Hollowness at low and high energies

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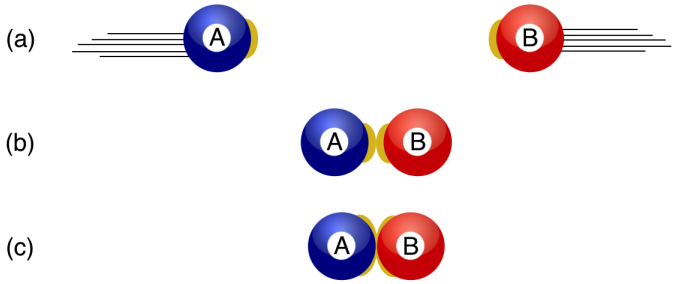
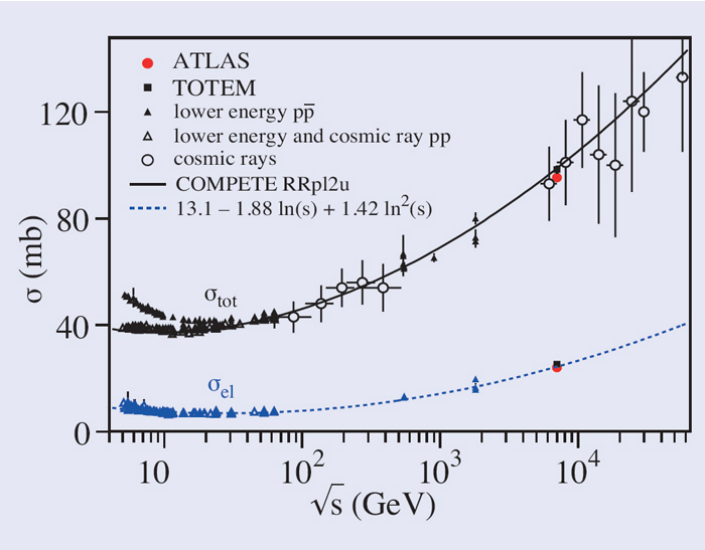
The Huygens accerelator (1650)

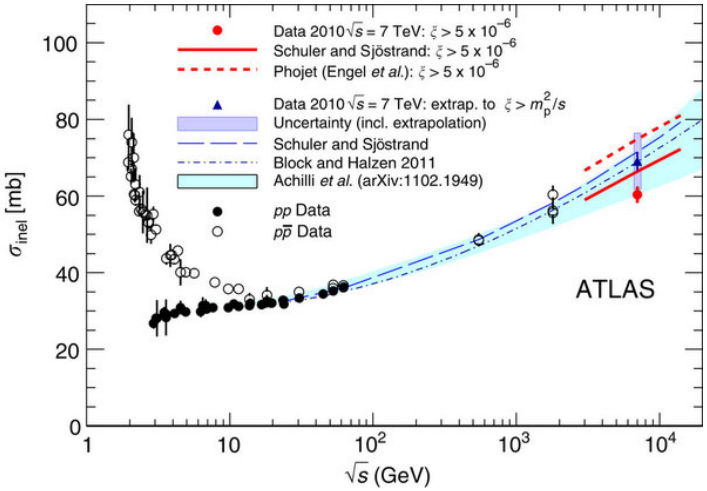
- Water canal is frictionless



- Head-on collision produces most damage
- Hollowness: slightly non-central (0.5fm) collisions are more inelastic at LHC (above 1 TeV)
- Hollowness also happens below $5 - 6GeV$

- Low energy nuclear reactions
- High energy NN scattering





- Inelastic transition → Regge Transition → Hollowness transition
- Larger , Blacker, Edgier

Neutron-nucleus scattering

- Partial wave expansion elastic scattering amplitude

$$f(\theta) = \sum_{l=0}^{pR} (2l+1) P_l(\cos\theta) \frac{S_l - 1}{2ip} \qquad S_l = \eta_l e^{2i\delta_l}$$

- Differential elastic cross section

$$\frac{d\sigma_{el}}{d\Omega} = |f(\theta)|^2 \quad \rightarrow \sigma_{el} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)|1 - S_l|^2$$

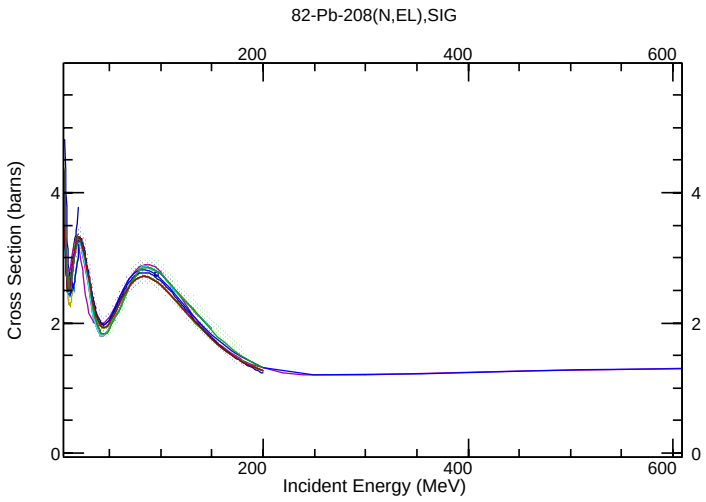
- Total Cross section and Optical theorem

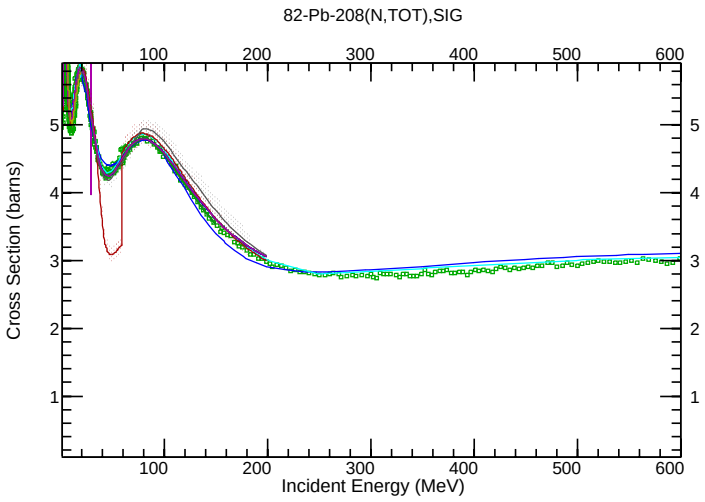
$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)(2 - 2\mathrm{Re} S_l) = \frac{4\pi}{p} \mathrm{Im} f(\theta = 0)$$

- Reaction (inelastic) cross section

$$\sigma_{\mathrm{in}} = \sigma_T - \sigma_{\mathrm{el}} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)(1 - |S_l|^2) = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l+1)(1 - \eta_l^2)$$

Neutron-Nucleus scattering





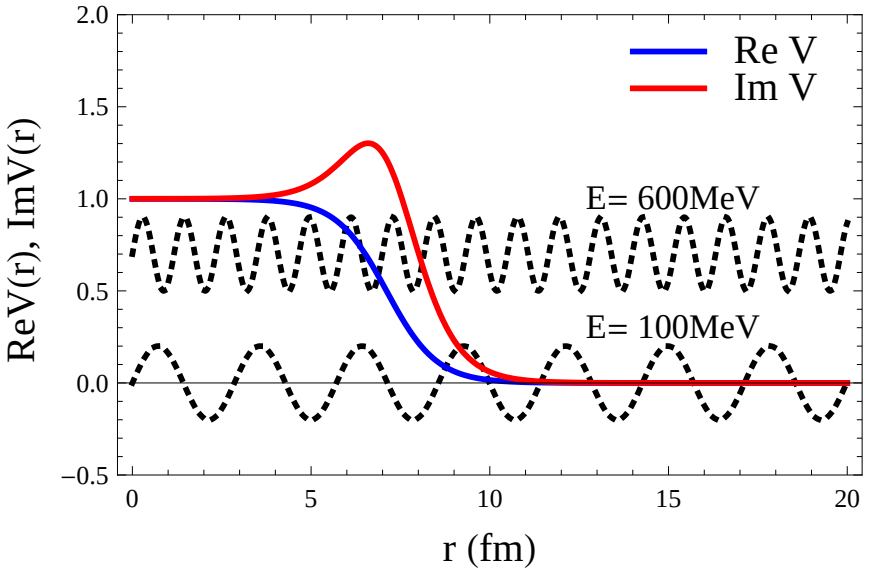
- Black disk $S_l = 0$ for $l \leq pR$ and $S_l = 1$ for $l > pR$

$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{pR} (2l+1)(2 - 2\mathrm{Re}S_l) = 2\pi(R + 1/p)^2 = 2\sigma_{\mathrm{el}}$$

- Using $R = r_0 A^{\frac{1}{3}} = 1.2(208)^{\frac{1}{3}}\,\mathrm{fm} = 7\mathrm{fm}$

$$\sigma_T = 2\pi R^2 = 3\mathrm{barn} \qquad \sigma_{\mathrm{el}} = \pi R^2 = 1.5\mathrm{barn}$$

Optical model and Optical Potential



- Optical model (Energy dependent complex potential)

$$V(r,E) = \mathrm{Re}V(r,E) + i\mathrm{Im}V(r,E)$$

- Woods-Saxon forms

$$F(r) = \frac{1}{1+e^{(r-R)/a}} \qquad R = r_0 A^{\frac{1}{3}} \qquad a = 0.7\mathrm{fm}$$

