

Asymptotic expansions for highly boosted Higgs production through the loop-tree duality

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27th September 2018

1st Workshop on High Energy Theory and Gender



highly-boosted Higgs boson production

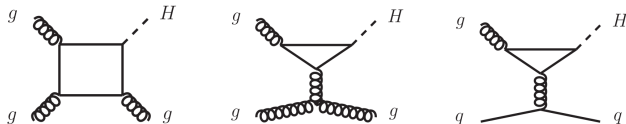
- effective **point-like ggH -coupling** not ruled out

[Grojean et al. '14]

$$\frac{m_t}{v} \bar{t}tH \rightarrow -\kappa_g \frac{\alpha_s}{12\pi v} G_{\mu\nu}^a G^{\mu\nu,a} H + \kappa_t \frac{m_t}{v} \bar{t}tH$$

- need to **consider Higgs + jet production at p_T sufficiently high for resolving the top loop** in order to disentangle possible BSM effects

LO result known for decades



[Ellis et al. '87]

[Baur, Glover '89]

NLO **▶** fix Higgs-top mass ratio, numerical integration

[Jones, Kerner, Luisoni '18]

▶ integration-by-parts identities + expansion in $\frac{m_H^2}{4m_t^2}$, $\frac{m_t^2}{s}$

[Lindert et al. '18]

goal: obtain independent NLO result using new regularization technique:

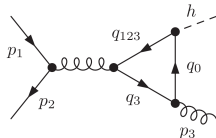
loop-tree duality (LTD)

[Catani et al. '08]

$q\bar{q} \rightarrow Hg$ amplitude

- **LO amplitude:** $\mathcal{M}_{\text{LO}} \sim$

$$\varepsilon_{\mu}^*(p_3) \bar{v}(p_2) \gamma_{\nu} u(p_1) F_{12} \left(g^{\mu\nu} - \frac{p_{12}^{\mu} p_3^{\nu}}{p_3 \cdot p_{12}} \right)$$



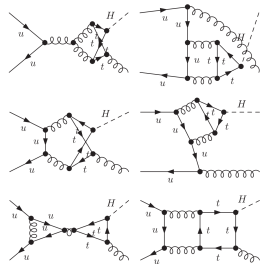
- straight-forward with dim. reg.: solutions for master integrals available

$$F_{12} = \int_{\ell} \frac{N(\ell \cdot p_3, \ell \cdot p_{12}, \ell^2, s, m_H^2)}{[\ell^2 - m_t^2 + i0][(\ell + p_3)^2 - m_t^2 + i0][(\ell + p_{123})^2 - m_t^2 + i0]}$$

- while at LO only one kinematic variable s ,
NLO amplitude depends on: s, t

$$\mathcal{M}_{\text{NLO}} \sim F_{12}^{(1)} \left(g^{\mu\nu} - \frac{p_{12}^{\mu} p_3^{\nu}}{p_3 \cdot p_{12}} \right) + \bar{F}_{12}^{(1)} \left(g^{\mu\nu} - \frac{\bar{p}_{12}^{\mu} \bar{p}_3^{\nu}}{p_3 \cdot \bar{p}_{12}} \right)$$

\Rightarrow four scales + two loops: **simplification through asymptotic expansion** necessary



Asymptotic expansions in the high- p_T limit

- high- p_T distribution measurement by CMS
- express p_T and pseudorapidity η through Mandelstam variables:

$$p_T = \frac{u t}{s} \quad \tanh \eta = \frac{t - u}{s - m_H^2}$$

- hierarchy used in literature:

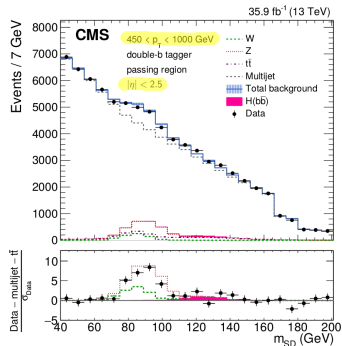
$$m_H^2 \ll m_t^2 \ll |s| \sim |t| \sim |u|$$

[Lindert et al. '18]

- a simple expansion in $s, t, u \propto p_T^2$ might not suffice at NLO

- first: **study expansions for the LO amplitude**

- ▶ **Expansion by Regions.** How to identify needed regions? [Beneke, Smirnov '98]
- ▶ general problem for expanding: integrand depends on four-momenta
- ⇒ will see: obtain integrands depending only on **euclidean loop three-momenta** when using the **loop-tree duality**



[CMS '17]

Loop-tree duality theorem (LTD)

