

# TMD parton densities from Parton Branching method and applications

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in collaboration with

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- Why TMDs are needed
  - TMDs for hadron-hadron collisions
- New developments
  - parton branching algorithm to solve evolution equations
    - benchmark tests
    - advantages for integrated PDFs
  - determination of TMD densities at NLO from HERA DIS data
- Application to DY production and high  $p_T$  dijets

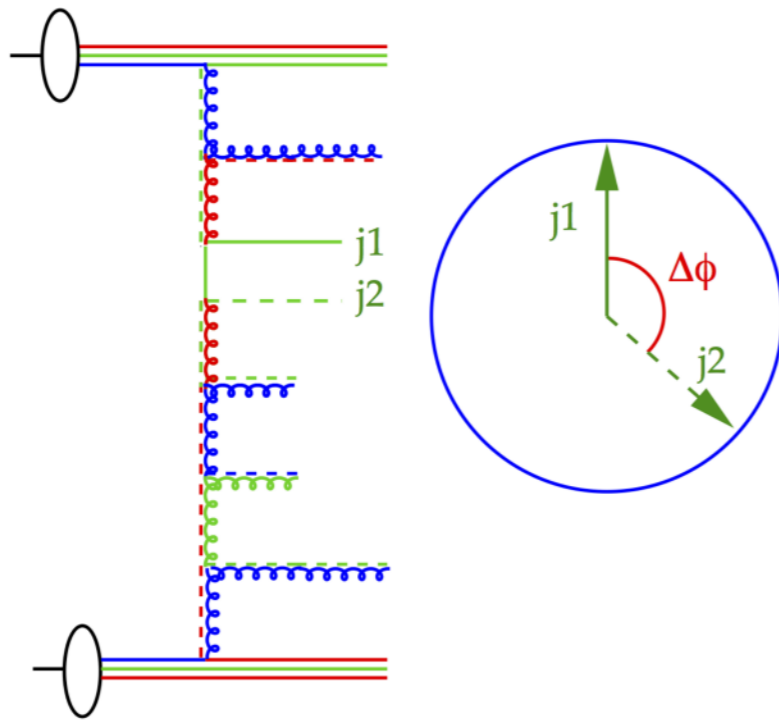
# TMD – what is it ?

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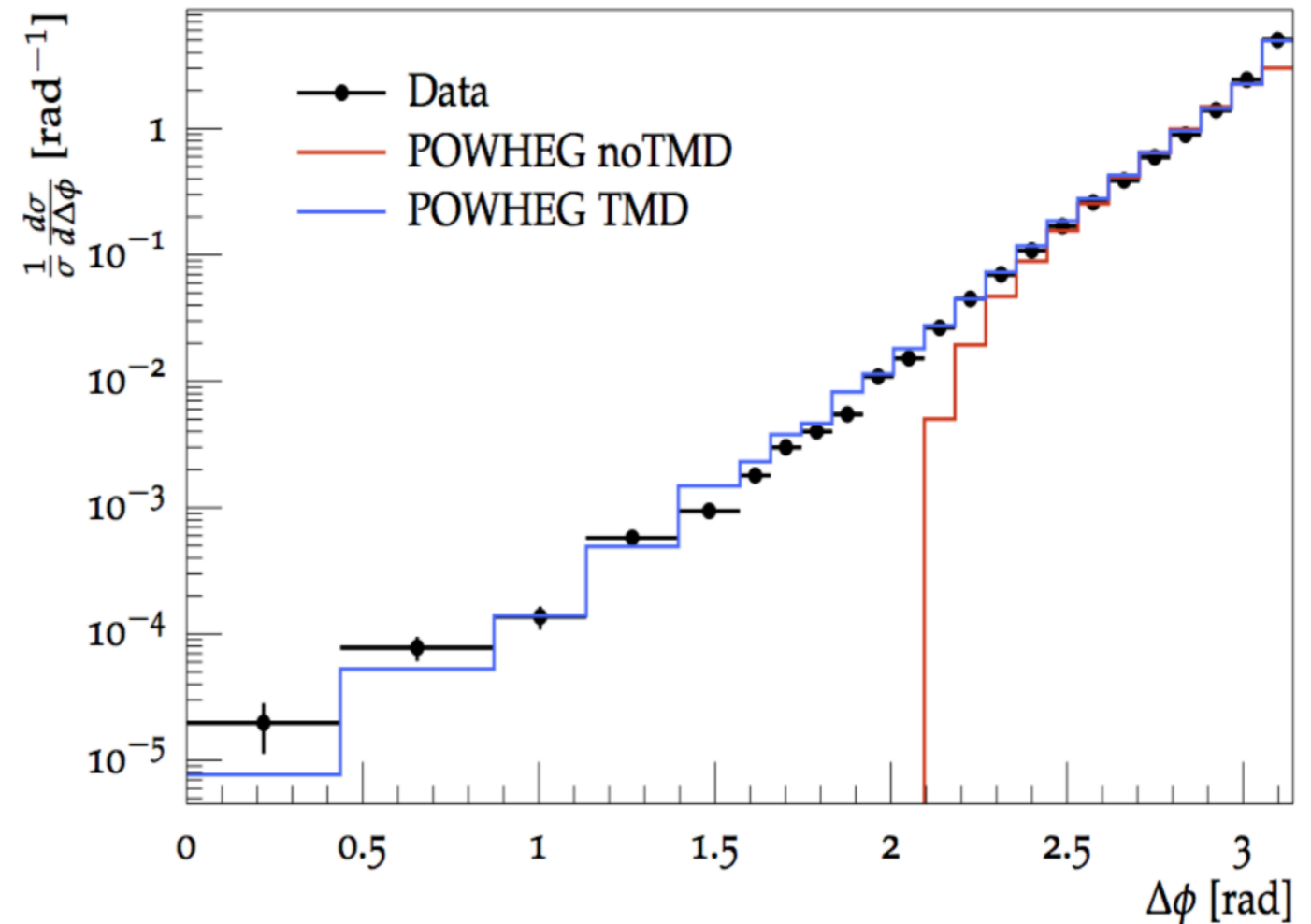
- TMDs (Transverse Momentum Dependent parton distribution)
  - at very small transverse momenta
    - typically for small  $q_t$  in DY production, or semi-inclusive DIS
  - at very small  $x$  – unintegrated PDFs
    - essentially only gluon densities (CCFM, BFKL etc)
- new approach: Parton Branching Method
  - cover all transverse momenta from small  $k_t$  to large  $k_t$  as well a large range in  $x$  and  $\mu^2$