- Real and imaginary potentials

$$\mathrm{Re}V(r) \sim F(r) \qquad \mathrm{Im}V(r) \sim F'(r)$$

Proton-Proton scattering

- First inelastic threshold (1π)

$$s = 4(p^2 + M_N^2) \qquad \sqrt{s_{\rm in}} = 2M_N + m_\pi \to E_L = 300{\rm MeV}$$

- Scattering amplitude

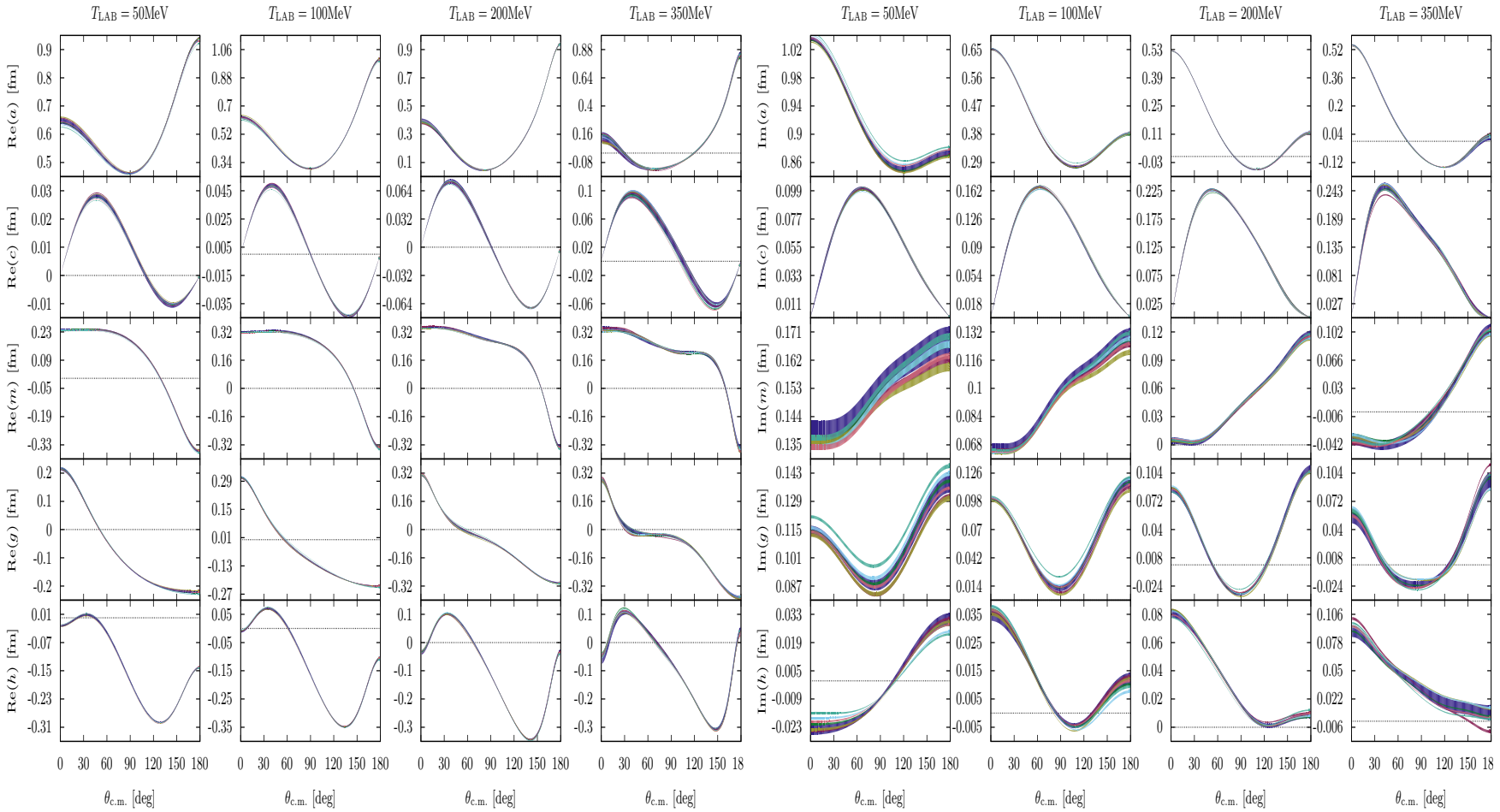
$$\begin{aligned} M &= a + m(\sigma_1 \cdot \mathbf{n})(\sigma_2 \cdot \mathbf{n}) + (g-h)(\sigma_1 \cdot \mathbf{m})(\sigma_2 \cdot \mathbf{m}) \\ &+ (g+h)(\sigma_1 \cdot \mathbf{l})(\sigma_2 \cdot \mathbf{l}) + c(\sigma_1 + \sigma_2) \cdot \mathbf{n} \end{aligned}$$

- Five complex a, m, g, h, c depend on energy and angle
- 24 measurable cross-sections and polarization asymmetries
- Complete set of experiments are needed at a GIVEN energy

$$0 < T_{\rm LAB} < 3{\rm GeV}$$

NN scattering below inelastic threshold

- Fit to 8000 np and pp scattering data from 1950-2013 below 350 MeV
- $\chi^2/\mathrm{DOF} = 1.04 \rightarrow$ NN potential with errors [Navarro,Amaro, E.R.A. 2013]



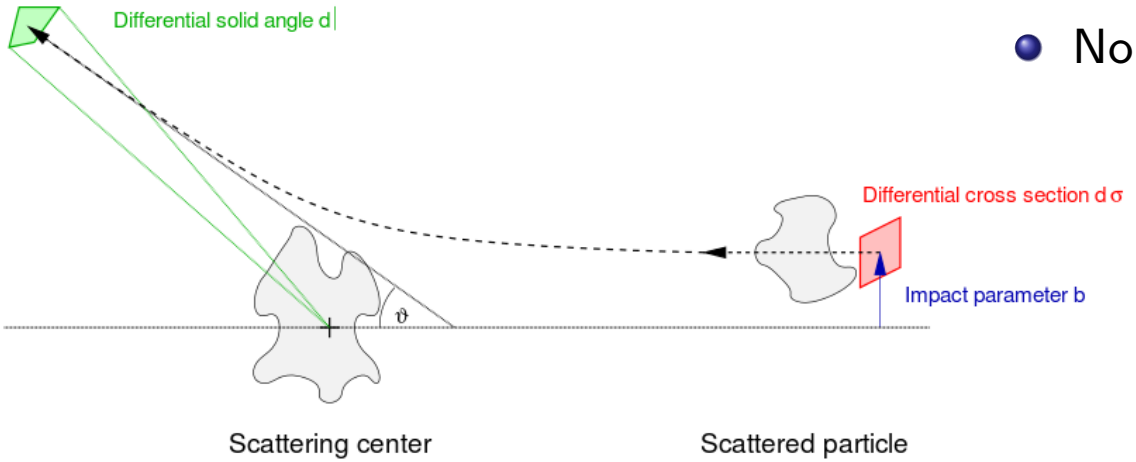
Finite range forces

- Meson exchange picture $\rightarrow$ Longest range $\equiv$ Lightest particle

$$r_c \sim \frac{\hbar}{m_\pi c} \sim 1.4\text{fm}$$

- Impact parameter

$$|\vec{L}| = |\vec{x} \wedge \vec{p}| \rightarrow bp \qquad L^2 = l(l+1)^2 \sim (l+1/2)^2 \rightarrow l + \frac{1}{2} = bp$$



- No scattering condition

$$\begin{aligned} V(r) &\sim 0 & r \gtrsim r_c \\ \delta_l(p) &\sim 0 & b \gtrsim r_c \\ \rightarrow l_{\max} + \frac{1}{2} &\sim pr_c \sim p/m_\pi \end{aligned}$$

- Truncation in the partial wave expansion

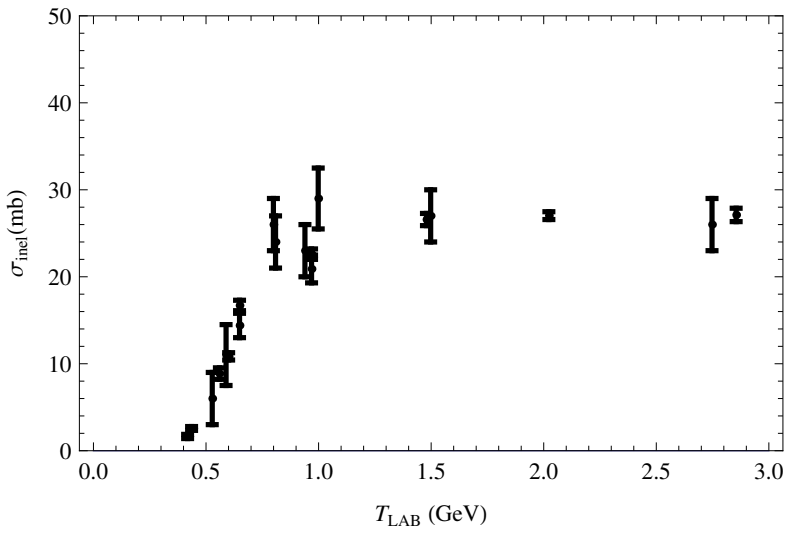
$$f(\theta,\phi) = \sum_{l=0}^{l_{\max}} (2l+1) \frac{e^{2i\delta_l} - 1}{2ip} P_l(\cos\theta) = \frac{e^{2i\delta_0} - 1}{2ip} + 3\frac{e^{2i\delta_1} - 1}{2ip}\cos\theta + \ldots$$

