s_{\rm SL}$. We conclude that, if supersymmetry is realized in Nature, and if indeed $a^s_{\rm SL} \sim 0.01$, then most likely source of the large CP violating contribution is δ^{RR}_{23}. This requires, however, that supersymmetry is not minimally flavor violating (see below).

VII. MINIMAL FLAVOR VIOLATION (MFV)

If supersymmetry breaking is gauge mediated, the squark mass matrices for $SU(2)_L$-doublet and $SU(2)_L$-singlet squarks have the following form at the scale of mediation m_M:

$$\begin{aligned}
\tilde M^2_{U_L}(m_M) &= \left(m^2_{\tilde Q_L} + D_{U_L}\right)\mathbf{1} + M_u M_u^\dagger,\\
\tilde M^2_{D_L}(m_M) &= \left(m^2_{\tilde Q_L} + D_{D_L}\right)\mathbf{1} + M_d M_d^\dagger,\\
\tilde M^2_{U_R}(m_M) &= \left(m^2_{\tilde U_R} + D_{U_R}\right)\mathbf{1} + M_u^\dagger M_u,\\
\tilde M^2_{D_R}(m_M) &= \left(m^2_{\tilde D_R} + D_{D_R}\right)\mathbf{1} + M_d^\dagger M_d,
\end{aligned} \tag{75}$$

where $D_{q_A} = (T_3)_{q_A} - (Q_{\rm EM})_{q_A} s^2_W m^2_Z \cos 2\beta$ are the D-term contributions. Here, the only source of the $SU(3)^3_q$ breaking are the SM Yukawa matrices.

This statement holds also when the renormalization group evolution is applied to find the form of these matrices at the weak scale. Taking the scale of the soft breaking terms $m_{\tilde{q}_A}$ to be somewhat higher than the electroweak breaking scale m_Z allows us to neglect the D_{q_A} and M_q terms in (75). Then we obtain

$$\begin{aligned}
\tilde{M}^2_{Q_L}(m_Z) &\sim m^2_{\tilde{Q}_L}\left(r_3\mathbf{1} + c_u Y_u Y_u^\dagger + c_d Y_d Y_d^\dagger\right), \\
\tilde{M}^2_{U_R}(m_Z) &\sim m^2_{\tilde{U}_R}\left(r_3\mathbf{1} + c_{uR} Y_u^\dagger Y_u\right), \\
\tilde{M}^2_{D_R}(m_Z) &\sim m^2_{\tilde{D}_R}\left(r_3\mathbf{1} + c_{dR} Y_d^\dagger Y_d\right).
\end{aligned} \tag{76}$$

Here r_3 represents the universal RGE contribution that is proportional to the gluino mass ($r_3 = \mathcal{O}(6) \times (M_3(m_M)/m_{\tilde{q}}(m_M))$) and the c-coefficients depend logarithmically on m_M/m_Z and can be of $\mathcal{O}(1)$ when m_M is not far below the GUT scale.

Models of gauge mediated supersymmetry breaking (GMSB) provide a concrete example of a large class of models that obey a simple principle called *minimal flavor violation* (MFV) [36]. This principle guarantees that low energy flavor changing processes deviate only very little from the SM predictions. The basic idea can be described as follows. The gauge interactions of the SM are universal in flavor space. The only breaking of this flavor universality comes from the three Yukawa matrices, Y_U, Y_D and Y_E. If this remains true in the presence of the new physics, namely Y_U, Y_D and Y_E are the only flavor non-universal parameters, then the model belongs to the MFV class.

Let us now formulate this principle in a more formal way, using the language of spurions that we presented in section III B. The Standard Model with vanishing Yukawa couplings has a large global symmetry (11,12). In this section we concentrate only on the quarks. The non-Abelian part of the flavor symmetry for the quarks is $SU(3)^3_q$ of Eq. (12) with the three generations of quark fields transforming as follows:

$$Q_L(3,1,1), \quad U_R(1,3,1), \quad D_R(1,1,3). \tag{77}$$

The Yukawa interactions,

$$\mathcal{L}_Y = \overline{Q_L} Y_D D_R H + \overline{Q_L} Y_U U_R H_c, \tag{78}$$

($H_c = i\tau_2 H^*$) break this symmetry. The Yukawa couplings can thus be thought of as spurions with the following transformation properties under $SU(3)^3_q$ [see Eq. (15)]:

$$Y_U \sim (3,\bar{3},1), \qquad Y_D \sim (3,1,\bar{3}). \tag{79}$$

When we say "spurions", we mean that we pretend that the Yukawa matrices are fields which transform under the flavor symmetry, and then require that all the Lagrangian terms, constructed from the SM fields, Y_D and Y_U, must be (formally) invariant under the flavor group $SU(3)_q^3$. Of course, in reality, $\mathcal{L}_Y$ breaks $SU(3)_q^3$ precisely because $Y_{D,U}$ are *not* fields and do not transform under the symmetry.

The idea of minimal flavor violation is relevant to extensions of the SM, and can be applied in two ways:

1. If we consider the SM as a low energy effective theory, then all higher-dimension operators, constructed from SM-fields and Y-spurions, are formally invariant under $G_{\rm global}$.

2. If we consider a full high-energy theory that extends the SM, then all operators, constructed from SM and the new fields, and from Y-spurions, are formally invariant under $G_{\rm global}$.

Exercise 10: *Use the spurion formalism to argue that, in MFV models, the $K_L \to \pi^0 \nu\bar{\nu}$ decay amplitude is proportional to $y_t^2 V_{td} V_{ts}^*$.*

Exercise 11: *Find the flavor suppression factors in the z_i^{bs} coefficients, if MFV is imposed, and compare to the bounds in Eq. (69).*

Examples of MFV models include models of supersymmetry with gauge-mediation or with anomaly-mediation of its breaking.

VIII. FLAVOR AT THE LHC

The LHC will study the physics of electroweak symmetry breaking. There are high hopes that it will discover not only the Higgs, but also shed light on the fine-tuning problem that is related to the Higgs mass. Here, we focus on the issue of how, through the study of new physics, the LHC can shed light on the new physics flavor puzzle.

A. Testing MFV and MLFV

If the LHC discovers new particles that couple to the SM fermions, then it will be able to test solutions to the new physics flavor puzzle such as MFV [37]. Much of its power to test such frameworks is based on identifying top and bottom quarks.

To understand this statement, we notice that the spurions Y_U and Y_D can always be written in terms of the two diagonal Yukawa matrices λ_u and λ_d and the CKM matrix V, see Eqs. (17,18). Thus, the only source of quark flavor changing transitions in MFV models is the CKM matrix. Next, note that to an accuracy that is better than $\mathcal{O}(0.05)$, we can write the CKM matrix as follows:

$$V = \begin{pmatrix} 1 & 0.23 & 0 \\ -0.23 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{80}$$

Exercise 12: *The approximation (80) should be intuitively obvious to top-physicists, but definitely counter-intuitive to bottom-physicists. (Some of them have dedicated a large part of their careers to experimental or theoretical efforts to determine V_{cb} and V_{ub}.) What does the approximation imply for the bottom quark? When we take into account that it is only good to $\mathcal{O}(0.05)$, what would the implications be?*

We learn that the third generation of quarks is decoupled, to a good approximation, from the first two. This, in turn, means that any new particle that couples to an odd number of the SM quarks (think, for example, of heavy quarks in vector-like representations of $G_{\rm SM}$), decay into either third generation quark, or to non-third generation quark, but not to both. For example, in Ref. [37], MFV models with additional charge $-1/3$, $SU(2)_{\rm L}$-singlet quarks – B' – were considered. A concrete test of MFV was proposed, based on the fact that the largest mixing effect involving the third generation is of order $|V_{cb}|^2 \sim 0.002$: Is the following prediction, concerning events of B' pair production, fulfilled:

$$\frac{\Gamma(B'\overline{B'} \to X q_{1,2} q_3)}{\Gamma(B'\overline{B'} \to X q_{1,2} q_{1,2}) + \Gamma(B'\overline{B'} \to X q_3 q_3)} \lesssim 10^{-3}. \tag{81}$$

If not, then MFV is excluded.