# Why TMDs ?

- Measurements with  $p_T > 200$  GeV
  - at least 2 jets



Di-jet azimuthal decorrelation,  $300 < p_T^{\text{leading}} < 400$  GeV



- NLO-dijet (Powheg) w/o PS cannot describe small  $\Delta\phi$
- NLO-dijet (Powheg) with TMDs describes spectrum at small and large  $\Delta\phi$
- Region of higher order emissions described by TMDs

# TMDs – how to determine ?

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- Transverse momentum effects are naturally coming from intrinsic  $k_t$  and parton showers
- **New: Parton Branching Method**
  - determine integrated PDF from parton branching solution of evolution eq.
    - check consistency with standard evolution on integrated PDFs
      - at LO, NLO and NNLO
      - advantages of Parton Branching Method
  - determine TMD:
    - since each branching is generated explicitly, energy-momentum conservation is fulfilled and transverse momentum distributions can be obtained

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[1] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Soft-gluon resolution scale in QCD evolution equations. Phys. Lett., B772:446–451, 2017.

[2] F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD Quark and Gluon Densities from Parton Branching Solution of QCD Evolution Equations. JHEP, 01:070, 2018.

[3] A. Bermudez Martinez, P. Connor, F. Hautmann, H. Jung, A. Lelek, V. Radescu, and R. Zlebcik. Collinear and TMD parton densities from fits to precision DIS measurements in the parton branching method. DESY-18-042, arXiv 1804.11152

see also Jadach, S., Placzek, W., Skrzypek, M., and Stoklosa, P. (2010). Comput. Phys. Commun., 181(2010), 398

# DGLAP evolution – solution with parton branching method

- differential form: 
$$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} P_+(z) f\left(\frac{x}{z}, \mu^2\right)$$

$$\Delta_s(\mu^2) = \exp\left(-\int^{z_M} dz \int_{\mu_0^2}^{\mu^2} \frac{\alpha_s}{2\pi} \frac{d\mu'^2}{\mu'^2} P^{(R)}(z)\right)$$

- differential form using  $f/\Delta_s$  with

$$\mu^2 \frac{\partial}{\partial \mu^2} \frac{f(x, \mu^2)}{\Delta_s(\mu^2)} = \int \frac{dz}{z} \frac{\alpha_s}{2\pi} \frac{P^{(R)}(z)}{\Delta_s(\mu^2)} f\left(\frac{x}{z}, \mu^2\right)$$

- integral form

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

  
 no – branching probability from  $\mu_0^2$  to  $\mu^2$

# DGLAP re-sums leading logs...

$$f(x, \mu^2) = f(x, \mu_0^2) \Delta_s(\mu^2) + \int \frac{dz}{z} \int \frac{d\mu'^2}{\mu'^2} \cdot \frac{\Delta_s(\mu^2)}{\Delta_s(\mu'^2)} P^{(R)}(z) f\left(\frac{x}{z}, \mu'^2\right)$$

- solve integral equation via iteration:

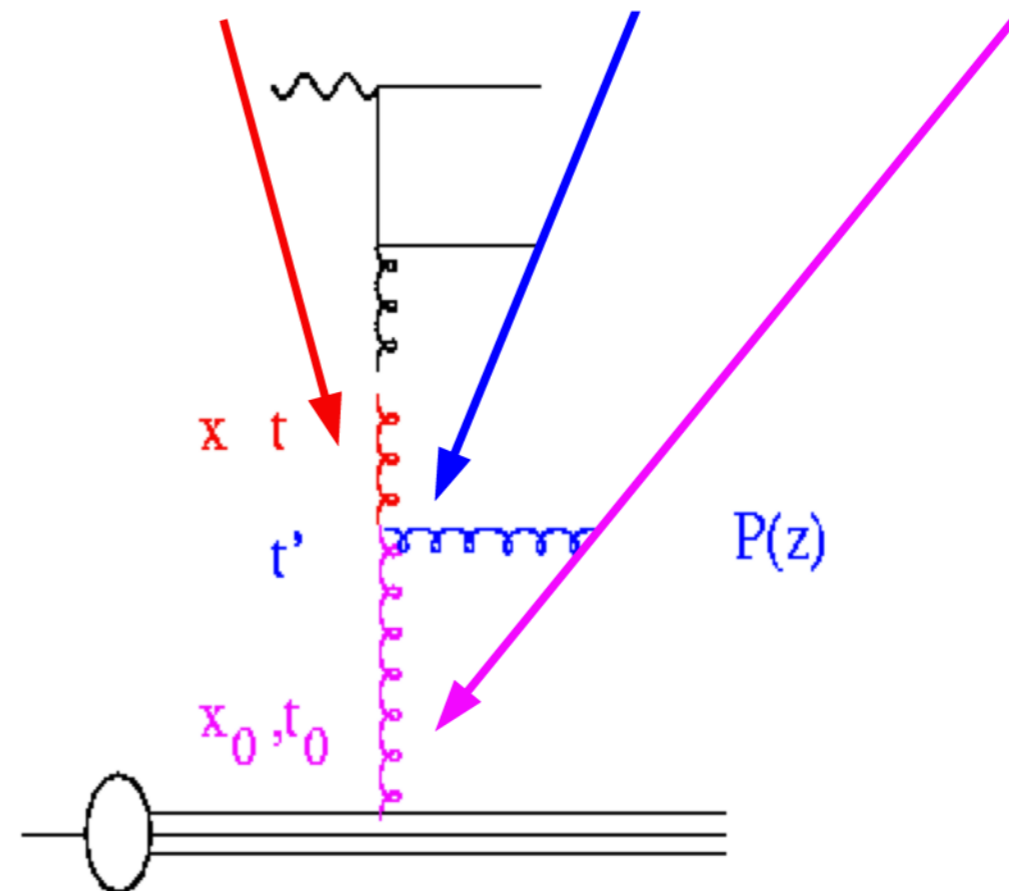
$$f_0(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2)$$

