

Parton distribution functions from Lattice QCD

Krzysztof Cichy
Adam Mickiewicz University, Poznań, Poland

in collaboration with:

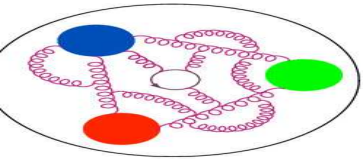
Constantia Alexandrou (Univ. of Cyprus, Cyprus Institute)
Martha Constantinou (Temple University, Philadelphia)
Karl Jansen (DESY Zeuthen)
Aurora Scapellato (Univ. of Cyprus, Univ. of Wuppertal)
Fernanda Steffens (Univ. of Bonn)



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Outline of the talk



1. Lattice QCD

- Why do we need this?
- Lattice formulation of QCD
- QCD simulations
- What can we compute?

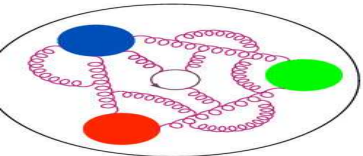
2. Parton distribution functions

- Basics
- Lattice calculation
- Results

3. Conclusions and prospects

Results on PDFs based on:

- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, “Reconstruction of light-cone parton distribution functions from lattice QCD simulations at the physical point”, Phys. Rev. Lett. 121 (2018) 112001
- C. Alexandrou, K. Cichy, M. Constantinou, K. Jansen, A. Scapellato, F. Steffens, “Transversity parton distribution functions from lattice QCD”, Phys. Rev. D (Rapid Communications), in press, arXiv: 1807.00232 [hep-lat]
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, H. Panagopoulos, F. Steffens, “A complete non-perturbative renormalization prescription for quasi-PDFs”, Nucl. Phys. B923 (2017) 394-415 (invited Frontiers Article)
- C. Alexandrou, K. Cichy, M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, “Updated Lattice Results for Parton Distributions”, Phys. Rev. D96 (2017) 014513
- C. Alexandrou, K. Cichy, V. Drach, E. Garcia-Ramos, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, “A Lattice Calculation of Parton Distributions”, Phys. Rev. D92 (2015) 014502



Outline of the talk

Lattice QCD

Need for lattice

Lattice formulation

Discretization

Discretization

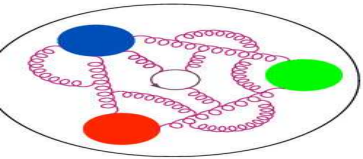
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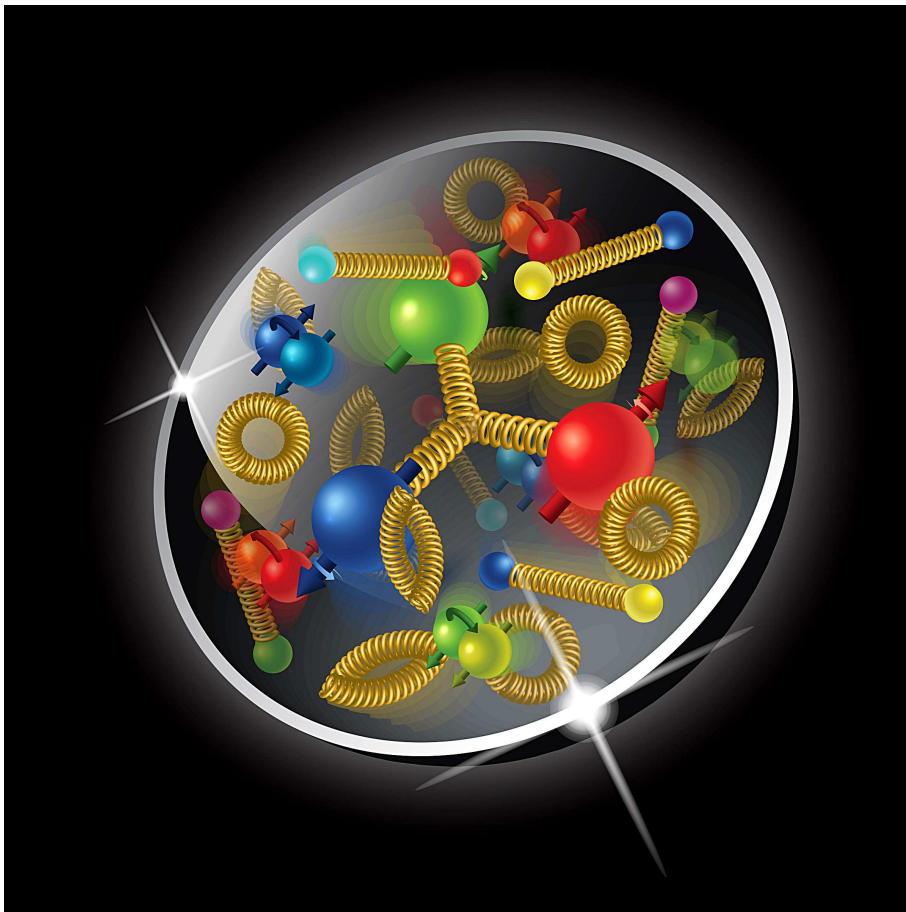
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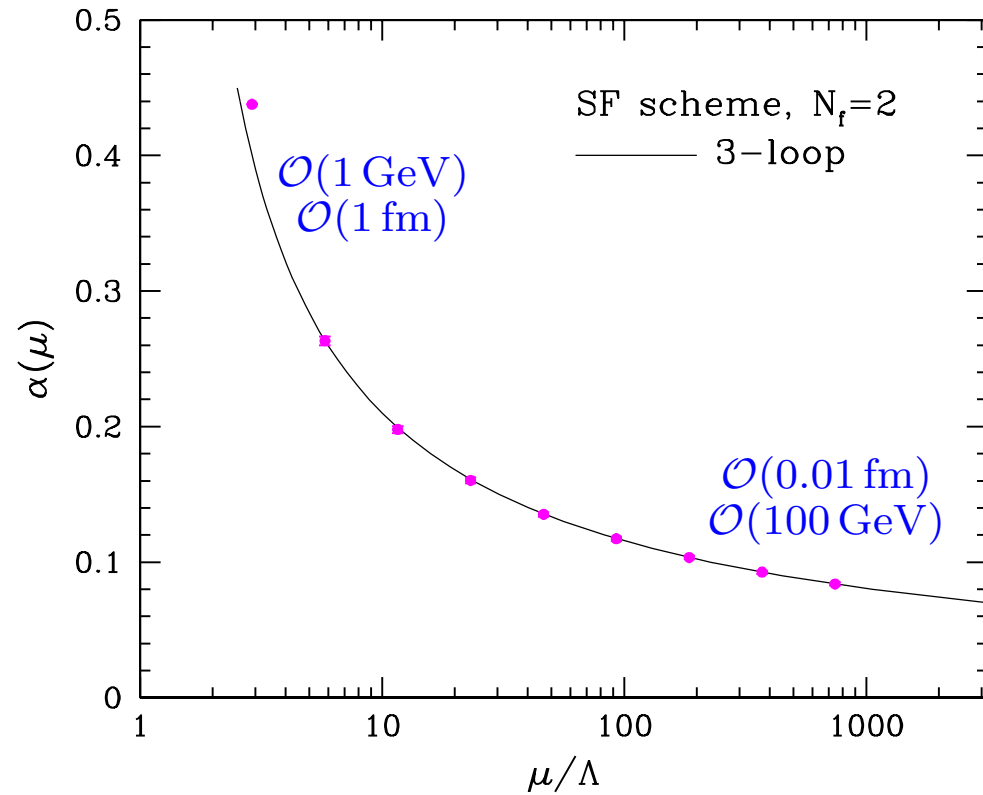
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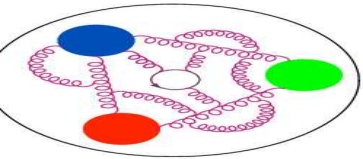


QCD and the need for the lattice

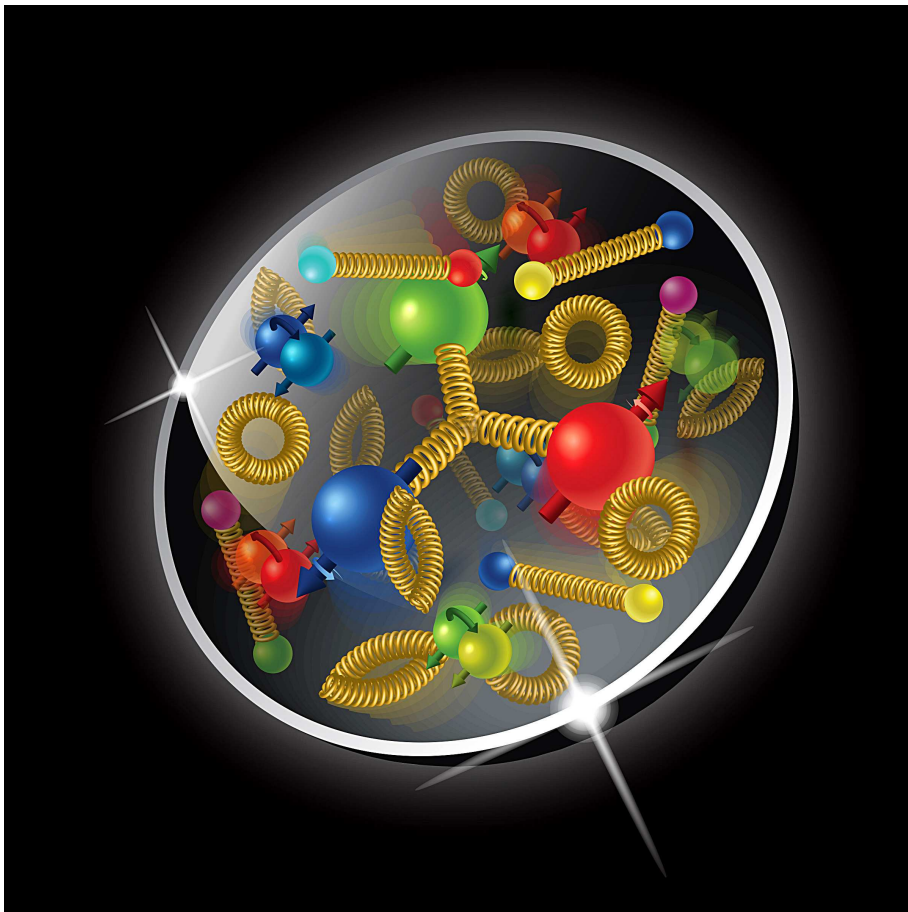


ALPHA Collaboration, 2004

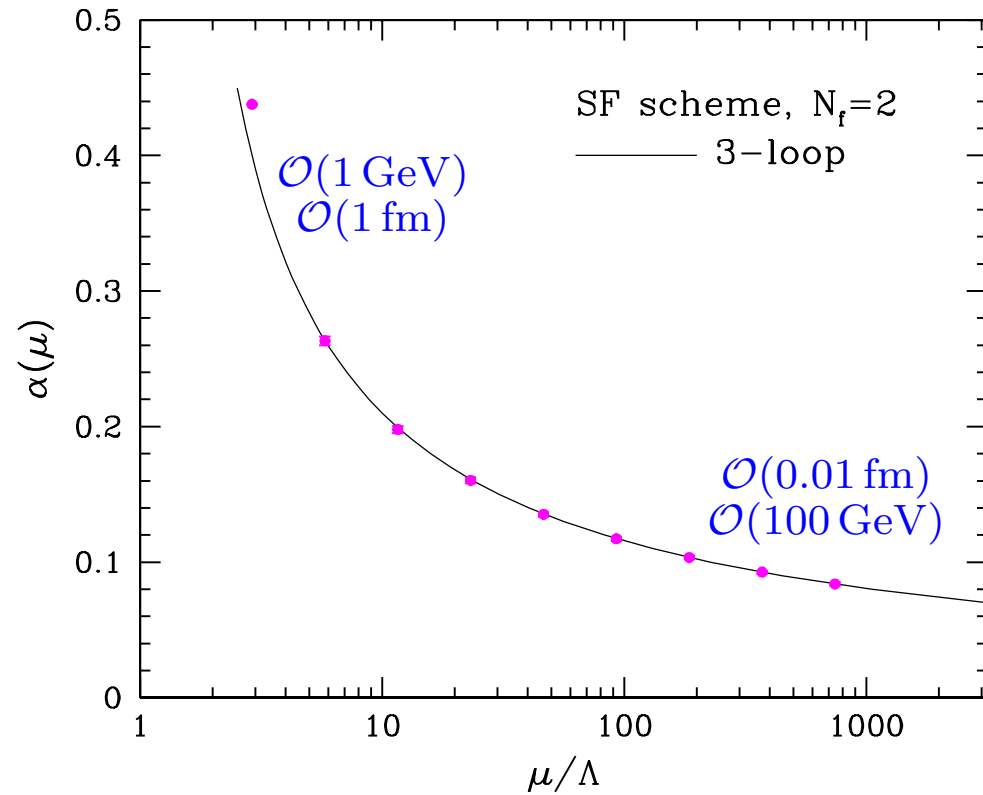




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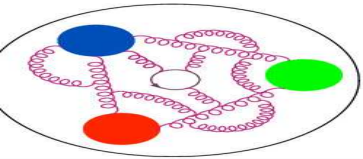
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NON-PERTURBATIVE REGIME



quantitative study needs LATTICE



QCD and the need for the lattice



Lagrangian of QCD:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \sum_{(f)=1}^{N_f} \bar{\psi}_{(f)} (i\gamma^\mu D_\mu - m_{(f)}) \psi_{(f)}$$

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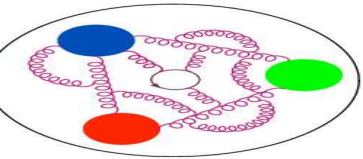
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Need to **quantize** the theory – **Euclidean** path integral:

$$\langle 0 | \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) | 0 \rangle = \frac{\int DA_\mu D\psi D\bar{\psi} \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) e^{-S[A_\mu, \psi, \bar{\psi}]}}{\int DA_\mu D\psi D\bar{\psi} e^{-S[A_\mu, \psi, \bar{\psi}]}}.$$

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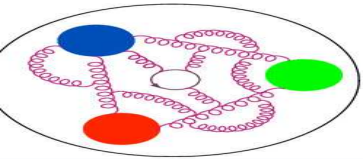
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Note:

- Minkowski path integral can not be used in practice – the phase factor e^{iS} would lead to oscillatory behaviour.
- Hence, it is replaced (analytical continuation) by a real valued exponential e^{-S} , formally one then evaluates a thermodynamic expectation value with respect to the Boltzmann factor e^{-S} .