One could similarly test various versions of minimal lepton flavor violation (MLFV) [38–42]. For example, if the only spurion that breaks that $SU(3)_L \times SU(3)_L$ lepton flavor symmetry is the charged lepton Yukawa matrix $Y^e(3,\bar{3})$, then there should be no lepton flavor changing processes at all. For example, in Ref. [43], such MLFV models with additional vector-like $SU(2)_L$-doublet leptons were considered. Concrete tests, based on measuring $N_{e^+e^-}/N_{\mu^+\mu^-}$ and $N_{e\mu}/(N_{ee}+N_{\mu\mu})$, were proposed. The simple version of MLFV will be excluded if $N_{e\mu} \neq 0$ or $N_{ee} \neq N_{\mu\mu}$. In that case, one would be led to consider models with either a low seesaw scale, such that neutrino-related spurions play a role, or non-MLFV models. If, on the other hand, the measurements are consistent with $N_{e\mu} = 0$ and $N_{ee} = N_{\mu\mu}$, it will support the existence of an approximate $U(1)_e \times U(1)_\mu$ symmetry.

B. Supersymmetric flavor at the LHC

One can think of analogous tests in the supersymmetric framework [44–50]. Here, there is also a generic prediction that, in each of the three sectors (Q_L, U_R, D_R), squarks of the first two generations are quasi-degenerate, and do not decay into third generation quarks. Squarks of the third generation can be separated in mass (though, for small $\tan\beta$, the degeneracy in the $\tilde{D}_R$ sector is threefold), and decay only to third generation quarks.

It is not necessary, however, that the mediation of supersymmetry breaking is MFV. Examples of natural and viable solutions to the supersymmetric flavor problem that are not MFV include the following:

1. The leading contribution to the soft supersymmetry breaking terms is gauge mediated, and therefore MFV, but there are subleading contributions that are gravity mediated and provide new sources of flavor and CP violation [44, 49]. The gravity mediated contributions could either have some structure (dictated, for example, by a Froggatt-Nielsen symmetry [44] or by localization in extra dimensions [51]) or be anarchical [50].

2. The first two sfermion generations are heavy, and their mixing with the third generation is suppressed (for a recent analysis, see [52]). These features can come, for example, from conformal dynamics [53].

Such framework have different predictions concerning the mass splitting between sfermion generations and the flavor decomposition of the sfermion mass eigenstates. Note that measurements of flavor changing neutral current processes are only sensitive to the products of the form

$$\delta_{ij} = \frac{\Delta\tilde{m}^2_{ij}}{\tilde{m}^2}\, K_{ij}K^*_{jj}, \tag{82}$$

where $\Delta\tilde{m}^2_{ij}$ is the mass-squared splitting between the sfermion generations i and j, $\tilde{m}^2$ is their average mass-squared, and K is the mixing matrix of gaugino couplings to these sfermions. On the other hand, the LHC experiments – ATLAS and CMS – can, at least in principle, measure the mass splitting and the mixing separately [46]. In Appendix D we describe in more details two methods to determine small mass splittings between sleptons at the LHC.

For example, Refs. [44–47] have considered a model of hybrid gauge- and gravity-mediation of syupersymmetry breaking, with mass hierarchy

$$m_{\tilde{G}} \ll m_{\tilde{\ell}_{1,2,3}} \ll m_{\chi_1^0} \ll m_{\tilde{\ell}_{4,5,6}} \ll m_{\chi_2^0}. \tag{83}$$

With the identification of the light, metastable slepton $\tilde{\ell}_1$, electrons and muons, five of the six slepton masses and two supersymmetric lepton mixing parameters can be measured at ATLAS. With this information, one could probe the following issues:

- Measuring $\Delta m^2_{\tilde{\ell}_2\tilde{\ell}_1}$ and K^e_{e2}:
  Are the mass splitting and alignment consistent with the $\mu \to e\gamma$ constraint? That would lead to solving the SUSY flavor problem.

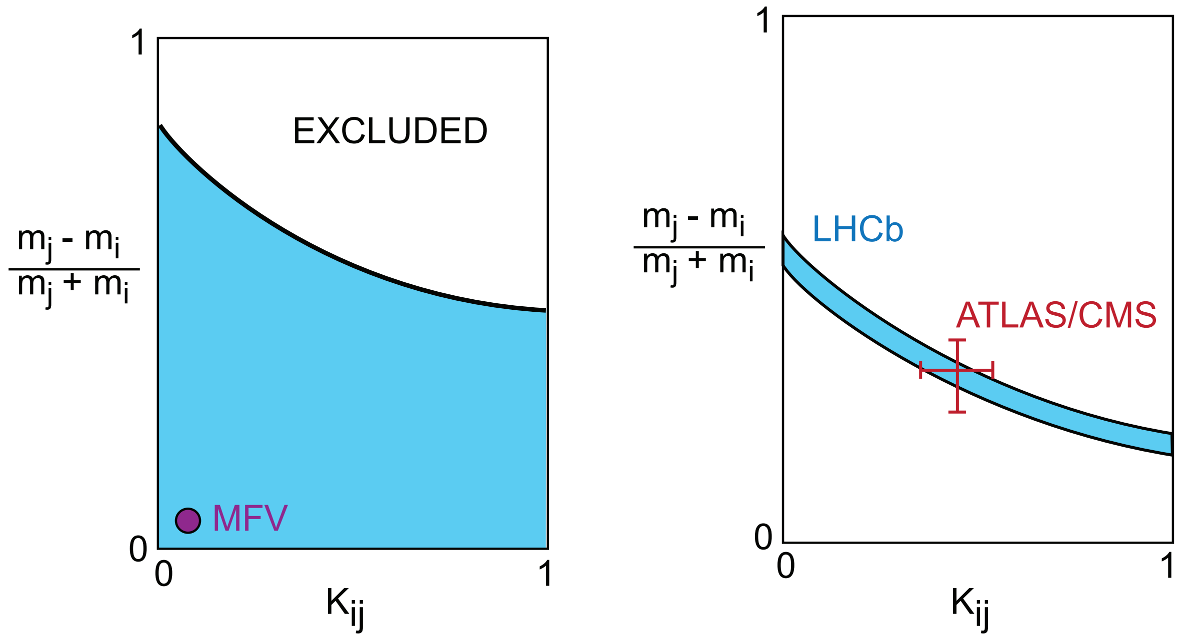
- Measuring $\Delta m^2_{\tilde{\ell}_2\tilde{\ell}_1}, \Delta m^2_{\tilde{\ell}_5\tilde{\ell}_4}, \ldots$:
  What is the messenger scale of gauge mediation M_m? That can probe physics at $M_m \sim 10^{15}$ GeV.

- Determine $|K_{e2}/K_{\mu 2}|$:
  Is the Froggatt-Nielsen mechanism at work? That would allow progress in understanding the SM flavor puzzle.

The present situation is depicted schematically in Fig. 5(a). Flavor factories have provided only upper bounds on deviations of FCNC processes, such as $\mu \to e\gamma$ or $D^0 - \overline{D}^0$ mixing, from the standard model predictions. In the supersymmetric framework, such bounds translate into an upper bound on a δ_{ij} parameter of Eq. (82), corresponding to the blue region in the figure. The supersymmetric flavor puzzle can be stated as the question of why the region in the upper right corner – where the flavor parameters are of order one – is excluded. MFV often puts us in the lower left corner of the plot, far from the experimental constraints. (This is particularly true for δ_{12} parameters.)

The optimal future situation is depicted schematically in Fig. 5(b). Imagine that a flavor factory does provide evidence for new physics, such as observation of $\Gamma(\mu \to e\gamma) \neq 0$ or CP violation in $D^0 - \overline{D}^0$ mixing. This will constrain the corresponding δ parameter, which is shown as the blue region in the Figure. If ATLAS/CMS measure the corresponding sfermion mass splitting and/or mixing, we will get a small allowed region in this flavor plane.

FIG. 5: Schematic description of the constraints in the plane of sfermion mass-squared splitting, $\Delta\tilde{m}^2_{ij}/\tilde{m}^2$, and mixing, $K_{ij}K^*_{jj}$: (a) Upper bounds from not observing any deviation from the SM predictions in present experiments; (b) Hypothetical future situation, where deviations have been observed in flavor factories (such as LHCb, a super-B factory, a $\mu \to e\gamma$ measurement, etc.) and the mass splitting and flavor decomposition have been measured by ATLAS/CMS.



If we have at our disposal such three consistent measurements (rate of FCNC process, spectrum and splitting), then we will understand the mechanism by which supersymmetry has its flavor violation suppressed. This will provide strong hints about the mechanism of supersymmetry breaking mediation.