Inelastic resonance transition

- The inelastic transition at $\sqrt{s} = M_N + M_\Delta$

$$NN \to \Delta N \to \pi NN \qquad NN \to \Delta\Delta \to 2\pi NN \qquad \ldots$$

- Partial wave analysis stops at 3 GeV

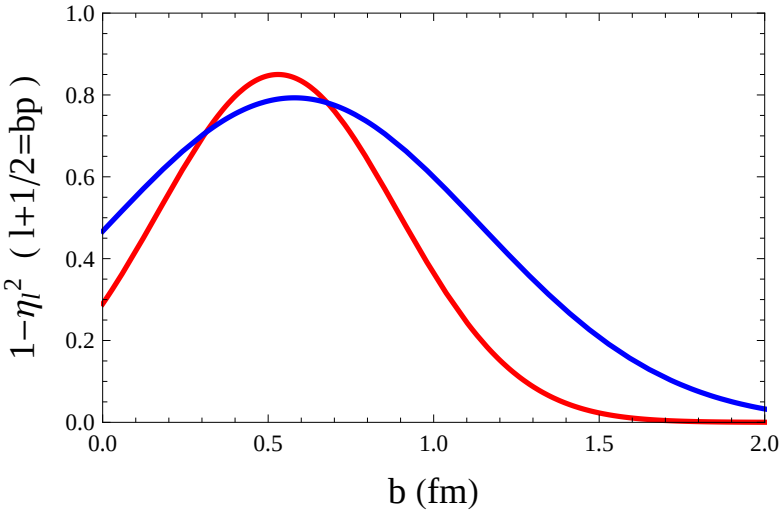


- We need a Complete set of measurements at higher energies !!!

Modeling absorption

- Hama,Kawaguchi (1966) Peripheral absorption

$$1.8 < T_L < 2.7\text{GeV}$$

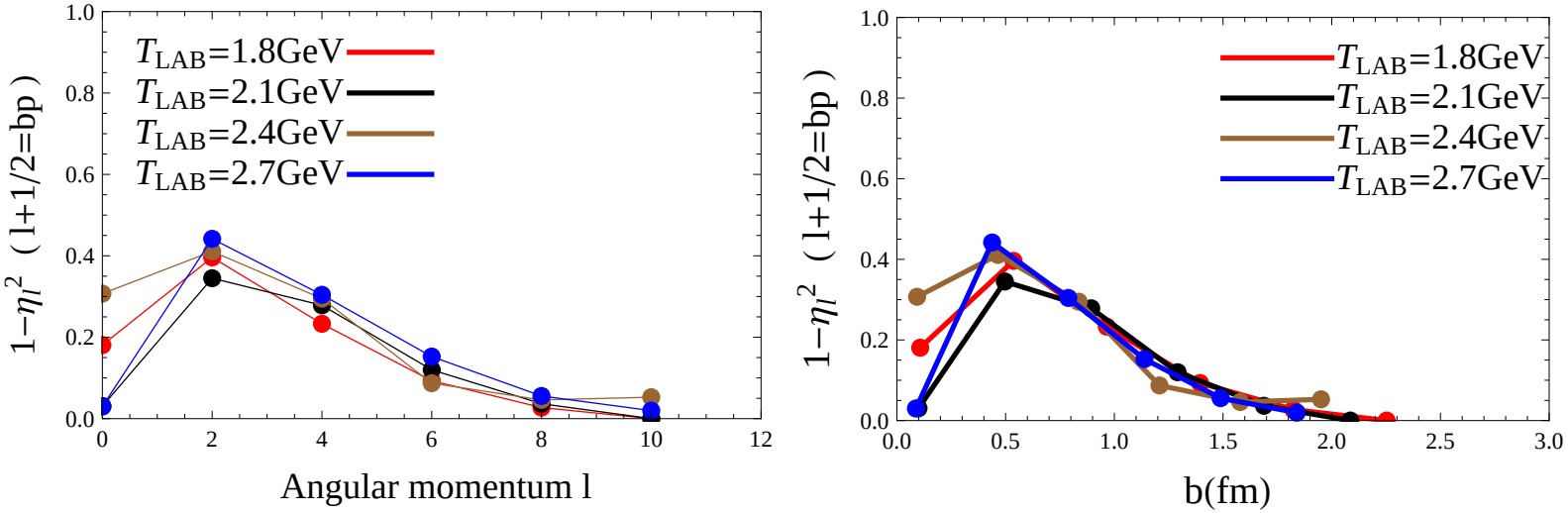


$$1 - \eta_l^2 = \alpha e^{-(l-l_0)^2/\gamma^2} \qquad l_0 \sim 2$$

Partial Wave Analysis

- Saturne Complete Experiments (Saclay 1998)

$$T_L = 1.8, 2.1, 2.4, 2.7\text{GeV}$$

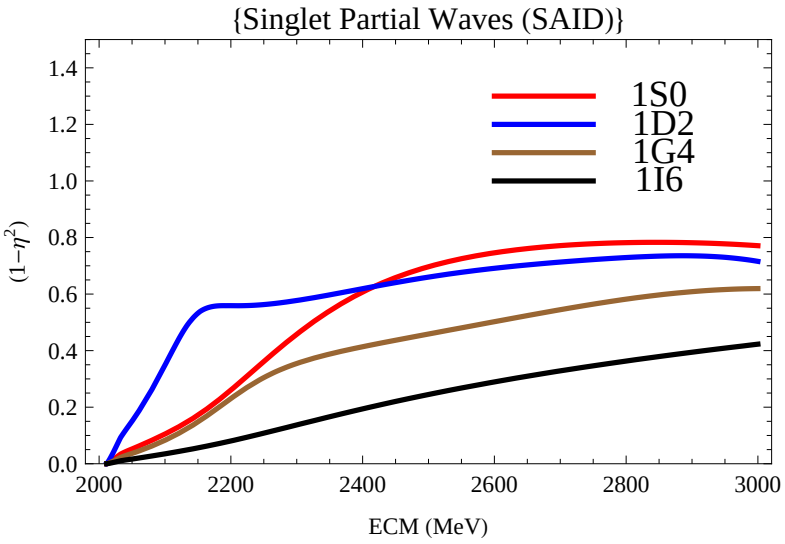


$$l + \frac{1}{2} = pb \qquad s = 4(p^2 + M^2)$$

Partial Wave Analysis

- Energy dependent analysis (SAID 2016)

$$1.8 < T_L < 2.7\text{GeV}$$



Regge transition

- From Pion Exchange to Pomeron Exchange
- Motivated by Regge theory (spin neglected)

[Barger,Phillips PLB (1972)]

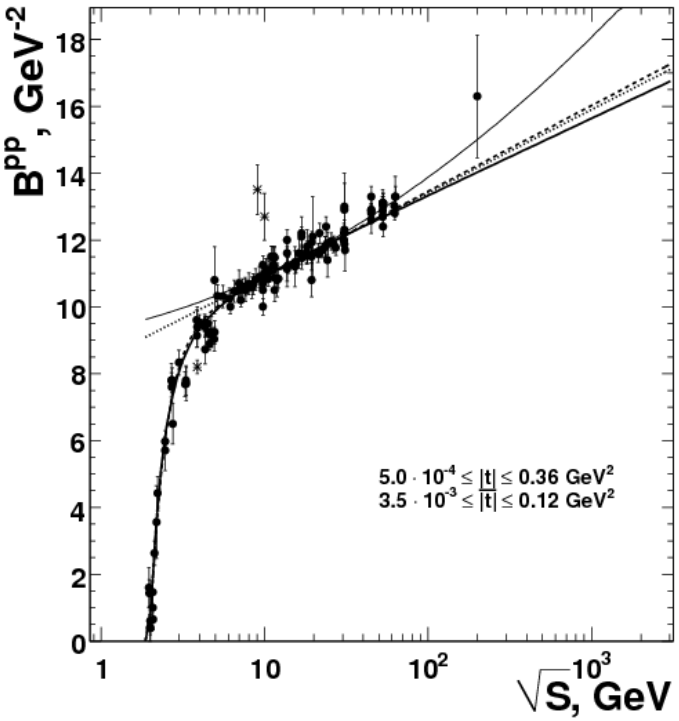
$$\mathcal{A}(s,t) \sim \sum_n A_n s^{\alpha_n(t)} \qquad \alpha_n(t) = \alpha_n(0) + \alpha_n'(0)t$$