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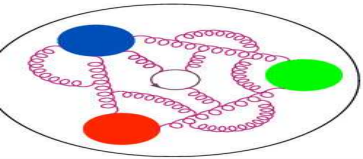
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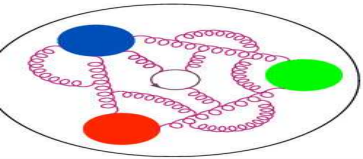
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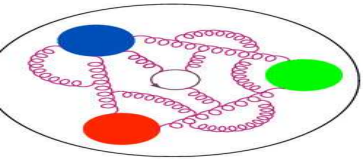
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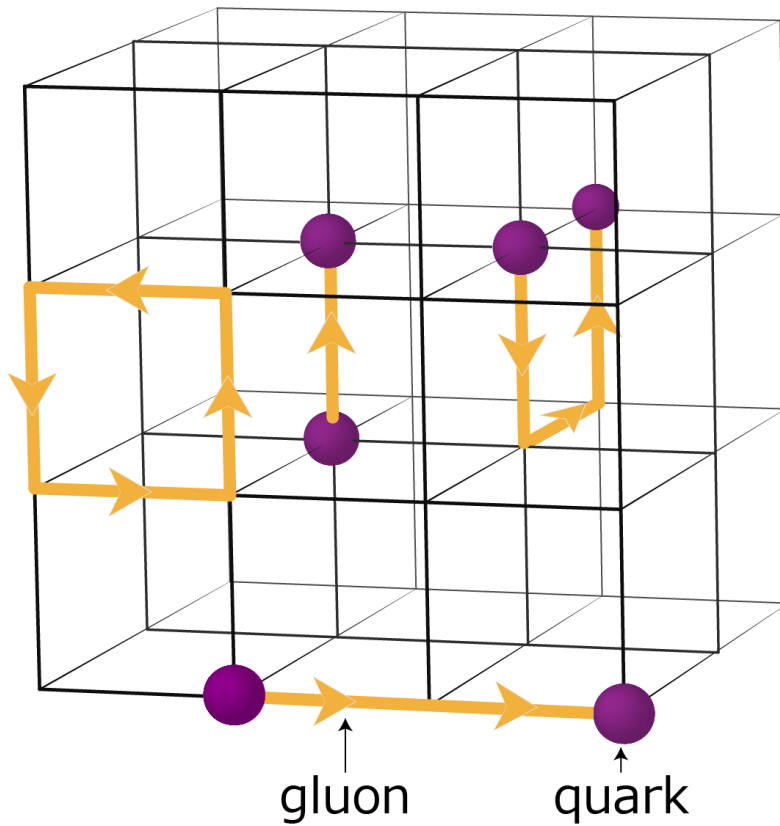
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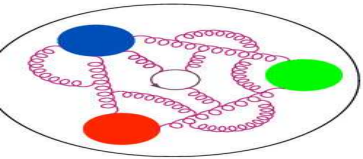


Lattice formulation

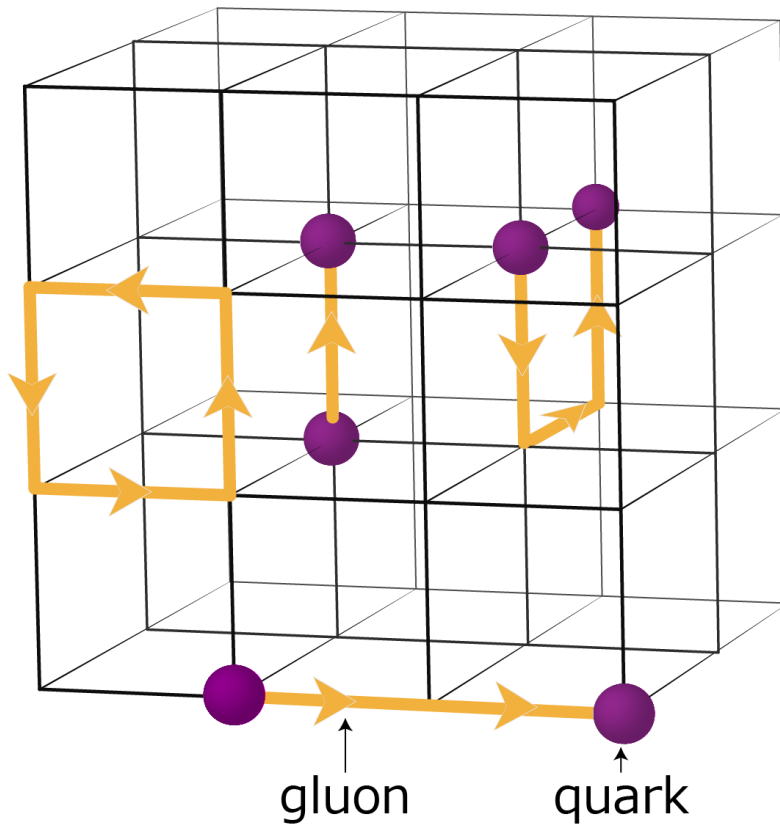
- We introduce a 4D hypercubic lattice:
 - ★ quark fields on lattice sites,
 - ★ gluon fields on lattice links.



Source: JICFuS, Tsukuba

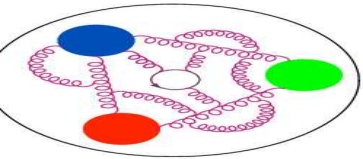


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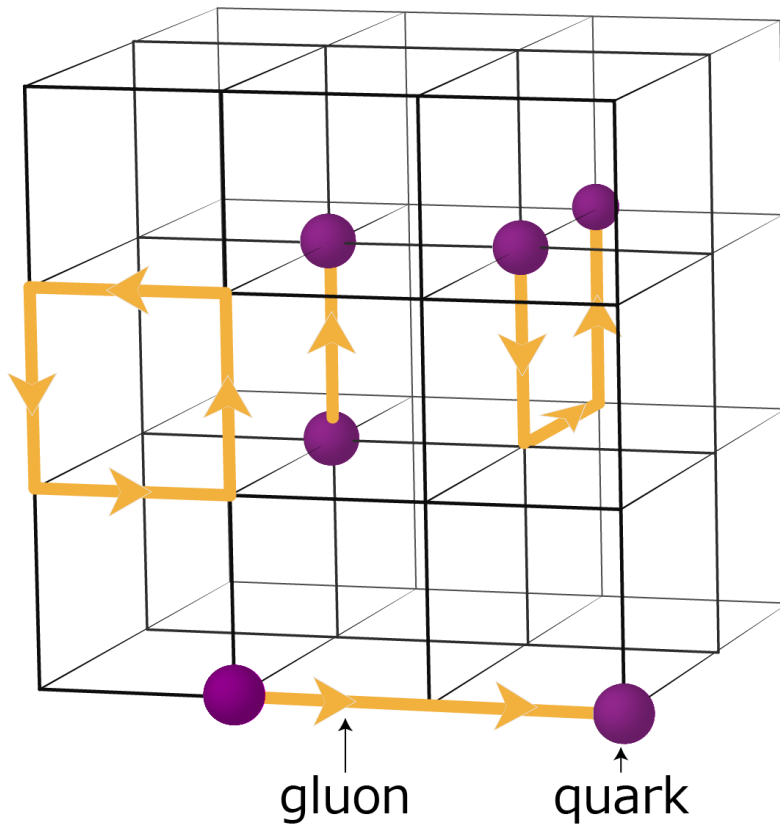


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- Gauge invariant objects:
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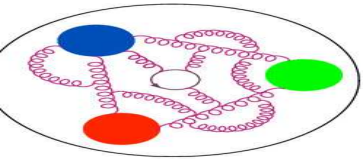


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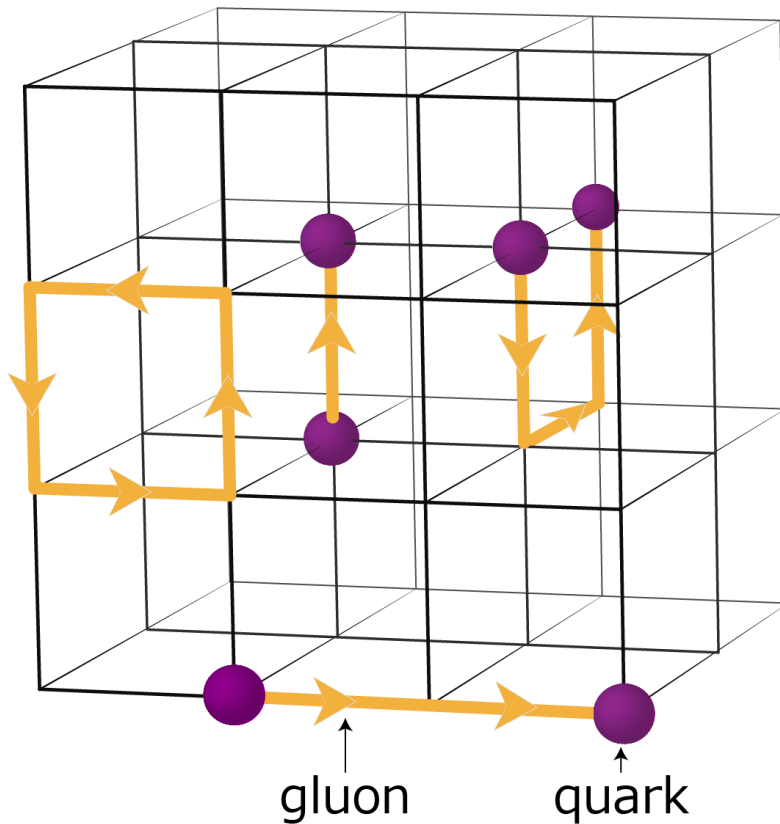


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- Lattice as a regulator:
 - ★ UV cut-off – inverse lat. spac. a^{-1} ,
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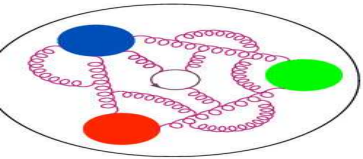


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- Remove the regulator:
 - ★ continuum limit $a \rightarrow 0$,
 - ★ infinite volume limit $L \rightarrow \infty$.

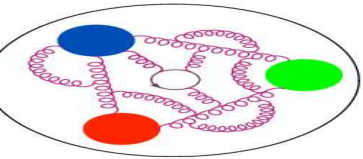


Discretization of the action

- gluonic part – “easy” – gauge action constructed from Wilson loops of size 1x1 (plaquettes) and 1x2 (rectangles):

$$S_G[U] = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\mu, \nu=1} \text{Re Tr}(1 - P_{x; \mu, \nu}^{1 \times 1}) + b_1 \sum_{\mu \neq \nu} \text{Re Tr}(1 - P_{x; \mu, \nu}^{1 \times 2}) \right),$$

where $\beta = 6/g_0^2$, g_0 is the bare coupling and the b_0, b_1 parameters are normalized according to: $b_0 = 1 - 8b_1$.



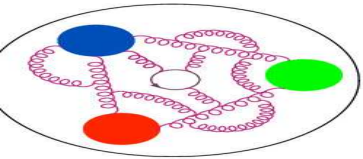
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$$-\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \mathcal{O}(a^2)$$



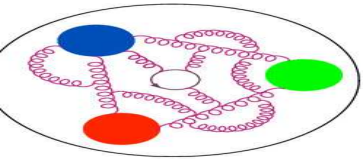
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- fermionic part – subtleties arise from the discretization of the derivative:
 - ★ fermion doubling problem



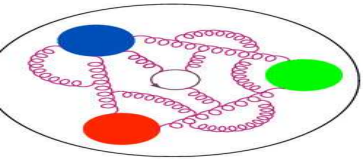
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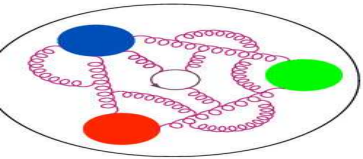


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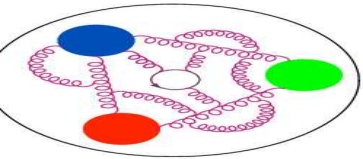
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- ★ fermion doubling problem
- ★ breaking of chiral symmetry
- ★ scaling towards the continuum limit
- ★ discretizations used in practice:

- ◇ clover fermions,
- ◇ twisted mass (TM) fermions,
- ◇ overlap fermions,
- ◇ domain wall fermions,
- ◇ staggered fermions,
- ◇ other less popular.

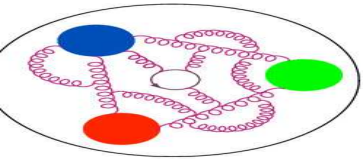




Simulating QCD on the lattice



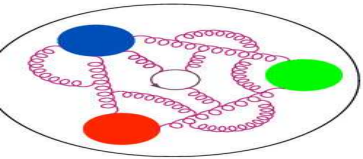
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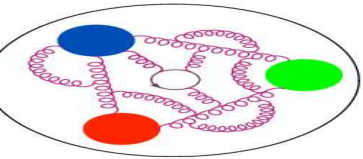
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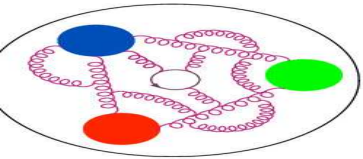
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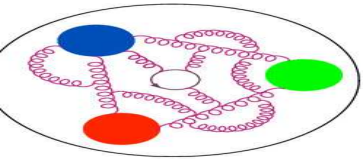
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- How many dimensional integral?
 - ★ typical lattice size: $48 \times 48 \times 48 \times 96$, $64 \times 64 \times 64 \times 128$,
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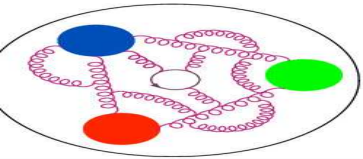
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- Hence, huge computational resources needed!
- QCD was one of the first branches of science that “asked” for such computational resources and thus inspired the development of supercomputers.



Systematic effects



Ultimately we are interested in continuum QCD.

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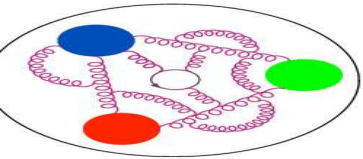
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the possibility to control ALL conceivable systematic effects:

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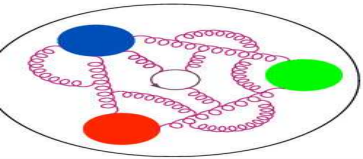
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The power of the lattice approach:

the possibility to control ALL conceivable systematic effects:

- discretization effects \leftrightarrow continuum limit

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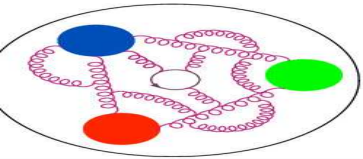
Discretization

QCD simulations

Parton distribution
functions (PDFs)

Results

Summary



Systematic effects



Ultimately we are interested in continuum QCD.