Cauchy residue theorem: $\int_{\ell_0} \rightarrow \sum$ residues

LTD theorem

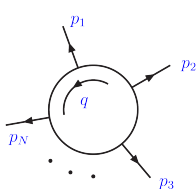
[Catani et al. '08]

$$\int_{\ell} N(\ell) \prod_i G_F(q_i) = - \sum_i \int_{\ell} N(\ell) \tilde{\delta}(q_i) \prod_{j \neq i} G_D(q_i; q_j),$$

$$\tilde{\delta}(q_i) = 2\pi i \theta(q_{i,0}) \delta(q_i^2 - m_t^2)$$

dual propagator:

$$G_D(q_i; q_j) = \frac{1}{q_j^2 - m_j^2 - i0\eta \cdot (q_j - q_i)}$$



$$= - \sum_{i=1}^N \text{Diagram with } \tilde{\delta}(q) \text{ and } \frac{1}{(q+p_i)^2 - i0\eta p_i}$$

The diagram on the right shows a circle with a dashed vertical line through its center labeled $\tilde{\delta}(q)$. A momentum q is indicated by a dashed arrow pointing upwards. External momenta p_{i-1} , p_i , and p_{i+1} are shown as arrows pointing outwards from the circle. The denominator $\frac{1}{(q+p_i)^2 - i0\eta p_i}$ is written to the right of the diagram.

- local cancellation of soft/collinear divergences (FDU) \Rightarrow calc. in $d = 4$
- successful applications to:
 - ▶ physical cross-section for $\gamma^* \rightarrow q\bar{q}(g)$
 - ▶ extended to massive particles
 - ▶ amplitude for $gg \rightarrow H$ and $H \rightarrow \gamma\gamma$

[Sborlini et al. '16]

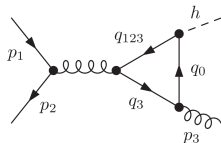
[Sborlini, Driencourt-Mangin, Rodrigo '16]

[Driencourt-Mangin, Rodrigo, Sborlini '17]

application of LTD on $q\bar{q} \rightarrow Hg$

- test applicability: need to extend usage of LTD to further processes
- asymptotic expansions (needed in the NLO calculation) expected to be easier in a **phase-space integral**

$$F_{12} \stackrel{LTD}{=} - \int_{\ell} \left[\frac{\tilde{\delta}(q_0) N(q_0)}{(2p_3 \cdot q_0 - i0)(2p_{123} \cdot q_0 + m_H^2 + i0)} + \frac{\tilde{\delta}(q_3) N(q_3 - p_3)}{(-2p_3 \cdot q_3 + i0)(2p_{12} \cdot q_3 + s + i0)} + \frac{\tilde{\delta}(q_{123}) N(-p_{123} + q_{123})}{(-2p_{12} \cdot q_{123} + s - i0)(-2p_{123} \cdot q_{123} + m_H^2 - i0)} \right]$$



note: integrand now only depends on **loop three-momenta**

expand dual propagators:

- $\frac{\tilde{\delta}(\ell)}{\pm m_H^2 - 2h \cdot \ell} = -\frac{\tilde{\delta}(\ell)}{2h \cdot \ell} \sum_{n=0}^{\infty} (-1)^n \left(\mp \frac{m_H^2}{2h \cdot \ell} \right)^n$
- in cmf: $\tilde{\delta}(\ell) p_{12} \cdot \ell = -\tilde{\delta}(\ell) \sqrt{s(m_t^2 + \ell^2)}$

⇒ need to separate the integral into two parts with distinct expansions.

However: **complexity reduced**

Conclusion

- large- p_T Higgs production necessary to exclude point-like ggH coupling
 - NLO calculation needs properly defined asymptotic expansion of the integrand
 - loop-tree duality allows to rewrite a loop integral in terms of a sum of phase-space integrals over the cut amplitude
- ⇒ the resulting expression can be expanded more straight-forwardly

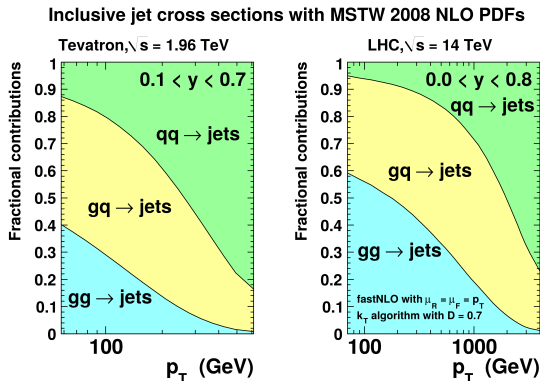
Outlook:

- reproduce LO result for $\bar{q}q \rightarrow Hg$ both using
 - ▶ dimensional regularization + expansion by regions and
 - ▶ LTD + direct expansions
- repeat for the other two LO contributions to large- p_T Higgs production: $gg \rightarrow Hg$ and $qg \rightarrow qH$
- extend to the NLO amplitude, will include local UV renormalization of the amplitude

backup slides

initial state for jet production

for jets with large p_T (\leftrightarrow Higgs with large p_T) the likelihood of having a gluon-gluon initial state decreases:



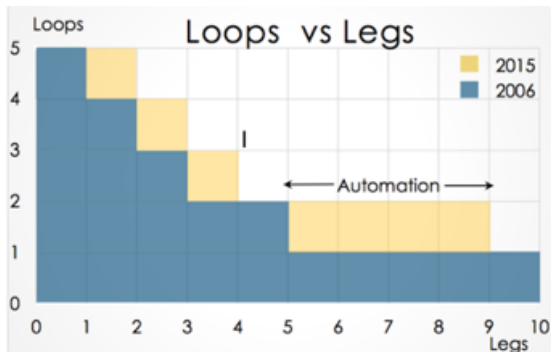
[taken from: Salam, Bautzen lectures, '09]

dimensional regularization result for F_{12}

$$\begin{aligned}
 F_{12} &= c_1 B_0(m_H^2, m_t^2, m_t^2) + c_2 \left(B_0(s, m_t^2, m_t^2) - B_0(m_H^2, m_t^2, m_t^2) \right) \\
 &\quad + c_3 C_0(0, s, m_H^2, m_t^2, m_t^2, m_t^2) \\
 &= \frac{1}{32\pi^2} \left[\left(1 - \frac{4m_t^2}{t+u} \right) \left(\log^2 \left(\frac{\sqrt{s(s-4m_t^2)} + 2m_t^2 - s}{2m_t^2} \right) \right. \right. \\
 &\quad \left. \left. - \log^2 \left(\frac{\sqrt{m_H^2(m_H^2-4m_t^2)} + 2m_t^2 - m_H^2}{2m_t^2} \right) \right) \right. \\
 &\quad \left. + \frac{4s}{t+u} \left(\sqrt{1 - 4\frac{m_t^2}{m_H^2}} \log \left(\frac{\sqrt{m_H^4 - 4m_H^2 m_t^2} + 2m_t^2 - m_H^2}{2m_t^2} \right) \right. \right. \\
 &\quad \left. \left. - \sqrt{1 - 4\frac{m_t^2}{s}} \log \left(\frac{\sqrt{s^2 - 4sm_t^2} + 2m_t^2 - s}{2m_t^2} \right) \right) \right] + 4
 \end{aligned}$$

difficulty of multi-loop or multi-leg calculations

amplitudes that contain up to one mass scale:



Mandelstam variables

$$s = m_H^2 - \frac{2p_T^2}{\tanh^2 \eta - 1} \left(1 + \sqrt{1 + \frac{m_H^2}{p_T^2} (1 - \tanh^2 \eta)} \right)$$

$$t = \frac{p_T^2}{\tanh \eta - 1} \left(1 + \sqrt{1 + \frac{m_H^2}{p_T^2} (1 - \tanh^2 \eta)} \right)$$

$$u = -\frac{p_T^2}{\tanh \eta + 1} \left(1 + \sqrt{1 + \frac{m_H^2}{p_T^2} (1 - \tanh^2 \eta)} \right)$$

expansion by regions of $q\bar{q} \rightarrow Hg$

expansion by regions

- Divide space of loop momentum into various regions and expand the integrand into a Taylor series w.r.t. the parameters considered small there.
- Integrate expanded integrand *over the whole integration domain* of the loop momenta.
- Set to zero any scaleless integral.

[Beneke, Smirnov '98]

- 'region': $\ell \sim m$ instead of $0 \leq \ell \leq \Lambda$.
- let's try for one of the integrals needed for $\bar{q}q \rightarrow Hg$ at LO:

$$\int_{\ell} \frac{1}{(\ell^2 - m_{\bar{t}}^2 + i0)(\ell^2 + 2\ell \cdot p_{12} + 2\ell \cdot p_3 + m_H^2 - m_{\bar{t}}^2 + i0)}$$

- How many regions? At least $\ell \sim m$ (soft) and $\ell \sim p_T$ (hard)
 - ▶ How about $\ell \sim \frac{m}{p_T}$ (ultrasoft)? Or $\ell_0 \sim \frac{m}{p_T}, |\ell| \sim m$ (potential)?
 - ▶ How to treat the scalar products?