If the sfermions are quasi-degenerate, then the mixing is determined by the small corrections to the unit mass-squared matrix. As mentioned above, the structure of such corrections may be dictated by the same symmetry or dynamics that gives the structure of the Yukawa couplings. If that is the case, then the measurement of the flavor decomposition might shed light on the Standard Model flavor puzzle.

We conclude that measurements at the LHC related to new particles that couple to the SM fermions are likely to teach us much more about flavor physics.

IX. CONCLUSIONS

(i) Measurements of CP violating B-meson decays have established that the Kobayashi-Maskawa mechanism is the dominant source of the observed CP violation.

(ii) Measurements of flavor changing B-meson decays have established the the Cabibbo-Kobayashi-Maskawa mechanism is a major player in flavor violation.

(iii) The consistency of all these measurements with the CKM predictions sharpens the new physics flavor puzzle: If there is new physics at, or below, the TeV scale, then its flavor structure must be highly non-generic.

(iv) If the recent evidence at D0 for a dimuon CP asymmetry in $B_{d,s}$ decays is confirmed, new physics will be called for.

(v) Measurements of neutrino flavor parameters have not only not clarified the standard model flavor puzzle, but actually deepened it. Whether they imply an anarchical structure, or a tribimaximal mixing, it seems that the neutrino flavor structure is very different from that of quarks.

(vi) If the LHC experiments, ATLAS and CMS, discover new particles that couple to the Standard Model fermions, then, in principle, they will be able to measure new flavor parameters. Consequently, the new physics flavor puzzle is likely to be understood.

(vii) If the flavor structure of such new particles is affected by the same physics that sets the flavor structure of the Yukawa couplings, then the LHC experiments (and future flavor factories) may be able to shed light also on the standard model flavor puzzle.

The huge progress in flavor physics in recent years has provided answers to many questions. At the same time, new questions arise. We look forward to the LHC era for more answers and more questions.

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APPENDIX A: THE CKM MATRIX

The CKM matrix V is a 3×3 unitary matrix. Its form, however, is not unique:

(i) There is freedom in defining V in that we can permute between the various generations. This freedom is fixed by ordering the up quarks and the down quarks by their masses, *i.e.* $(u_1, u_2, u_3) \to (u, c, t)$ and $(d_1, d_2, d_3) \to (d, s, b)$. The elements of V are written as follows:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}. \tag{A1}$$

(ii) There is further freedom in the phase structure of V. This means that the number of physical parameters in V is smaller than the number of parameters in a general unitary 3×3 matrix which is nine (three real angles and six phases). Let us define P_q ($q = u, d$) to be diagonal unitary (phase) matrices. Then, if instead of using V_{qL} and V_{qR} for the rotation (21) to the mass basis we use $\tilde{V}_{qL}$ and $\tilde{V}_{qR}$, defined by $\tilde{V}_{qL} = P_q V_{qL}$ and $\tilde{V}_{qR} = P_q V_{qR}$, we still maintain a legitimate mass basis since $M_q^{\rm diag}$ remains unchanged by such transformations. However, V does change:

$$V \to P_u V P_d^*. \tag{A2}$$

This freedom is fixed by demanding that V has the minimal number of phases. In the three generation case V has a single phase. (There are five phase differences between the elements of P_u and P_d and, therefore, five of the six phases in the CKM matrix can be removed.) This is the Kobayashi-Maskawa phase $\delta_{\rm KM}$ which is the single source of CP violation in the quark sector of the Standard Model [1].

The fact that V is unitary and depends on only four independent physical parameters can be made manifest by choosing a specific parametrization. The standard choice is [54]

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{A3}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The θ_{ij}'s are the three real mixing parameters while δ is the Kobayashi-Maskawa phase. It is known experimentally that $s_{13} \ll s_{23} \ll s_{12} \ll 1$. It is convenient to choose an approximate expression where this hierarchy is manifest. This is the Wolfenstein parametrization, where the four mixing parameters are (λ, A, ρ, η) with

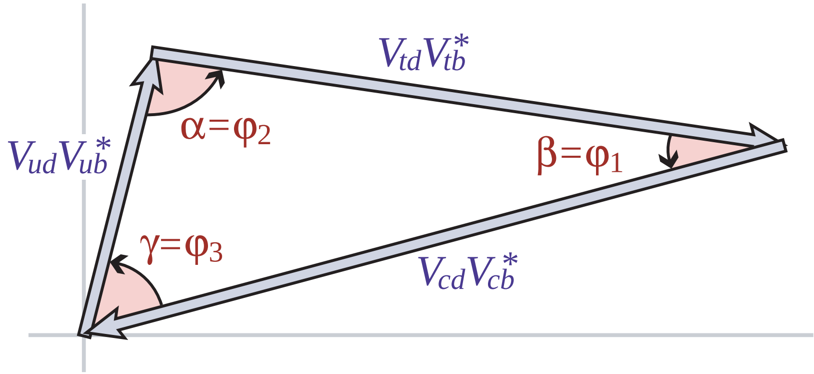

FIG. 6: Graphical representation of the unitarity constraint $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ as a triangle in the complex plane.

$\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter and η representing the CP violating phase [55, 56]:

$$V = \begin{pmatrix} 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda+\frac{1}{2}A^2\lambda^5[1-2(\rho+i\eta)] & 1-\frac{1}{2}\lambda^2-\frac{1}{8}\lambda^4(1+4A^2) & A\lambda^2 \\ A\lambda^3[1-(1-\frac{1}{2}\lambda^2)(\rho+i\eta)] & -A\lambda^2+\frac{1}{2}A\lambda^4[1-2(\rho+i\eta)] & 1-\frac{1}{2}A^2\lambda^4 \end{pmatrix}. \quad \text{(A4)}$$

A very useful concept is that of the *unitarity triangles*. The unitarity of the CKM matrix leads to various relations among the matrix elements, *e.g.*

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0, \quad \text{(A5)}$$

$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0, \quad \text{(A6)}$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad \text{(A7)}$$

Each of these three relations requires the sum of three complex quantities to vanish and so can be geometrically represented in the complex plane as a triangle. These are "the unitarity triangles", though the term "unitarity triangle" is usually reserved for the relation (A7) only. The unitarity triangle related to Eq. (A7) is depicted in Fig. 6.

The rescaled unitarity triangle is derived from (A7) by (a) choosing a phase convention such that $(V_{cd}V_{cb}^*)$ is real, and (b) dividing the lengths of all sides by $|V_{cd}V_{cb}^*|$. Step (a) aligns one side of the triangle with the real axis, and step (b) makes the length of this side 1. The form of the triangle is unchanged. Two vertices of the rescaled unitarity triangle are thus fixed at (0,0) and (1,0). The coordinates of the remaining vertex correspond to the Wolfenstein parameters (ρ, η). The area of the rescaled unitarity triangle is $|\eta|/2$.

Depicting the rescaled unitarity triangle in the (ρ, η) plane, the lengths of the two complex sides are

$$R_u \equiv \left|\frac{V_{ud}V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2+\eta^2}, \quad R_t \equiv \left|\frac{V_{td}V_{tb}}{V_{cd}V_{cb}}\right| = \sqrt{(1-\rho)^2+\eta^2}. \tag{A8}$$

The three angles of the unitarity triangle are defined as follows [57, 58]:

$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right], \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right], \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]. \tag{A9}$$

They are physical quantities and can be independently measured by CP asymmetries in B decays. It is also useful to define the two small angles of the unitarity triangles (A6,A5):

$$\beta_s \equiv \arg\left[-\frac{V_{ts}V_{tb}^*}{V_{cs}V_{cb}^*}\right], \quad \beta_K \equiv \arg\left[-\frac{V_{cs}V_{cd}^*}{V_{us}V_{ud}^*}\right]. \tag{A10}$$

The λ and A parameters are very well determined at present, see Eq. (37). The main effort in CKM measurements is thus aimed at improving our knowledge of ρ and η:

$$\rho = 0.14^{+0.03}_{-0.02}, \quad \eta = 0.35 \pm 0.02. \tag{A11}$$

The present status of our knowledge is best seen in a plot of the various constraints and the final allowed region in the $\rho - \eta$ plane. This is shown in Fig. 2.

