Hollowness at low and high energies

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The Huygens accelerator (1650)

- Water canal is frictionless

- Head-on collision produces most damage

- Hollowness: slightly non-central (0.5fm) collisions are more inelastic at LHC (above 1 TeV)

- Hollowness also happens below $5 - 6 GeV$
- Low energy nuclear reactions
- High energy NN scattering

Inelastic transition $\rightarrow$ Regge Transition $\rightarrow$ Hollowness transition

Larger, Blacker, Edgier
Neutron-nucleus scattering

- Partial wave expansion elastic scattering amplitude

\[ f(\theta) = \sum_{l=0}^{pR} (2l + 1) P_l(\cos \theta) \frac{S_l - 1}{2ip} \quad S_l = \eta_l e^{2i\delta_l} \]

- Differential elastic cross section

\[ \frac{d\sigma_{el}}{d\Omega} = |f(\theta)|^2 \quad \rightarrow \sigma_{el} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)|1 - S_l|^2 \]

- Total Cross section and Optical theorem

\[ \sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)(2 - 2\text{Re}S_l) = \frac{4\pi}{p} \text{Im} f(\theta = 0) \]

- Reaction (inelastic) cross section

\[ \sigma_{in} = \sigma_T - \sigma_{el} = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)(1 - |S_l|^2) = \frac{4\pi}{p^2} \sum_{l=0}^{\infty} (2l + 1)(1 - \eta_l^2) \]
Neutron-Nucleus scattering

- Black disk $S_l = 0$ for $l \leq pR$ and $S_l = 1$ for $l > pR$

$$\sigma_T = \frac{4\pi}{p^2} \sum_{l=0}^{pR} (2l + 1)(2 - 2\text{Re}S_l) = 2\pi(R + 1/p)^2 = 2\sigma_{el}$$

- Using $R = r_0 A^{1/3} = 1.2(208)^{1/3} \text{fm} = 7\text{fm}$

$$\sigma_T = 2\pi R^2 = 3\text{barn} \quad \sigma_{el} = \pi R^2 = 1.5\text{barn}$$
Optical model and Optical Potential

Optical model (Energy dependent complex potential)

\[ V(r, E) = \text{Re}V(r, E) + i\text{Im}V(r, E) \]

Woods-Saxon forms

\[ F(r) = \frac{1}{1 + e^{(r-R)/a}} \quad R = r_0A^{\frac{1}{3}} \quad a = 0.7\text{fm} \]

Real and imaginary potentials

\[ \text{Re}V(r) \sim F(r) \quad \text{Im}V(r) \sim F'(r) \]
Proton-Proton scattering

- First inelastic threshold \((1\pi)\)
  
  \[
  s = 4(p^2 + M_N^2) \quad \sqrt{s_{\text{in}}} = 2M_N + m_\pi \quad \rightarrow \quad E_L = 300\text{MeV}
  \]

- Scattering amplitude
  
  \[
  M = a + m(\sigma_1 \cdot n)(\sigma_2 \cdot n) + (g - h)(\sigma_1 \cdot m)(\sigma_2 \cdot m) \\
  + \quad (g + h)(\sigma_1 \cdot l)(\sigma_2 \cdot l) + c(\sigma_1 + \sigma_2) \cdot n
  \]

- Five complex \(a, m, g, h, c\) depend on energy and angle
- 24 measurable cross-sections and polarization asymmetries
- Complete set of experiments are needed at a GIVEN energy

\[
0 < T_{\text{LAB}} < 3\text{GeV}
\]
NN scattering below inelastic threshold

- Fit to 8000 np and pp scattering data from 1950-2013 below 350 MeV
- $\chi^2/DOF = 1.04 \rightarrow$ NN potential with errors [Navarro, Amaro, E.R.A. 2013]
Finite range forces

- Meson exchange picture \(\rightarrow\) Longest range \(\equiv\) Lightest particle

\[
rc \sim \frac{\hbar}{m_\pi c} \sim 1.4\text{fm}
\]

- Impact parameter

\[
|\vec{L}| = |\vec{x} \wedge \vec{p}| \rightarrow bp \quad L^2 = l(l + 1)^2 \sim (l + 1/2)^2 \rightarrow l + \frac{1}{2} = bp
\]

- No scattering condition

\[
V(r) \sim 0 \quad r \gtrsim rc
\]

\[
\delta_l(p) \sim 0 \quad b \gtrsim rc
\]

\[
\rightarrow l_{\text{max}} + \frac{1}{2} \sim prc \sim p/m_\pi
\]

- Truncation in the partial wave expansion

\[
f(\theta, \phi) = \sum_{l=0}^{l_{\text{max}}} (2l + 1) \frac{e^{2i\delta_l} - 1}{2ip} P_l(\cos \theta) = \frac{e^{2i\delta_0} - 1}{2ip} + 3 \frac{e^{2i\delta_1} - 1}{2ip} \cos \theta + \ldots
\]
Inelastic resonance transition

- The inelastic transition at $\sqrt{s} = M_N + M_\Delta$
  
  $NN \rightarrow \Delta N \rightarrow \pi NN \quad NN \rightarrow \Delta\Delta \rightarrow 2\pi NN \quad \ldots$

- Partial wave analysis stops at 3 GeV

- We need a Complete set of measurements at higher energies !!!