from  $t'$  to  $t$   
w/o branching

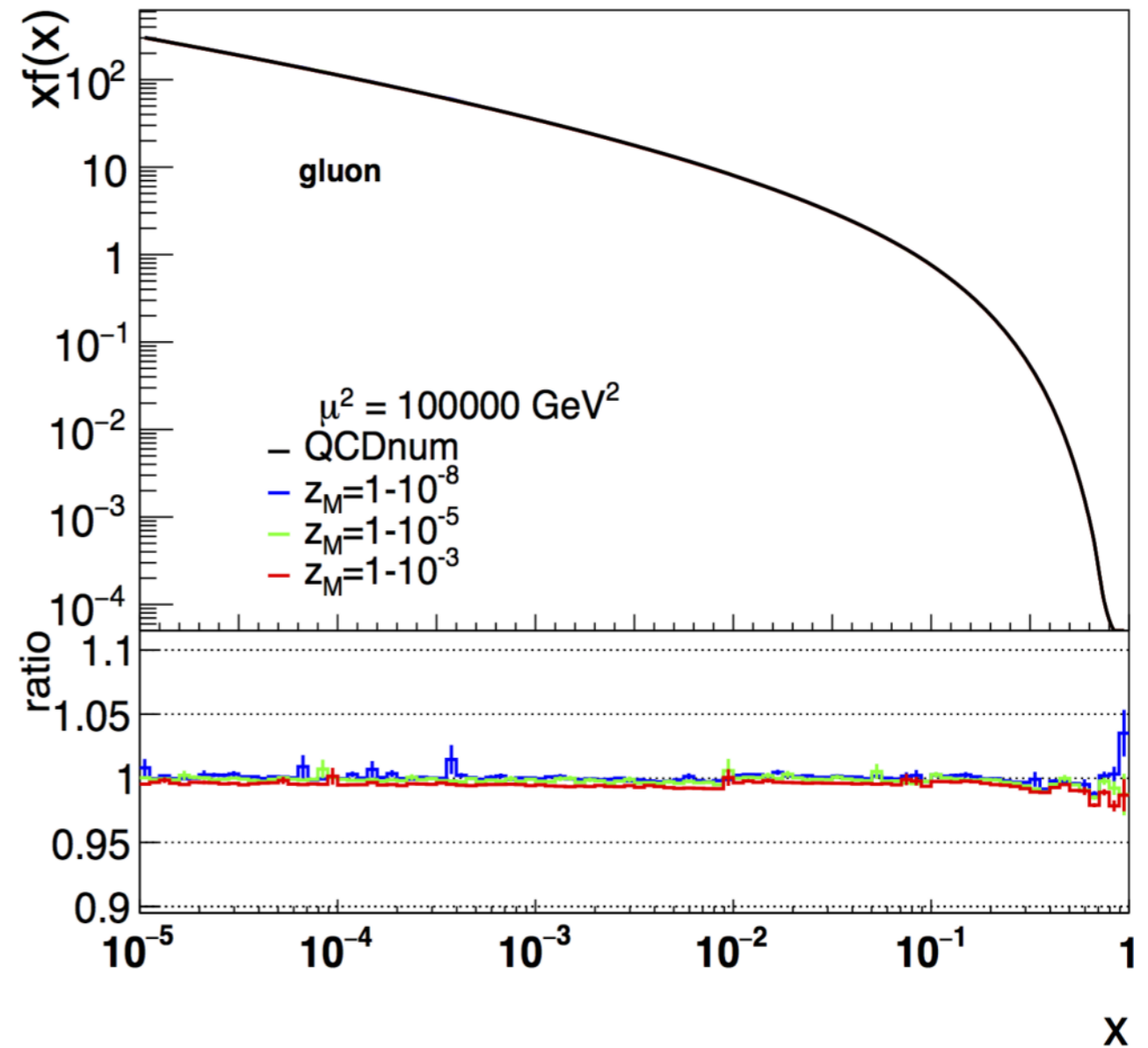
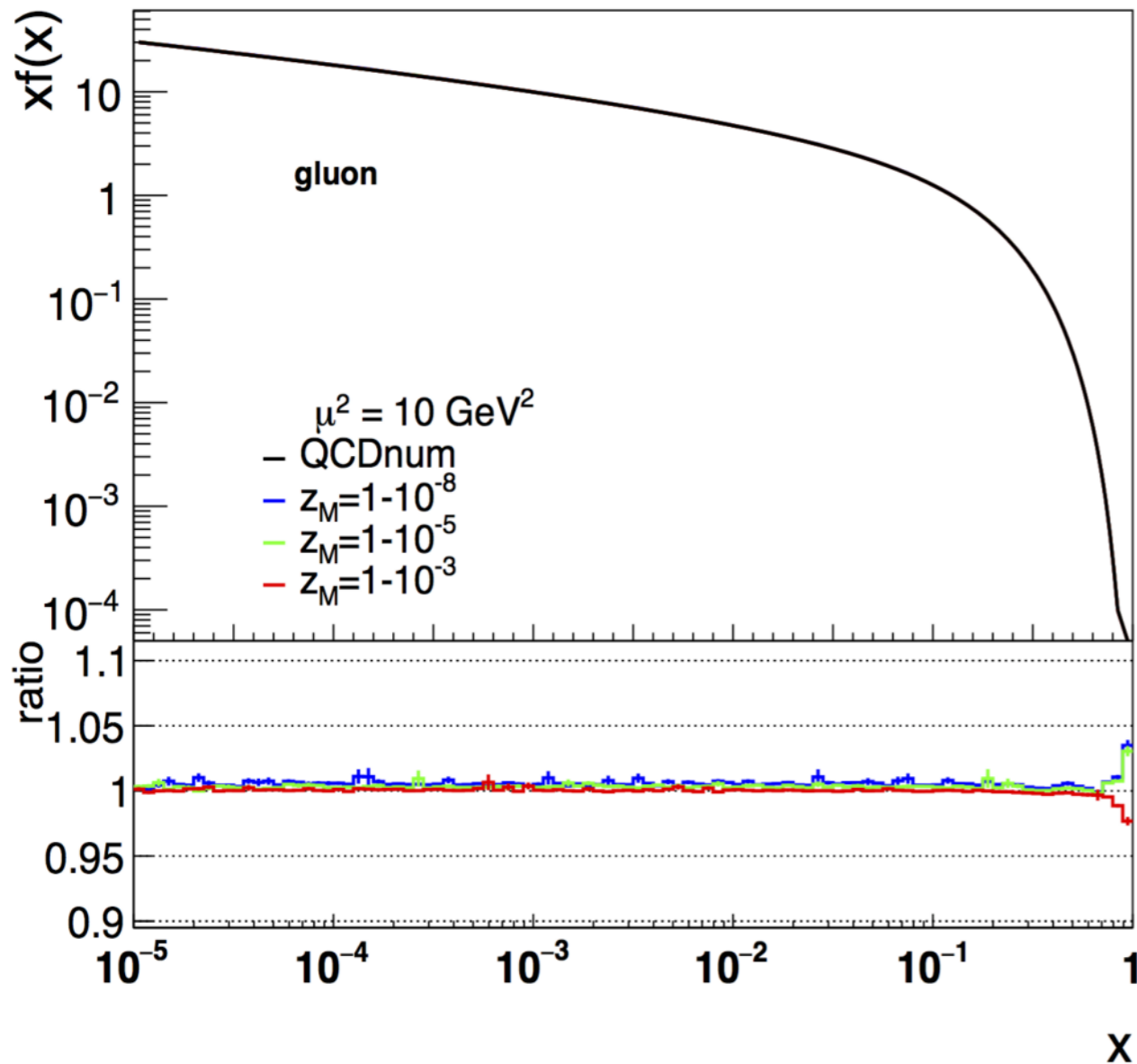
branching at  $t'$

from  $t_0$  to  $t'$   
w/o branching

$$f_1(x, \mu^2) = f(x, \mu_0^2) \Delta(\mu^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta(\mu^2)}{\Delta(\mu'^2)} \int \frac{dz}{z} P^{(R)}(z) f(x/z, \mu_0^2) \Delta(\mu'^2)$$