- Two terms and a relative phase

$$A(s,t) = i\left(\sqrt{A}e^{Bt/2} + \sqrt{C}e^{Dt/2+i\phi}\right)$$

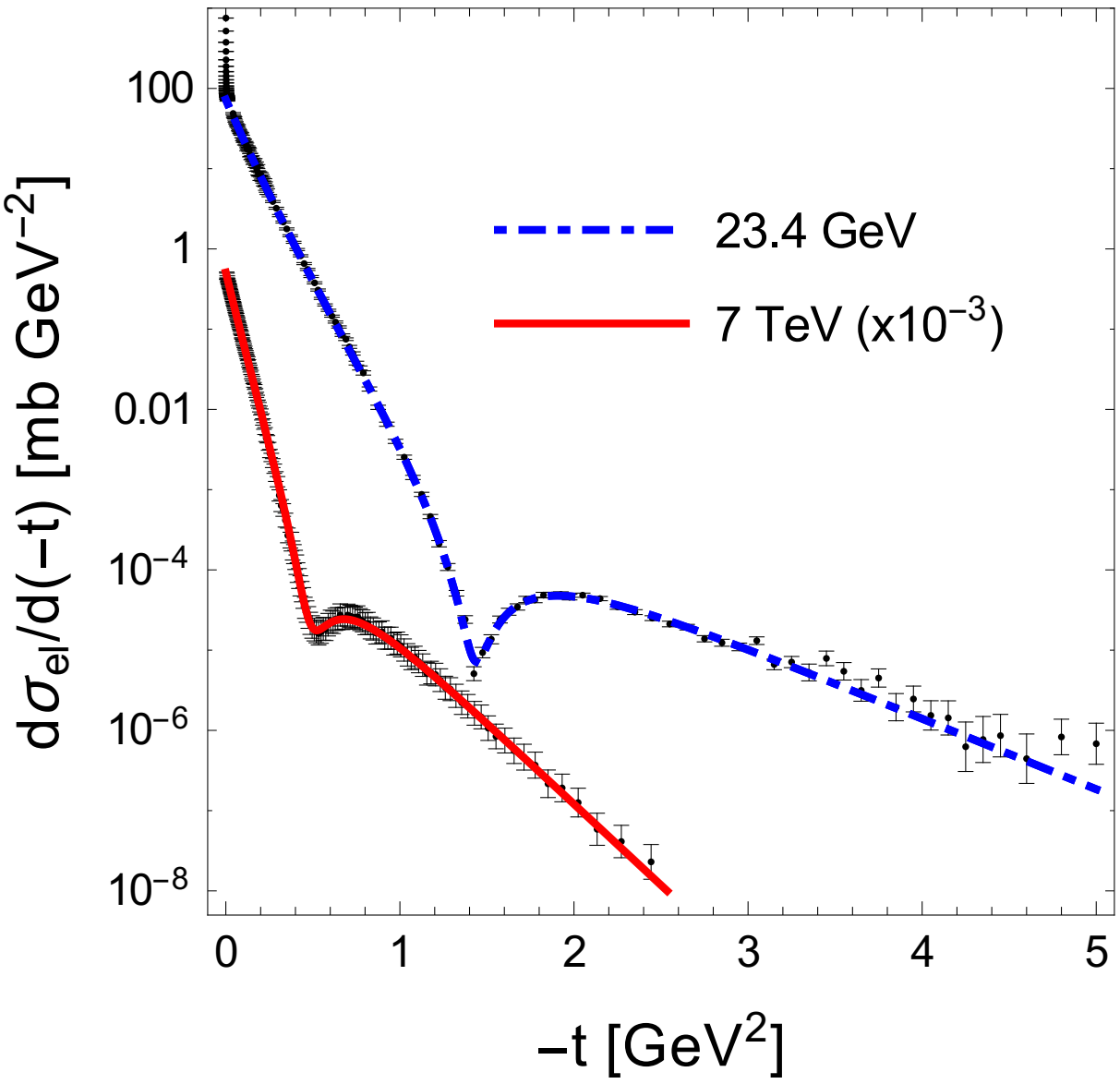
The nuclear slope

- Onset of diffraction (Okorokov, 2015)



$$B(s) = (8.00 \pm 0.06) + 2(0.309 \pm 0.010)\log(s/s_0) \qquad \chi^2/\nu = 234/98$$

From ISR to TOTEM



Parametrization of the scattering amplitude

S Parametrization by [Fagundes 2013], based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

$$\frac{f(s,t)}{p} = \sum_n c_n(s) F_n(t) s^{\alpha_n(t)} = \frac{i\sqrt{A} e^{\frac{Bt}{2}}}{\left(1 - \frac{t}{t_0}\right)^4} + i\sqrt{C} e^{\frac{Dt}{2} + i\phi}$$

s-dependent (real) parameters are fitted separately to all known differential pp cross sections for $\sqrt{s} = 23.4$, 30.5, 44.6, 52.8, 62.0, and 7000 GeV with $\chi^2/\mathrm{d.o.f} \sim 1.2 - 1.7$

$$\frac{d\sigma_{\mathrm{el}}}{dt} = \frac{\pi}{p^2} |f(s,t)|^2, \quad \sigma_T = \frac{4\pi}{p} \mathrm{Im} f(s,0)$$

Eikonal approximation

$$f(s,t) = \sum_{l=0}^{\infty}(2l+1)f_l(p)P_l(\cos\theta)$$
$$= \frac{p^2}{\pi}\int d^2b\, h(\vec{b},s)\, e^{i\vec{q}\cdot\vec{b}} = 2p^2\int_0^{\infty} bdbJ_0(bq)h(b,s)$$

$t=-\vec{q}^2$, $q=2p\sin(\theta/2)$, $bp=l+1/2+\mathcal{O}(s^{-1})$, $P_l(\cos\theta)\to J_0(qb)$, hence the amplitude in the impact-parameter representation becomes

$$h(b,s) = \frac{i}{2p}\left[1-e^{i\chi(b)}\right] = f_l(p)+\mathcal{O}(s^{-1})$$

The eikonal approximation works well for $b<2\ \mathrm{fm}$ and $\sqrt{s}>20\ \mathrm{GeV}$

Procedure: $f(s,t)\to h(b,s)\to\chi(b)$...

Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

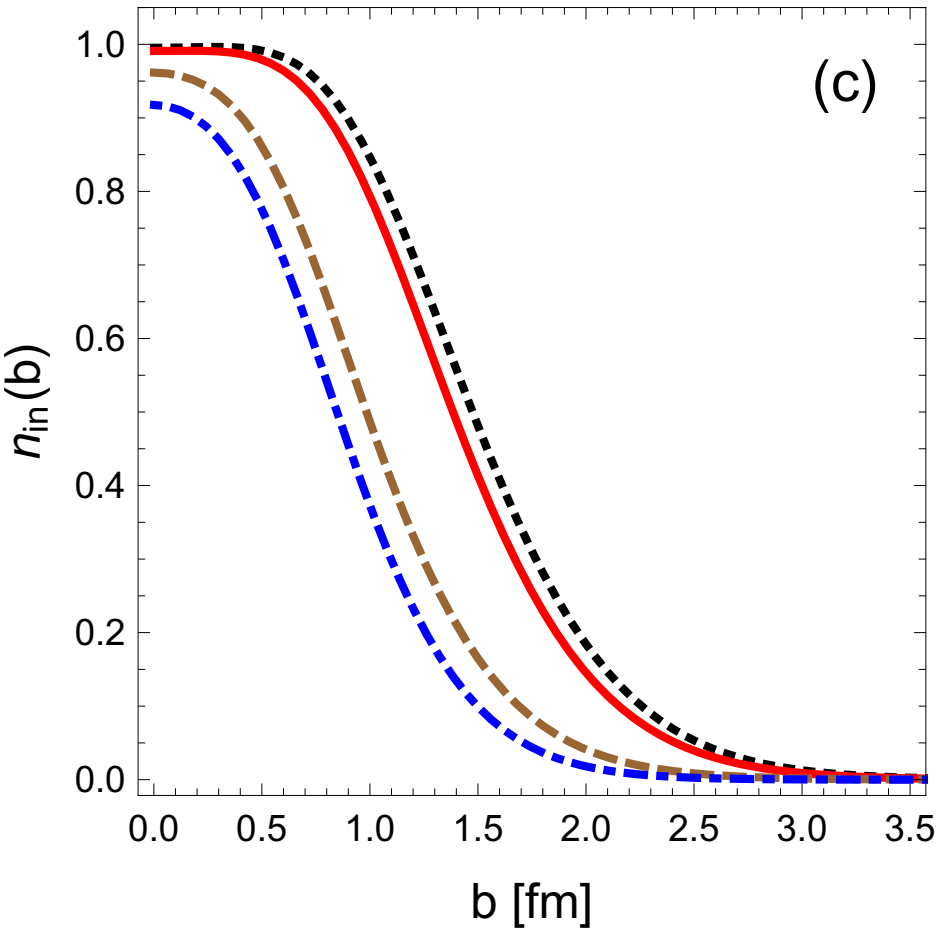
$$\begin{aligned}
\sigma_T &= \frac{4\pi}{p}\mathrm{Im} f(s,0) = 4p\int d^2b \mathrm{Im} h(\vec{b},s) = 2\int d^2b\left[1-\mathrm{Re}\, e^{i\chi(b)}\right] \\
\sigma_{\mathrm{el}} &= \int d\Omega |f(s,t)|^2 = 4p^2\int d^2b|h(\vec{b},s)|^2 = \int d^2b|1-e^{i\chi(b)}|^2 \\
\sigma_{\mathrm{in}} &\equiv \sigma_T - \sigma_{\mathrm{el}} = \int d^2b n_{\mathrm{in}}(b) = \int d^2b\left[1-e^{-2\mathrm{Im}\chi(b)}\right]
\end{aligned}$$

