TMD gluon distributions for multiparton processes

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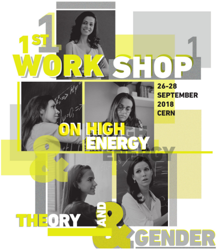


Based on arXiv:1809.08968 (MB, P. Kotko and K. Kutak)

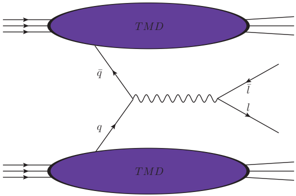
Outline

- Introduction
- TMD gluon distributions
- Color-flow Feynman rules
- Example
- General method
- Results
- Summary

Introduction

- Collinear factorization is not sufficient to describe observables sensitive to the transverse component of incoming parton's momentum

Example: Drell-Yan



- Other approaches: Parton showers, TMD factorization, k_T-factorization, Color Glass Condensate
- TMD factorization is useful in dilute-dense collisions, where low-x (longitudinal momentum fraction) gluons dominate

$$d\sigma = \sum_i \mathcal{F}^{(i)}_{a/h_1}(x_1, k_T) \otimes \mathcal{F}^{(i)}_{b/h_2}(x_2, k_T) \otimes H^{(i)}_{ab\to cd}$$

TMD gluon distributions

- TMDs describe the structure of hadrons in 3D momentum space
- Not universal, their operator definition depends on the process under consideration
- Fourier Transform of hadronic ME of bilocal field operators

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T \cdot \vec{\xi}_T} \langle P| \mathrm{Tr} \left\{ \hat{F}^{i+}(0)\, \mathcal{U}_{C_1} \hat{F}^{i+}(\xi)\, \mathcal{U}_{C_2} \right\} |P\rangle ,$$

- Wilson Lines must be inserted to ensure gauge invariance

$$\mathcal{U}_C = \mathcal{P} \exp \left\{ -ig \int_C dz_\mu \hat{A}^\mu(z) \right\} .$$

- Shape of the links is determined by the hard process   Mulders et al. 2004

TMD gluon distributions

$$\mathcal{F}(x, k_T) = 2 \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle P| \,\mathrm{Tr}\left\{\hat{F}^{i+}(0)\,\mathcal{U}_{C_1}\hat{F}^{i+}(\xi)\,\mathcal{U}_{C_2}\right\}|P\rangle \,,$$

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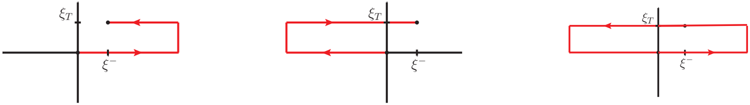
- Shape of the links is determined by the hard process

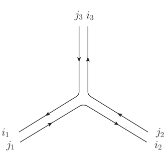


$\mathcal{U}^{[+]}$ $\qquad$ $\mathcal{U}^{[-]}$ $\qquad$ $\mathcal{U}^{[\Box]} = \mathcal{U}^{[-]\dagger}\mathcal{U}^{[+]}$

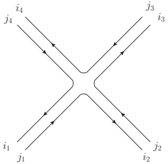
- For arbitrary process the procedure is known $\quad$ Bomhof, Mulders, Pijlman 2006

Color-flow Feynman rules

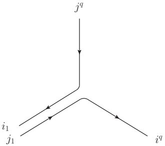
- Vertices (color part)



$\sim\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\delta^{i_1}_{j_3}$

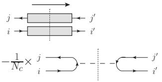


$\sim\delta^{i_2}_{j_1}\delta^{i_3}_{j_2}\delta^{i_4}_{j_3}\delta^{i_1}_{j_4}$



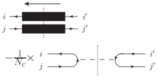
$\sim\delta^{i^q}_{j_1}\delta^{i_1}_{j^q}$

- Wilson lines



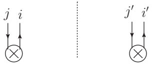
$\left(\mathcal{U}^{[+]}\right)_{i'i}\left(\mathcal{U}^{[+]\dagger}\right)^{jj'}-\frac{1}{N_c}\delta^j_i\delta^{j'}_{i'}$

Outgoing gluon



$\left(\mathcal{U}^{[-]\dagger}\right)^{ii'}\left(\mathcal{U}^{[-]}\right)_{j'j}-\frac{1}{N_c}\delta^i_j\delta^{i'}_{j'}$