# Validation of method at **NLO**: $z_M$ - dependence



- No dependence on  $z_M$  if  $z_M$  is large enough:
  - approximation is of  $\mathcal{O}(1 - z_M)$
- Very good agreement with **NLO** - QCDnum



# Transverse Momentum Dependence

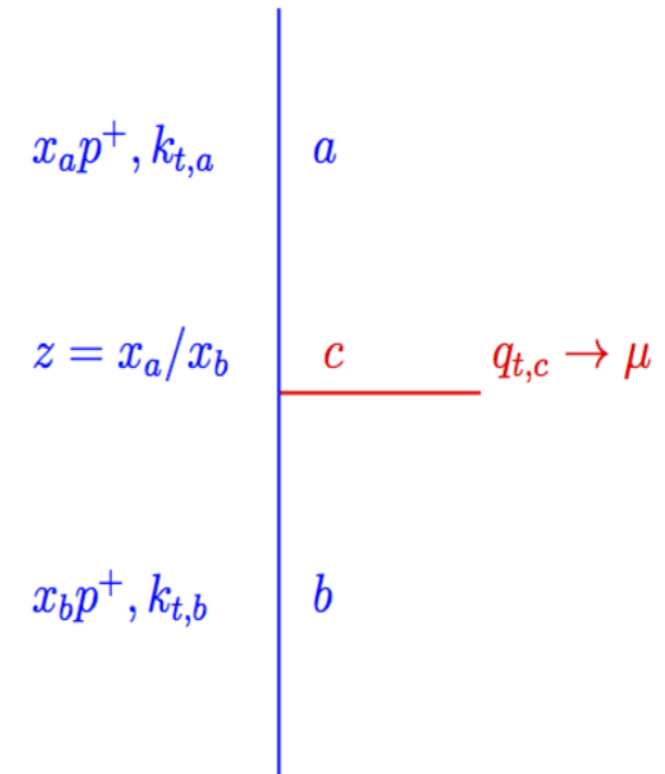
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- Parton Branching evolution generates every single branching:
  - kinematics can be calculated at every step
- Give physics interpretation of evolution scale:
  - in high energy limit:  $p_T$  -ordering:

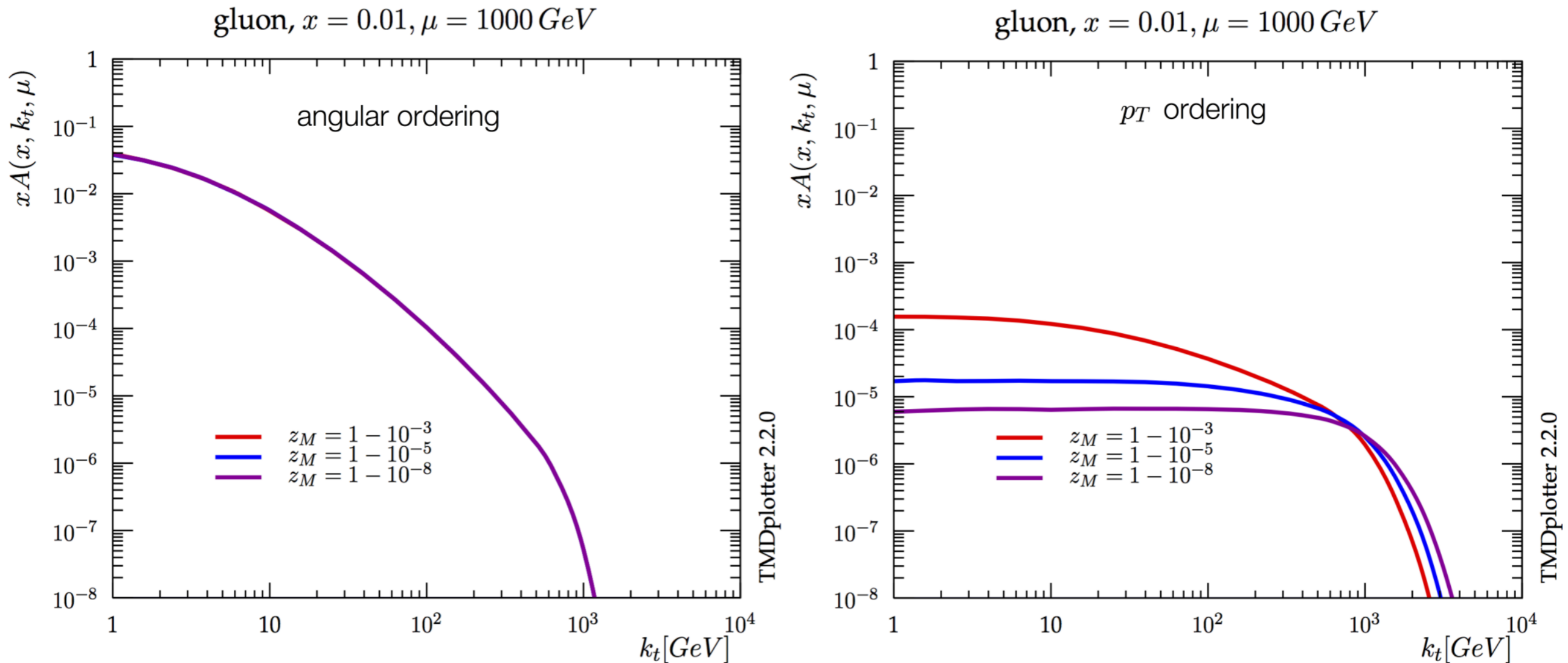
$$\mu = q_T$$

- angular ordering:

$$\mu = q_T / (1-z)$$



# Transverse Momentum: $z_M$ - dependence



- $p_T$  – ordering ( $\mu = q_T$ ) shows significant dependence on  $z_M$ : unstable result because of soft gluon contribution
- angular ordering ( $\mu = q_T/(1-z)$ ) is independent of  $z_M$ : stable results since soft gluons are suppressed (angular ordering)

# PDFs from Parton Branching method: fit to HERA data

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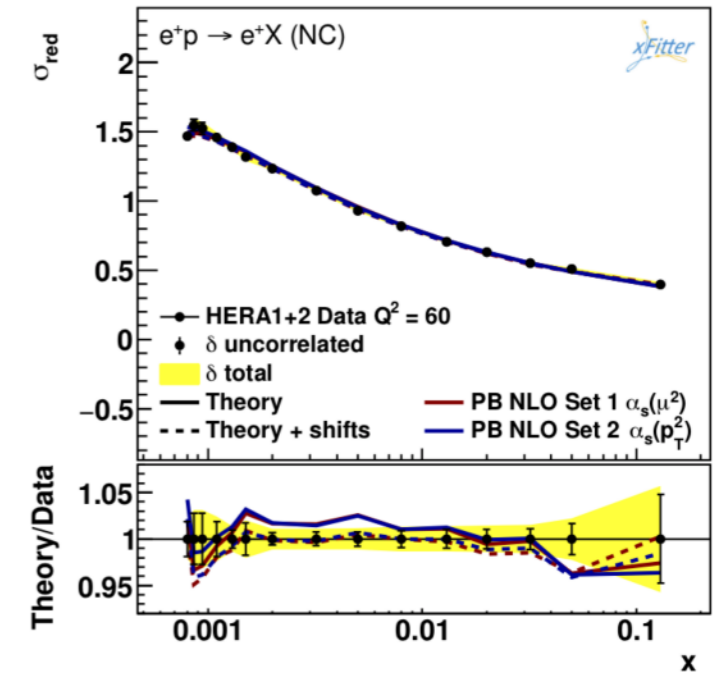
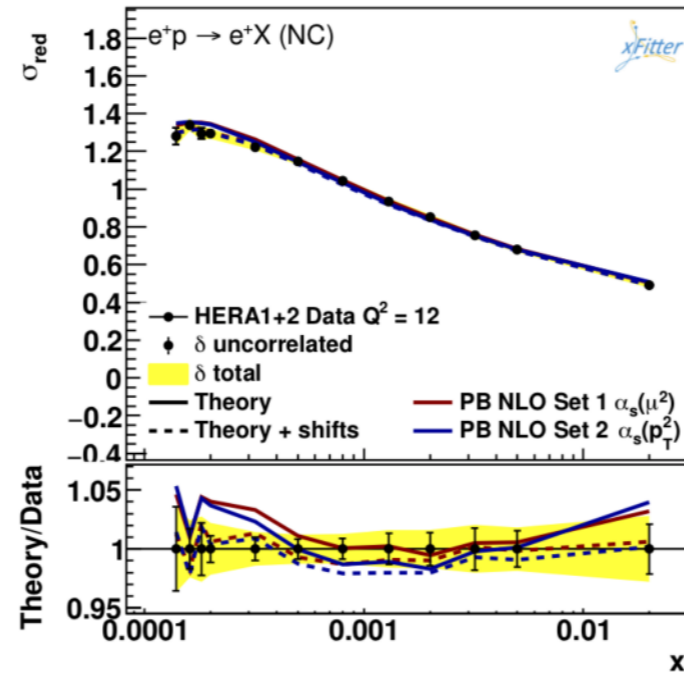
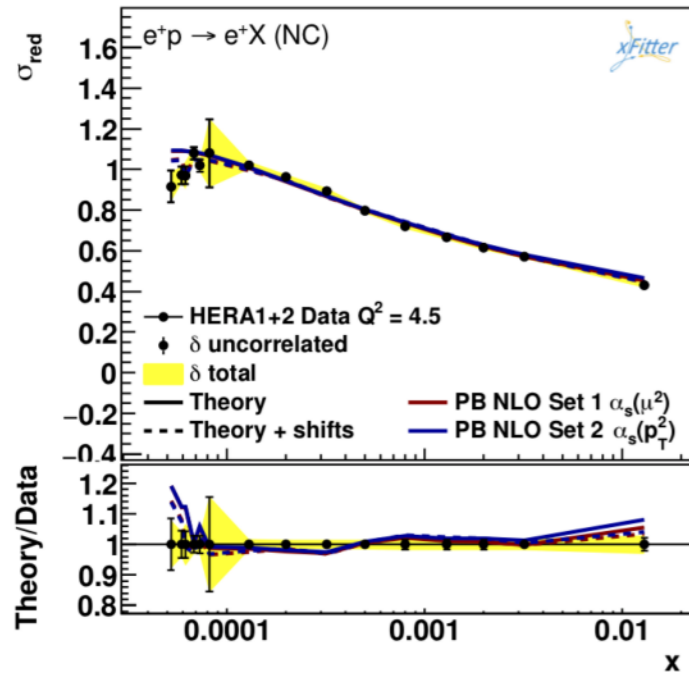
- Convolution of kernel with starting distribution

