Lattice QCD for Flavor Physics

Elvira Gámiz
Introduction

# Flavour-violating and CP-violating processes allow us to test high energy scales.

* Unveiling New Physics effects
* Constraining NP models.
* Tests not limited by available energy by available precision.

# SM predictions for those observables depend on only a few parameters → can overconstrain the value of those parameters.
Introduction: Lattice QCD

\[
\text{Experiment} = (\text{known factors}) \times (V_{CKM}) \times (\text{matrix elements})
\]

Lattice QCD: Numerical evaluation of QCD path integral (rely only on first principles) using Monte Carlo methods.

Parametrize MEs in terms of decay constants, form factors, bag parameters, ...
**Introduction: Lattice QCD**

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Phenomenology (in particular *flavour physics*) needs precise lattice QCD calculations →

* Control and reliably estimate systematic errors.

* Not everything can be calculated precisely using lattice techniques, only some processes:

    with estable (or almost stable) hadrons, masses and amplitudes with no more than one initial (final) state hadron
Introduction: Lattice QCD

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** This includes quark masses and \( \alpha_s \), hadron spectrum, weak decays (leptonic, semileptonic, mixing)...

Introduction

... and most CKM matrix elements

\[ V_{CKM} = \begin{pmatrix}
|V_{ud}| & |V_{us}| & |V_{ub}| \\
\pi \rightarrow \ell \nu & K \rightarrow \ell \nu & B \rightarrow \tau \nu \\
K \rightarrow \pi \ell \nu & B \rightarrow \pi \tau \nu, B_s \rightarrow K \ell \nu & \Lambda_b \rightarrow p \ell \nu \\
|V_{cd}| & |V_{cs}| & |V_{cb}| \\
D \rightarrow \ell \nu & D_s \rightarrow \ell \nu & B_{(s)} \rightarrow D_{(s)}, D_{(s)}^* \ell \nu \\
D \rightarrow \pi \ell \nu & D \rightarrow K \ell \nu \\
|V_{td}| & |V_{ts}| & |V_{tb}| \\
\langle B_d^0 | \bar{B}_d^0 \rangle & \langle B_s^0 | \bar{B}_s^0 \rangle \\
B \rightarrow \pi \ell \ell & B \rightarrow K \ell \ell
\end{pmatrix} \]

\[(\rho, \eta) \quad \langle K^0 | \bar{K}^0 \rangle \]
Introduction: Lattice QCD

Development of new methods is allowing to increase the scope of LQCD calculations:

* Baryons
* Nonleptonic decays ($K \to \pi\pi\ldots$)
* Resonances
* Scattering
* Long-distance effects
* QED effects ...
Introduction: Lattice QCD

Combined chiral-continuum extrapolation

MILC $N_f = 2 + 1 + 1$

Many lattice collaborations doing now simulations with physical light-quark masses; PACS-CS, BMW, MILC, RBC/UKQCD, ETM...
Introduction: Lattice QCD

Combined chiral-continuum extrapolation

Many lattice collaborations doing now simulations with **physical light-quark masses**: PACS-CS, BMW, MILC, RBC/UKQCD, ETM...

# Next generation of gauge configurations: isospin-breaking ($N_f = 1 + 1 + 1 + 1$) and QED+QCD BMW, QCDSF, RBC, MILC ...
Leptonic Decays
Leptonic Kaon decays

# Decay constants come from simple matrix element

\[ \langle 0 | \bar{q}_1 \gamma_\mu \gamma_5 q_2 | P(p) \rangle = i f_P p_\mu \left( (m_{q_1} + m_{q_2}) \langle 0 | \bar{q}_1 \gamma_5 q_2 | P(p = 0) \rangle = f_P M_P^2 \right) \]

→ precise calculations on the lattice (even higher precision for ratios due to cancellation of stats. and systs.)
Leptonic Kaon decays: $K(\pi) \rightarrow \ell\nu$

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→ precise calculations on the lattice (even higher precision for ratios due to cancellation of stats. and syts. )