The power of the lattice approach:

the possibility to control ALL conceivable systematic effects:

- discretization effects \leftrightarrow continuum limit
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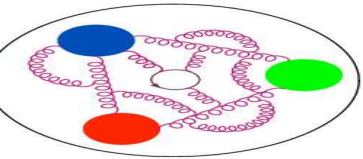
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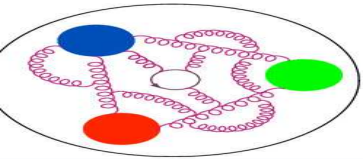
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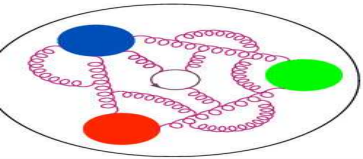
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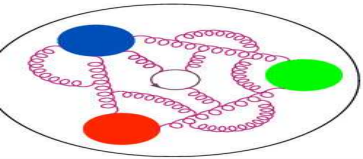
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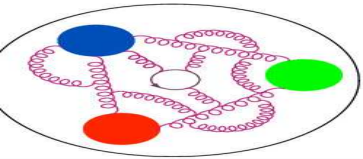
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- renormalization \leftrightarrow non-perturbative procedures

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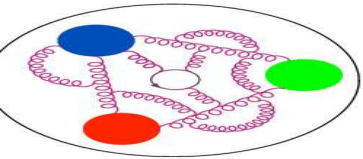
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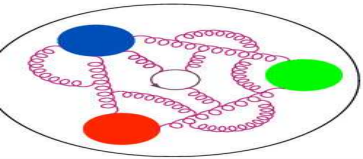
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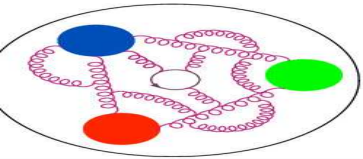
Some of the aspects of QCD that can be studied on the lattice:

- QCD parameters: α_s , Λ_{QCD} , quark masses etc.
- hadron spectrum: meson and baryon masses, exotic hadrons
- hadron structure: nucleon charges, EM form factors, **parton distribution functions**, GPDs, nucleon spin content
- QCD thermodynamics: QCD phase diagram, deconfinement, chiral symmetry restoration
- Standard Model parameters: CKM matrix
- constraints on effective theories: χ PT, HQET



Some collaborations in LQCD:

Alpha, BMW, CLS, CP-PACS, **ETMC**, HALQCD, hotQCD, JLQCD, LHC, LSD, Mainz, MILC, NME, NPLQCD, QCDSF, PNDME, RBC, RQCD, SWME, tmFT, TWQCD, UKQCD, USQCD, WHOT-QCD
in total $\approx 500 - 600$ physicists



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PDFs

Quasi-PDFs

Renormalization

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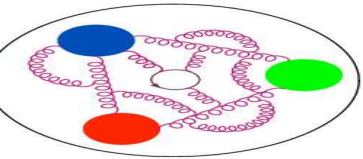
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PDFs

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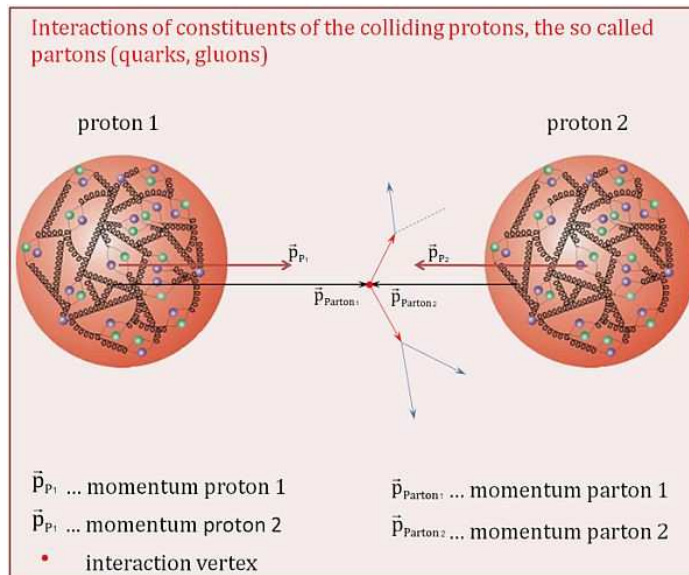
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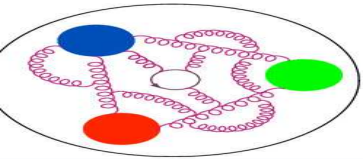
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Summary

- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.



Source: LHC, CERN



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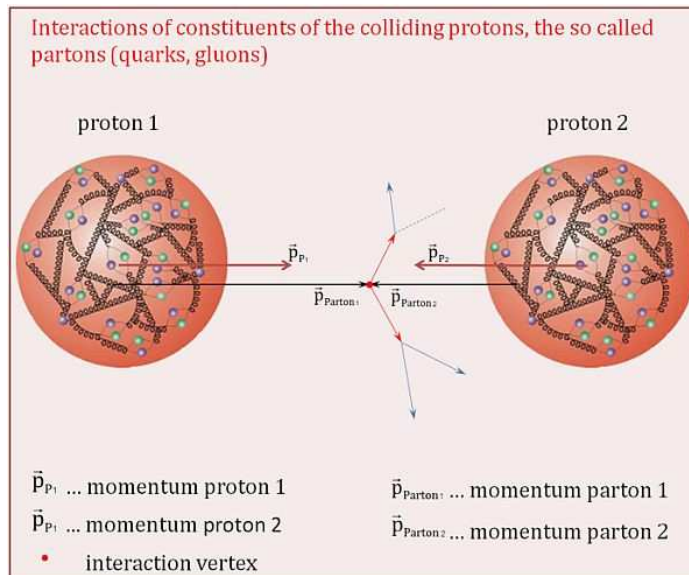
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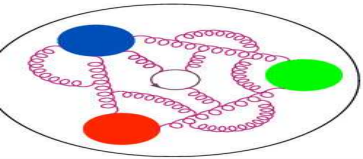
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- Hadrons are complicated systems with properties resulting from the strong dynamics of quarks and gluons inside them.
- This dynamics is characterized in terms of, among others, parton distribution functions (PDFs).
- PDFs are essential in making predictions for collider experiments.



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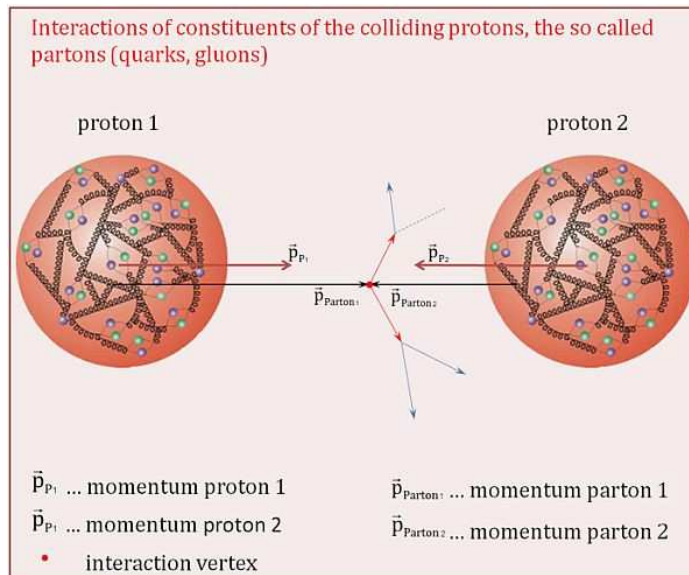
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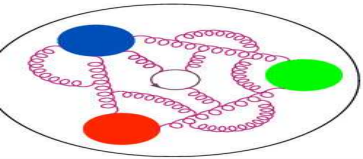
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$$\sigma_{AB} = \sum_{a,b=q,g} \sigma_{ab} \otimes f_{a|A}(x_1, Q^2) \otimes f_{b|B}(x_2, Q^2)$$



Source: LHC, CERN



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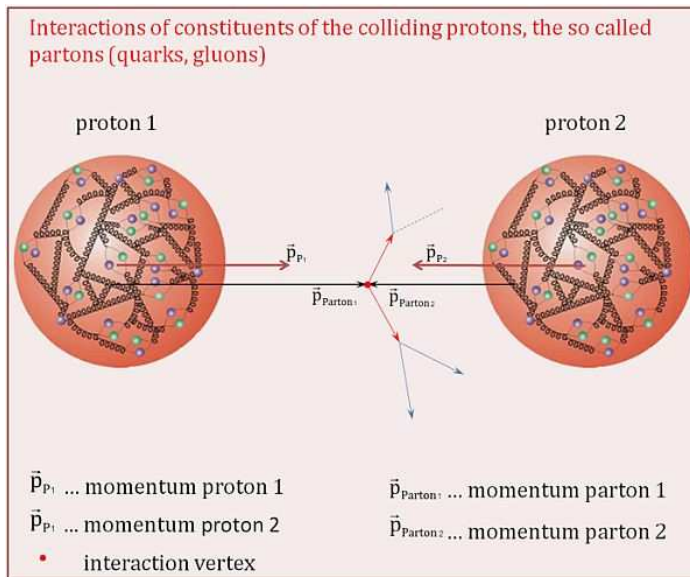
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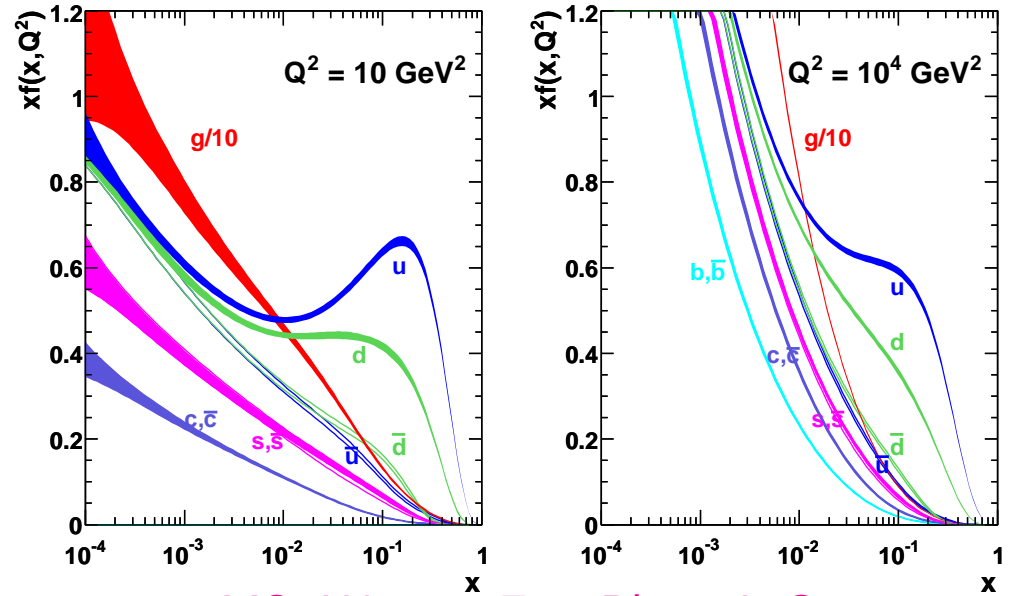
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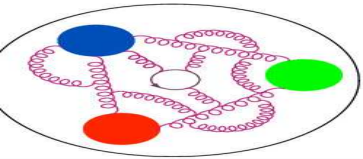
MSTW 2008 NLO PDFs (68% C.L.)



Source: LHC, CERN



MSTW2008, Eur. Phys. J. C63, 189



PDFs – why is it difficult on the lattice?



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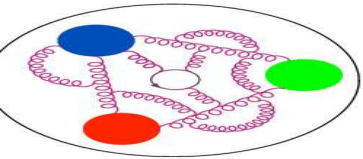
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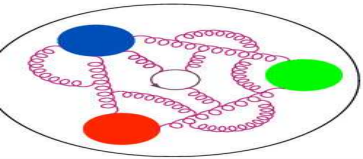
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- But: PDFs given in terms of non-local light-cone correlators – intrinsically Minkowskian – problem for the lattice!



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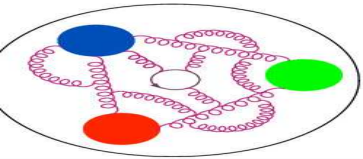
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where: $\xi^- = \frac{\xi^0 - \xi^3}{\sqrt{2}}$ and $\mathcal{A}(\xi^-, 0)$ is the Wilson line from 0 to ξ^- .

- This expression is light-cone dominated – needs $\xi^2 = \vec{x}^2 + t^2 \sim 0$ – very hard due to non-zero lattice spacing!



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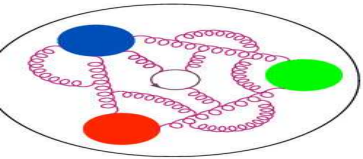
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- Accessible on the lattice – moments of the distributions, but ...

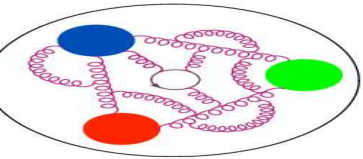


Quasi-PDFs



- Quasi-PDF approach:

*X. Ji, Parton Physics on a Euclidean Lattice, Phys. Rev. Lett. **110** (2013) 262002*



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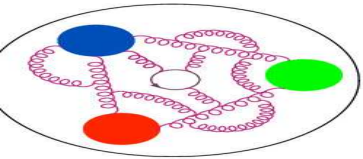


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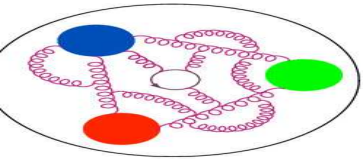
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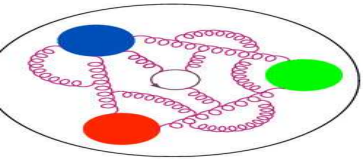
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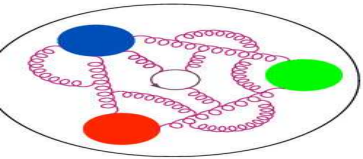
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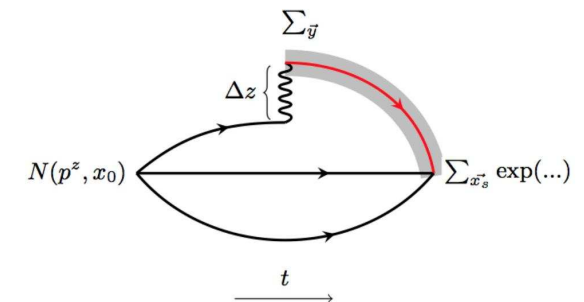
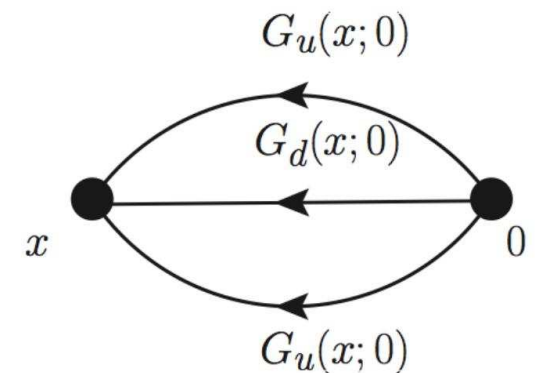
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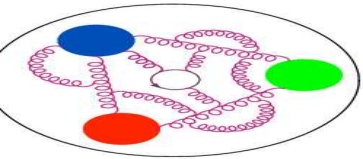
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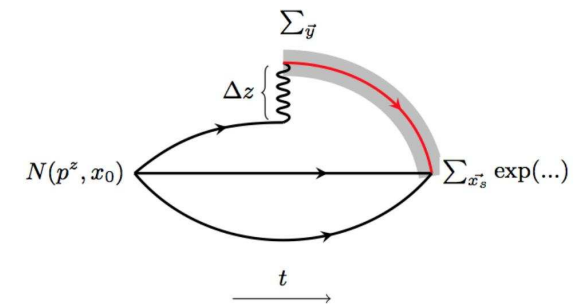
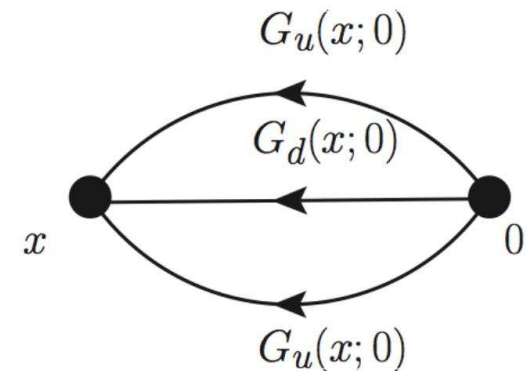
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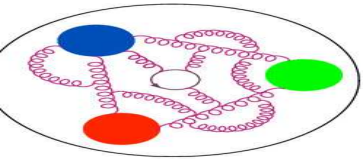
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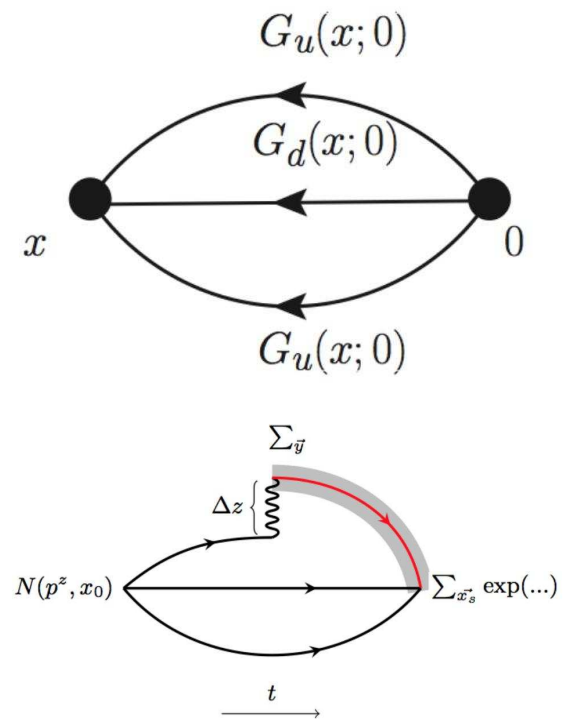
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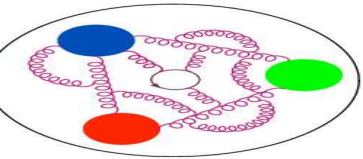
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- The highly non-trivial aspect:
how to relate $\tilde{q}(x, \mu^2, P_3)$ to the light-front PDF $q(x, \mu^2)$ (infinite momentum frame)
 \Rightarrow **Large Momentum Effective Theory (LaMET)**





Renormalization



Bare matrix elements $\langle N | \bar{\psi}(z) \Gamma \mathcal{A}(z, 0) \psi(0) | N \rangle$ contain divergences that need to be removed:

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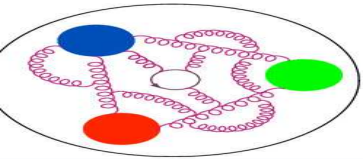
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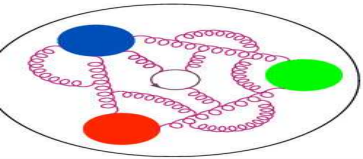
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- power divergence related to the Wilson line; resums into a multiplicative exponential factor, $\exp(-\delta m |z|/a + c|z|)$
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 c – arbitrary scale (fixed by the renormalization prescription).

