Goldstone theorem in non-Hermitian QFT

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Why non-Hermitian QFT?

- Hermitian conjugation $\rightarrow$ Unitarity & Real eigenvalues.
- Ensures diagonalization
- Mathematical property

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$\Rightarrow$ Unbroken $\mathcal{PT}$-symmetry:

- $\mathcal{PT}$-symmetric
- Eigenfunction of $\hat{H}$ is eigenfunction of $\mathcal{PT}$

Unbroken $\mathcal{PT}$-symmetries

Real energies:

$$
\begin{aligned}
\mathcal{PT}\left(\hat{H}\phi_i\right) = E_i^\star\left(\mathcal{PT}\phi_i\right) &= E_i^\star e^{i\alpha_i}\phi_i \\
\hat{H}\left(\mathcal{PT}\phi_i\right) = e^{i\alpha_i}\hat{H}\phi_i &= E_i e^{i\alpha_i}\phi_i
\end{aligned}
$$

Unbroken $\mathcal{PT}$-symmetries

Real energies:

$$\mathcal{PT}\left(\hat{H}\phi_i\right) = E_i^\star\left(\mathcal{PT}\phi_i\right) = E_i^\star e^{i\alpha_i}\phi_i$$
$$\hat{H}\left(\mathcal{PT}\phi_i\right) = e^{i\alpha_i}\hat{H}\phi_i = E_i e^{i\alpha_i}\phi_i$$

Possitive inner product

$$\langle\phi_i^{\mathcal{PT}}|\phi_j\rangle = \pm\delta_{ij} \quad , \quad \langle\phi_i^{\mathcal{C}'\mathcal{PT}}|\phi_j\rangle = \delta_{ij}$$

Hermitian theories $\rightarrow$ Noether's theorem

$$\delta\mathcal{L} = \left(\partial^\mu \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i} - \frac{\partial\mathcal{L}}{\partial\phi_i}\right)\delta\phi_i + \delta\phi_i^\dagger\left(\partial^\mu \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i^\dagger} - \frac{\partial\mathcal{L}}{\partial\phi_i^\dagger}\right) + \partial_\mu j^\mu$$

Hermitian theories → Noether's theorem

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j^μ is conserved if:

$$0 = \left(\partial^\mu \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i} - \frac{\partial\mathcal{L}}{\partial\phi_i}\right) \quad \text{and} \quad 0 = \left(\partial^\mu \frac{\partial\mathcal{L}}{\partial\partial^\mu\phi_i^\dagger} - \frac{\partial\mathcal{L}}{\partial\phi_i^\dagger}\right)$$

Symmetries and Conservation Laws

$$\left(\partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi_i} - \frac{\partial \mathcal{L}}{\partial \phi_i}\right) \neq \left(\partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi_i^\dagger} - \frac{\partial \mathcal{L}}{\partial \phi_i^\dagger}\right)^\dagger$$

One of the equations of motion should be choosen

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$$\delta \mathcal{L} = \left(\partial^\mu \frac{\partial \mathcal{L}}{\partial \partial^\mu \phi_i^\dagger} - \frac{\partial \mathcal{L}}{\partial \phi_i^\dagger}\right) \Rightarrow \partial_\mu j^\mu = 0$$

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Non-Hermitian context:

A continuous symmetry, with conserved current that doesn't leave the vacuum invariant has a Nambu-Goldstone boson.

Goldstone's Theorem

$$
\begin{aligned}
\mathcal{L} &= \partial_\mu \phi_1^\star \partial^\mu \phi_1 + \partial_\mu \phi_2^\star \partial^\mu \phi_2 + m_1^2 |\phi_1|^2 - m_2^2 |\phi_2|^2 \\
&\quad - \mu^2 \left(\phi_1^\star \phi_2 - \phi_2^\star \phi_1 \right) - \frac{g}{4} |\phi_1|^4
\end{aligned}
$$

Goldstone's Theorem

$$\begin{aligned}\mathcal{L} &= \partial_\mu \phi_1^\star \partial^\mu \phi_1 + \partial_\mu \phi_2^\star \partial^\mu \phi_2 + m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2 \\ &\quad -\mu^2\left(\phi_1^\star\phi_2 - \phi_2^\star\phi_1\right) - \frac{g}{4}|\phi_1|^4\end{aligned}$$

Conserved current:

$$\begin{aligned}\phi_1 &\rightarrow e^{i\alpha}\phi_1 \\ \phi_2 &\rightarrow e^{-i\alpha}\phi_2\end{aligned}$$

$$\mathcal{L} = \partial_\mu\phi_1^\star\partial^\mu\phi_1 + \partial_\mu\phi_2^\star\partial^\mu\phi_2 + m_1^2|\phi_1|^2 - m_2^2|\phi_2|^2$$
$$-\mu^2\left(\phi_1^\star\phi_2 - \phi_2^\star\phi_1\right) - \frac{g}{4}|\phi_1|^4$$

Conserved current:

$$\phi_1 \rightarrow e^{i\alpha}\phi_1$$
$$\phi_2 \rightarrow e^{-i\alpha}\phi_2$$

$$j^\mu = i\Big((\phi_1^\star\partial^\mu\phi_1 - \partial^\mu\phi_1^\star\phi_1) - (\phi_2^\star\partial^\mu\phi_2 - \partial^\mu\phi_2^\star\phi_2) \Big)$$

Goldstone's Theorem

$$\begin{aligned}\mathcal{L} &= \partial_\mu \phi_1^\star \partial^\mu \phi_1 + \partial_\mu \phi_2^\star \partial^\mu \phi_2 + m_1^2 |\phi_1|^2 - m_2^2 |\phi_2|^2 \\ &\quad -\mu^2 \left(\phi_1^\star \phi_2 - \phi_2^\star \phi_1\right) - \frac{g}{4} |\phi_1|^4\end{aligned}$$

non-trivial vacuum:

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = \sqrt{2 \frac{m_1^2 m_2^2 - \mu^4}{g m_2^2}} \begin{pmatrix} 1 \\ \frac{\mu^2}{m_2^2} \end{pmatrix}$$

Mass matrix in terms of fluctuations:

$$M^2 = \begin{pmatrix} \frac{m_1^2 m_2^2 - 2\mu^4}{m_2^2} & \frac{m_1^2 m_2^2 - \mu^4}{m_2^2} & \mu^2 & 0 \\ \frac{m_1^2 m_2^2 - \mu^4}{m_2^2} & \frac{m_1^2 m_2^2 - 2\mu^4}{m_2^2} & 0 & \mu^2 \\ -\mu^2 & 0 & m_2^2 & 0 \\ 0 & -\mu^2 & 0 & m_2^2 \end{pmatrix}$$

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Massless mode:

$$G = \sqrt{\frac{2m_2^4}{m_2^4 - \mu^4}} \left(\mathrm{Im}\hat{\phi}_1 - \frac{\mu^2}{m_2^2} \mathrm{Im}\hat{\phi}_2 \right)$$

Conclusion

- Conserved current needed for the Goldstone theorem
- Goldstone theorem works in non-Hermitian context

- Higgs model in non-Hermitian context
- Multi-Higgs doublet models