APPENDIX B: CPV IN B DECAYS TO FINAL CP EIGENSTATES

We define decay amplitudes of B (which could be charged or neutral) and its CP conjugate $\overline{B}$ to a multi-particle final state f and its CP conjugate $\overline{f}$ as

$$A_f = \langle f|\mathcal{H}|B\rangle \quad , \quad \overline{A}_f = \langle f|\mathcal{H}|\overline{B}\rangle \quad , \quad A_{\overline{f}} = \langle \overline{f}|\mathcal{H}|B\rangle \quad , \quad \overline{A}_{\overline{f}} = \langle \overline{f}|\mathcal{H}|\overline{B}\rangle \ , \tag{B1}$$

where $\mathcal{H}$ is the Hamiltonian governing weak interactions. The action of CP on these states introduces phases ξ_B and ξ_f according to

$$\begin{aligned} CP\ |B\rangle \ &= \ e^{+i\xi_B}\ |\overline{B}\rangle \quad , \quad CP\ |f\rangle = e^{+i\xi_f}\ |\overline{f}\rangle\ , \\ CP\ |\overline{B}\rangle \ &= \ e^{-i\xi_B}\ |B\rangle \quad , \quad CP\ |\overline{f}\rangle = e^{-i\xi_f}\ |f\rangle\ , \end{aligned} \tag{B2}$$

so that $(CP\ )^2 = 1$. The phases ξ_B and ξ_f are arbitrary and unphysical because of the flavor symmetry of the strong interaction. If CP is conserved by the dynamics, $[CP\ , \mathcal{H}] = 0$, then A_f and $\overline{A}_{\overline{f}}$ have the same magnitude and an arbitrary unphysical relative phase

$$\overline{A}_{\overline{f}} = e^{i(\xi_f - \xi_B)}\, A_f\ . \tag{B3}$$

A state that is initially a superposition of B^0 and $\overline{B}^0$, say

$$|\psi(0)\rangle = a(0)|B^0\rangle + b(0)|\overline{B}^0\rangle \ , \tag{B4}$$

will evolve in time acquiring components that describe all possible decay final states $\{f_1, f_2, \ldots\}$, that is,

$$|\psi(t)\rangle = a(t)|B^0\rangle + b(t)|\overline{B}^0\rangle + c_1(t)|f_1\rangle + c_2(t)|f_2\rangle + \cdots \ . \tag{B5}$$

If we are interested in computing only the values of $a(t)$ and $b(t)$ (and not the values of all $c_i(t)$), and if the times t in which we are interested are much larger than the typical strong interaction scale, then we can use a much simplified formalism [59]. The simplified time evolution is determined by a 2×2 effective Hamiltonian $\mathcal{H}$ that is not Hermitian, since otherwise the mesons would only oscillate and not decay. Any complex matrix, such as $\mathcal{H}$, can be written in terms of Hermitian matrices M and Γ as

$$\mathcal{H} = M - \frac{i}{2}\Gamma \ . \tag{B6}$$

M and Γ are associated with $(B^0, \overline{B}^0) \leftrightarrow (B^0, \overline{B}^0)$ transitions via off-shell (dispersive) and on-shell (absorptive) intermediate states, respectively. Diagonal elements of M and Γ are associated with the flavor-conserving transitions $B^0 \to B^0$ and $\overline{B}^0 \to \overline{B}^0$ while off-diagonal elements are associated with flavor-changing transitions $B^0 \leftrightarrow \overline{B}^0$.

The eigenvectors of $\mathcal{H}$ have well defined masses and decay widths. We introduce complex parameters $p_{L,H}$ and $q_{L,H}$ to specify the components of the strong interaction eigenstates, B^0 and $\overline{B}^0$, in the light (B_L) and heavy (B_H) mass eigenstates:

$$|B_{L,H}\rangle = p_{L,H}|B^0\rangle \pm q_{L,H}|\overline{B}^0\rangle \tag{B7}$$

with the normalization $|p_{L,H}|^2 + |q_{L,H}|^2 = 1$. If either CP or CPT is a symmetry of $\mathcal{H}$ (independently of whether T is conserved or violated) then $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$, and solving the eigenvalue problem for $\mathcal{H}$ yields $p_L = p_H \equiv p$ and $q_L = q_H \equiv q$ with

$$\left(\frac{q}{p}\right)^2 = \frac{M_{12}^* - (i/2)\Gamma_{12}^*}{M_{12} - (i/2)\Gamma_{12}} \ . \tag{B8}$$

From now on we assume that CPT is conserved. If either CP or T is a symmetry of $\mathcal{H}$ (independently of whether CPT is conserved or violated), then M_{12} and Γ_{12} are relatively real, leading to

$$\left(\frac{q}{p}\right)^2 = e^{2i\xi_B} \quad \Rightarrow \quad \left|\frac{q}{p}\right| = 1 \ , \tag{B9}$$

where ξ_B is the arbitrary unphysical phase introduced in Eq. (B2).

The real and imaginary parts of the eigenvalues of $\mathcal{H}$ corresponding to $|B_{L,H}\rangle$ represent their masses and decay-widths, respectively. The mass difference Δm_B and the width difference $\Delta\Gamma_B$ are defined as follows:

$$\Delta m_B \equiv M_H - M_L, \quad \Delta\Gamma_B \equiv \Gamma_H - \Gamma_L. \tag{B10}$$

Note that here Δm_B is positive by definition, while the sign of $\Delta\Gamma_B$ is to be experimentally determined. The average mass and width are given by

$$m_B \equiv \frac{M_H + M_L}{2}, \quad \Gamma_B \equiv \frac{\Gamma_H + \Gamma_L}{2}. \tag{B11}$$

It is useful to define dimensionless ratios x and y:

$$x \equiv \frac{\Delta m_B}{\Gamma_B}, \quad y \equiv \frac{\Delta\Gamma_B}{2\Gamma_B}. \tag{B12}$$

Solving the eigenvalue equation gives

$$(\Delta m_B)^2 - \frac{1}{4}(\Delta\Gamma_B)^2 = (4|M_{12}|^2 - |\Gamma_{12}|^2), \quad \Delta m_B \Delta\Gamma_B = 4\mathcal{R}e(M_{12}\Gamma_{12}^*). \tag{B13}$$

All CP-violating observables in B and $\overline{B}$ decays to final states f and $\overline{f}$ can be expressed in terms of phase-convention-independent combinations of A_f, $\overline{A}_f$, $A_{\overline{f}}$ and $\overline{A}_{\overline{f}}$, together with, for neutral-meson decays only, q/p. CP violation in charged-meson decays depends only on the combination $|\overline{A}_{\overline{f}}/A_f|$, while CP violation in neutral-meson decays is complicated by $B^0 \leftrightarrow \overline{B}^0$ oscillations and depends, additionally, on $|q/p|$ and on $\lambda_f \equiv (q/p)(\overline{A}_f/A_f)$.

For neutral D, B, and B_s mesons, $\Delta\Gamma/\Gamma \ll 1$ and so both mass eigenstates must be considered in their evolution. We denote the state of an initially pure $|B^0\rangle$ or $|\overline{B}^0\rangle$ after an elapsed proper time t as $|B^0{}_{\rm phys}(t)\rangle$ or $|\overline{B}^0_{\rm phys}(t)\rangle$, respectively. Using the effective Hamiltonian approximation, we obtain

$$\begin{aligned} |B^0{}_{\rm phys}(t)\rangle &= g_+(t)\,|B^0\rangle - \frac{q}{p}\, g_-(t)|\overline{B}^0\rangle, \\ |\overline{B}^0_{\rm phys}(t)\rangle &= g_+(t)\,|\overline{B}^0\rangle - \frac{p}{q}\, g_-(t)|B^0\rangle\,, \end{aligned} \tag{B14}$$

where

$$g_\pm(t) \equiv \frac{1}{2}\left(e^{-im_H t - \frac{1}{2}\Gamma_H t} \pm e^{-im_L t - \frac{1}{2}\Gamma_L t}\right). \tag{B15}$$