# Many $N_f = 2 + 1, 2 + 1 + 1$ calculations → good test of lattice QCD

Most recent (and precise) FNAL/MILC, 1712.09262

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1950(15)_{\text{stat}}^{+4}_{-17}f_{\pi^+},PDG[3]_{\text{EMscheme}} = 1.1950\left(\frac{+15}{-23}\right)0.15\%\text{ error}$$

(FLAG $N_f = 2 + 1 + 1$ average = $f_{K^+}/f_{\pi^+} = 1.193(3)$)
Leptonic Kaon decays: Extraction of $|V_{us}|$

Most recent (and precise) FNAL/MILC, 1712.09262

$$\frac{f_{K^+}}{f_{\pi^+}} = 1.1950 \pm 0.015$$

Using exp. data rates for $K_{l2}/\pi_{l2}$ and radiative correction factors from Rosner, Stone, Van de Water, 1509.02220

$$\frac{|V_{us}|}{|V_{ud}|} = 0.2310(4) f_{K^+}/f_{\pi} \times (2)_{e x p t} \times (2)_{E M}$$

$$\Gamma(K^+ (\pi^+) \rightarrow l^+ \nu_l (\gamma)) = \text{(known)} \left( 1 + \delta_{EM,K (\pi)}^l \right) |V_{us (ud)}|^2 f_{K^+ (\pi^+)}^2$$

($\delta_{EM}^l$ includes structure dependent EM corrections, currently estimated phenomenologically within ChPT Cirigliano et al 1107.6001)
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# Improvements underway, but eventually require QED and strong isospin-breaking on the lattice

* **RM123 approach:** expand lattice path-integral in powers of $\alpha_{em}, (m_d - m_u)/\Lambda_{QCD}$

First results **Giusti et al, 1711.06537** $\delta R_{K\pi}^{lat} = -1.22(16)\%$ agree with pheno estimates.
Leptonic Kaon decays: Extraction of $|V_{us}|$

With $\frac{|V_{us}|}{|V_{ud}|} = 0.2310(4) f_K / f_\pi (2)_{expt} (2)_{EM}$ and $|V_{ud}| = 0.97420(21)$ from superallowed $\beta$ decays Hardy & Towner 1807.01146 + Marciano & Sirlin PRL71(1993) radiative corrections:

$$\rightarrow |V_{us}| = 0.22453(48)(5)|V_{ud}|$$

Compatible with unitarity
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* Only $K \rightarrow \ell\nu$ data $\pm f_K$ from lattice $\rightarrow |V_{us}|$ with larger errors
Semileptonic Kaon decays $K \rightarrow \pi \ell \nu$

Direct determination of $|V_{us}|$ (no input needed for $|V_{ud}|$)

$$\Gamma_{K_{l3(\gamma)}} \propto |V_{us}|^2 |f_+^{K^0\pi^-}(0)|^2 \left(1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi}\right)$$

Test different current (vector vs axial-vector) as with leptonic decays.
Semileptonic Kaon decays $K \rightarrow \pi \ell \nu$

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Test different current (vector vs axial-vector) as with leptonic decays.

# New: $N_f = 2 + 1 + 1$ FNAL/MILC 1809.02827

* Includes FV corrections at one-loop, and strong isospin-breaking at two-loops ($f_+^{K^0\pi^-}$) 0.19% error

Grey bands: FLAG 2016 averages
Semileptonic Kaon decays $K \rightarrow \pi \ell \nu$

Direct determination of $|V_{us}|$ (no input needed for $|V_{ud}|$)

$$\Gamma_{K_{13(\gamma)}} \propto |V_{us}|^2 |f^{K^0\pi^-}_+ (0)|^2 \left( 1 + \delta_{EM}^{K\ell} + \delta_{SU(2)}^{K\pi} \right)$$

Test different current (vector vs axial-vector) as with leptonic decays.