The inelasticity profile

$$n_{\mathrm{in}}(b) = 4p\mathrm{Im} h(b,s) - 4p^2|h(b,s)|^2$$

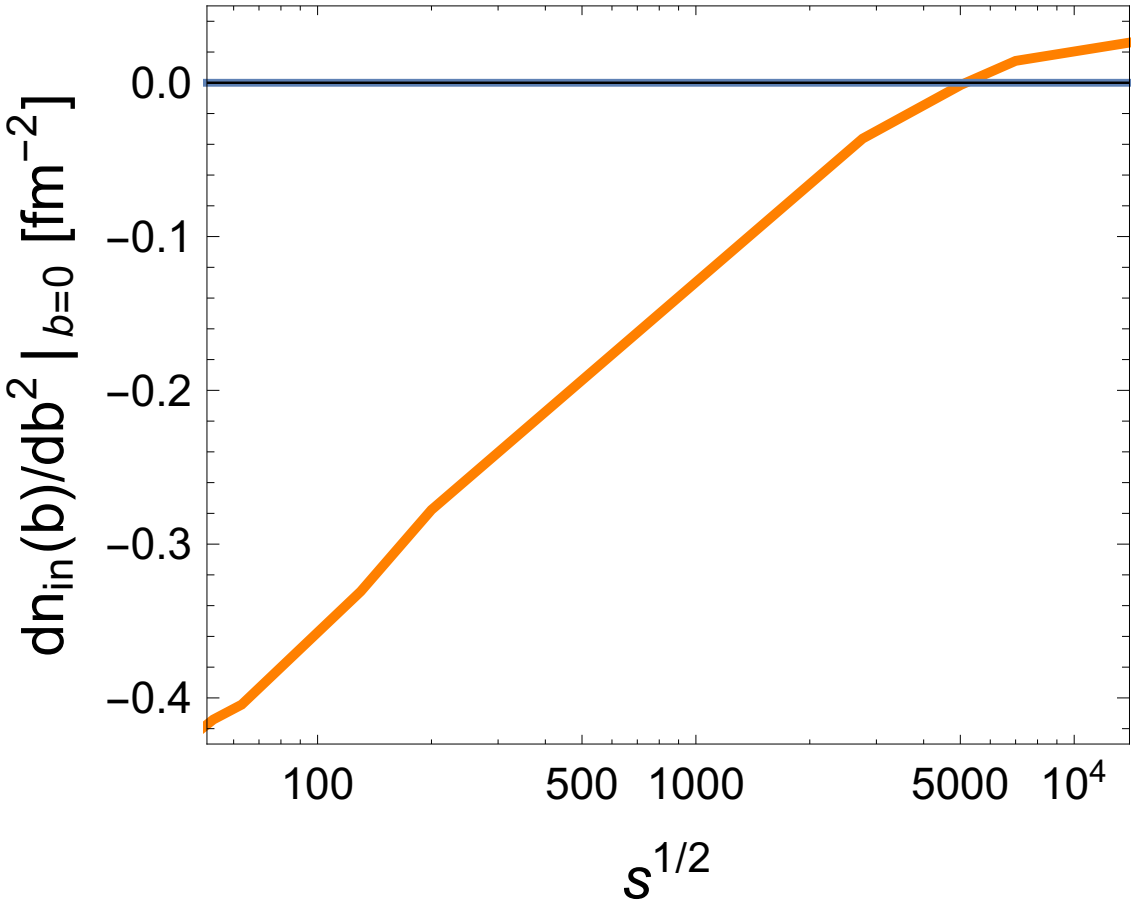
satisfies $n_{\mathrm{in}}(b) \leq 1$ (unitarity)

Dip in the inelasticity profile



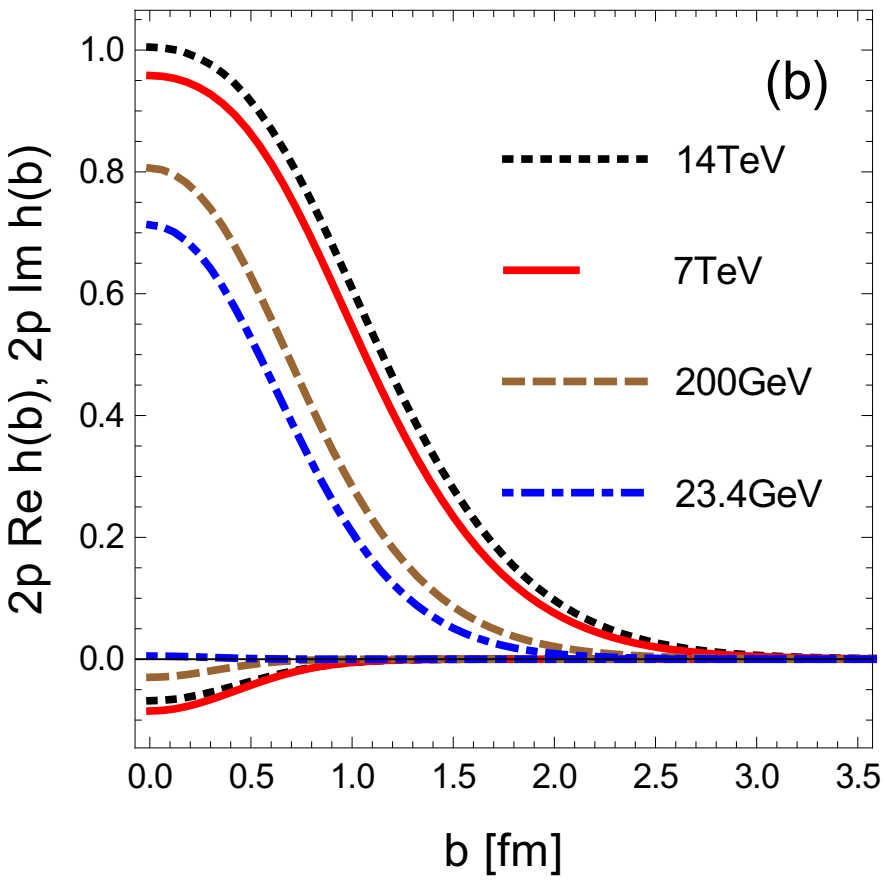
From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV

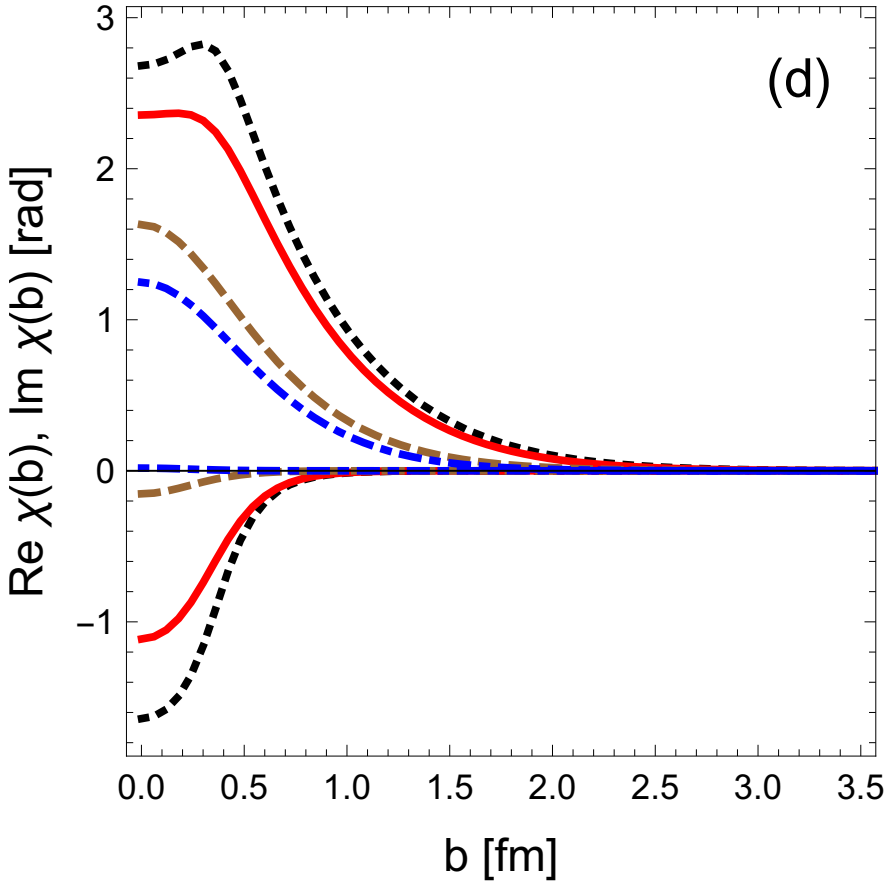
Slope of the inelasticity profile



Transition around $\sqrt{s} = 5$ TeV

Amplitude and eikonal phase





$$2p\,h(b) = i\left[1 - e^{i\chi(b)}\right]$$

(top curves - Im, bottom - Re)

No classical folding of absorptive parts

The hollowness effect cannot be reproduced by by folding of uncorrelated proton structures. We would then get, small r