One obtains the following time-dependent decay rates:

$$\frac{d\Gamma[{B^0}_{\rm phys}(t) \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|A_f|^2 + |(q/p)\overline{A}_f|^2\right)\cosh(y\Gamma t) + \left(|A_f|^2 - |(q/p)\overline{A}_f|^2\right)\cos(x\Gamma t)$$
$$+\ 2\,\mathcal{R}e((q/p)A_f^*\overline{A}_f)\sinh(y\Gamma t) - 2\,\mathcal{I}m((q/p)A_f^*\overline{A}_f)\sin(x\Gamma t)\ , \quad \text{(B16)}$$

$$\frac{d\Gamma[\overline{B}^0_{\rm phys}(t) \to f]/dt}{e^{-\Gamma t}\mathcal{N}_f} = \left(|(p/q)A_f|^2 + |\overline{A}_f|^2\right)\cosh(y\Gamma t) - \left(|(p/q)A_f|^2 - |\overline{A}_f|^2\right)\cos(x\Gamma t)$$
$$+\ 2\,\mathcal{R}e((p/q)A_f\overline{A}_f^*)\sinh(y\Gamma t) - 2\,\mathcal{I}m((p/q)A_f\overline{A}_f^*)\sin(x\Gamma t)\ , \quad \text{(B17)}$$

where $\mathcal{N}_f$ is a common normalization factor. Decay rates to the CP-conjugate final state $\overline{f}$ are obtained analogously, with $\mathcal{N}_f = \mathcal{N}_{\overline{f}}$ and the substitutions $A_f \to A_{\overline{f}}$ and $\overline{A}_f \to \overline{A}_{\overline{f}}$ in Eqs. (B16,B17). Terms proportional to $|A_f|^2$ or $|\overline{A}_f|^2$ are associated with decays that occur without any net $B \leftrightarrow \overline{B}$ oscillation, while terms proportional to $|(q/p)\overline{A}_f|^2$ or $|(p/q)A_f|^2$ are associated with decays following a net oscillation. The $\sinh(y\Gamma t)$ and $\sin(x\Gamma t)$ terms of Eqs. (B16,B17) are associated with the interference between these two cases. Note that, in multi-body decays, amplitudes are functions of phase-space variables. Interference may be present in some regions but not others, and is strongly influenced by resonant substructure.

One possible manifestation of CP-violating effects in meson decays [60] is in the interference between a decay without mixing, $B^0 \to f$, and a decay with mixing, $B^0 \to \overline{B}^0 \to f$ (such an effect occurs only in decays to final states that are common to B^0 and $\overline{B}^0$, including all CP eigenstates). It is defined by

$$\mathcal{I}m(\lambda_f) \neq 0\ , \quad \text{(B18)}$$

with

$$\lambda_f \equiv \frac{q}{p}\frac{\overline{A}_f}{A_f}\ . \quad \text{(B19)}$$

This form of CP violation can be observed, for example, using the asymmetry of neutral meson decays into final CP eigenstates f_{CP}

$$\mathcal{A}_{f_{CP}}(t) \equiv \frac{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\ ] - d\Gamma/dt[{B^0}_{\rm phys}(t) \to f_{CP}\ ]}{d\Gamma/dt[\overline{B}^0_{\rm phys}(t) \to f_{CP}\ ] + d\Gamma/dt[{B^0}_{\rm phys}(t) \to f_{CP}\ ]}\ . \quad \text{(B20)}$$

For $\Delta\Gamma = 0$ and $|q/p| = 1$ (which is a good approximation for B mesons), $\mathcal{A}_{f_{CP}}$ has a particularly simple form [61–63]:

$$\mathcal{A}_f(t) = S_f\sin(\Delta m t) - C_f\cos(\Delta m t),$$
$$S_f \equiv \frac{2\,\mathcal{I}m(\lambda_f)}{1+|\lambda_f|^2}, \quad C_f \equiv \frac{1-|\lambda_f|^2}{1+|\lambda_f|^2}\ , \quad \text{(B21)}$$

Consider the $B \to f$ decay amplitude A_f, and the CP conjugate process, $\overline{B} \to \overline{f}$, with decay amplitude $\overline{A}_{\overline{f}}$. There are two types of phases that may appear in these decay amplitudes. Complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the CP-conjugate amplitude. Thus their phases appear in A_f and $\overline{A}_{\overline{f}}$ with opposite signs. In the Standard Model, these phases occur only in the couplings of the $W^\pm$ bosons and hence are often called "weak phases". The weak phase of any single term is convention dependent. However, the difference between the weak phases in two different terms in A_f is convention independent. A second type of phase can appear in scattering or decay amplitudes even when the Lagrangian is real. Their origin is the possible contribution from intermediate on-shell states in the decay process. Since these phases are generated by CP-invariant interactions, they are the same in A_f and $\overline{A}_{\overline{f}}$. Usually the dominant rescattering is due to strong interactions and hence the designation "strong phases" for the phase shifts so induced. Again, only the relative strong phases between different terms in the amplitude are physically meaningful.

The 'weak' and 'strong' phases discussed here appear in addition to the 'spurious' CP-transformation phases of Eq. (B3). Those spurious phases are due to an arbitrary choice of phase convention, and do not originate from any dynamics or induce any CP violation. For simplicity, we set them to zero from here on.

It is useful to write each contribution a_i to A_f in three parts: its magnitude $|a_i|$, its weak phase ϕ_i, and its strong phase δ_i. If, for example, there are two such contributions, $A_f = a_1 + a_2$, we have

$$\begin{aligned} A_f &= |a_1| e^{i(\delta_1+\phi_1)} + |a_2| e^{i(\delta_2+\phi_2)}, \\ \overline{A}_{\overline{f}} &= |a_1| e^{i(\delta_1-\phi_1)} + |a_2| e^{i(\delta_2-\phi_2)}. \end{aligned} \tag{B22}$$

Similarly, for neutral meson decays, it is useful to write

$$M_{12} = |M_{12}| e^{i\phi_M} \quad , \quad \Gamma_{12} = |\Gamma_{12}| e^{i\phi_\Gamma} \; . \tag{B23}$$

Each of the phases appearing in Eqs. (B22,B23) is convention dependent, but combinations such as $\delta_1 - \delta_2$, $\phi_1 - \phi_2$, $\phi_M - \phi_\Gamma$ and $\phi_M + \phi_1 - \overline{\phi}_1$ (where $\overline{\phi}_1$ is a weak phase contributing to $\overline{A}_f$) are physical.

In the approximations that only a single weak phase contributes to decay, $A_f = |a_f| e^{i(\delta_f+\phi_f)}$, and that $|\Gamma_{12}/M_{12}| = 0$, we obtain $|\lambda_f| = 1$ and the CP asymmetries in decays to a final CP eigenstate f [Eq. (B20)] with eigenvalue $\eta_f = \pm 1$ are given by

$$\mathcal{A}_{f_{CP}}(t) = \mathcal{I}m(\lambda_f)\ \sin(\Delta m t) \ \text{ with } \ \mathcal{I}m(\lambda_f) = \eta_f \sin(\phi_M + 2\phi_f). \tag{B24}$$

Note that the phase so measured is purely a weak phase, and no hadronic parameters are involved in the extraction of its value from $\mathcal{I}m(\lambda_f)$.

APPENDIX C: SUPERSYMMETRIC FLAVOR VIOLATION

1. Mass insertions

Supersymmetric models provide, in general, new sources of flavor violation. We here present the formalism of mass insertions. We do that for the charged sleptons, but the formalism is straightforwardly adapted for squarks.

The supersymmetric lepton flavor violation is most commonly analyzed in the basis in which the charged lepton mass matrix and the gaugino vertices are diagonal. In this basis, the slepton masses are not necessarily flavor-diagonal, and have the form

$$\widetilde{\ell}^*_{Mi}(M^2_{\widetilde{\ell}})^{MN}_{ij}\widetilde{\ell}_{Nj} = (\widetilde{\ell}^*_{Li}\ \widetilde{\ell}^*_{Rk}) \begin{pmatrix} M^2_{Lij} & A_{il}v_d \\ A_{jk}v_d & M^2_{Rkl} \end{pmatrix} \begin{pmatrix} \widetilde{\ell}_{Lj} \\ \widetilde{\ell}_{Rl} \end{pmatrix}, \tag{C1}$$

where $M, N = L, R$ label chirality, and $i, j, k, l = 1, 2, 3$ are generational indices. M^2_L and M^2_R are the supersymmetry breaking slepton masses-squared. The A parameters enter in the trilinear scalar couplings $A_{ij}\phi_d\widetilde{\ell}_{Li}\widetilde{\ell}^*_{Rj}$, where ϕ_d is the down-type Higgs boson, and $v_d = \langle\phi_d\rangle$. We neglect small flavor-conserving terms involving $\tan\beta = v_u/v_d$.