![Graph showing $\sigma_{inel}(\text{mb})$ vs $T_{LAB}$ (GeV)]
Modeling absorption

- Hama, Kawaguchi (1966) Peripheral absorption

\[ 1.8 < T_L < 2.7 \text{GeV} \]

\[ 1 - \eta_i^2 = \alpha e^{-(l-l_0)^2/\gamma^2} \quad l_0 \sim 2 \]
Partial Wave Analysis

- Saturne Complete Experiments (Saclay 1998)

\[ T_L = 1.8, 2.1, 2.4, 2.7 \text{GeV} \]

\[ l + \frac{1}{2} = p_b \quad s = 4(p^2 + M^2) \]
Partial Wave Analysis

- Energy dependent analysis (SAID 2016)

\[ 1.8 < T_L < 2.7 \text{GeV} \]
Regge transition

- From Pion Exchange to Pomeron Exchange
- Motivated by Regge theory (spin neglected)

\[ A(s,t) \sim \sum_n A_n s^{\alpha_n(t)} \quad \alpha_n(t) = \alpha_n(0) + \alpha'_n(0)t \]

- Two terms and a relative phase

\[ A(s,t) = i \left( \sqrt{A} e^{Bt/2} + \sqrt{C} e^{Dt/2 + i\phi} \right) \]
The nuclear slope

- Onset of diffraction (Okorokov, 2015)

\[ B(s) = \left(8.00 \pm 0.06\right) + 2\left(0.309 \pm 0.010\right) \log\left(\frac{s}{s_0}\right) \quad \chi^2/\nu = 234/98 \]
From ISR to TOTEM

- 23.4 GeV
- 7 TeV \( \times 10^{-3} \)

\[ \frac{d\sigma_{el}}{d(-t)} \text{ [mb GeV}^{-2} \]

\(-t [GeV^2]\)

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Various Faces of QCD
Parametrization of the scattering amplitude

S Parametrization by [Fagundes 2013], based on [Barger-Phillips 1974], motivated by the Regge asymptotics:

\[
\frac{f(s, t)}{p} = \sum_n c_n(s) F_n(t) s^\alpha n(t) = \frac{i\sqrt{A} e^{\frac{Bt}{2}}}{\left(1 - \frac{t}{t_0}\right)^4} + i\sqrt{C} e^{\frac{Dt}{2}} + i\phi
\]

\(s\)-dependent (real) parameters are fitted separately to all known differential pp cross sections for \(\sqrt{s} = 23.4, 30.5, 44.6, 52.8, 62.0, \) and 7000 GeV with \(\chi^2/\text{d.o.f} \sim 1.2 - 1.7\)

\[
\frac{d\sigma_{el}}{dt} = \frac{\pi}{p^2} |f(s, t)|^2, \quad \sigma_T = \frac{4\pi}{p} \text{Im} f(s, 0)
\]
Eikonal approximation

\[ f(s, t) = \sum_{l=0}^{\infty} (2l + 1) f_l(p) P_l(\cos \theta) \]

\[ = \frac{p^2}{\pi} \int d^2 b \, h(b, s) \, e^{i \vec{q} \cdot \vec{b}} = 2p^2 \int_0^\infty b db J_0(bq) h(b, s) \]

\[ t = -q^2, \quad q = 2p \sin(\theta/2), \quad bp = l + 1/2 + \mathcal{O}(s^{-1}), \quad P_l(\cos \theta) \to J_0(qb), \] hence the amplitude in the impact-parameter representation becomes

\[ h(b, s) = \frac{i}{2p} \left[ 1 - e^{i \chi(b)} \right] = f_l(p) + \mathcal{O}(s^{-1}) \]

The eikonal approximation works well for \( b < 2 \text{ fm} \) and \( \sqrt{s} > 20 \text{ GeV} \)

Procedure: \( f(s, t) \to h(b, s) \to \chi(b) \ldots \)
Eikonal approximation 2

The standard formulas for the total, elastic, and total inelastic cross sections read

\[ \sigma_T = \frac{4\pi}{p} \text{Im} f(s, 0) = 4p \int d^2 b \text{Im} h(\vec{b}, s) = 2 \int d^2 b \left[ 1 - \text{Re} e^{i\chi(b)} \right] \]

\[ \sigma_{el} = \int d\Omega |f(s, t)|^2 = 4p^2 \int d^2 b |h(\vec{b}, s)|^2 = \int d^2 b |1 - e^{i\chi(b)}|^2 \]

\[ \sigma_{in} \equiv \sigma_T - \sigma_{el} = \int d^2 b n_{in}(b) = \int d^2 b \left[ 1 - e^{-2\text{Im}\chi(b)} \right] \]