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\begin{align*}
0.19\% \text{ error}
\end{align*}

Using exp. average from \textbf{M. Moulson} 1704.04104 $\Rightarrow$ $|V_{us}| = 0.22333(43) f_+(0)(42)_{\exp}$
First row CKM unitarity

Unitarity: Neglect $|V_{ub}|$, so $|V_{us}| = \sqrt{1 - |V_{ud}|^2}$

Inclusive hadronic $\tau$ decays: $D > 4$ condensates from the lattice Hudspith et al 1702.01767, replacing OPE expansion by lattice HVP functions and optimizing weight functions RBC/UKQCD Boyle et al 1803.07228 see Taku Izubuchi talk at CKM18
First row CKM unitarity

$|V_{us}|$ from $K_{l3}$: FNAL/MILC 1809.02827 + Moulson 1704.04104

$|V_{us}|/|V_{ud}|$ from $K_{l2}$: FNAL/MILC 1712.09262 + PDG16

$|V_{ud}| 0^+ \rightarrow 0^+$: superallowed $\beta$ decays:

Hardy & Towner + RC from Marciano & Sirlin

$|V_{cd}|_{\text{unitarity}}$: $V_{cd}$ from lattice FNAL/MILC 1712.09262 $f_D$ and PDG16 + CKM unit.
| $V_{us}$ | from $K_{l3}$: FNAL/MILC 1809.02827 + Moulson 1704.04104 

| $V_{us}$ | $|V_{ud}|$ from $K_{l2}$: FNAL/MILC 1712.09262 + PDG16 

$|V_{ud}|$ $0^+ \rightarrow 0^+$: superallowed $\beta$ decays: 
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$|V_{cd}|$ unitarity: $V_{cd}$ from lattice 
FNAL/MILC 1712.09262 $f_D$ and PDG16 + CKM unit. 

* $|V_{us}|$ from $K_{l3}$ and from superallowed $\beta$ decays 

$$\Delta_u \equiv |V_{ud}|_{\text{Marciano&Sirlin}}^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00104(27)V_{us}(41)V_{ud} \quad 2.1\sigma \text{ tension}$$
$$\Delta_u \equiv |V_{ud}|_{\text{Seng et al.}}^2 + |V_{us}|^2 + |V_{ub}|^2 - 1 = -0.00209(27)V_{us}(29)V_{ud} \sim 5\sigma \text{ tension}$$
First row CKM unitarity

\[ |V_{us}| \text{ from } K_{l3}: \text{FNAL/MILC 1809.02827} + \text{Moulson 1704.04104} \]
\[ |V_{us}| / |V_{ud}| \text{ from } K_{l2}: \text{FNAL/MILC 1712.09262} + \text{PDG16} \]
\[ |V_{ud}| 0^+ \rightarrow 0^+: \text{superallowed } \beta \text{ decays:} \]
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\[ |V_{cd}| \text{unitarity}: V_{cd} \text{ from lattice FNAL/MILC} 1712.09262 f_D \text{ and PDG16} + \text{CKM unit.} \]

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* \( |V_{us}| \) from \( K_{l3} \) and \( |V_{us}| / |V_{ud}| \) from \( K_{l2} \)

\[ \Delta_u = -0.0151(38)_{f_{\pi^+}(0)} (36)_{f_{K^\pm}} / f_{\pi^\pm} (36)_{\exp(27)_{\text{EM}}} 2.2\sigma \text{ tension} \]
**Heavy-Light mesons decay constants**

**Key for precision:** relativistic description of heavy quarks

Plots from **FNAL/MILC, 1712.09262**

- **$f_{D_s}/f_{D^+}$** : $0.14\%$
- **$f_{B_s}/f_{B^+}$** : $0.4\%$

0.27% – 0.18% error

0.7% – 0.57% error

Errors in ratios, $f_{D_s}/f_{D^+}$ : $0.14\%$ $f_{B_s}/f_{B^+}$ : $0.4\%$
Heavy-Light decay constants: CKM matrix

With **FNAL/MILC, 1712.09262** heavy-light decay constants

* Using **PDG16** experimental averages + known short- and long-distance EW corrections + uncertainty from unknown meson-structure-dependent EM corrections:

\[ |V_{cs}|_{SM,f_{Ds}} = 0.997(2)f_{Ds}(16)_{expt}(6)_{EM} \quad |V_{cd}| = 0.2144(5)f_{D}(49)_{expt}(13)_{EM} \]