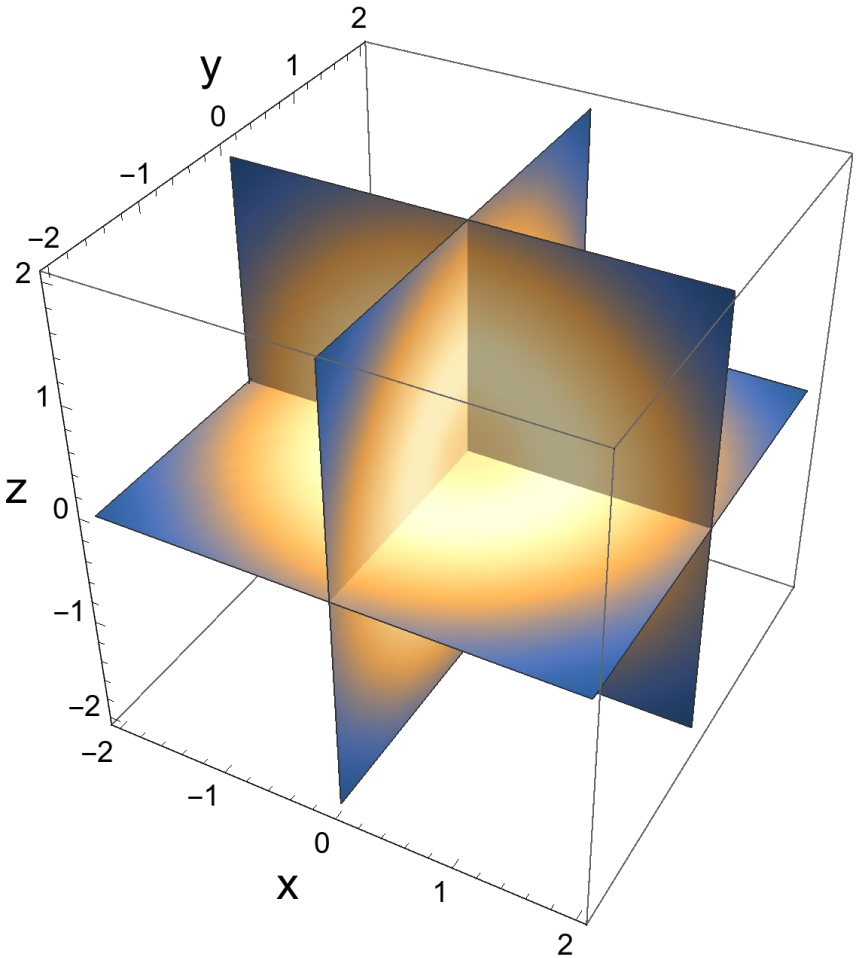
$$\begin{aligned} W(r) &= \int d^3y \rho(\vec{y}+\vec{r}/2)\rho(\vec{y}-\vec{r}/2) \\ &= \int d^3y \rho(\vec{y})^2 - \frac{1}{4}\int d^3y [\vec{r}\cdot\nabla\rho(\vec{y})]^2 + \ldots \end{aligned}$$

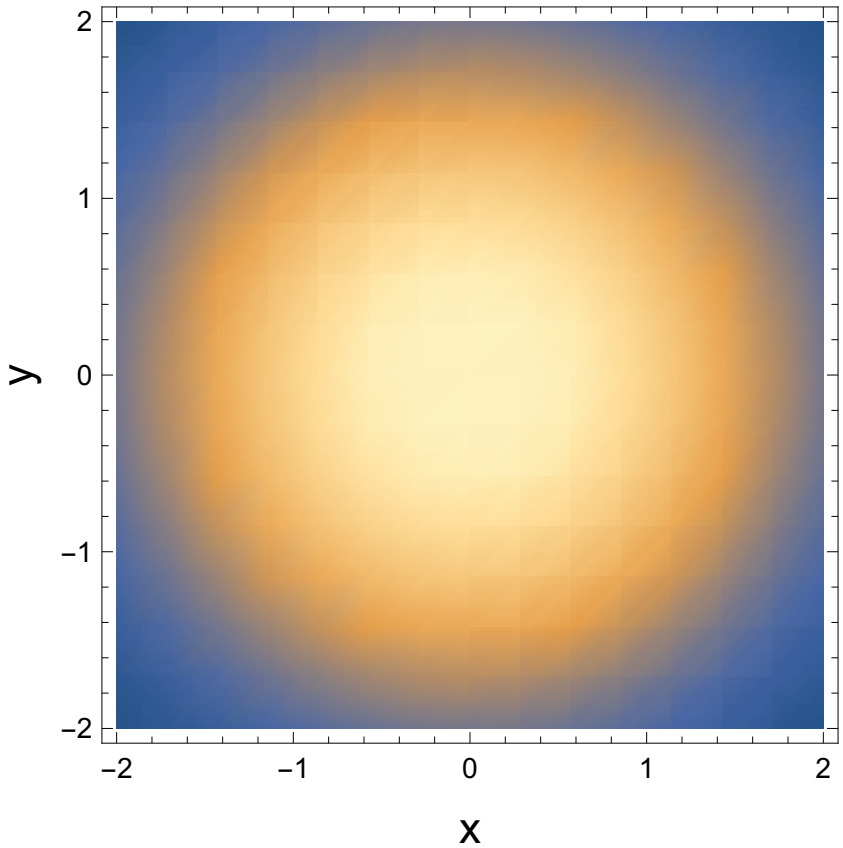
$\rightarrow$ $W(r)$ would necessarily have a local maximum at $r=0$, in contrast to the phenomenological result

$\rightarrow$ not possible to obtain hollowness classically by folding the absorptive parts from uncorrelated constituents

2D vs 3D opacity

Projection of 3D on 2D covers up the hollow: $f(x,y,z)$ vs $\int_{-\infty}^{\infty} dz f(x,y,z)$



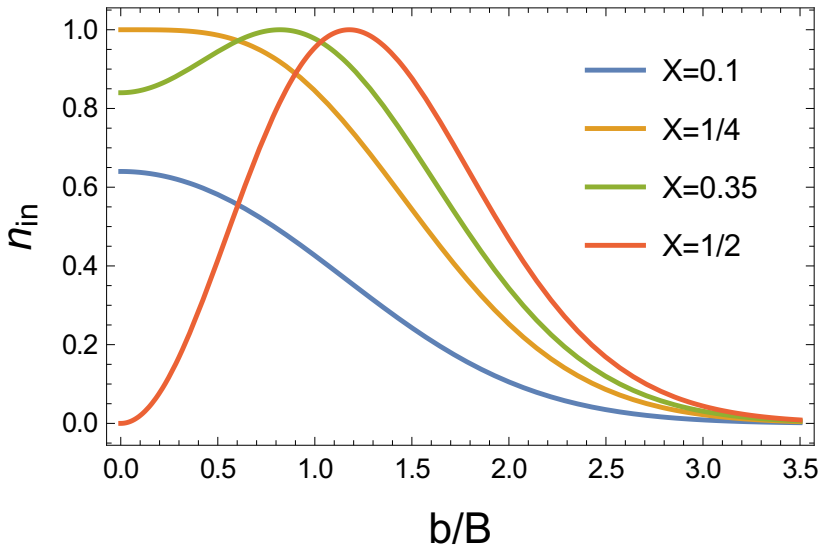


The hollow is covered up

Aspects of unitarity: model of [Dremin 2014]

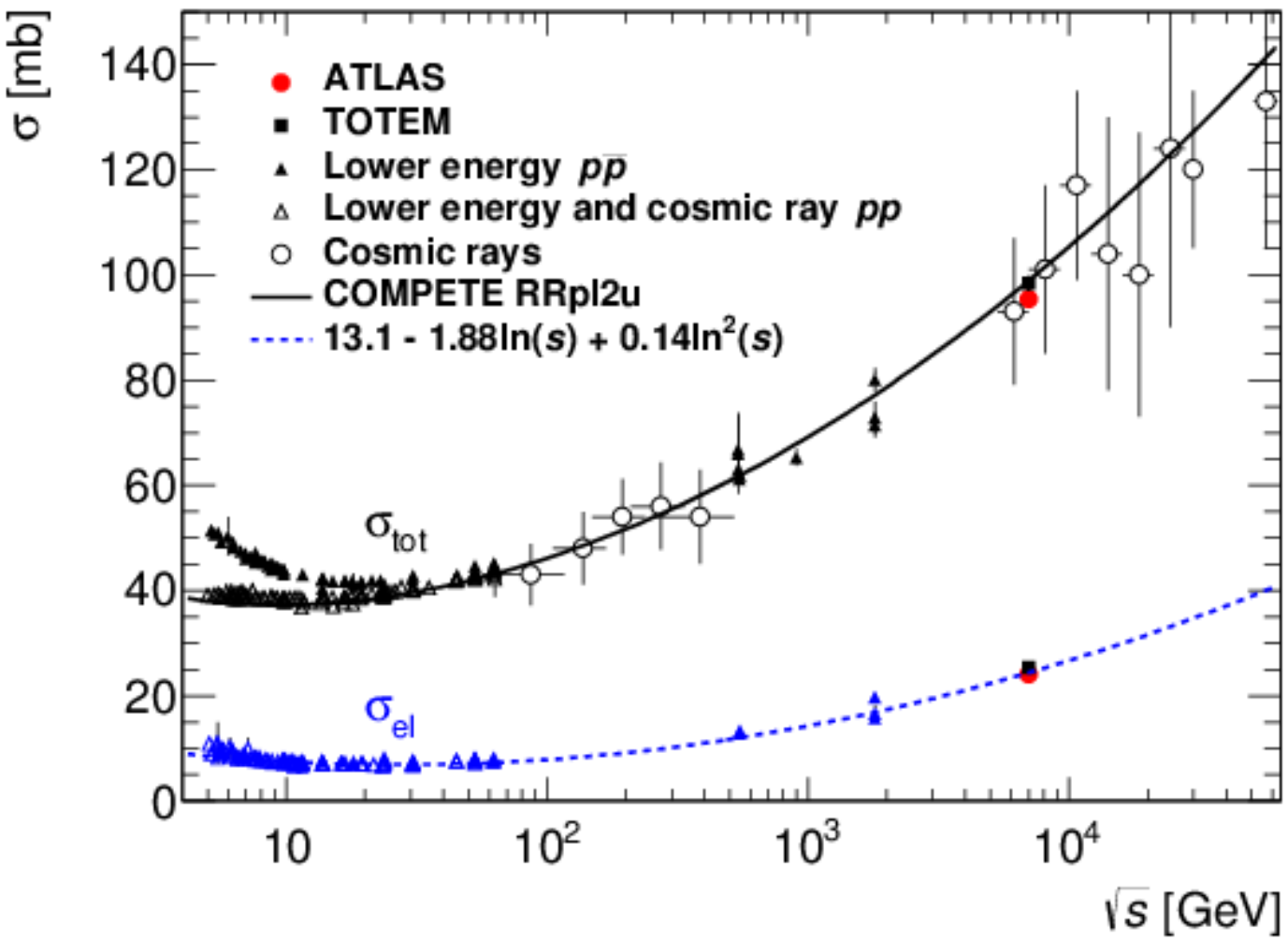
$$2p\mathrm{Im}h(b) \equiv k(b) = 4Xe^{-b^2/(2B)}, \;\; \mathrm{Re}h(b) = 0, \;\; X = \sigma_{el}/\sigma_T$$