Outline of the talk

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PDFs

Quasi-PDFs

Renormalization

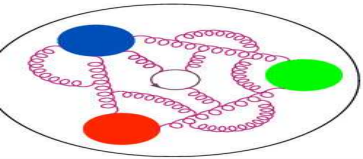
Matching

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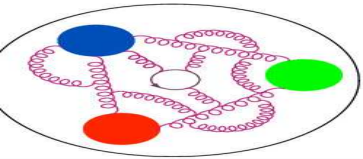
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→ mixing of $\Gamma = \gamma_3$ and $\Gamma = \mathbf{1}$, important guidance to non-pert. renormalization!



Renormalization



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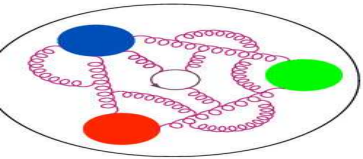
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Non-perturbative renormalization scheme: **RI'-MOM**.

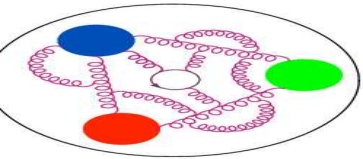
G. Martinelli et al., Nucl. Phys. B445 (1995) 81



Matching of quasi-PDFs and PDFs



To relate the quasi-PDFs to the usual PDFs, one uses the fact that the IR region of the distributions is untouched when going from a finite to an infinite momentum. In other words, if $q(x, \mu)$ is the usual PDF defined through light-cone correlations, then one should have:



Matching of quasi-PDFs and PDFs

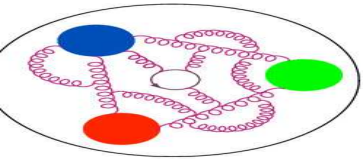


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$$q(x, \mu) = q_{bare}(x) \left\{ 1 + \frac{\alpha_s}{2\pi} Z_F(\mu) \right\} + \frac{\alpha_s}{2\pi} \int_x^1 q^{(1)}(x/y, \mu) q_{bare}(y) \frac{dy}{y} + \mathcal{O}(\alpha_s^2),$$

$$\tilde{q}(x, \Lambda, P_3) = q_{bare}(x) \left\{ 1 + \frac{\alpha_s}{2\pi} \tilde{Z}_F(\Lambda, P_3) \right\} + \frac{\alpha_s}{2\pi} \int_{x/x_c}^1 \tilde{q}^{(1)}(x/y, \Lambda, P_3) q_{bare}(y) \frac{dy}{y} + \mathcal{O}(\alpha_s^2),$$

where: q_{bare} – bare distribution, Z_F, \tilde{Z}_F – wave function corrections, $q^{(1)}, \tilde{q}^{(1)}$ – vertex corrections.



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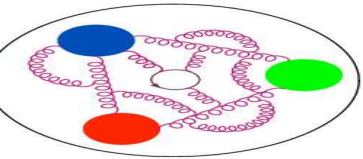
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Explicit formulae for 1-loop perturbative matching:

- transverse momentum cutoff scheme to $\overline{\text{MS}}$ matching
[X. Xiong et al., PRD 90 \(2014\) 014051](#)
- $\overline{\text{MS}}$ to $\overline{\text{MS}}$ matching [W. Wang, S. Zhao, R. Zhu, arXiv:1708.02458 \[hep-ph\]](#)
- RI to $\overline{\text{MS}}$ matching [I.W. Stewart, Y. Zhao, arXiv:1709.04933 \[hep-ph\]](#)
- treatment of the UV log divergence in wave function corrections [T. Izubuchi et al., arXiv:1801.03917 \[hep-ph\]](#), [C. Alexandrou et al., arXiv:1803.02685, 1807.00232 \[hep-lat\]](#)



Summary of the procedure



The procedure to obtain light-cone PDFs from the lattice computation can be summarized as follows:

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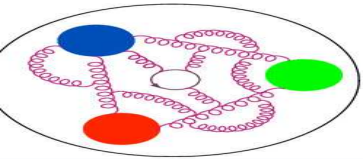
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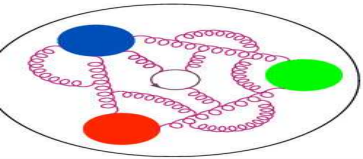
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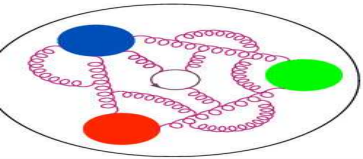
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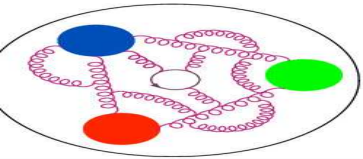
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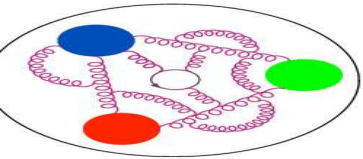
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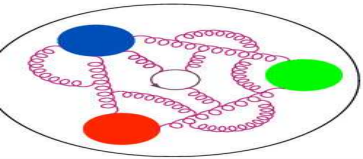
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$$\tilde{q}(x, \mu^2, P_3) = \int \frac{dz}{4\pi} e^{ixP_3z} \langle N | \bar{\psi}(z) \gamma^z \mathcal{A}(z, 0) \psi(0) | N \rangle.$$



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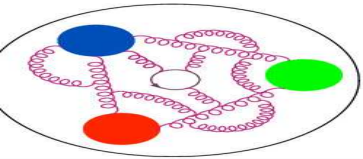
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7. Apply target mass corrections to eliminate residual m_N/P_3 effects.

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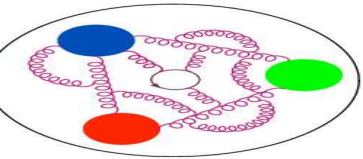
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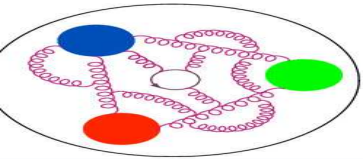
Summary

- fermions: $N_f = 2$ twisted mass fermions + clover term
- gluons: Iwasaki gauge action, $\beta = 2.1$

$\beta=2.10,$	$c_{\text{SW}}=1.57751,$	$a=0.0938(3)(2)$ fm
$48^3 \times 96$	$a\mu = 0.0009$	$m_N = 0.932(4)$ GeV
$L = 4.5$ fm	$m_\pi = 0.1304(4)$ GeV	$m_\pi L = 2.98(1)$



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001
C. Alexandrou et al., arXiv: 1807.00232 [hep-lat]



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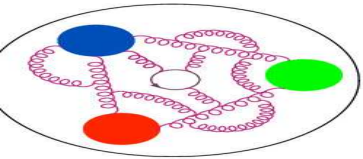
Matched PDFs

Final PDFs

Systematics

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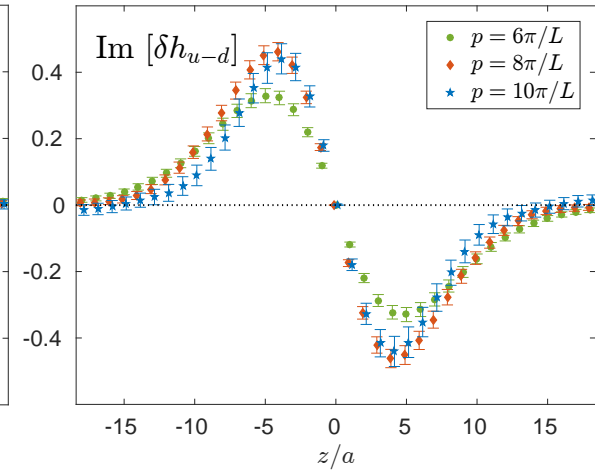
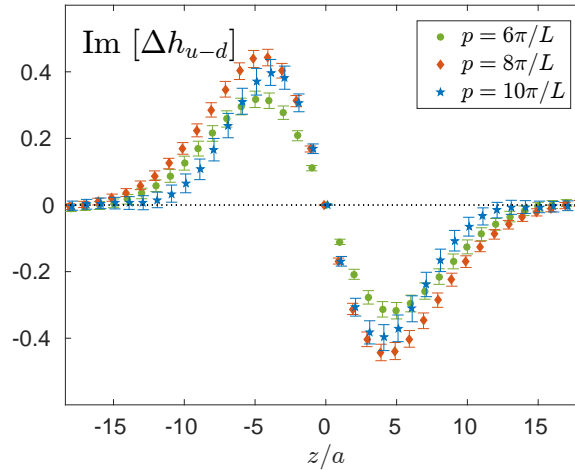
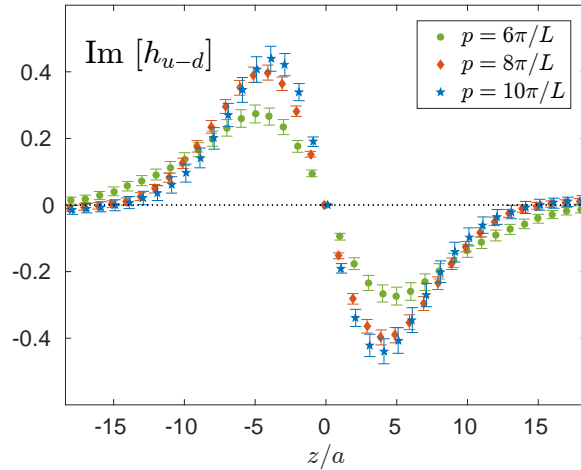
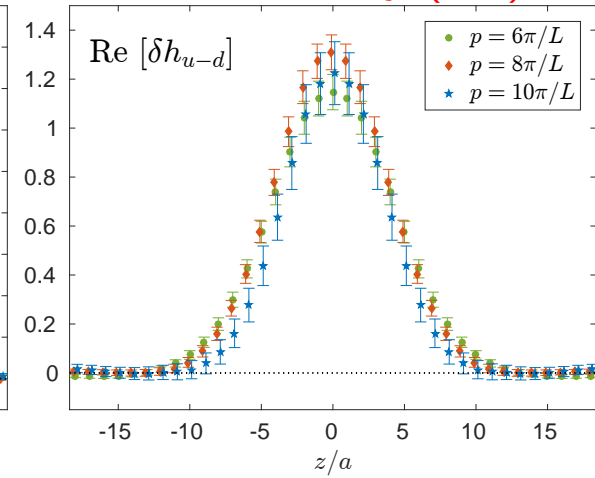
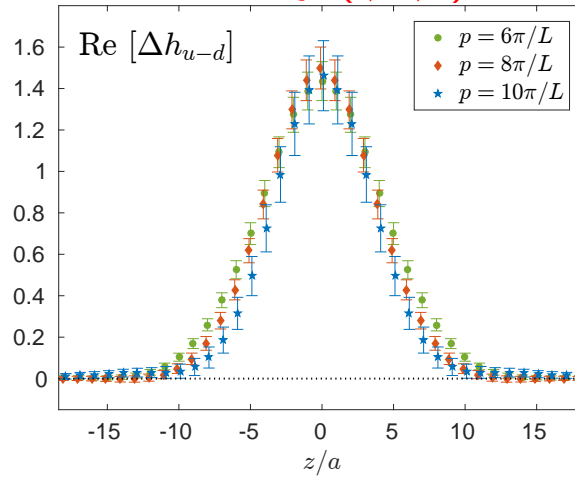
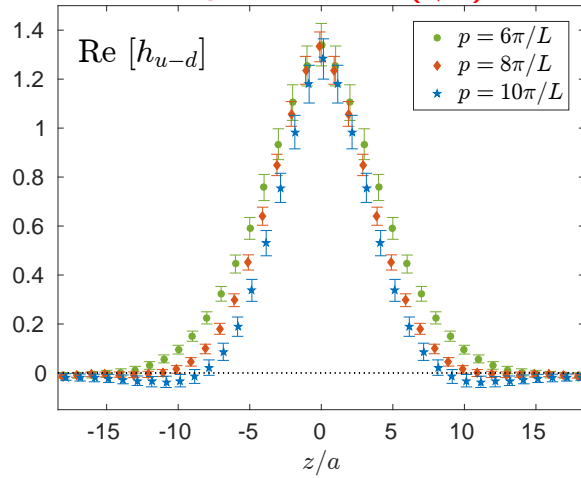


Bare matrix elements at $t_s = 12a$

unpolarized (γ_0)

helicity ($\gamma_5 \gamma_3$)

transversity (σ_{3i})