Incoming gluon

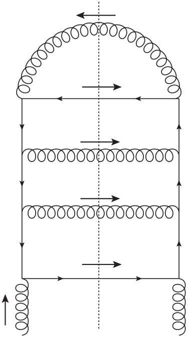


$\left(\hat{F}^{+i}(\xi)\right)^j_i\left(\hat{F}^{+i}(0)\right)^{j'}_{i'}$

Field strength operators

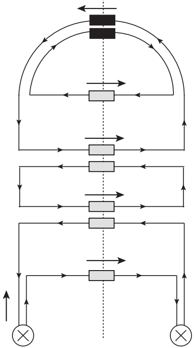
$gg \to q\bar{q}gg$

Leading N_c diagram



$\rightsquigarrow$

Leading N_c color-flow diagram



$+ \ \ldots$

$$N_c \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger} F(0)\,\mathcal{U}^{[+]}\right\} \mathrm{Tr}\mathcal{U}^{[\Box]}\, \mathrm{Tr}\mathcal{U}^{[\Box]\dagger}$$

This expression enters the TMD definition

General method for multi-partons

- Color decomposition

$$\mathcal{M}_{j_1\dots j_n}^{i_1\dots i_n}\left(k_1,\dots,k_n\right) = 2^{-n/2}\sum_{\pi\in S_{n-1}}\delta_{j_{\pi(2)}}^{i_1}\delta_{j_{\pi(3)}}^{i_{\pi(2)}}\delta_{j_{\pi(4)}}^{i_{\pi(3)}}\dots\delta_{j_1}^{i_{\pi(n)}}\mathcal{A}\left(1,\pi\left(2\right),\dots,\pi\left(n\right)\right),$$

$$|\mathcal{M}|^2 = \vec{\mathcal{A}}^{\dagger}\,\mathbf{C}\,\vec{\mathcal{A}}$$

$$\vec{\mathcal{A}} = \begin{pmatrix} \mathcal{A}(1,2,3,\dots,n-1,n) \\ \vdots \\ \mathcal{A}(1,n,n-1,\dots,3,2) \end{pmatrix}$$

Basis structures

$$\mathcal{F}_{gg}^{(1)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]}{N_c} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \frac{1}{N_c} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\right] \mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(4)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(5)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\Box]}\mathcal{U}^{[+]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(6)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c} \frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]\dagger}\right]}{N_c} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle,$$

$$\mathcal{F}_{gg}^{(7)}(x,k_T) = 2\int \frac{d\xi^- d^2\xi_T}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\Box]}\right]}{N_c} \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\Box]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle.$$

Gluon TMDs for 4 gluons

$$d\sigma \sim \vec{\mathcal{A}}^{\dagger}\, \mathbf{\Phi}_{gg\to gg}\, \vec{\mathcal{A}}$$

$$\mathbf{\Phi}_{gg\to gg} = \begin{pmatrix} \Phi_1 & \Phi_2 \\ \Phi_2 & \Phi_1 \end{pmatrix},$$

$$\Phi_1 = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}_{\rm gg}^{(1)} - 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} + N_c^2\mathcal{F}_{\rm gg}^{(6)}\right),$$

$$\Phi_2 = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}_{\rm gg}^{(2)} - 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} + N_c^2\mathcal{F}_{\rm gg}^{(6)}\right).$$

Gluon TMDs for 5 gluons

$$\mathbf{\Phi}_{gg\to ggg} = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4^* \\ \Phi_2 & \Phi_1 & \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_3 \\ \Phi_2 & \Phi_3 & \Phi_1 & \Phi_2 & \Phi_4^* & \Phi_3 \\ \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_1 & \Phi_3 & \Phi_2 \\ \Phi_3 & \Phi_2 & \Phi_4^* & \Phi_3 & \Phi_1 & \Phi_2 \\ \Phi_4^* & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}$$

$$\Phi_1 = \frac{1}{4N_c^2}\left((N_c^2+2)\mathcal{F}_{\rm gg}^{(1)} - 4\mathcal{F}_{\rm gg}^{(2)} - 4\mathcal{F}_{\rm gg}^{(3)} + 3N_c^2\mathcal{F}_{\rm gg}^{(6)} + 2\mathcal{F}_{\rm gg}^{(7)}\right),$$

$$\Phi_2 = -\frac{1}{2N_c^2}\left(-2\mathcal{F}_{\rm gg}^{(1)} + 4\mathcal{F}_{\rm gg}^{(2)} + 2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)} - 2N_c^2\mathcal{F}_{\rm gg}^{(6)} - 2\mathcal{F}_{\rm gg}^{(7)}\right)$$

$$\vdots$$

Gluon TMDs for 6 gluons

$$\mathbf{\Phi}_{gg\to gggg} = \begin{pmatrix} T_1 & T_2 & T_3 & T_4 \\ T_2 & T_1 & T_5 & T_6 \\ T_3^\intercal & T_5 & T_1 & T_7 \\ T_4^\intercal & T_6^\intercal & T_7 & T_1 \end{pmatrix},$$