The inelasticity profile

\[ n_{in}(b) = 4p\text{Im} h(b, s) - 4p^2 |h(b, s)|^2 \]

satisfies \( n_{in}(b) \leq 1 \) (unitarity)
Dip in the inelasticity profile

From top to bottom: $\sqrt{s} = 14000, 7000, 200, 23.4$ GeV
Slope of the inelasticity profile

Transition around $\sqrt{s} = 5$ TeV
Amplitude and eikonal phase

\[ 2p h(b) = i \left[ 1 - e^{i\chi(b)} \right] \]

(top curves - Im, bottom - Re)
No classical folding of absorptive parts

The hollowness effect cannot be reproduced by folding of uncorrelated proton structures. We would then get, small $r$

$$W(r) = \int d^3 y \rho(\vec{y} + \vec{r}/2) \rho(\vec{y} - \vec{r}/2)$$

$$= \int d^3 y \rho(\vec{y})^2 - \frac{1}{4} \int d^3 y [\vec{r} \cdot \nabla \rho(\vec{y})]^2 + \ldots$$

→ $W(r)$ would necessarily have a local maximum at $r = 0$, in contrast to the phenomenological result

→ not possible to obtain hollowness classically by folding the absorptive parts from uncorrelated constituents
2D vs 3D opacity

Projection of 3D on 2D covers up the hollow: $f(x, y, z)$ vs $\int_{-\infty}^{\infty} dz f(x, y, z)$

The hollow is covered up
Aspects of unitarity: model of [Dremin 2014]

\[ 2p \text{Im} h(b) \equiv k(b) = 4X e^{-b^2/(2B)} \], \quad \text{Re} h(b) = 0, \quad X = \sigma_{el}/\sigma_T \]

\[ n_{in}(b) = 2k(b) - k(b)^2 = 8X e^{-b^2/(2B^2)} - 16X^2 e^{-b^2/B^2} \]

- \( X > 1/4 \): \( n_{in}(b) \) has a maximum at \( b_0 = \sqrt{2B} \log(4X) > 0 \), with \( k(b_0) = 1 \)
- \( X = 1/2 \): black disk limit
- \( W(r) \) develops a dip when \( X > \sqrt{2}/8 = 0.177 \)
Cross sections

Ratio goes above $1/4$ as energy increases!
If $2ph(b) = k(b)$ is not necessarily Gaussian but purely imaginary, then

$$n_{in}(b) = 2k(b) - k(b)^2$$

$$\frac{dn_{in}(b)}{db^2} = 2\frac{dk(b)}{db^2}[1 - k(b)]$$

hence the minimum of $n(b)$ moves away from the origin when $k(0) > 1$

The real part of the amplitude, which is $\sim 10\%$, brings in corrections at the level of $1\%$
Inverse scattering problem

- Are low energy nuclear reactions and high NN interactions that different?

![Graph showing inverse scattering](image)

- \(\sqrt{s} = 7 \text{ TeV}\)
- \(\sqrt{s} = 23-62 \text{ GeV}\)

- \(\text{ImW}(r) / \text{ImW}(0)\)
- \(\text{ReV}(r), \text{ImV}(r)\)

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The phase ambiguity

- We measure the modulus at finite $t$

$$f(s, t) = \text{Re} f(s, t) + i \text{Im} f(s, t)$$

$$\frac{d\sigma_{el}}{dt} = \frac{\pi}{p^2} |f(s, t)|^2$$

- $$f(s, t) = |f(s, t)| \frac{\rho(s, t) + i}{\sqrt{1 + \rho(s, t)^2}}$$

$$\rho(s, t) = \frac{\text{Re} f(s, t)}{\text{Im} f(s, t)}$$

- Fixed-t dispersion relation

$$\text{Re} f(s, t) = \frac{1}{\pi} \int_{4M^2}^{\infty} ds' \frac{\text{Im} f(s', t)}{s' - s}.$$

- Regge Compatible with fixed-t dispersion relations

$$\frac{1}{\pi} \int_{0}^{\infty} ds' s'^\alpha \left[ \frac{1}{s' - s} \pm \frac{1}{s' + s} \right] = \frac{(-s)^\alpha \pm (s)^\alpha}{\sin \pi \alpha}.$$
Regge theory fits data

- Double pomeron pole
Regge theory may produce hollowness

- High energies
Regge theory may also produce hollowness

- The diffractive correlation (Goulianos 1996) at 500 GeV

\[
\frac{\sigma_{el}}{\sigma_{tot}} = \frac{(1 + \rho^2)\sigma_{tot}}{4\pi B}
\]
Conclusions

- Some models predict that there is a **hollowness transition** at about 1000+ GeV
- Quantum effect, rise of $2p\text{Im}h(b)$ above 1
- Not possible to obtain classically by folding the absorptive parts from uncorrelated constituents (incompatible with geometric models)
- $2D \rightarrow 3D$ magnifies the effect (flat in 2D means hollow in 3D)
- Similar hollowness effect in low-energy pp scattering
- There are models (COMPETE, PDG) which DO NOT produce hollowness.