C. Alexandrou et al.: Phys. Rev. Lett. 121 (2018) 112001 and 1807.00232 [hep-lat]

STATISTICS:

$$P_3 = \frac{6\pi}{L} - 4800 \text{ meas.}$$

$$P_3 = \frac{8\pi}{L} - 38250 \text{ meas.}$$

$$P_3 = \frac{10\pi}{L} - 72990 \text{ meas.}$$

$$P_3 = \frac{6\pi}{L} - 6240 \text{ meas.}$$

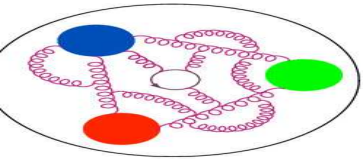
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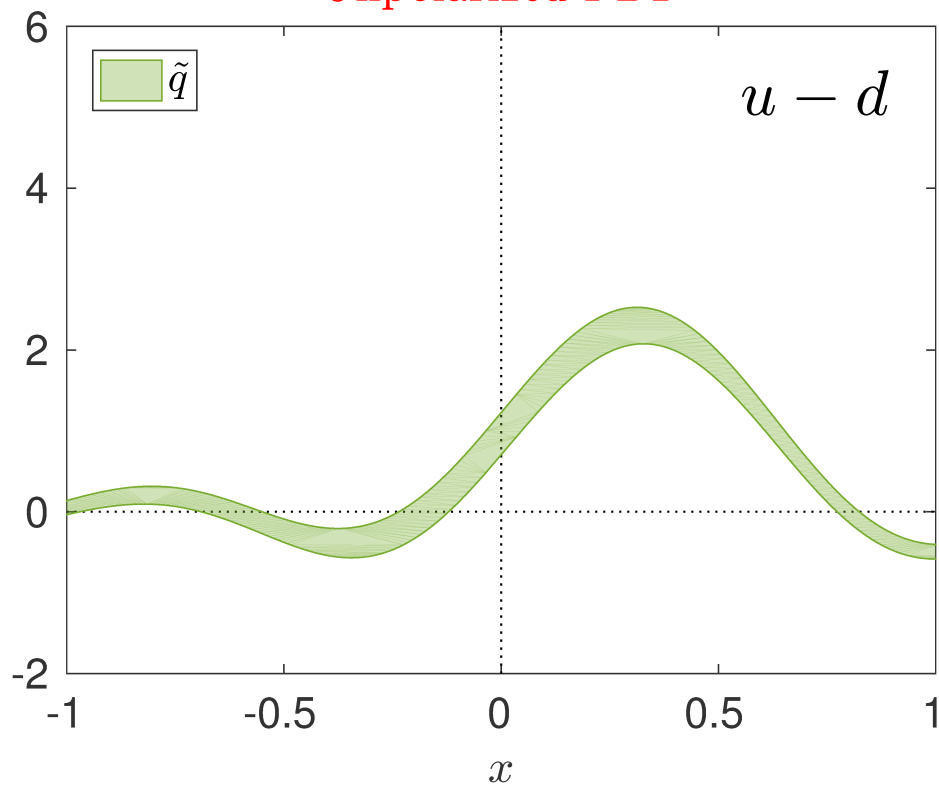


Quasi-PDFs

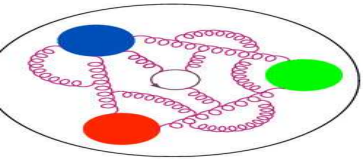


Nucleon momentum $\frac{10\pi}{48}$

Unpolarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

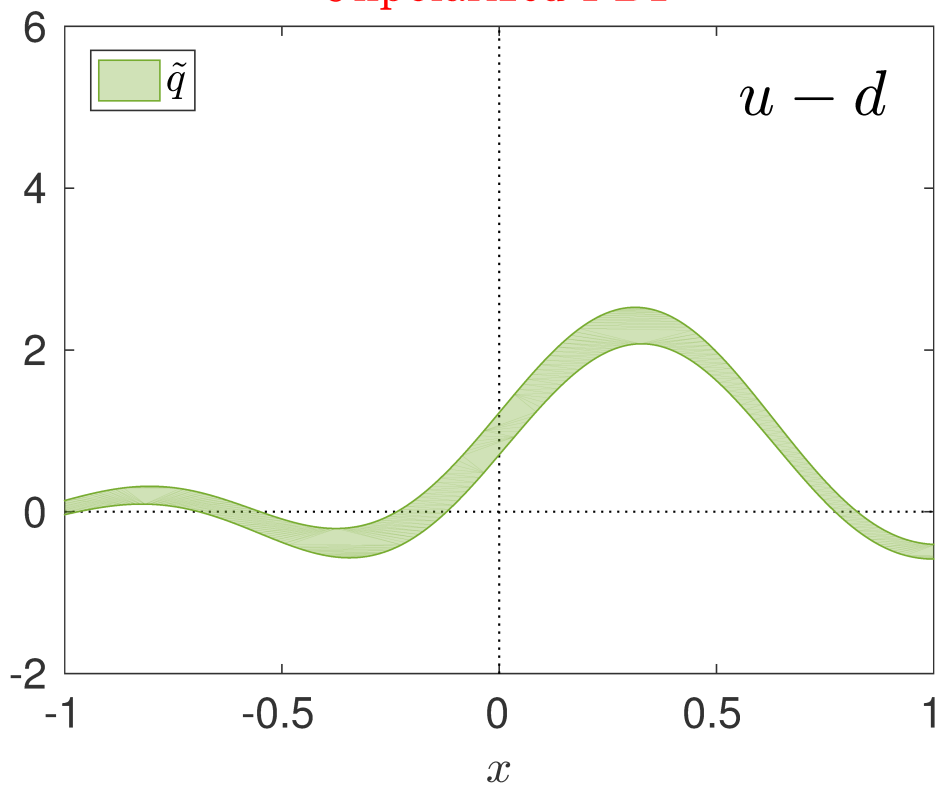


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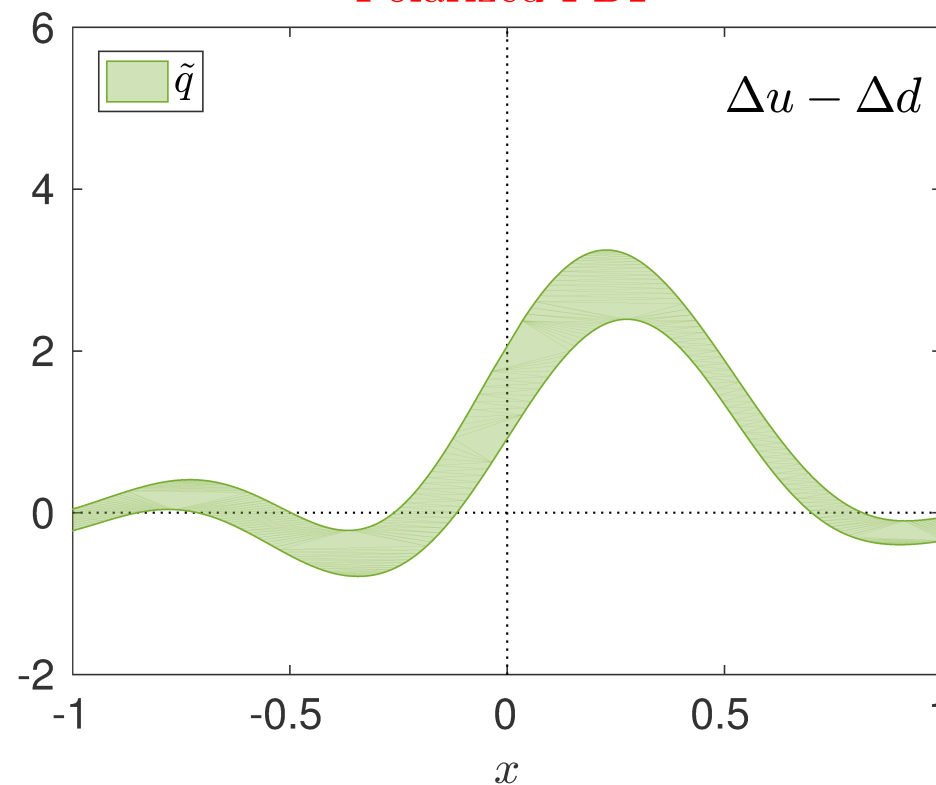


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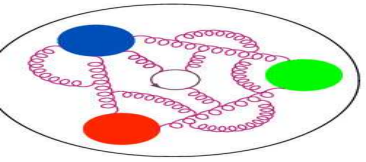
Unpolarized PDF



Polarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

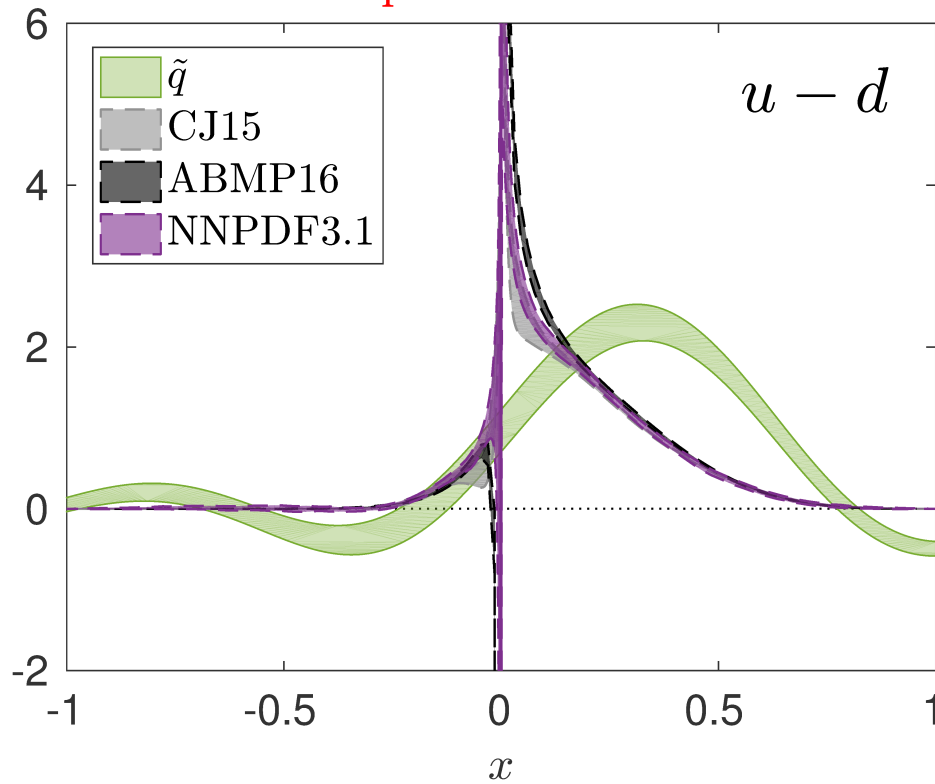


Quasi-PDFs + pheno

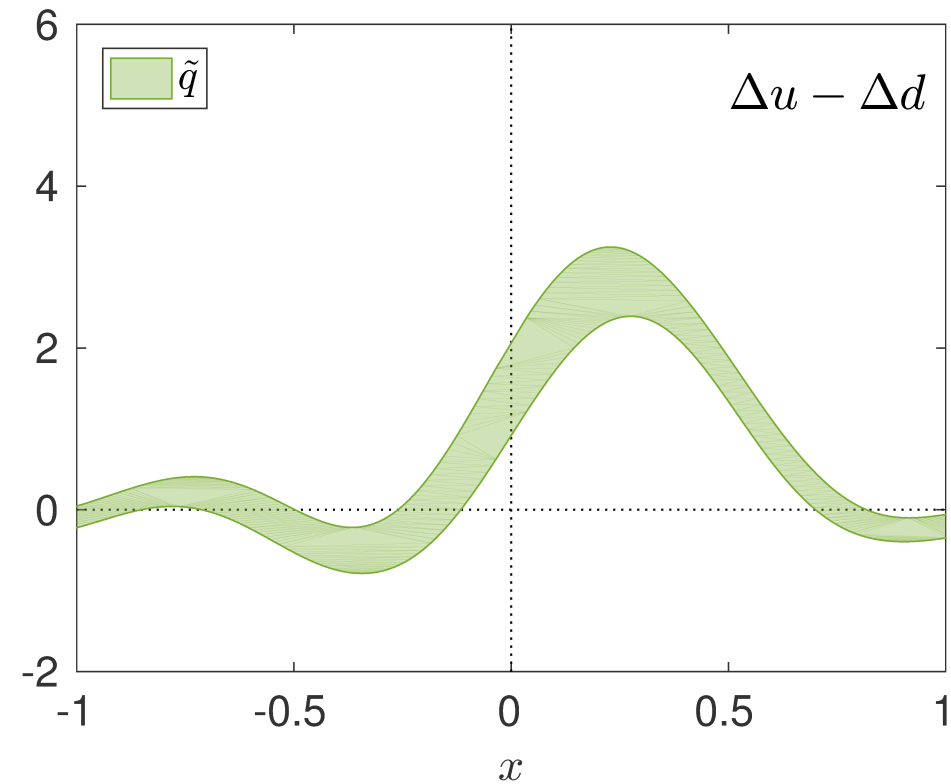


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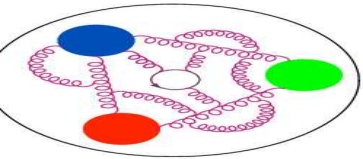
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C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

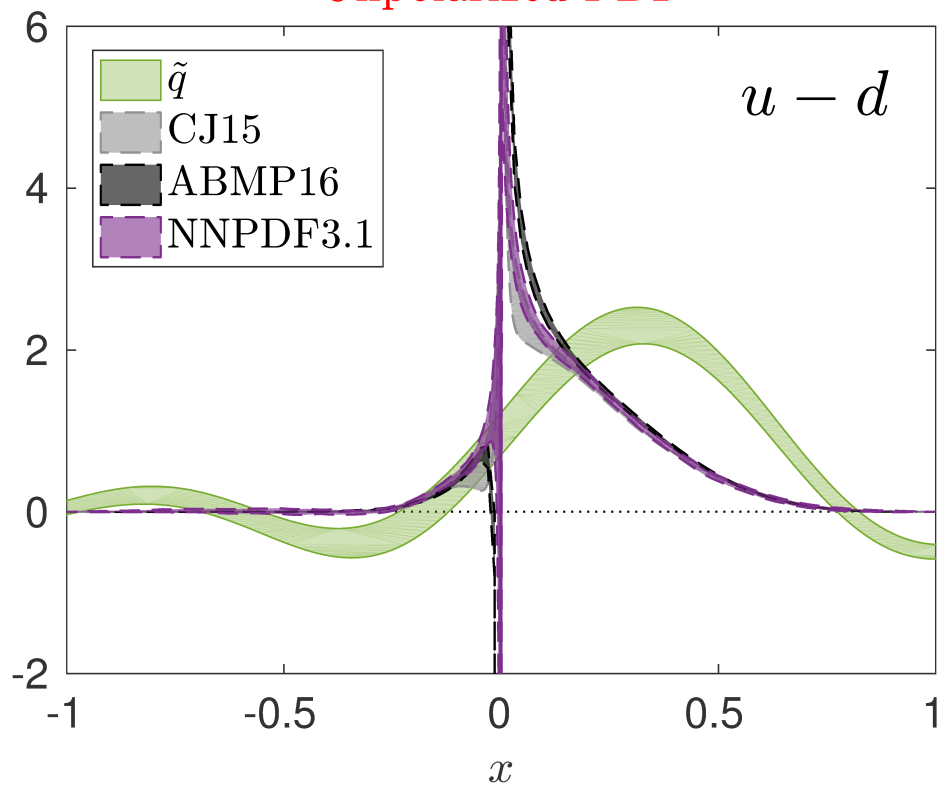


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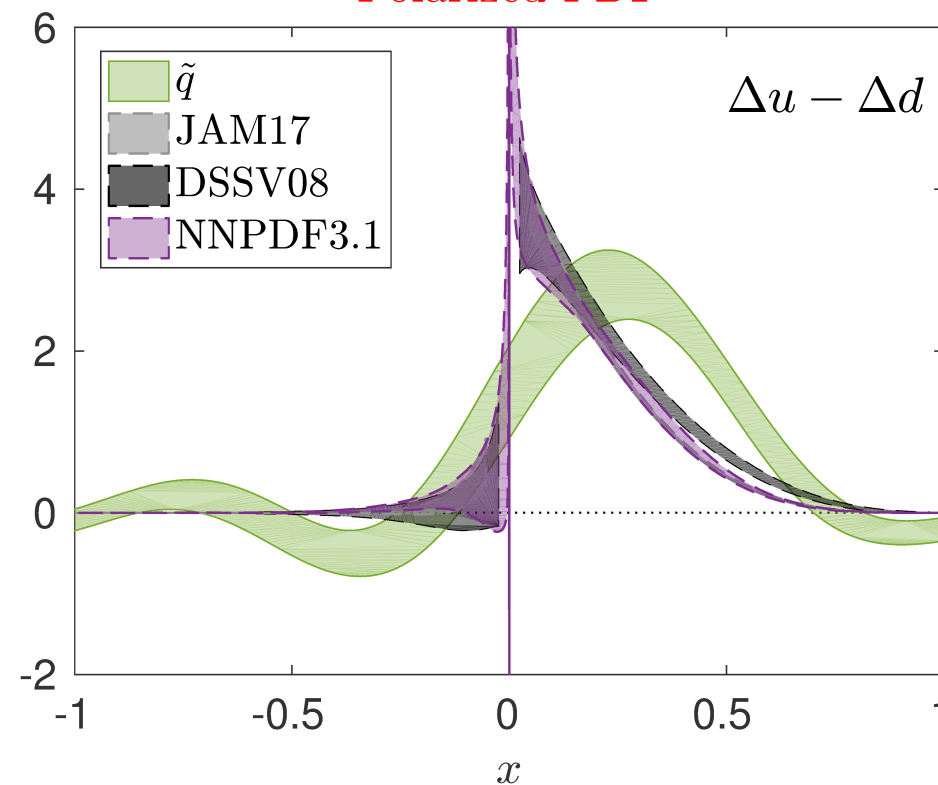