EM error comes from unknown structure-dependent corrections and it is based on analogous corrections for pions and kaons: need a direct calculation.
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\[ |V_{ub}| = 4.07(3)_{f_{B^+}}^{(37)}_{expt} \cdot 10^{-3} \]

with the large uncertainty agrees with both inclusive and exclusive determinations.
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* Predictions for SM rare decays:

\[ 10^9 \cdot \bar{B}(B_s \rightarrow \mu^+ \mu^-)_{SM} = 3.64(4) f_{B_s} (8)_{CKM} (7)_{other} = 3.64(11) \]
\[ 10^{11} \cdot \bar{B}(B \rightarrow \mu^+ \mu^-)_{SM} = 1.00(1) f_{B^0} (2)_{CKM} (2)_{other} = 1.00(3) \]
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\[
10^{11} \cdot B(B \to \mu^+\mu^-)_{SM} = 1.00(1)f_{B^0}(2)_{CKM(2)}_{other} = 1.00(3)
\]

Given the current and projected experimental uncertainties on the $D$ and $B$ meson leptonic decay rates, better lattice-QCD calculations of the decay constants are not needed in the near future.
Second row CKM unitarity

With $|V_{cd,cs}|$ and $|V_{cb}|_{incl+excl} = 41.40(77) \cdot 10^{-3}$ from FNAL/MILC, 1712.09262

$$\Delta_c \equiv |V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 - 1 = 0.049(2)|V_{cd}|(32)|V_{cs}|(0)|V_{cb}|$$

compatible with three-generation CKM unitarity within $1.5\sigma$

* Precision limited by exp. error on leptonic $D^{+}_{(s)}$ decay widths: BES-III, Belle II
$B$ anomalies
Exclusive vs inclusive $|V_{ub}|$ and $|V_{cb}|$

Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ at the $\sim 3\sigma$ level.
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Long-standing tension between exclusive and inclusive determinations of the CKM matrix elements $|V_{cb}|$ and $|V_{ub}|$ at the $\sim 3\sigma$ level.

![Graph showing the relationship between $|V_{ub}|$ and $|V_{cb}|$](image)

**LQCD inputs**

* $B \to \pi \ell \nu$: $f_+(q^2)$ ($f_0(q^2)$)

* $\Lambda_b \to p \mu \nu/\Lambda_b \to \Lambda_c \mu \nu$:
  Six form factors for each channel

<table>
<thead>
<tr>
<th>$V_{ub}$ [$10^{-3}$]</th>
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$|V_{cb}|$ from exclusive $B$ decays ($w = v_B \cdot v_D$ velocity transfer to the leptonic pair)

\[
\frac{d\Gamma(B \to D^* \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2
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\frac{d\Gamma(B \to D \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{3/2} |\mathcal{G}(w)|^2
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Exclusive determination of $|V_{cb}|$

$|V_{cb}|$ extracted from exclusive $B$ decays ($w = v_B \cdot v_D$ is the velocity transfer to the leptonic pair)

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* How to parametrize the $\omega$ dependence?
Exclusive determination of $|V_{cb}|$

$|V_{cb}|$ extracted from exclusive $B$ decays ($w = v_B \cdot v_D$ is the velocity transfer to the leptonic pair)

$$\frac{d\Gamma(B \rightarrow D^* l\nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |F(w)|^2$$

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* How to parametrize the $\omega$ dependence?

Model-independent $z$-expansion: map $\omega$ onto a complex variable $z$ via the conformal transformation $z = (\sqrt{\omega + 1} - \sqrt{\omega_0 + 1})/(\sqrt{\omega + 1} + \sqrt{\omega_0 + 1})$

* Coefficients in the $z$-expansion are subject to unitarity bounds based on analyticity.
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BCL (Bourrely-Caprini-Lellouch), 0807.2722


CLN (Caprini-Lellouch-Neubert), hep-ph/9712417: + model-dependent NLO HQET constraints to reduce the error:

heavy-quark corrections (neglecting $(\Lambda/m_c)^2$) underestimated?