$$n_{in}(b) = 2k(b) - k(b)^2 = 8Xe^{-b^2/(2B^2)} - 16X^2e^{-b^2/B^2}$$



- $X > 1/4$: $n_{in}(b)$ has a maximum at $b_0 = \sqrt{2}B\log(4X) > 0$, with $k(b_0) = 1$
- $X = 1/2$: black disk limit
- $W(r)$ develops a dip when $X > \sqrt{2}/8 = 0.177$

Cross sections



Ratio goes above 1/4 as energy increases!

Aspects of unitarity 2

If $2ph(b) = k(b)$ is not necessarily Gaussian but purely imaginary, then

$$n_{in}(b) = 2k(b) - k(b)^2$$

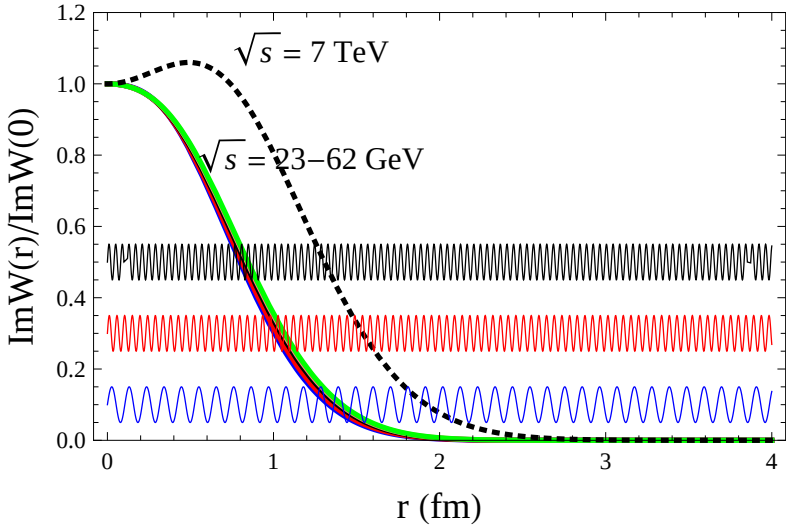
$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}[1 - k(b)]$$

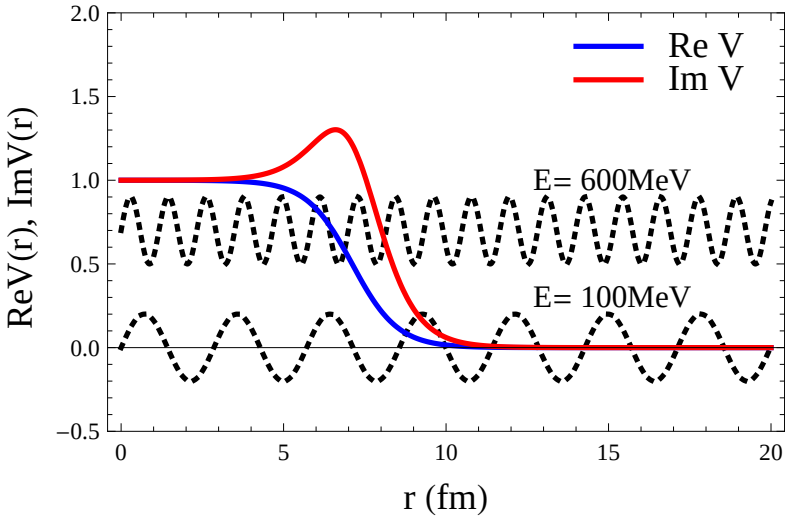
hence the minimum of $n(b)$ moves away from the origin when $k(0) > 1$

The real part of the amplitude, which is $\sim 10\%$, brings in corrections at the level of 1%

Inverse scattering problem

- Are low energy nuclear reactions and high NN interactions that different ?





The phase ambiguity

- We measure the modulus at finite t

$$f(s,t) = \mathrm{Re}\, f(s,t) + i\, \mathrm{Im}\, f(s,t) \qquad \frac{d\sigma_{el}}{dt} = \frac{\pi}{p^2} |f(s,t)|^2$$

- $$f(s,t) = |f(s,t)| \frac{\rho(s,t) + i}{\sqrt{1+\rho(s,t)^2}} \qquad \rho(s,t) = \frac{\mathrm{Re}\, f(s,t)}{\mathrm{Im}\, f(s,t)}$$

- Fixed-t dispersion relation

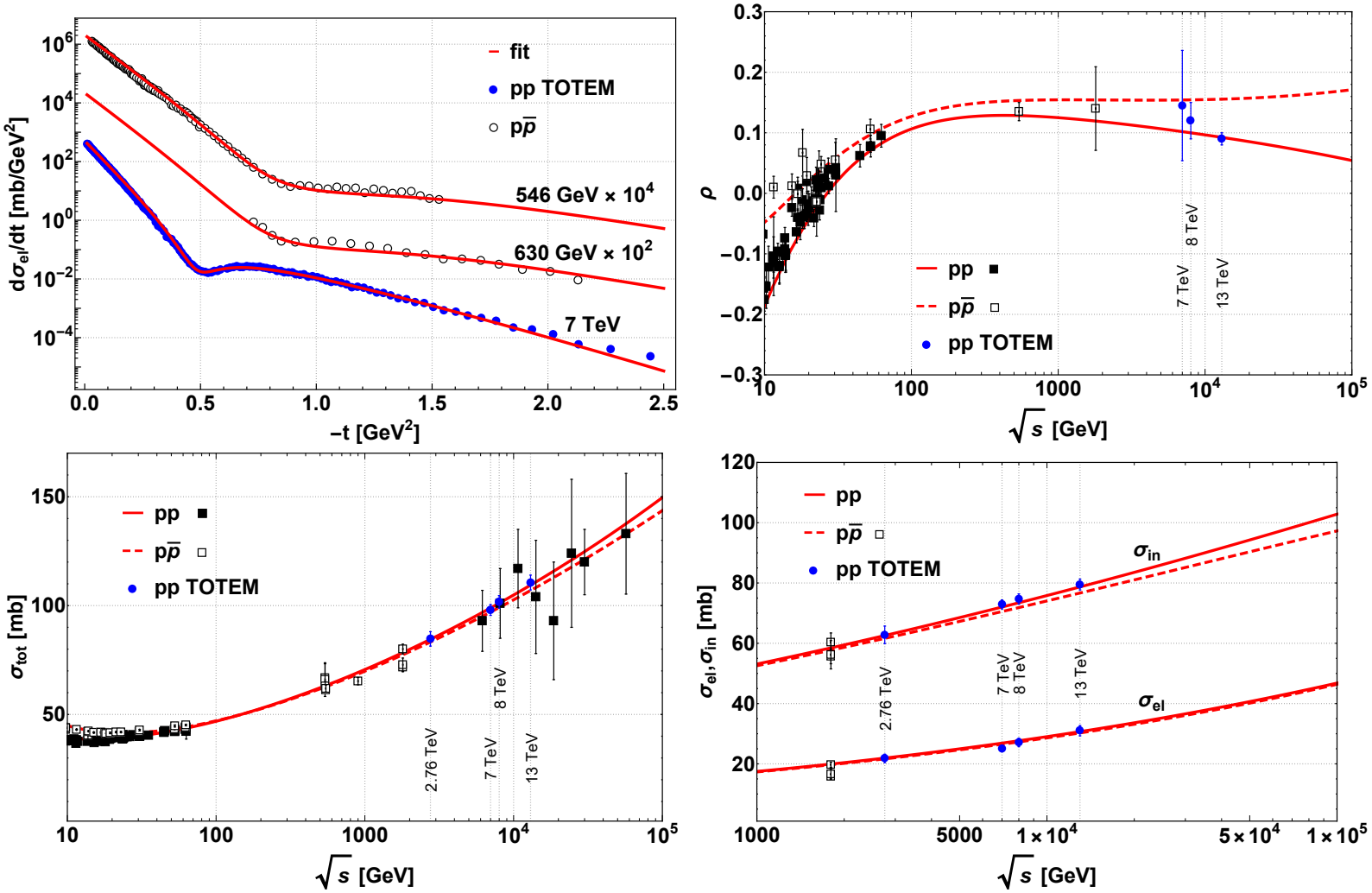
$$\mathrm{Re}\, f(s,t) = \frac{1}{\pi} \int_{4M^2}^{\infty} ds' \frac{\mathrm{Im}\, f(s',t)}{s'-s}.$$