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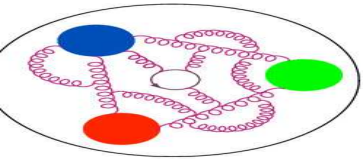
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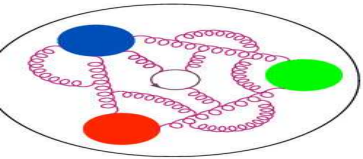


Matching to light-front PDFs



The matching formula can be expressed as:

$$q(x, \mu) = \int_{-\infty}^{\infty} \frac{d\xi}{|\xi|} C \left(\xi, \frac{\mu}{xP_3} \right) \tilde{q} \left(\frac{x}{\xi}, \mu, P_3 \right)$$



Matching to light-front PDFs

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C – matching kernel:

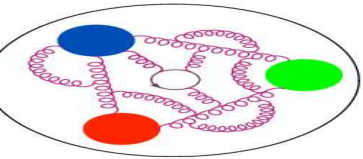
$$C \left(\xi, \frac{\xi\mu}{xP_3} \right) = \delta(1 - \xi) + \frac{\alpha_s}{2\pi} C_F \begin{cases} \left[\frac{1 + \xi^2}{1 - \xi} \ln \frac{\xi}{\xi - 1} + 1 + \frac{3}{2\xi} \right]_+ & \xi > 1, \\ \left[\frac{1 + \xi^2}{1 - \xi} \ln \frac{x^2 P_3^2}{\xi^2 \mu^2} (4\xi(1 - \xi)) - \frac{\xi(1 + \xi)}{1 - \xi} + 2\iota(1 - \xi) \right]_+ & 0 < \xi < 1, \\ \left[-\frac{1 + \xi^2}{1 - \xi} \ln \frac{\xi}{\xi - 1} - 1 + \frac{3}{2(1 - \xi)} \right]_+ & \xi < 0, \end{cases}$$

[T. Izubuchi et al., arXiv:1801.03917 [hep-ph], C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001]

$\iota=0$ for γ_0 and $\iota=1$ for $\gamma_3/\gamma_5\gamma_3$.

Plus prescription at $\xi=1$:

$$\int \frac{d\xi}{|\xi|} \left[C \left(\xi, \frac{\xi\mu}{xP_3} \right) \right]_+ \tilde{q} \left(\frac{x}{\xi} \right) = \int \frac{d\xi}{|\xi|} C \left(\xi, \frac{\xi\mu}{xP_3} \right) \tilde{q} \left(\frac{x}{\xi} \right) - \tilde{q}(x) \int d\xi C \left(\xi, \frac{\mu}{xP_3} \right).$$

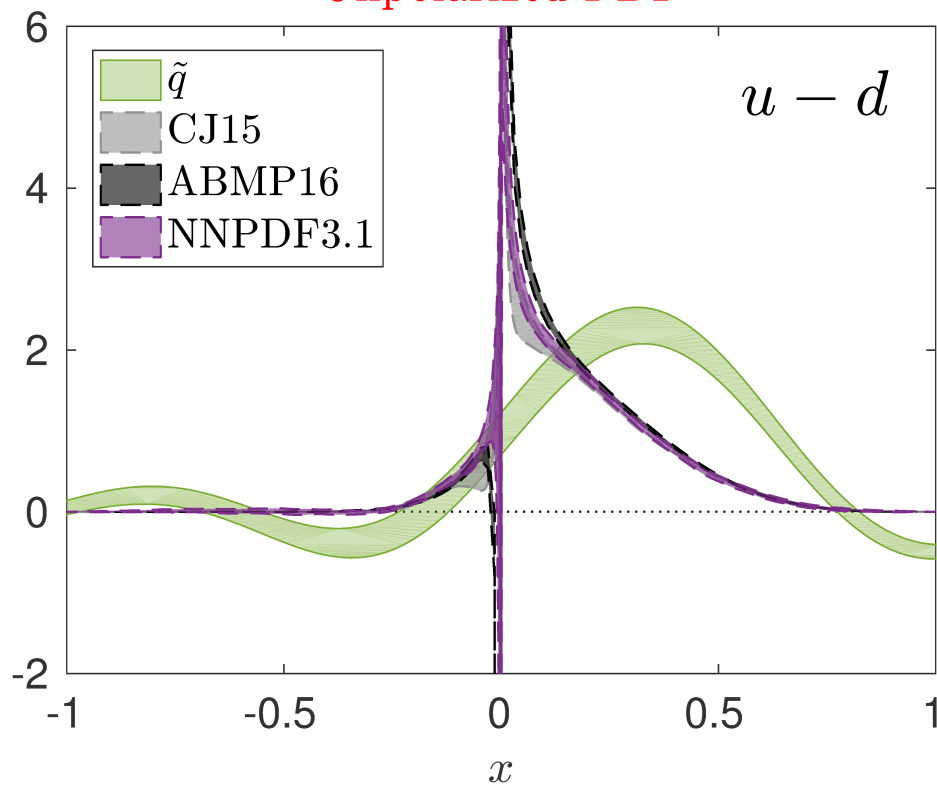


Matched PDFs

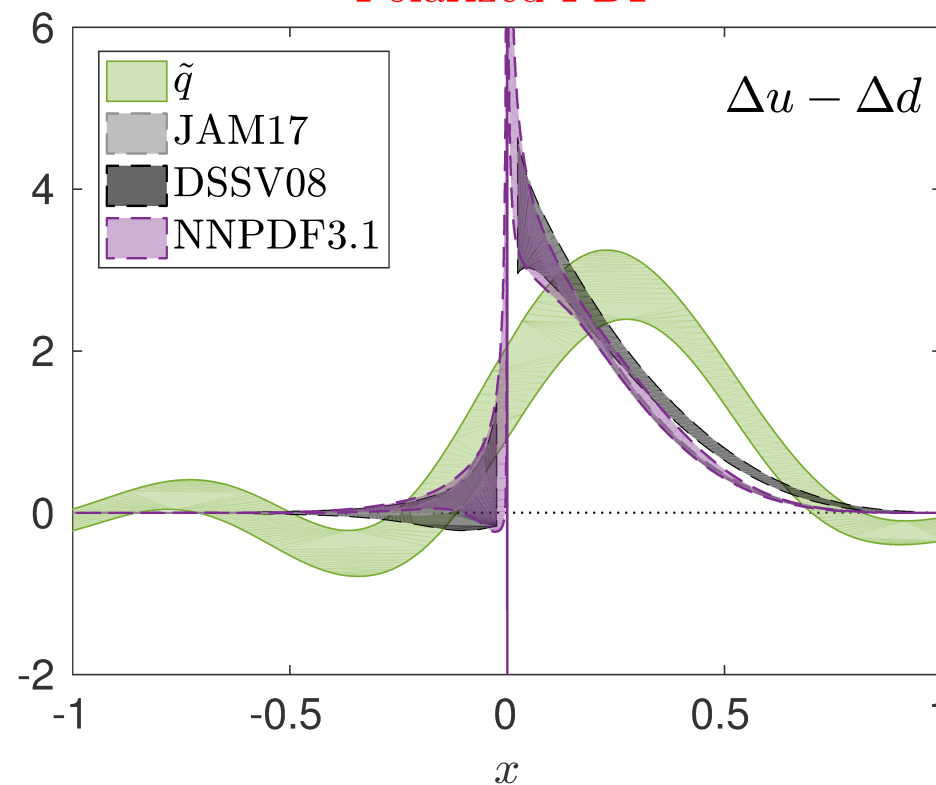


Nucleon momentum $\frac{10\pi}{48}$

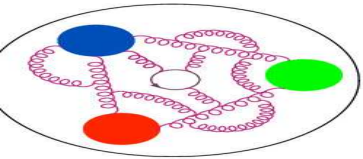
Unpolarized PDF



Polarized PDF



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

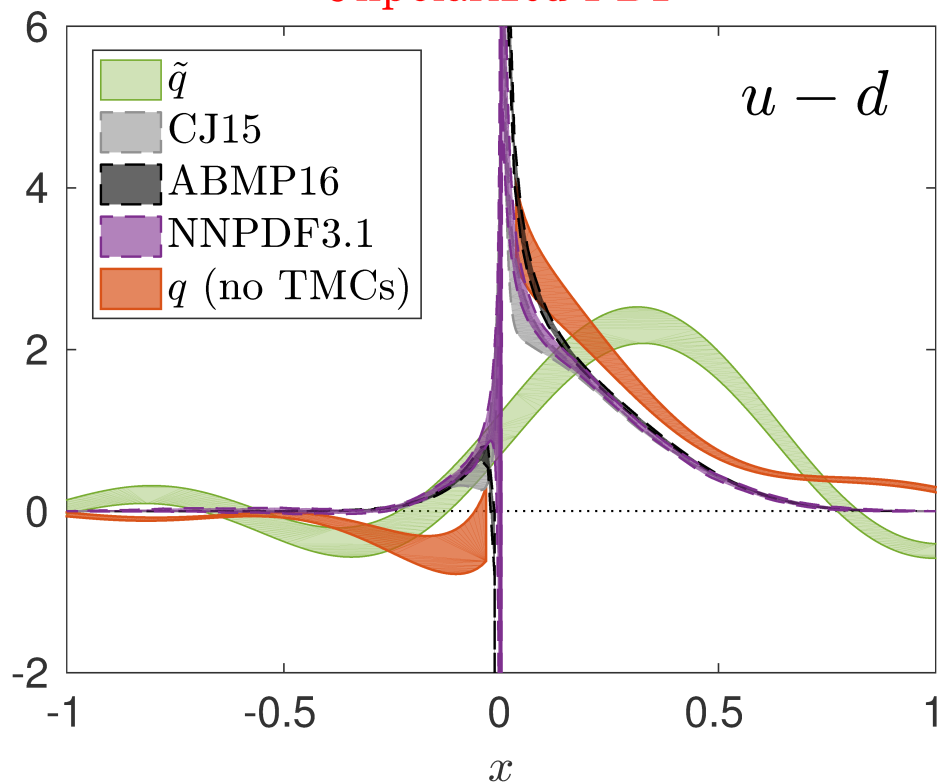


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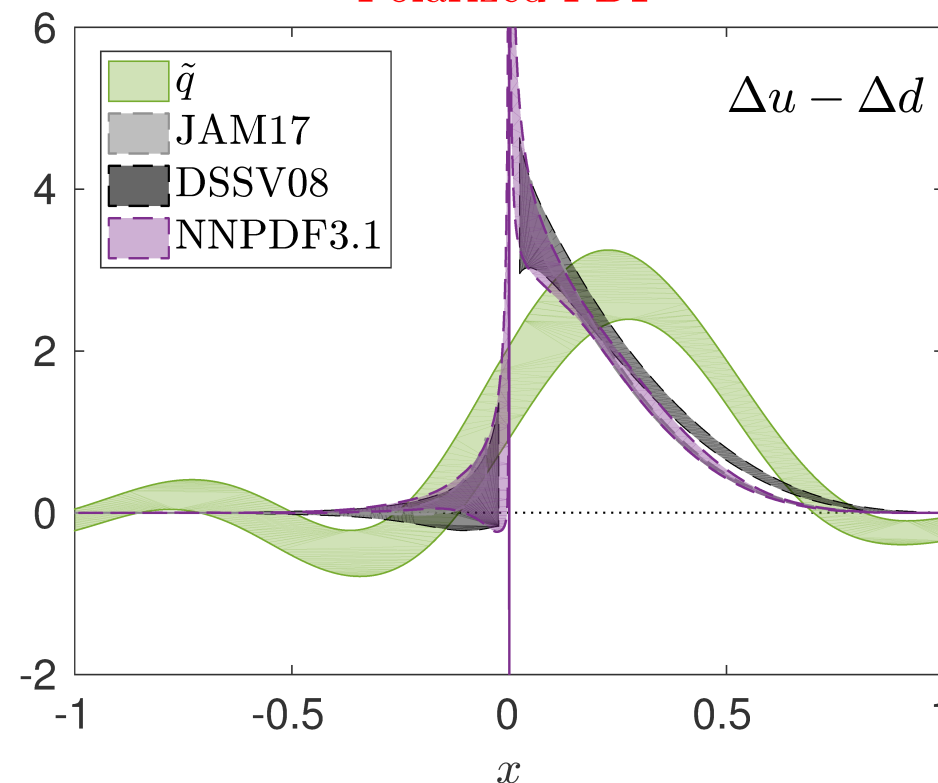


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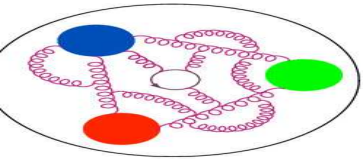
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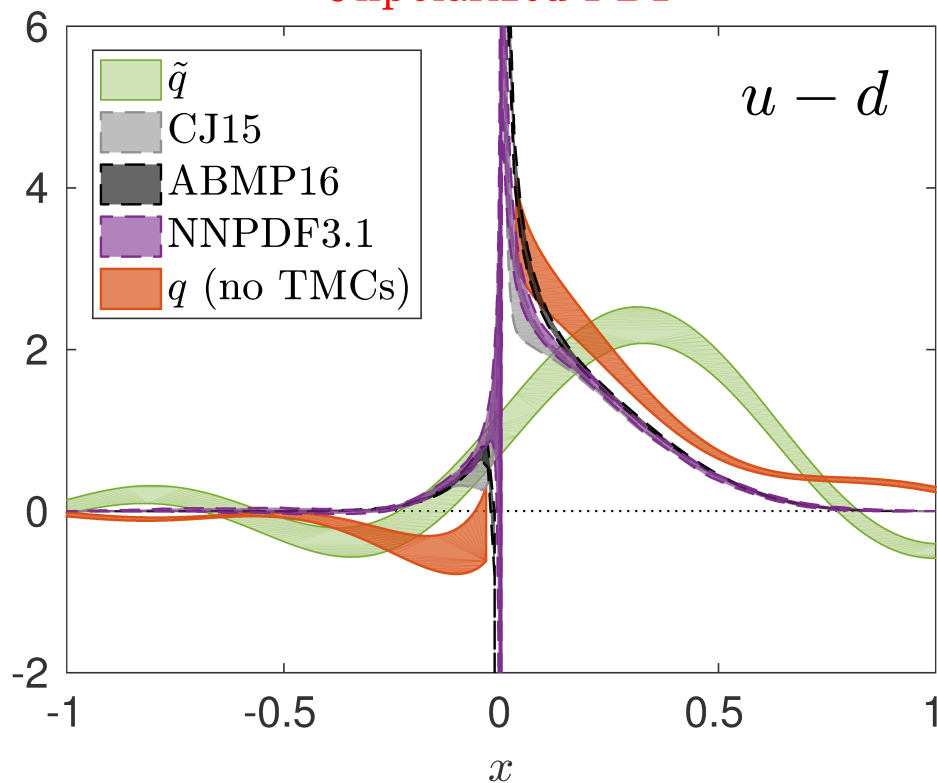