Bigi\&Gambino, 1703.06124 Grinstein\&Kobach, 1703.08170
Form factors for $B \rightarrow D \ell \nu$ with $\ell = e, \mu, \tau$

Exclusive $B \rightarrow D$ decays without disregarding lepton masses (SM)

$$ \frac{d\Gamma(B \rightarrow D\ell\nu)}{dw} = (\text{known}) \cdot |V_{cb}|^2 |f_+^2(q^2)|^2 + (\text{known}) \cdot |V_{cb}|^2 m_\ell^2 f_0^2(q^2)|^2 $$

Combined BGL fit to experimental and lattice data on different $q^2$ regions

Bigi&Gambino, 1606.08030

HPQCD, 1505.03925 FNAL/MILC, 1503.07237

$|V_{cb}| = 40.49(97) \cdot 10^{-3}$

* LQCD form factor error ($\sim 1.2\%$) smaller than experiment.

In acceptable agreement with either $B \rightarrow D^*$ exclusive and inclusive determinations.

Nice agreement with BCL FLAG global fit ($|V_{cb}| = 40.1(1.0) \cdot 10^{-3}$)
Form factors for $B \rightarrow D \ell \nu$ with $\ell = e, \mu, \tau$

Exclusive $B \rightarrow D$ decays without disregarding lepton masses (SM)

$$\frac{d\Gamma(B \rightarrow D\ell\nu)}{d\omega} = (\text{known}) \cdot |V_{cb}|^2 |f_+^2(q^2)|^2 + (\text{known}) \cdot |V_{cb}|^2 |m_\ell f_0^2(q^2)|^2$$

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Bigi & Gambino, 1606.08030

HPQCD, 1505.03925 FNAL/MILC, 1503.07237

$$|V_{cb}| = 40.49(97) \cdot 10^{-3}$$

* LQCD form factor error ($\sim 1.2\%$) smaller than experiment.

In acceptable agreement with either $B \rightarrow D^*$ exclusive and inclusive determinations.

* HFLAV quotes $|V_{cb}| = 39.18(1.04) \cdot 10^{-3}$ when using only the data at zero recoil from a CLN fit.
$B \to D\ell\nu$: Lepton Flavor Universality tests

Lattice $f_{+}^{B \to D}(q^2)$ and $f_{0}^{B \to D}(q^2)$ form factors can be used to calculate:

$$R(D) \equiv \frac{B(B \to D\tau\nu_{\tau})}{B(B \to D\ell\nu)} \text{FLAG2016,only lattice} = 0.300(8)$$
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Theory results $\sim 2.3\sigma$ lower than experimental average
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\]

*Global fit experiment+lattice with BGL parametrization*

\[
R(D) = 0.299(3), \; 2.3\sigma \quad \text{Bigi & Gambino PRD94,094008(2016)}
\]

[similar results from Bernlochner, Ligeti, Papucci, Robinson 1703.05330, Jaiswal, Nandi, Patra 1707.09977]

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* On-going improvements on LQCD form factors:

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[N_f = 2 + 1 C. Monahan and O. Witzel talks at CKM18]

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[See $N_{f} = 2 + 1 + 1$ C. Monahan and O. Witzel talks at CKM18]

* Soft-photon corrections to $R(D)$, could amplify $R(D^{+0})$ by $\leq 5.5\%(3.6\%)$ Boer, Kitahara, Nisandzic 1803.05881

** Understand how QED effects are handled in experiment (PHOTOS)

Theory results $\sim 2.3\sigma$ lower than experimental average

See $N_{f} = 2 + 1 + 1$ T. Kitahara talk at CKM18
$B \to D^* \ell \nu$ at zero recoil

\[ \frac{d\Gamma(B \to D^* \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |F(w)|^2 \]

Until now, lattice data for $F(\omega)$ only available at zero recoil $\omega = 1$ ($q_{max}^2$)

* One form factor needed $F(1) = h_{A_1}(1)$, extracted from double ratio

\[ R_{A_1}(1) = \frac{\langle D^* | A_1 | \bar{B} \rangle \langle \bar{B} | A_1 | D^* \rangle}{\langle D^* | V_0 | D^* \rangle \langle \bar{B} | V_0 | \bar{B} \rangle} = |h_{A_1}(1)|^2 \]