$$\begin{aligned} x f_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\ &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right) \end{aligned}$$

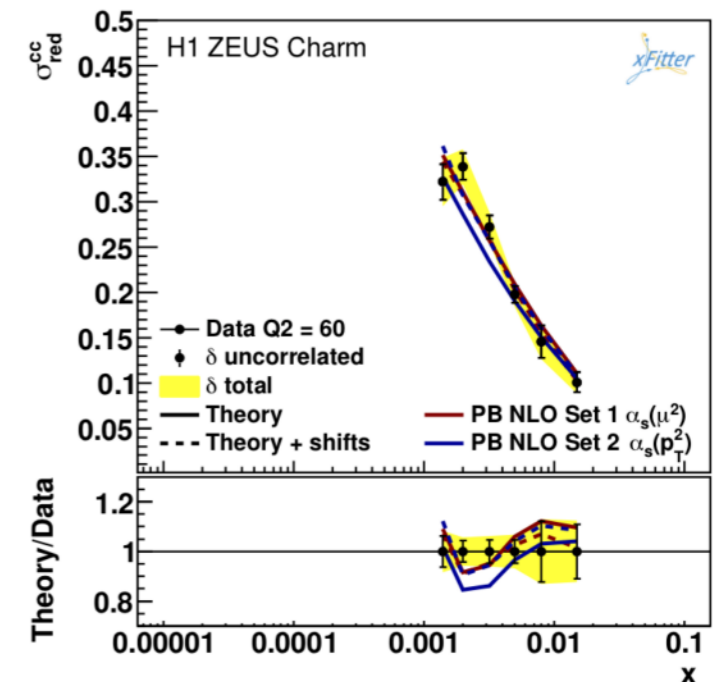
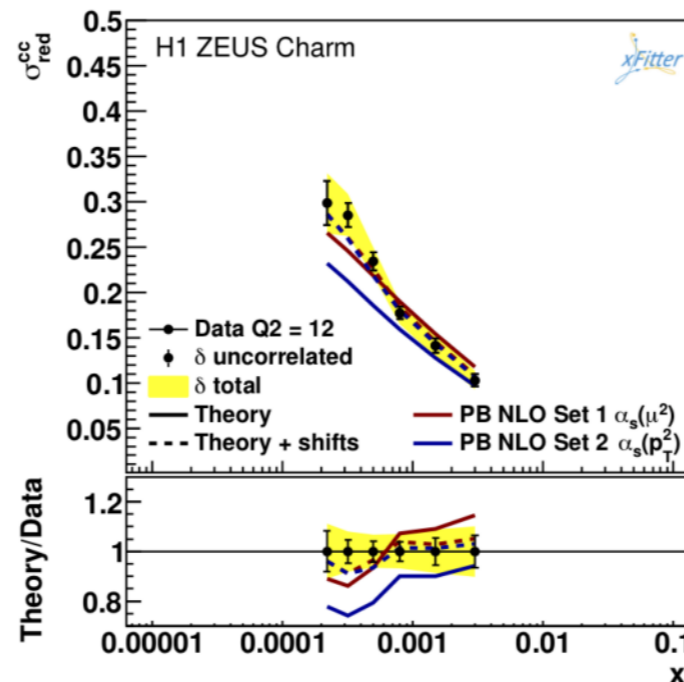
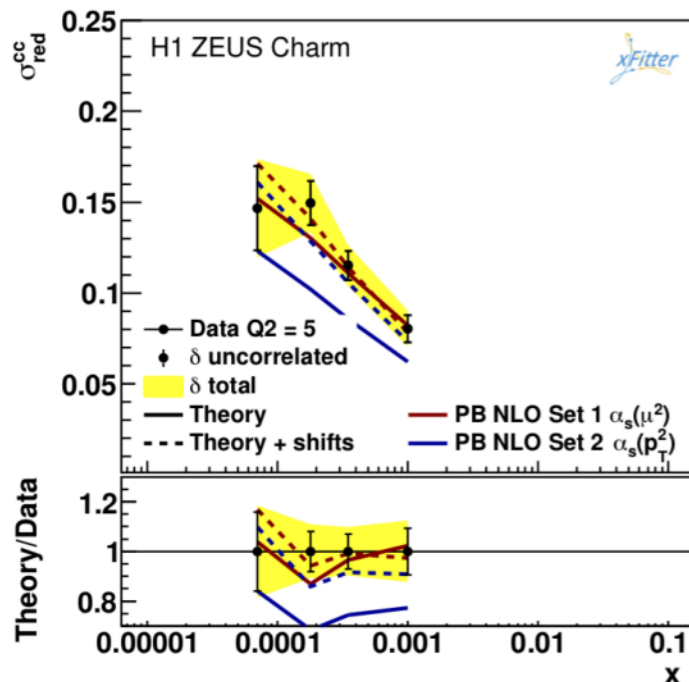
- NLO fit to combined HERA 1+2 inclusive DIS (neutral current, charged current) data
  - with in total 1145 data points
    - $3.5 < Q^2 < 50000 \text{ GeV}^2$
    - $4 \cdot 10^{-5} < x < 0.65$
  - using starting distribution as in HERAPDF2.0
  - $\chi^2/ndf = 1.2$
  - using `xFitter` frame
  - ➔ collinear PDFs and TMDs from PB method with full experimental and model dependent systematic uncertainties