In this basis, charged LFV takes place through one or more slepton mass insertion. Each mass insertion brings with it a factor of

$$\delta^{MN}_{ij} \equiv (M^2_{\widetilde{\ell}})^{MN}_{ij}/\tilde{m}^2, \tag{C2}$$

where $\tilde{m}^2$ is the representative slepton mass scale. Physical processes therefore constrain

$$(\delta^{MN}_{ij})_{\text{eff}} \sim \max\left[\delta^{MN}_{ij}, \delta^{MP}_{ik}\delta^{PN}_{kj}, \ldots, (i \leftrightarrow j)\right]. \tag{C3}$$

For example,

$$(\delta^{LR}_{12})_{\text{eff}} \sim \max\left[A_{12}v_d/\tilde{m}^2, M^2_{L1k}A_{k2}v_d/\tilde{m}^4, A_{1k}v_dM^2_{Rk2}/\tilde{m}^4, \ldots, (1 \leftrightarrow 2)\right]. \tag{C4}$$

Note that contributions with two or more insertions may be less suppressed than those with only one.

It is useful to express the δ_{ij}^{MN} mass insertions in terms of parameters in the mass basis. We can write, for example,

$$\delta_{ij}^{LL} = \frac{1}{\tilde{m}^2} \sum_\alpha K_{i\alpha}^L K_{j\alpha}^{L*} \Delta\tilde{m}_{L\alpha}^2. \tag{C5}$$

Here, we ignore $L-R$ mixing, so that $K_{i\alpha}^L$ is the mixing angle in the coupling of a neutralino to $\ell_{Li} - \widetilde{\ell}_{L\alpha}$ (with $\ell_i = e, \mu, \tau$ denoting charged lepton mass eigenstates and $\widetilde{\ell}_\alpha = \widetilde{\ell}_1, \widetilde{\ell}_2, \widetilde{\ell}_3$ denoting charged slepton mass eigenstates), and $\Delta\tilde{m}_{L\alpha}^2 = m_{\widetilde{\ell}_{L\alpha}}^2 - \tilde{m}^2$. Using the unitarity of the mixing matrix K^L, we can write

$$\tilde{m}^2 \delta_{ij}^{LL} = \sum_\alpha K_{i\alpha}^L K_{j\alpha}^{L*} (\Delta\tilde{m}_{L\alpha}^2 + \tilde{m}^2) = (M_{\widetilde{\ell}}^2)_{ij}^{LL}, \tag{C6}$$

thus reproducing the definition (C2).

In many cases, a two generation effective framework is useful. To understand that, consider a case where (no summation over i, j, k)

$$\begin{aligned} |K_{ik}^L K_{jk}^{L*}| &\ll |K_{ij}^L K_j^{L*}|, \\ |K_{ik}^L K_{jk}^{L*} \Delta m_{\widetilde{\ell}_{Lk}\widetilde{\ell}_{Li}}^2| &\ll |K_{ij}^L K_j^{L*} \Delta m_{\widetilde{\ell}_{Lj}\widetilde{\ell}_{Li}}^2|, \end{aligned} \tag{C7}$$

where $\Delta m_{\widetilde{\ell}_j \widetilde{\ell}_i}^2 = m_{\widetilde{\ell}_{Lj}}^2 - m_{\widetilde{\ell}_{Li}}^2$. Then, the contribution of the intermediate $\widetilde{\ell}_k$ can be neglected and, furthermore, to a good approximation $K_{ii}^L K_{ji}^{L*} + K_{ij}^L K_{jj}^{L*} = 0$. For these cases, we obtain

$$\delta_{ij}^{LL} = \frac{\Delta m_{\widetilde{\ell}_{Lj}\widetilde{\ell}_{Li}}^2}{\tilde{m}^2} K_{ij}^L K_{jj}^{L*}. \tag{C8}$$

2. Neutral meson mixing

We consider the squark-gluino box diagram contribution to $D^0 - \overline{D}^0$ mixing amplitude that is proportional to $K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}$, where K^u is the mixing matrix of the gluino couplings to left-handed up quarks and their up squark partners. (In the language of the mass insertion approximation, we calculate here the contribution that is $\propto [(\delta_{LL}^u)_{12}]^2$.) We work in the mass basis for both quarks and squarks.

The contribution is given by

$$M_{12}^D = -i\frac{4\pi^2}{27} \alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD} \sum_{i,j} (K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*})(11\tilde{I}_{4ij} + 4\tilde{m}_g^2 I_{4ij}). \tag{C9}$$

where

$$\tilde{I}_{4ij} \equiv \int \frac{d^4p}{(2\pi)^4} \frac{p^2}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)}$$
$$= \frac{i}{(4\pi)^2}\left[\frac{\tilde{m}_g^2}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)} + \frac{\tilde{m}_i^4}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^4}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right], \quad \text{(C10)}$$

$$I_{4ij} \equiv \int \frac{d^4p}{(2\pi)^4} \frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_i^2)(p^2-\tilde{m}_j^2)}$$
$$= \frac{i}{(4\pi)^2}\left[\frac{1}{(\tilde{m}_i^2-\tilde{m}_g^2)(\tilde{m}_j^2-\tilde{m}_g^2)} + \frac{\tilde{m}_i^2}{(\tilde{m}_i^2-\tilde{m}_j^2)(\tilde{m}_i^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_i^2}{\tilde{m}_g^2} + \frac{\tilde{m}_j^2}{(\tilde{m}_j^2-\tilde{m}_i^2)(\tilde{m}_j^2-\tilde{m}_g^2)^2}\ln\frac{\tilde{m}_j^2}{\tilde{m}_g^2}\right]. \quad \text{(C11)}$$

We now follow the discussion in refs. [26, 30]. To see the consequences of the super-GIM mechanism, let us expand the expression for the box integral around some value $\tilde{m}_q^2$ for the squark masses-squared:

$$I_4(\tilde{m}_g^2,\tilde{m}_i^2,\tilde{m}_j^2) = I_4(\tilde{m}_g^2,\tilde{m}_q^2+\delta\tilde{m}_i^2,\tilde{m}_q^2+\delta\tilde{m}_j^2)$$
$$= I_4(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2) + (\delta\tilde{m}_i^2+\delta\tilde{m}_j^2)I_5(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2)$$
$$+ \frac{1}{2}\left[(\delta\tilde{m}_i^2)^2+(\delta\tilde{m}_j^2)^2+2(\delta\tilde{m}_i^2)(\delta\tilde{m}_j^2)\right] I_6(\tilde{m}_g^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2,\tilde{m}_q^2) + \cdots \quad \text{(C12)}$$

where

$$I_n(\tilde{m}_g^2,\tilde{m}_q^2,\ldots,\tilde{m}_q^2) \equiv \int \frac{d^4p}{(2\pi)^4}\frac{1}{(p^2-\tilde{m}_g^2)^2(p^2-\tilde{m}_q^2)^{n-2}}, \quad \text{(C13)}$$

and similarly for $\tilde{I}_{4ij}$. Note that $I_n \propto (\tilde{m}_q^2)^{n-2}$ and $\tilde{I}_n \propto (\tilde{m}_q^2)^{n-3}$. Thus, using $x \equiv \tilde{m}_g^2/\tilde{m}_q^2$, it is customary to define

$$I_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-2}} f_n(x), \qquad \tilde{I}_n \equiv \frac{i}{(4\pi)^2(\tilde{m}_q^2)^{n-3}} \tilde{f}_n(x). \quad \text{(C14)}$$

The unitarity of the mixing matrix implies that

$$\sum_i (K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = \sum_j (K_{2i}^u K_{1i}^{u*} K_{2j}^u K_{1j}^{u*}) = 0. \quad \text{(C15)}$$

We learn that the terms that are proportional f_4, $\tilde{f}_4$, f_5 and $\tilde{f}_5$ vanish in their contribution to M_{12}. When $\delta\tilde{m}_i^2 \ll \tilde{m}_q^2$ for all i, the leading contributions to M_{12} come from f_6 and $\tilde{f}_6$. We learn that for quasi-degenerate squarks, the leading contribution is quadratic in the small mass-squared difference. The functions $f_6(x)$ and $\tilde{f}_6(x)$ are given by

$$f_6(x) = \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(1-x)^5},$$
$$\tilde{f}_6(x) = \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(1-x)^5}. \tag{C16}$$

For example, with $x = 1$, $f_6(1) = -1/20$ and $\tilde{f}_6 = +1/30$; with $x = 2.33$, $f_6(2.33) = -0.015$ and $\tilde{f}_6 = +0.013$.