**FNAL/MILC 1403.0635, HPQCD 1711.11013**
$B \rightarrow D^* \ell \nu \text{ at zero recoil}$

$$\frac{d\Gamma(B \rightarrow D^* \ell \nu)}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} |\mathcal{F}(w)|^2$$

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**FNAL/MILC 1403.0635, HPQCD 1711.11013**

* Suppression factor at zero recoil limits experimental measurements:
Experimental data extrapolated to $\omega = 1$ using CLN parametrization

$$\rightarrow \tilde{\eta}_{EW} |V_{cb}| \mathcal{F}(\omega = 1)$$
$B \rightarrow D^* \ell \nu$ at zero recoil

Experimental data extrapolated to $\omega = 1$ using CLN parametrization.

$$|V_{cb}|_{FLAG2016}^{B\rightarrow D^*} = 39.27(56)_{\text{lat}}(49)_{\text{exp.}} \cdot 10^{-3} \quad 2.7\sigma \text{ from inclus.}$$

$$|V_{cb}|_{incl.} = 42.2(8) \cdot 10^{-3} \quad \text{HFLAV2016}$$
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# For the first time, Belle 1702.01521 provided unfolded fully-differential decay rate and associated covariance matrix: strong dependence on param.

Bigi, Gambino & Schacht 1703.06124,1707.09509 Grinstein & Kobach 1703.08170
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Belle 1809.03290 update $B^0 \rightarrow D^*\ell\nu$ untagged, extrapolate to get $\eta_{EW}|V_{cb}|F(1)$

Belle+LQCD: $|V_{cb}|_{\text{CLN}} = 38.4(0.2)(0.6)(0.6) \cdot 10^{-3} = 38.4(0.9) \cdot 10^{-3}$

Belle+LQCD: $|V_{cb}|_{\text{BGL}} = 42.5(0.3)(0.7)(0.6) \cdot 10^{-3} = 42.5(1.0) \cdot 10^{-3}$

(using FNAL/MILC 2014 result for $F(1)$ and $\eta_{EW} = 1.0066$ Sirlin NPB196(1982))

* Exp. data does not discriminate between parametrizations

* Large difference not necessarily true for other exp. data.
$B \rightarrow D^* \ell \nu$: Lepton Flavor Universality tests

No lattice inputs available to calculate $R(D^*) \equiv \frac{\mathcal{B}(B \rightarrow D^* \tau \nu \tau)}{\mathcal{B}(B \rightarrow D^* \ell \nu)}$

(Need 4 form-factors at all $\omega$)

* The SM calculation used until recently, Fajfer et al 1203.2654, based on exp. $B \rightarrow D^*$ measurements which use CLN parametrization.

$R(D^*)_{\text{SM}} = 0.252(3) \rightarrow 3.6\sigma$ discrep. with exper.

$\sim 3\sigma$ SM-experiment disagreement
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* Bigi, Gambino & Schacht JHEP 1711 (2017) 061 use BGL param. instead with Belle 2017 data

$$R(D^*) = 0.260(8) (3.6 \rightarrow 2.7\sigma)$$

Similar PRD 95 (2017) 115008, JHEP 1712 (2017) 060

Systematic errors larger due to accounting for unknown NNLO corrections.

$\sim 3\sigma$ SM-experiment disagreement
\[ B \rightarrow D^* \ell \nu: \text{ Lepton Flavor Universality tests} \]

Needs reliable LQCD form factors at non-zero recoil

A LQCD calculation of the form factors would:

* significantly reduce uncertainties in \( R(D^*) \) (especially \( P_1(1) \)) and \( |V_{cb}| \)

* give information about the shape of the form factors \( \rightarrow \) stabilize fits

**CLN vs BGL** parametrization

Some preliminary lattice data suggest **CLN** better description than **BGL**?