- Regge Compatible with fixed-t dispersion relations

$$\frac{1}{\pi} \int_0^{\infty} ds' s'^{\alpha} \left[\frac{1}{s'-s} \pm \frac{1}{s'+s} \right] = \frac{(-s)^{\alpha} \pm (s)^{\alpha}}{\sin \pi\alpha}.$$

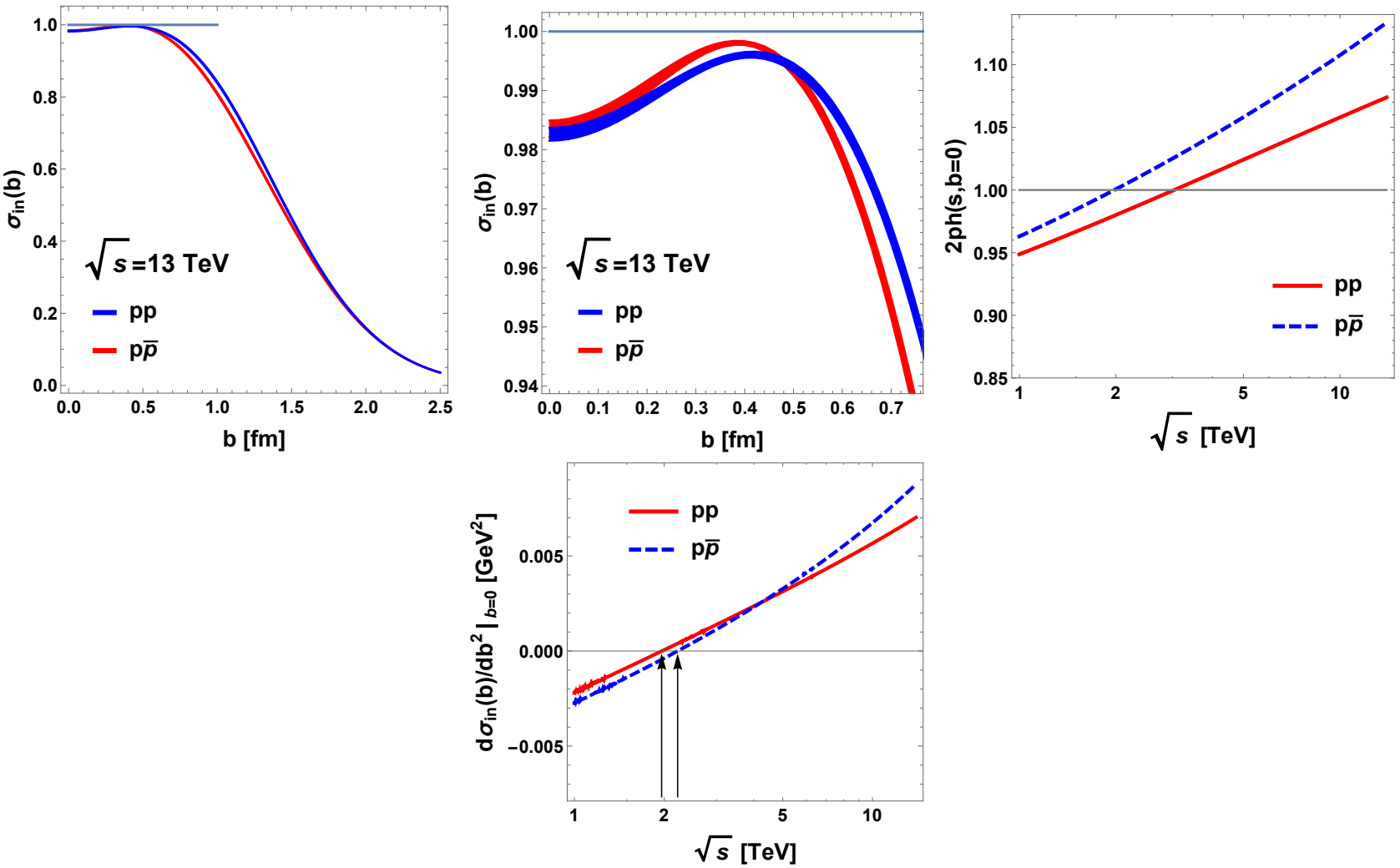
Regge theory fits data

- Double pomeron pole



Regge theory may produce hollowness

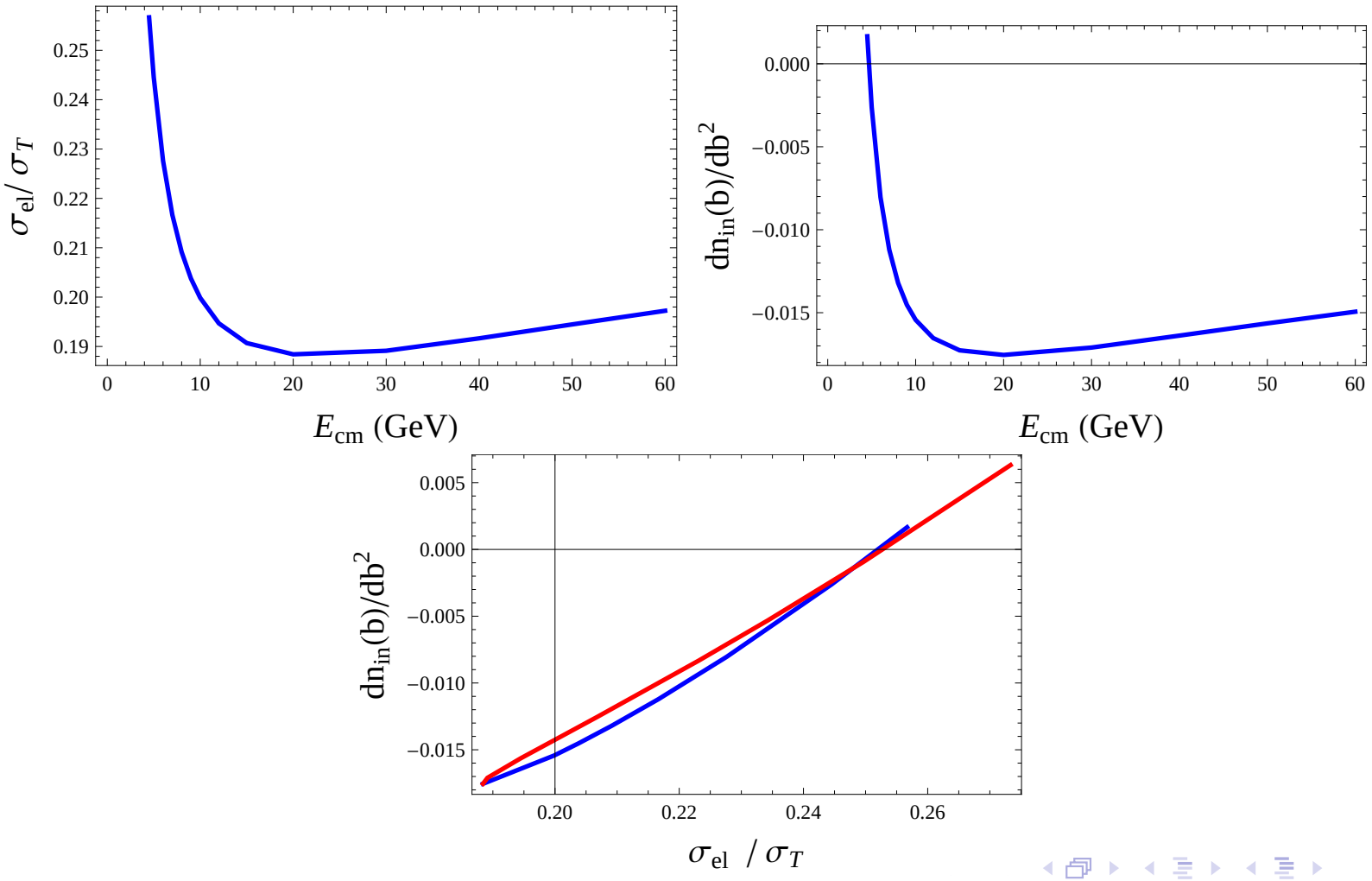
- High energies



Regge theory may also produce hollowness

- The diffractive correlation (Goulianos 1996) ¡ 500 GeV

$$\frac{\sigma_{\rm el}}{\sigma_{\rm tot}} = \frac{(1+\rho^2)\sigma_{\rm tot}}{4\pi B}$$



Conclusions

- Some models predict that there is a hollowness transition at about 1000+ GeV
- Quantum effect, rise of $2p\mathrm{Im}h(b)$ above 1
- Not possible to obtain classically by folding the absorptive parts from uncorrelated constituents (incompatible with geometric models)
- 2D $\rightarrow$ 3D magnifies the effect (flat in 2D means hollow in 3D)e
- Similar hollowness effect in low-energy pp scattering
- There are models (COMPETE,PDG) which DO NOT produce hollowness.