To further simplify things, let us consider a two generation case. Then

$$M_{12}^D \propto 2(K_{21}^u K_{11}^{u*})^2(\delta\tilde{m}_1^2)^2 + 2(K_{22}^u K_{12}^{u*})^2(\delta\tilde{m}_2^2)^2 + (K_{21}^u K_{11}^{u*} K_{22}^u K_{12}^{u*})(\delta\tilde{m}_1^2 + \delta\tilde{m}_2^2)^2$$
$$= (K_{21}^u K_{11}^{u*})^2(\tilde{m}_2^2 - \tilde{m}_1^2)^2. \tag{C17}$$

We thus rewrite Eq. (C9) for the case of quasi-degenerate squarks:

$$M_{12}^D = \frac{\alpha_s^2 m_D f_D^2 B_D \eta_{\rm QCD}}{108\tilde{m}_q^2}[11\tilde{f}_6(x) + 4x f_6(x)]\frac{(\Delta\tilde{m}_{21}^2)^2}{\tilde{m}_q^4}(K_{21}^u K_{11}^{u*})^2. \tag{C18}$$

For example, for $x = 1$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.17$. For $x = 2.33$, $11\tilde{f}_6(x) + 4x f_6(x) = +0.003$.

APPENDIX D: SMALL SLEPTON MASS SPLITTINGS AT THE LHC

A review of mass measurements techniques proposed for the LHC can be found in Ref. [64]. We here present two specific ideas of how to measure small mass splittings between sleptons with large selectron and smuon components, first when the lightest charged slepton is metastable and therefore visible [46], and the second for the case where the LSP is an invisible neutralino [65].

1. Shifted peak [46]

Consider supersymmetric models with two sleptons, $\tilde{\ell}_{1,2}$, with $\Delta m \equiv m_2 - m_1 \ll m_1$, with a metastable ℓ_1. Such a spectrum is predicted by supersymmetric models that explain the masses and mixings of the standard models charged leptons and neutrinos in terms of broken flavor symmetries.

The method to measure Δm relies on the decays of a mother particle, in this case the neutralino χ_1^0, to $\tilde{\ell}_{1,2}$. If we use direct decays to $\tilde{\ell}_1$ to reconstruct the neutralino, the invariant mass distribution of $m^2_{\tilde{\ell}\ell}$ will be peaked at the correct neutralino mass. Some of the time, however, χ_1^0 decays to $\tilde{\ell}_2$, which subsequently decays to $\tilde{\ell}_1$. The leptons produced in the $\tilde{\ell}_2 \to \tilde{\ell}_1$ decay are relatively soft, with energies typically of order Δm. They may therefore be lost, implying that the decays $\chi_1^0 \to \tilde{\ell}_1$ and $\chi_1^0 \to \tilde{\ell}_2$ have the same topology. Rather than blurring the picture, however, it turns out that by attempting to reconstruct the χ_1^0 in both cases, one finds two peaks: one at the neutralino mass M, and one slightly shifted by an amount $E_{\rm shift} \sim \Delta m$. Thus, measuring a shift in the neutralino mass peak will tell us that there are in fact two slepton states lighter than the neutralino. Furthermore, Δm can be determined in terms of $E_{\rm shift}$, m_1 and M.

More specifically, the neutralino can decay directly to $\tilde{\ell}_1$,

$$\chi_1^0 \to \tilde{\ell}_1^{\pm} \ell_1^{\mp}. \tag{D1}$$

It can also decay to $\tilde{\ell}_2$,

$$\chi_1^0 \to \tilde{\ell}_2^{\pm} \ell_2^{\mp}, \tag{D2}$$

followed by one of the two three-body decays

$$\tilde{\ell}_2^{\pm} \;\to\; \tilde{\ell}_1^{\pm} X^{\pm\mp}, \tag{D3}$$

$$\tilde{\ell}_2^{\pm} \;\to\; \tilde{\ell}_1^{\mp} X^{\pm\pm}, \tag{D4}$$

where $X^{\pm\mp}$ contains two opposite-sign (OS) leptons, and $X^{\pm\pm}$ contains two same-sign (SS) leptons. We denote these lepton pairs by X to emphasize the fact that they are too soft to pass our cuts. Thus, the observed particles are the hard lepton from Eq. (D1) or (D2), and the long-lived slepton $\tilde{\ell}_1$ from Eq. (D1) and (D3) or (D4).

We can thus construct distributions for the following invariant mass squared:

$$m^2_{\tilde{\ell}\ell_1} \equiv (p_{\tilde{\ell}_1} + p_{\ell_1})^2, \quad m^2_{\tilde{\ell}\ell_2} \equiv (p_{\tilde{\ell}_1} + p_{\ell_2})^2. \tag{D5}$$

Obviously,

$$m_{\tilde{\ell}\ell_1} = M. \tag{D6}$$

However, because of the missing soft leptons, $m_{\tilde{\ell}\ell_2} \neq M$, so we define the shift of the peak of the $m_{\tilde{\ell}\ell_2}$ distribution from M:

$$\sqrt{m^2_{\tilde{\ell}\ell_2}|_{\rm peak}} = M - E_{\rm shift}. \tag{D7}$$

One obtains [46]:

$$E_{\text{shift}} \approx \frac{M^2 + m_1^2}{2Mm_1}\Delta m. \tag{D8}$$

Thus, E_{shift} is enhanced compared to Δm. According to Ref. [46], with an integrated luminosity of $\mathcal{O}(1\ \text{fb}^{-1})$, mass splitting as small as 1-2 GeV can be discovered and measured.

2. Shifted edge [65]

Consider the cascade decay

$$\chi_2^0 \to \tilde{\ell}^\pm \ell^\mp \to \chi_1^0 \ell^\pm \ell^\mp, \tag{D9}$$

with mass ordering $m_{\chi_2^0} > m_{\tilde{\ell}} > m_{\chi_1^0}$. The dilepton mass spectrum has a prominent kinematic edge at

$$m_{\ell\ell}^2 = \frac{(m_{\chi_2^0}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}}^2 - m_{\chi_1^0}^2)}{m_{\tilde{\ell}}^2}. \tag{D10}$$

Such an edge may be measured at the LHC experiments with per mille precision.

Exercise 13: *Prove Eq. (D10).*

Consider now the case of two quasidegenerate sleptons with mass splitting $\Delta m_{\tilde{\ell}}$, and average mass $m_{\tilde{\ell}}$. The variation of the edge position with the slepton mass is given by

$$\frac{dm_{\ell\ell}^2}{m_{\tilde{\ell}}^2} = \frac{m_{\chi_1^0}^2 m_{\chi_2^0}^2}{m_{\tilde{\ell}}^4} - 1. \tag{D11}$$

Thus, when $m_{\tilde{\ell}} = \sqrt{m_{\chi_1^0} m_{\chi_2^0}}$, the shift in the edge vanishes to leading order in $\Delta m_{\tilde{\ell}}/m_{\tilde{\ell}}$. The fractional shift in the invariant mass edge is given by

$$\frac{\Delta m_{\ell\ell}}{m_{\ell\ell}} = \frac{\Delta m_{\tilde{\ell}}}{m_{\tilde{\ell}}}\left[\frac{m_{\chi_1^0}^2 m_{\chi_2^0}^2 - m_{\tilde{\ell}}^4}{(m_{\chi_2^0}^2 - m_{\tilde{\ell}}^2)(m_{\tilde{\ell}}^2 - m_{\chi_1^0}^2)}\right], \tag{D12}$$

Note that under favorable circumstances the fractional shift in the edge could be significantly enhanced compared to the fractional splitting. Specifically, the enhancement could be strong when $m_{\chi_2^0}$ approaches $m_{\chi_1^0}$, though the benefit may be diluted by the fact that the leptons will be softer and thus harder to identify and measure.

Now consider the case that one of the sleptons is dominantly a smuon $\tilde{\mu}$, and the other is dominantly a selectron $\tilde{e}$. Then, the shift in the edge can be experimentally established by measuring $m_{\mu\mu}$ and m_{ee} separately. According to Ref. [65], a fractional splitting of order a few per mille can be discovered with 30 fb^{-1}.

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[1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[2] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963).

[3] A. B. Carter and A. I. Sanda, Phys. Rev. Lett. **45**, 952 (1980); Phys. Rev. D **23**, 1567 (1981).

[4] I. I. Y. Bigi and A. I. Sanda, Nucl. Phys. B **193**, 85 (1981).

[5] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996) [arXiv:hep-ph/9512380].

[6] Y. Grossman, A. L. Kagan and Z. Ligeti, Phys. Lett. B **538**, 327 (2002) [arXiv:hep-ph/0204212].

[7] H. Boos, T. Mannel and J. Reuter, Phys. Rev. D **70**, 036006 (2004) [arXiv:hep-ph/0403085].

[8] H. n. Li and S. Mishima, JHEP **0703**, 009 (2007) [arXiv:hep-ph/0610120].