**S. Hashimoto**, plenary talk at Lattice2018, **T. Kaneko** poster at Lattice2018
$B \rightarrow D^*\ell\nu$ beyond zero recoil

**Preliminary:** $N_f = 2 + 1$ FNAL/MILC, A. Vaquero talk at CKM18 (blinded analysis)

with **BGL** parametrization, compare to **Belle 1702.01521**

Red **Lattice** points on the right are synthetic data.

Value at zero recoil won’t change much from **FNAL/MILC 1403.0635**, main new information from the slope.

Final results by the end of the year! ($R(D^*)$ may come a bit later)
Inclusive vs Exclusive determinations of $|V_{cb(ub)}|$

Parameterization dependence?

* LQCD form factors at non-zero recoil (in progress): FNAL/MILC, A. Vaquero talk at CKM18, JLQCD, T. Kaneko Lattice18

* $B_{(s)} \rightarrow D_{(s)}^{(*)}$ on-going calculations:

LANL/SNU 1711.01777, 1711.01786, RBC/UKQCD

O. Witzel at CKM18, HPQCD C. Monahan at CKM18
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O. Witzel at CKM18, **HPQCD** C. Monahan at CKM18

**Tension persists**

* Improvements from LQCD in progress: **HPQCD, FNAL/MILC, RBC/UKQCD, JLQCD, ETMC**
[see C. Monahan talk at CKM18]

* Alternative calculations with LQCD results available $B_s \rightarrow K\ell\nu, \Lambda_b \rightarrow p(\Lambda_c), B \rightarrow \tau\nu ...$

** Better leptonic exp. data Belle II: 3-5%**
Inclusive vs Exclusive determinations of \( |V_{cb(ub)}| \)

**Parametrization dependence?**

**Tension persists**

New ideas to calculate inclusive semileptonic decays on the lattice:

**S. Hashimoto Lattice18,**
**More $b \to c \ell \nu$ LFU tests**

Can help to shed light on existing tensions  

$$R(X \to Y) = \frac{\mathcal{B}(X \to Y \tau \nu)}{\mathcal{B}(X \to Y \mu \nu)}$$

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Theoretical (Lattice) Prediction</th>
<th>Experimental meas.</th>
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</thead>
<tbody>
<tr>
<td>$R(B_s \to D_s)$</td>
<td>$0.314(6)$, HPQCD 1703.09728</td>
<td>LHCb</td>
</tr>
<tr>
<td></td>
<td>[FNAL/MILC 1202.6346, ETMC 1310.5238]</td>
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</tr>
<tr>
<td>$f_{+}^{B_s \to K(0)} / f_{+}^{B_s \to D_s(0)}$ for $</td>
<td>V_{ub}</td>
<td>/</td>
</tr>
<tr>
<td></td>
<td>in progress: FNAL/MILC, RBC/UKQCD...</td>
<td></td>
</tr>
<tr>
<td>$R(B_s \to D_s^*)$</td>
<td>in progress (longer term): similar (easier) to $R(D^*)$</td>
<td>LHCb</td>
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<tr>
<td>$R(B_c \to J/\psi)$</td>
<td>SM (SR &amp; models): 0.25-0.28*</td>
<td>$0.71 \pm 0.25$</td>
</tr>
<tr>
<td></td>
<td>in progress: HPQCD, A. Lytle</td>
<td>LHCb 1711.05623</td>
</tr>
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<td>“Instant workshop on B-mesons anomalies”</td>
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<tr>
<td>$R(\Lambda_b \to \Lambda_c)$</td>
<td>$0.333 \pm 0.010$ **</td>
<td>LHCb</td>
</tr>
<tr>
<td>$R(\Lambda_b \to \Lambda_c^*)$</td>
<td>in progress: S. Meinel, G. Rendon</td>
<td>LHCb</td>
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</tbody>
</table>


Neutral-current $b$ decays

Flavor-changing neutral currents $b \rightarrow q$ transitions are potentially sensitive to NP effects $B \rightarrow K^* \gamma$, $B \rightarrow K^{(*)} \ell^+ \ell^-$, $B \rightarrow \pi \ell^+ \ell^-$
Neutral-current $b$ decays: LFU tests