where for example

$$T_1 = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4^* \\ \Phi_2 & \Phi_1 & \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_3 \\ \Phi_2 & \Phi_3 & \Phi_1 & \Phi_2 & \Phi_4^* & \Phi_3 \\ \Phi_3 & \Phi_4^* & \Phi_2 & \Phi_1 & \Phi_3 & \Phi_2 \\ \Phi_3 & \Phi_2 & \Phi_4^* & \Phi_3 & \Phi_1 & \Phi_2 \\ \Phi_4^* & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}, \quad T_2 = \begin{pmatrix} \Phi_2 & \Phi_5 & \Phi_3 & \Phi_6 & \Phi_7 & \Phi_8^* \\ \Phi_5 & \Phi_2 & \Phi_7 & \Phi_8^* & \Phi_3 & \Phi_6 \\ \Phi_3 & \Phi_7 & \Phi_4^* & \Phi_8^* & \Phi_9 & \Phi_{10} \\ \Phi_6 & \Phi_8^* & \Phi_8^* & \Phi_{10} & \Phi_{10} & \Phi_{11} \\ \Phi_7 & \Phi_3 & \Phi_9 & \Phi_{10} & \Phi_4^* & \Phi_8^* \\ \Phi_8^* & \Phi_6 & \Phi_{10} & \Phi_{11} & \Phi_8^* & \Phi_{10} \end{pmatrix},$$

with

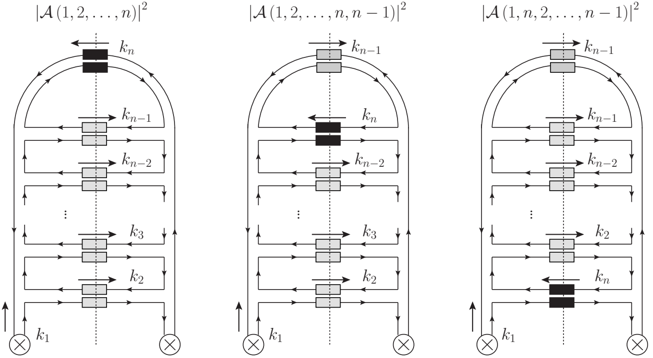
$$\Phi_1 = \frac{2N_c^2(\mathcal{F}_{\rm gg}^{(1)} - 4\mathcal{F}_{\rm gg}^{(3)} + 2\mathcal{F}_{\rm gg}^{(6)} + \mathcal{F}_{\rm gg}^{(7)}) + N_c^4(\mathcal{F}_{\rm gg}^{(1)} + 7\mathcal{F}_{\rm gg}^{(6)}) + 4(-2\mathcal{F}_{\rm gg}^{(3)} + \mathcal{F}_{\rm gg}^{(4)} + \mathcal{F}_{\rm gg}^{(5)})}{8N_c^4}$$

$$\vdots$$

Summary

- We formulated color-flow Feynman rules for gauge links
- We calculated TMD gluon distributions for processes with 5 and 6 partons $\rightarrow$ sufficient to calculate 3 an 4 jet production
- Color decompositions were used, which is the most efficient method
- We found new basic TMD gluon distributions
- We analysed the large N_c limit for multigluon processes

Large N_c analysis for arbitrary number of gluons

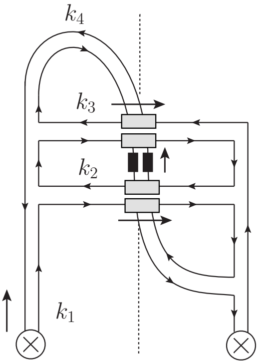


1) $\frac{N_c^{n-3}}{N_c^{n-2}}\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[-]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\mathrm{Tr}\mathcal{U}^{[\Box]\dagger} \rightsquigarrow \mathcal{F}_{gg}^{(1)}$

2) $\frac{N_c^{n-4}}{N_c^{n-2}}\mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[+]\dagger}F(0)\,\mathcal{U}^{[+]}\right\}\mathrm{Tr}\mathcal{U}^{[\Box]\dagger}\mathrm{Tr}\mathcal{U}^{[\Box]} \rightsquigarrow \mathcal{F}_{gg}^{(6)}$

3) $\mathcal{F}_{gg}^{(1)}$

Large N_c analysis for arbitrary number of gluons



$$N_c \mathrm{Tr}\left\{F(\xi)\,\mathcal{U}^{[\Box]}\right\} \mathrm{Tr}\left\{F(0)\,\mathcal{U}^{[\Box]\dagger}\right\} = N_c^2 \mathcal{F}_{gg}^{(2)} .$$