[9] M. Gronau and J. L. Rosner, Phys. Lett. B **672**, 349 (2009) [arXiv:0812.4796 [hep-ph]].

[10] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex], online update at http://www.slac.stanford.edu/xorg/hfag

[11] C. Amsler *et al.* [Particle Data Group], Phys. Lett. B **667**, 1 (2008).

[12] CKMfitter Group (J. Charles et al.), Eur. Phys. J. **C41**, 1 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

[13] Y. Nir, Nucl. Phys. Proc. Suppl. **117**, 111 (2003) [arXiv:hep-ph/0208080].

[14] Y. Grossman, Y. Nir and M. P. Worah, Phys. Lett. B **407**, 307 (1997) [hep-ph/9704287].

[15] Y. Grossman, Y. Nir and G. Raz, Phys. Rev. Lett. **97**, 151801 (2006) [arXiv:hep-ph/0605028].

[16] M. Bona *et al.* [UTfit Collaboration], JHEP **0803**, 049 (2008) [arXiv:0707.0636 [hep-ph]].

[17] G. C. Branco, L. Lavoura and J. P. Silva, *CP violation,* Clarendon Press, Oxford (1999).

[18] I. I. Y. Bigi and N. G. Uraltsev, Nucl. Phys. B **592**, 92 (2001) [arXiv:hep-ph/0005089].

[19] A. F. Falk, Y. Grossman, Z. Ligeti and A. A. Petrov, Phys. Rev. D **65**, 054034 (2002) [arXiv:hep-ph/0110317].

[20] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir and A. A. Petrov, Phys. Rev. D **69**, 114021 (2004) [arXiv:hep-ph/0402204].

[21] V. M. Abazov *et al.* [The D0 Collaboration], arXiv:1005.2757 [hep-ex].

[22] A. Lenz and U. Nierste, JHEP **0706**, 072 (2007) [arXiv:hep-ph/0612167].

[23] S. Laplace, Z. Ligeti, Y. Nir and G. Perez, Phys. Rev. D **65**, 094040 (2002) [arXiv:hep-ph/0202010].

[24] Z. Ligeti, M. Papucci, G. Perez and J. Zupan, arXiv:1006.0432 [hep-ph].

[25] K. Blum, Y. Hochberg and Y. Nir, arXiv:1007.1872 [hep-ph].

[26] G. Raz, Phys. Rev. D **66**, 037701 (2002) [arXiv:hep-ph/0205310].

[27] G. Isidori, Y. Nir and G. Perez, Ann. Rev. Nucl. Part. Sci., in press [arXiv:1002.0900 [hep-ph]].

[28] N. Arkani-Hamed and S. Dimopoulos, JHEP **0506**, 073 (2005) [arXiv:hep-th/0405159].

[29] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B **388**, 588 (1996) [arXiv:hep-ph/9607394].

[30] Y. Nir and G. Raz, Phys. Rev. D **66**, 035007 (2002) [arXiv:hep-ph/0206064].

[31] K. Blum, Y. Grossman, Y. Nir and G. Perez, Phys. Rev. Lett. **102**, 211802 (2009) [arXiv:0903.2118 [hep-ph]].

[32] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993) [arXiv:hep-ph/9304307].

[33] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B **420**, 468 (1994) [arXiv:hep-ph/9310320].

[34] M. Ciuchini, E. Franco, D. Guadagnoli, V. Lubicz, M. Pierini, V. Porretti and L. Silvestrini, Phys. Lett. B **655**, 162 (2007) [arXiv:hep-ph/0703204].

[35] Y. Nir, JHEP **0705**, 102 (2007) [arXiv:hep-ph/0703235].

[36] G. D'Ambrosio, G. F. Giudice, G. Isidori and A. Strumia, Nucl. Phys. B **645**, 155 (2002) [arXiv:hep-ph/0207036].

[37] Y. Grossman, Y. Nir, J. Thaler, T. Volansky and J. Zupan, Phys. Rev. D **76**, 096006 (2007) [arXiv:0706.1845 [hep-ph]].

[38] V. Cirigliano, B. Grinstein, G. Isidori and M. B. Wise, Nucl. Phys. B **728**, 121 (2005) [arXiv:hep-ph/0507001].

[39] V. Cirigliano and B. Grinstein, Nucl. Phys. B **752**, 18 (2006) [arXiv:hep-ph/0601111].

[40] V. Cirigliano, G. Isidori and V. Porretti, Nucl. Phys. B **763**, 228 (2007) [arXiv:hep-ph/0607068].

[41] G. C. Branco, A. J. Buras, S. Jager, S. Uhlig and A. Weiler, JHEP **0709**, 004 (2007) [arXiv:hep-ph/0609067].

[42] M. C. Chen and H. B. Yu, Phys. Lett. B **672**, 253 (2009) [arXiv:0804.2503 [hep-ph]].

[43] E. Gross, D. Grossman, Y. Nir and O. Vitells, Phys. Rev. D **81**, 055013 (2010)

[arXiv:1001.2883 [hep-ph]].

[44] J. L. Feng, C. G. Lester, Y. Nir and Y. Shadmi, Phys. Rev. D **77**, 076002 (2008) [arXiv:0712.0674 [hep-ph]].

[45] J. L. Feng, I. Galon, D. Sanford, Y. Shadmi and F. Yu, Phys. Rev. D **79**, 116009 (2009) [arXiv:0904.1416 [hep-ph]].

[46] J. L. Feng, S. T. French, C. G. Lester, Y. Nir and Y. Shadmi, Phys. Rev. D **80**, 114004 (2009) [arXiv:0906.4215 [hep-ph]].

[47] J. L. Feng *et al.*, JHEP **1001**, 047 (2010) [arXiv:0910.1618 [hep-ph]].

[48] G. Hiller and Y. Nir, JHEP **0803**, 046 (2008) [arXiv:0802.0916 [hep-ph]].

[49] G. Hiller, Y. Hochberg and Y. Nir, JHEP **0903**, 115 (2009) [arXiv:0812.0511 [hep-ph]].

[50] G. Hiller, Y. Hochberg and Y. Nir, JHEP **1003**, 079 (2010) [arXiv:1001.1513 [hep-ph]].

[51] Y. Nomura, M. Papucci and D. Stolarski, Phys. Rev. D **77**, 075006 (2008) [arXiv:0712.2074 [hep-ph]]; JHEP **0807**, 055 (2008) [arXiv:0802.2582 [hep-ph]].

[52] G. F. Giudice, M. Nardecchia and A. Romanino, Nucl. Phys. B **813**, 156 (2009) [arXiv:0812.3610 [hep-ph]].

[53] A. E. Nelson and M. J. Strassler, JHEP **0009**, 030 (2000) [arXiv:hep-ph/0006251]; JHEP **0207**, 021 (2002) [arXiv:hep-ph/0104051].

[54] L. Chau and W. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[55] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).

[56] A. J. Buras, M. E. Lautenbacher, and G. Ostermaier, Phys. Rev. D **50**, 3433 (1994) [arXiv:hep-ph/9403384].

[57] C. Dib, I. Dunietz, F. J. Gilman and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

[58] J. L. Rosner, A. I. Sanda and M. P. Schmidt, EFI-88-12-CHICAGO [Presented at Workshop on High Sensitivity Beauty Physics, Batavia, IL, Nov 11-14, 1987].

[59] V. Weisskopf and E. P. Wigner, Z. Phys. **63**, 54 (1930); Z. Phys. **65**, 18 (1930). [See Appendix A of P. K. Kabir, "The CP Puzzle: Strange Decays of the Neutral Kaon", Academic Press (1968).]

[60] Y. Nir, SLAC-PUB-5874 [Lectures given at 20th Annual SLAC Summer Institute on Particle Physics (Stanford, CA, 1992)].

[61] I. Dunietz and J. L. Rosner, Phys. Rev. D **34**, 1404 (1986).

[62] Ya. I. Azimov, N. G. Uraltsev, and V. A. Khoze, Sov. J. Nucl. Phys. **45**, 878 (1987) [Yad. Fiz.

45, 1412 (1987)].

[63] I. I. Bigi and A. I. Sanda, Nucl. Phys. B **281**, 41 (1987).

[64] A. J. Barr and C. G. Lester, arXiv:1004.2732 [hep-ph].

[65] B. C. Allanach, J. P. Conlon and C. G. Lester, Phys. Rev. D **77**, 076006 (2008) [arXiv:0801.3666 [hep-ph]].