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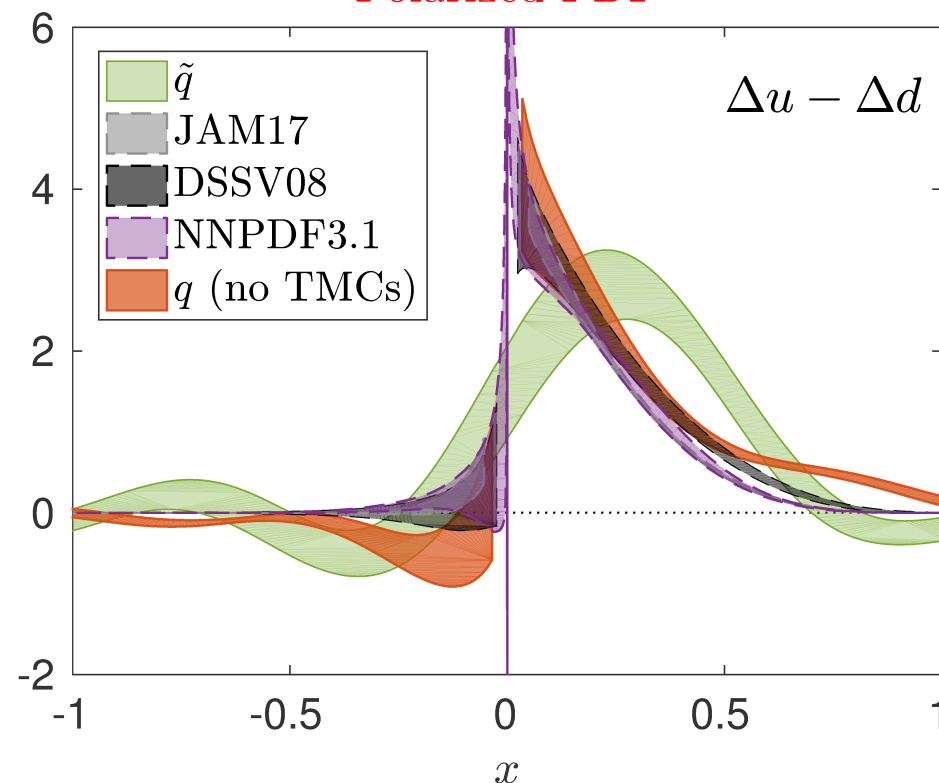


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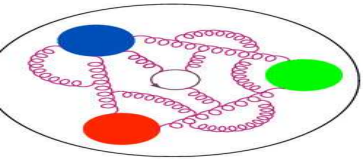
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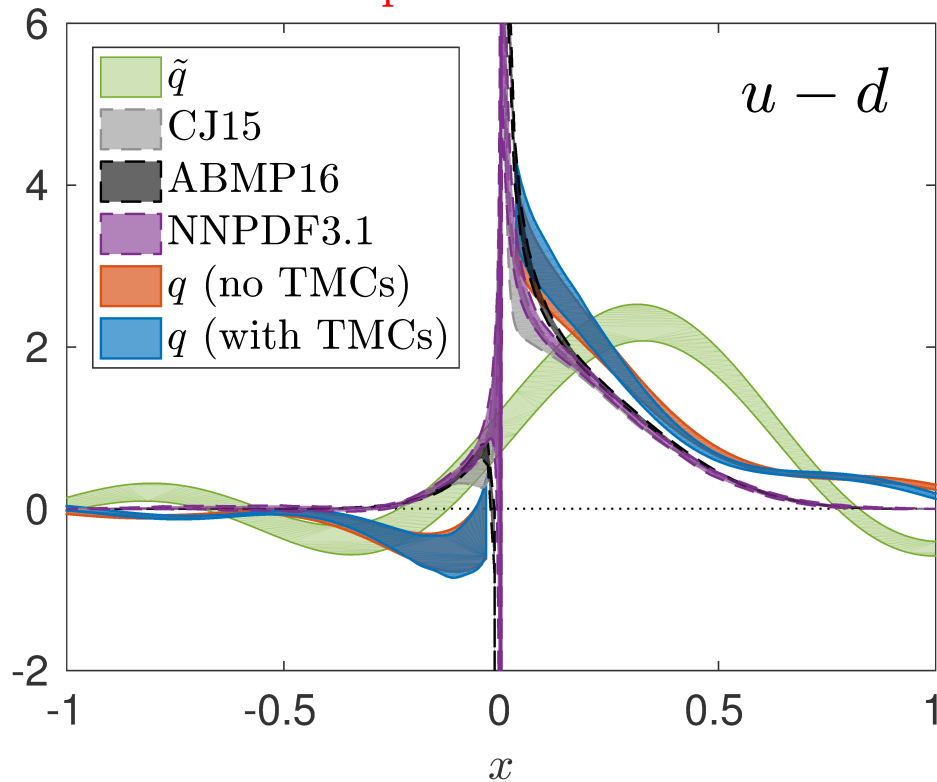
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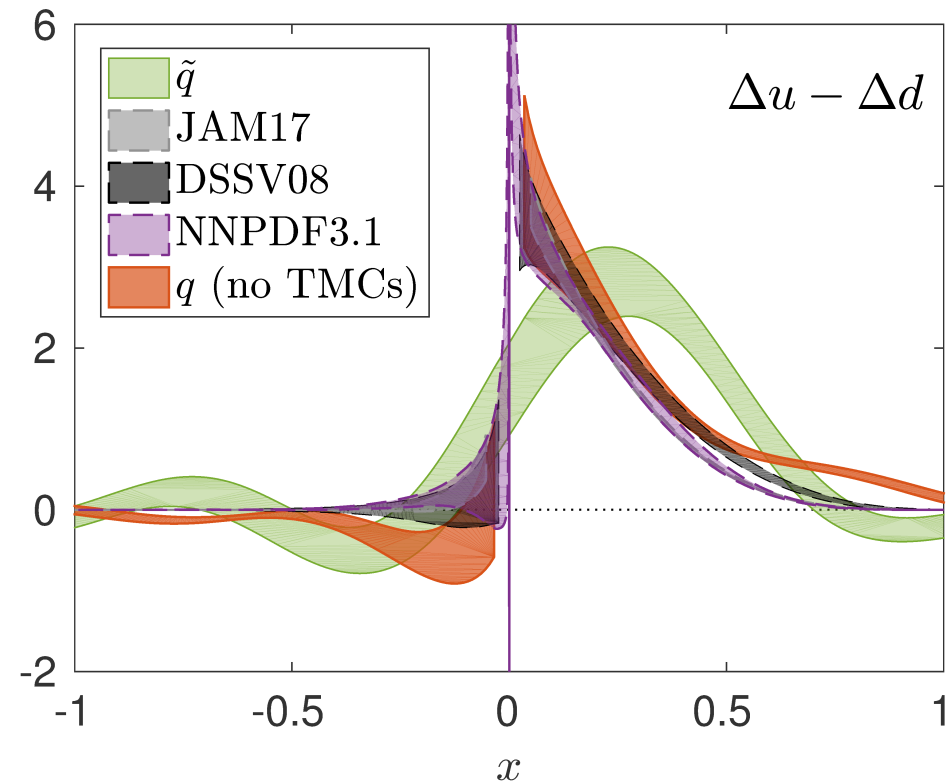
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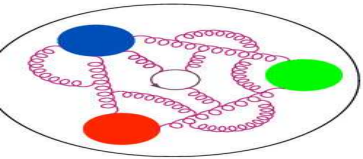
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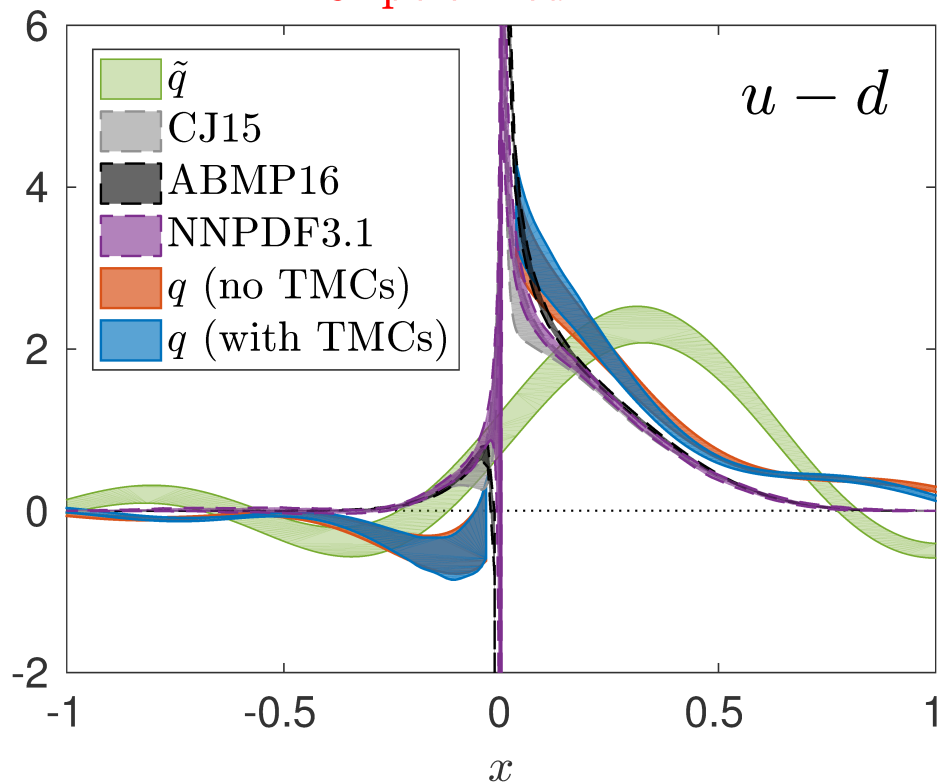
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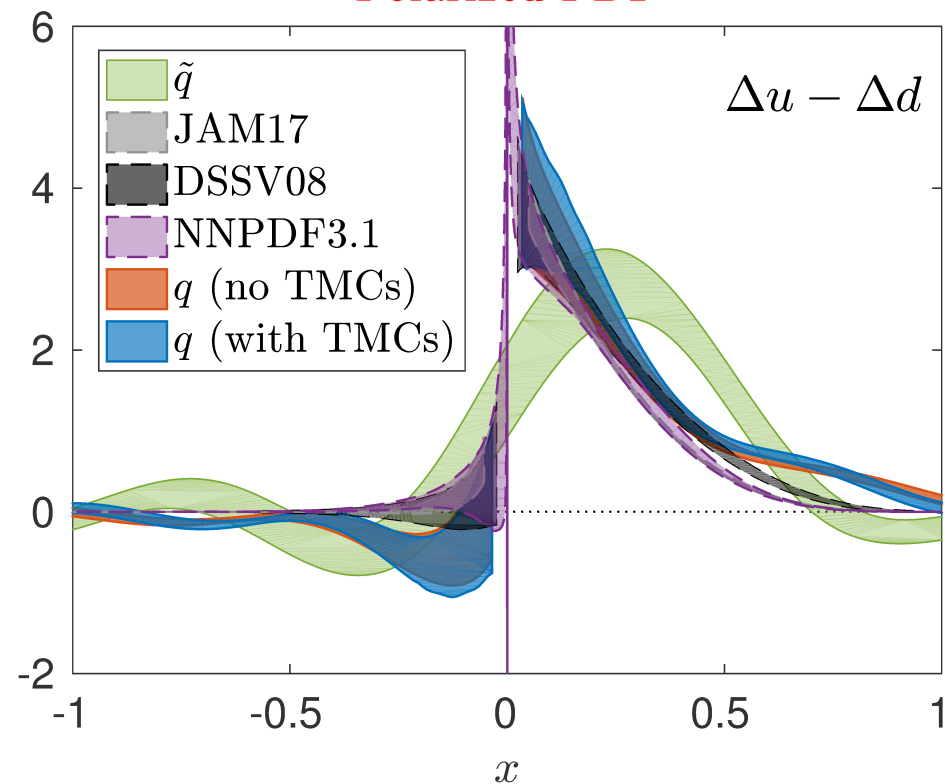
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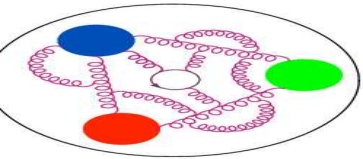
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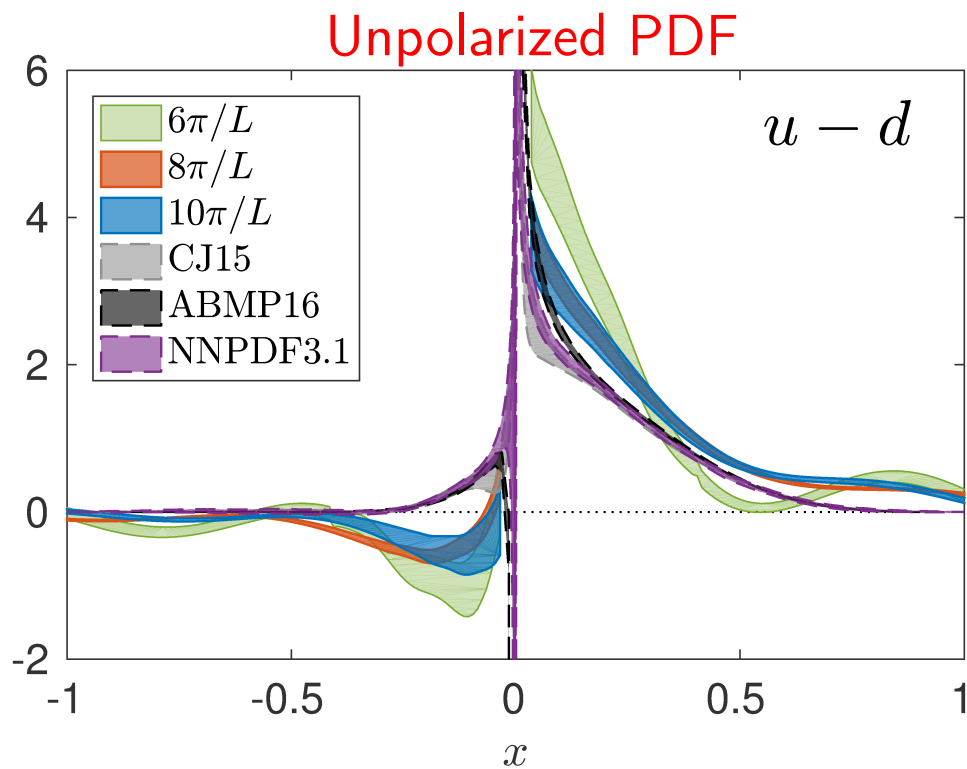


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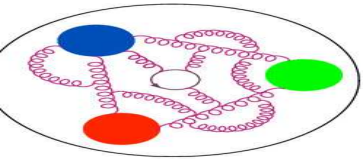