Flavor-changing neutral currents $b \to q$ transitions are potentially sensitive to NP effects $B \to K^*\gamma$, $B \to K(\ast)\ell^+\ell^-$, $B \to \pi\ell^+\ell^-$

2.6σ with SM for $q^2 \in [1.1 - 6.0]$

* Update of $R_K$ by LHCb in review G. Pomery talk at CKM18

$R_H(q_{min}^2, q_{max}^2) = \frac{\int_{q_{min}^2}^{q_{max}^2} dq^2 dB(B \to H\mu^+\mu^-)}{\int_{q_{min}^2}^{q_{max}^2} dq^2 dB(B \to He^+e^-)}$

LHCb measurement of $R_{K^*}$, R. Ajai et al 1705.05802: 2 − 2.5σ tension with SM.

SM predictions for these ratios (and other observables such as $P_5'$) pretty insensitive to form factors and non-factor contributions.
Neutral-current $b$ decays

$\Lambda_b \rightarrow \Lambda \mu^+\mu^-$ LHCb

FNAL/MILC, 1509.06235

1503.07138 vs SM (LQCD) Detmold & Meinel 1602.01399

Horgan, Liu, Meinel and Wingate, 1310.3722, 1310.3887

SM predictions (with LQCD and other form factors) systematically higher than experiment.
Neutral-current $b$ decays

* Lattice on-going (and planned) calculations should reduce form factors errors considerably: For $B \to K\ell\ell$ FNAL/MILC, Z. Gelzer talk at Lattice18

* Theoretical framework for semileptonic $B$ decays to unstable vector meson final states exists Briceño et al 1406.5965, Agadjanov et al 1605.03386, Hansen, Meyer, Robaina 1704.08993

** Pilot studies of form factors for $B_s \to K^*(\to K\pi)\ell\nu$, $B \to K^*(\to K\pi)\ell\ell,...$ underway Leskovec, Meinel

** On-going work for $B_s \to \phi$
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* Long-distance effects (and, in particular, charm loops) under control?

** New ideas to study those effects on the lattice S. Hashimoto, K. Nakayama talks at Lattice18.

similar formulation to LD effects in $K \to \pi\ell\ell$ A. Jüttner talk at CKM18

[exploratory study results in RBC/UKQCD 1608.07585,1507.03094]
Summary and outlook

* Subpercent error achieved for quantities such as decay constants and quark masses.

** Relativistic description of $c$ and $b$ quarks allow an important reduction of errors

* First complete calculations of weak $\Lambda_{b,c}$ decay form factors (some tension with experiment)
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** Strong isospin breaking: eventually, $N_f = 1 + 1 + 1 + 1$

** Structure-dependent QED corrections

** Lots of effort to include QED in lattice simulations A. Patella 1702.03857 (kaons, $(g - 2)_\mu$...), Giusti et al 1711.06537, T. Blum et al. 1801.07224
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* Increasing number of quantities: $(g-2)_\mu$, $K \rightarrow \pi\pi$, nucleon matrix elements, resonances ... will become high precision. New quantities will be studied (inclusive decays, $K \rightarrow \pi\ell\ell(\nu\bar{\nu})$, ...)
Summary and outlook

Some tensions need theory (lattice) and experimental improvements to clarify:

* $|V_{cb}|_{\text{incl.}} - |V_{cb}|_{\text{excl.}}$: $B \rightarrow D^* \ell \nu$ at non-zero recoil available soon.

** Also relevant for $R(D^*)$.

* $R(D), R_K, R_{K^*}$...

** Form factors for $B(s) \rightarrow K^* \ell^+ \ell^-$, $B \rightarrow K \ell^+ \ell^-$, $B(s) \rightarrow \pi \ell^+ \ell^-$.

** LQCD predictions for related quantities: $R_\pi$, $B(B \rightarrow K(\pi)\nu \bar{\nu})$, $B_s \rightarrow K \ell \nu$,...

* First-row CKM unitarity.

** Leptonic vs semileptonic extraction of $|V_{us}|$

* Inclusive vs exclusive $|V_{ub}|$

* Consistency in tree-level and loop extractions of $|V_{td,t\bar{s}}|$

...