Momentum dependence of final PDF

Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$



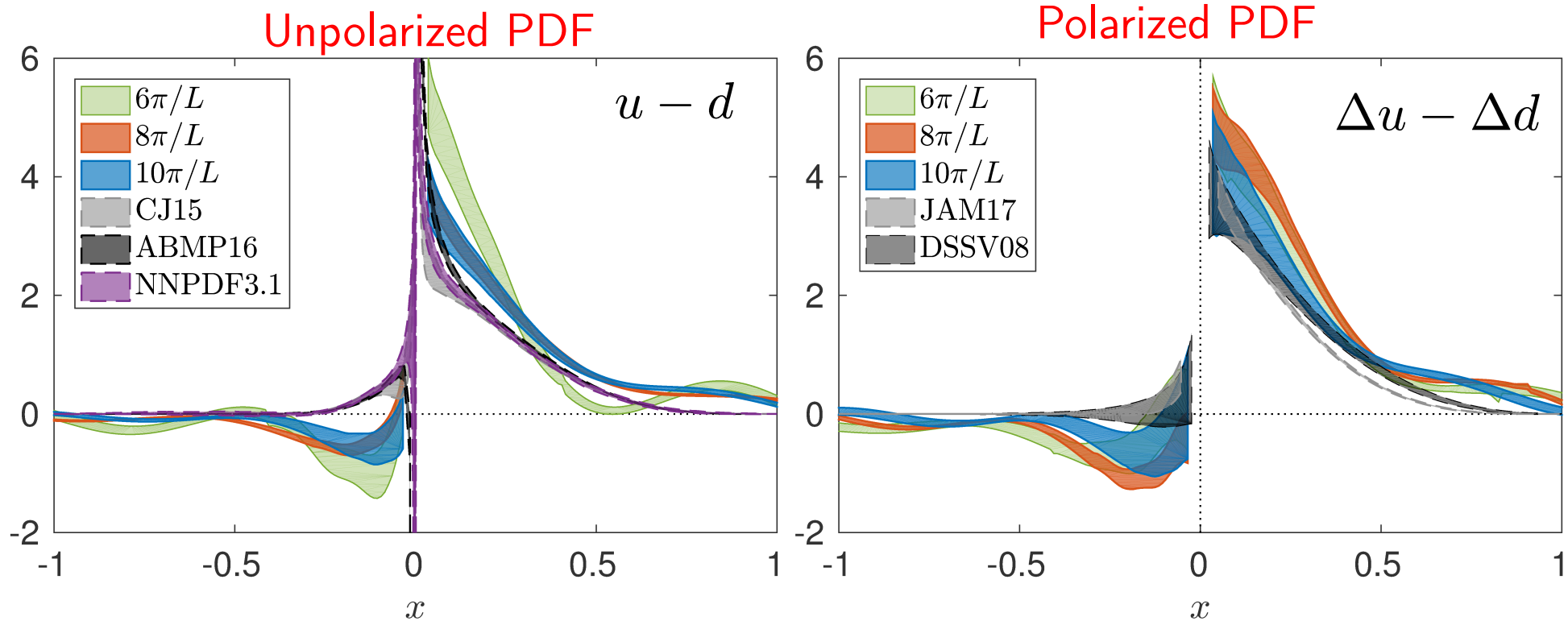
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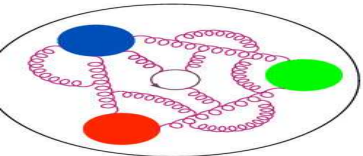
Momentum dependence of final PDF



Nucleon momenta $\frac{\{6,8,10\}\pi}{48}$



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Comparison with non-physical pion mass

Physical vs. non-physical pion mass – 135 vs. 375 MeV
unpolarized PDF

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Bare ME

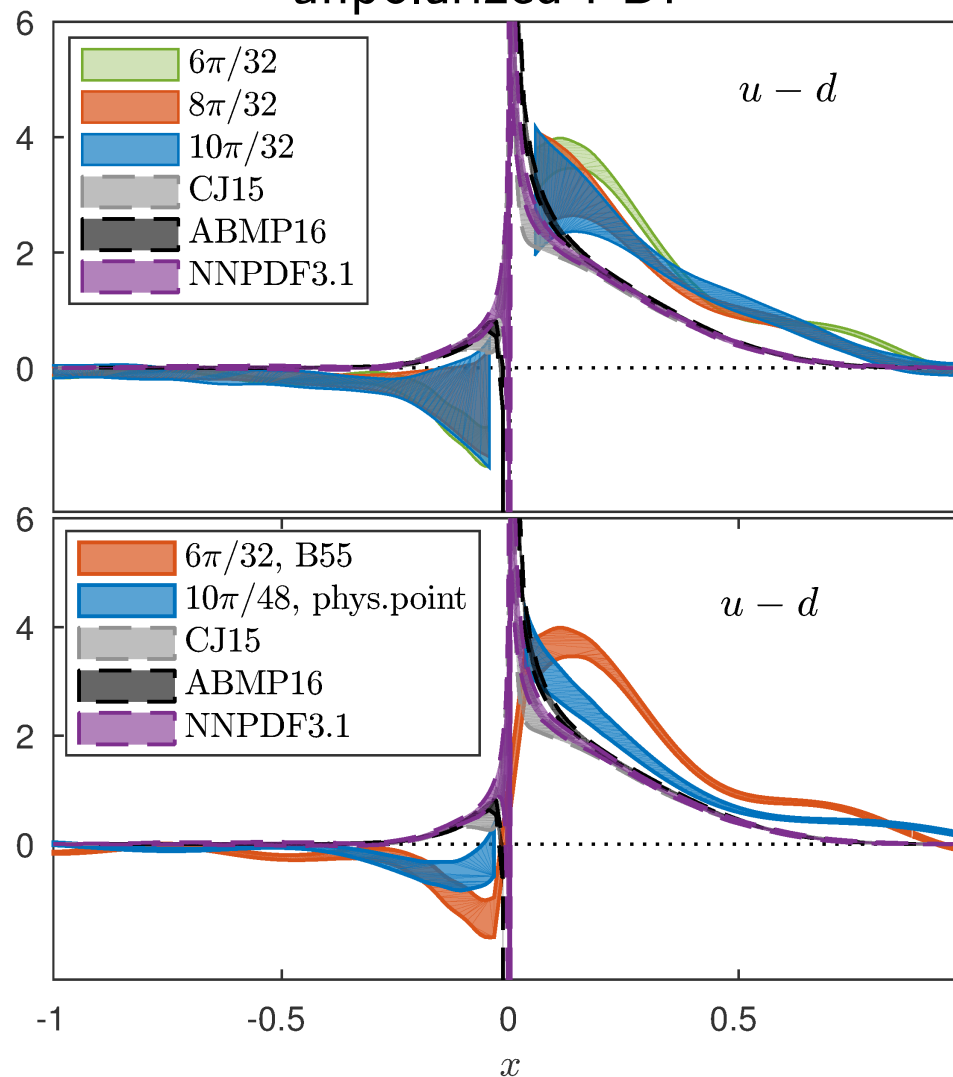
Matching

Matched PDFs

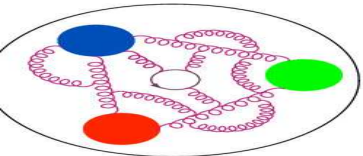
Final PDFs

Systematics

Summary



C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001



Transversity PDF

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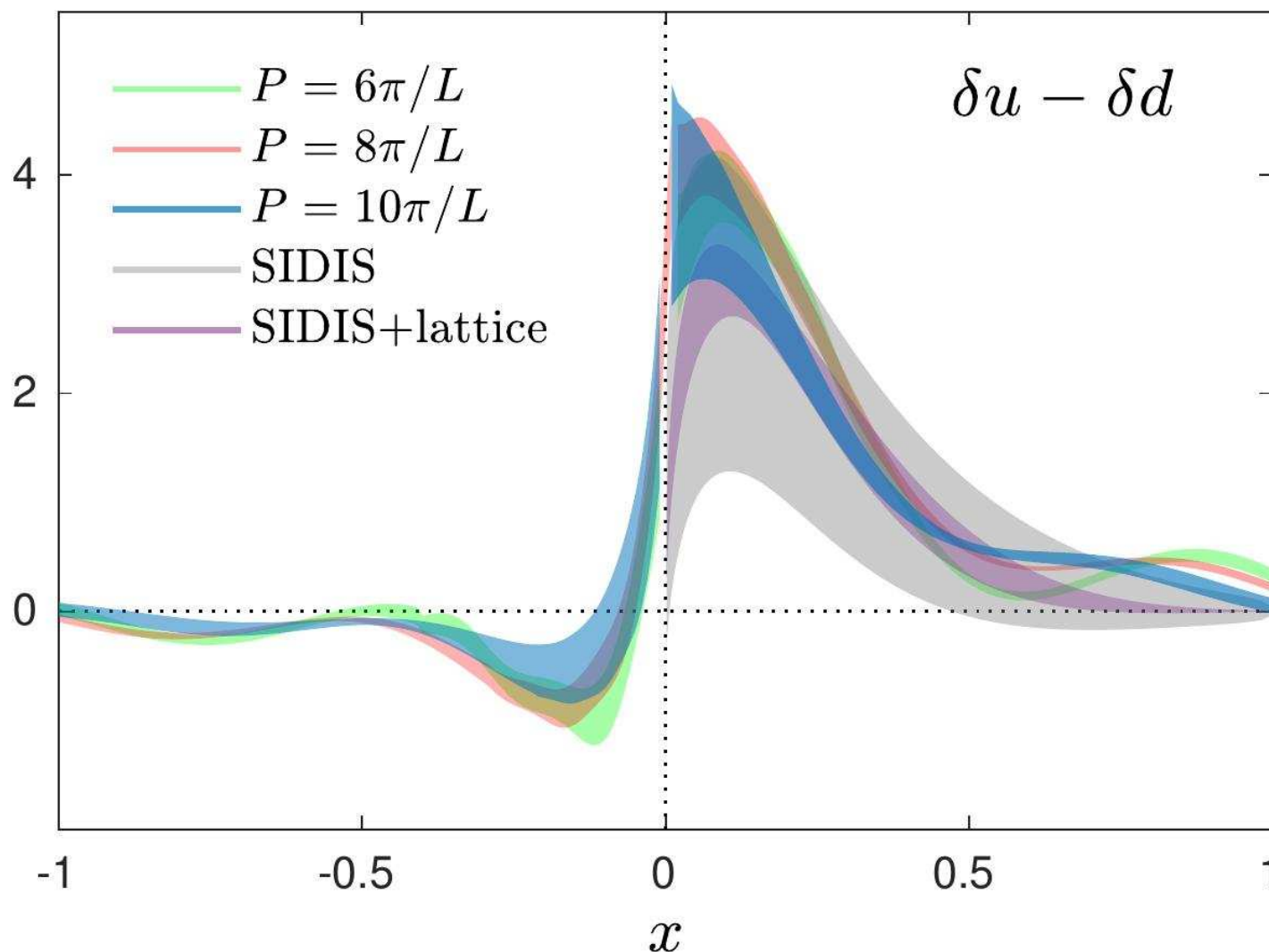
Matching

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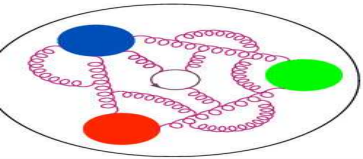
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Systematics



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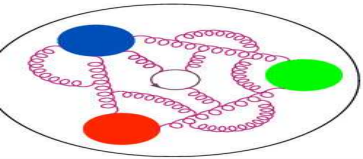
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Systematics

Summary

Different systematic effects still need to be addressed:



Systematics



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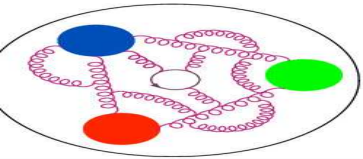
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Systematics



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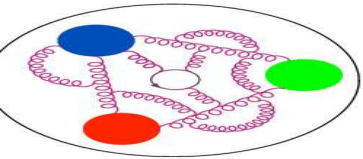
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Summary

Different systematic effects still need to be addressed:

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- cut-off effects ✓✗



Systematics



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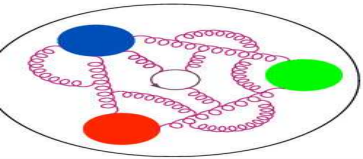
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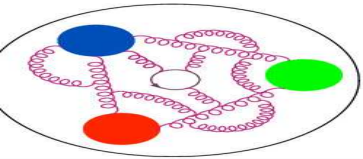
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Systematics



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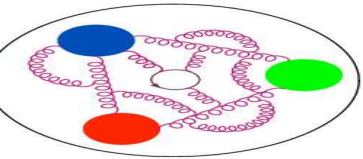
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Systematics



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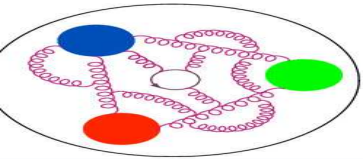
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Systematics



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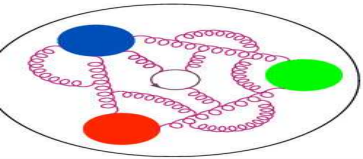
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Systematics



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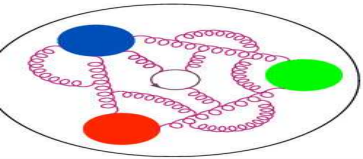
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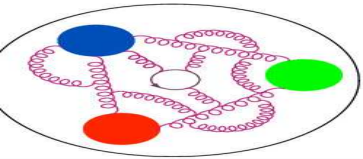
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Biggest challenge:

Reach large momenta at large source-sink separations



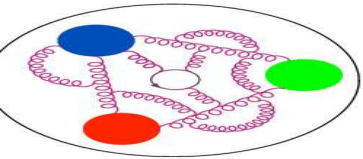
A guide to light-cone PDFs from lattice QCD: an overview of approaches, techniques and results

Krzysztof Cichy¹, Martha Constantinou² 

¹ *Faculty of Physics, Adam Mickiewicz University, Umultowska 85, 61-614 Poznań, Poland*

² *Department of Physics, Temple University, Philadelphia, PA 19122 - 1801, USA*

- around 100 pages, next Monday on arXiv
- discusses in detail quasi-distributions:
nucleon: non-singlet quark qPDFs, qGPDs, qTMDs, singlet qPDFs, gluon qPDFs; pion: qPDFs, qDAs
- reviews also other approaches:
hadronic tensor, auxiliary scalar quark, auxiliary heavy quark, auxiliary light quark, pseudo-distributions, “OPE without OPE”, lattice cross sections



Conclusions and prospects



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Lattice QCD

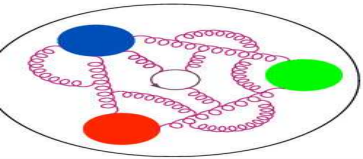
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C. Alexandrou et al., Phys. Rev. Lett. 121 (2018) 112001

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Conclusions and prospects



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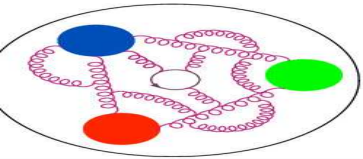
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Conclusions and prospects



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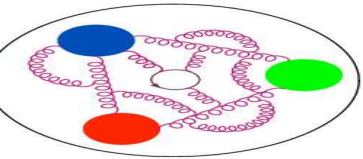
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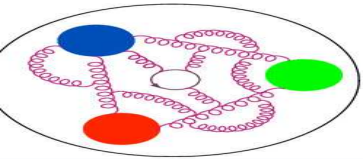
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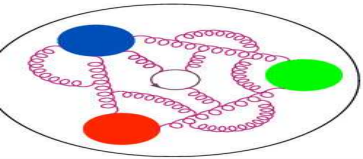
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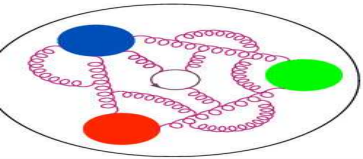
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Conclusions and prospects



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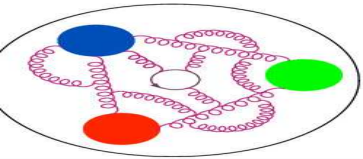
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Thank you for your attention!