# Fit to DIS structure functions: $F_2$ and $F_2^C$

## Inclusive DIS

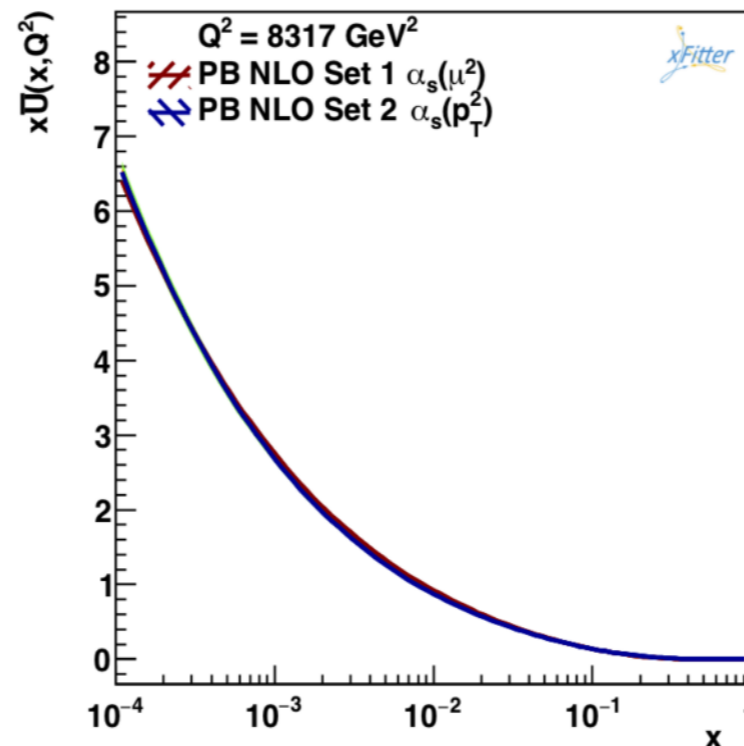
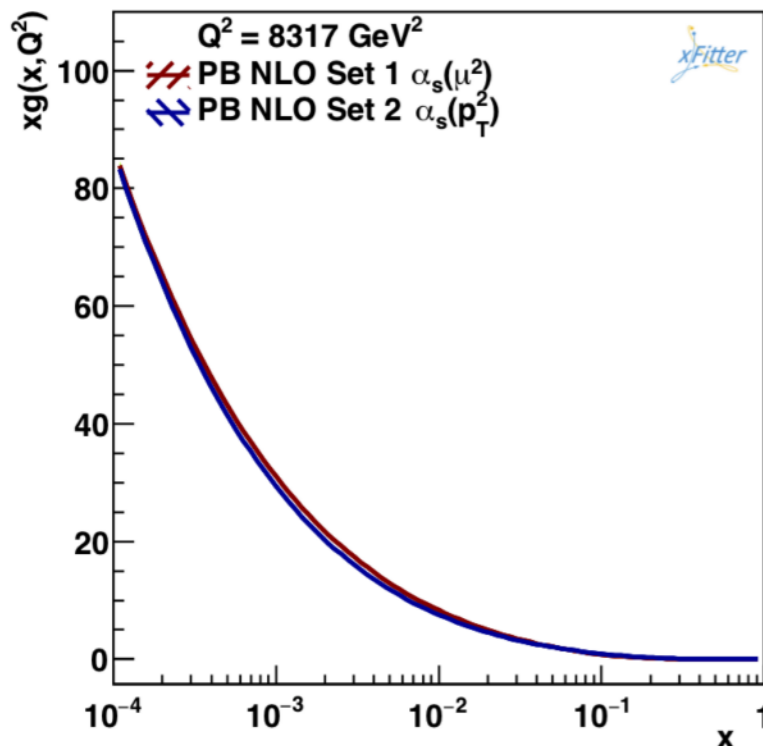
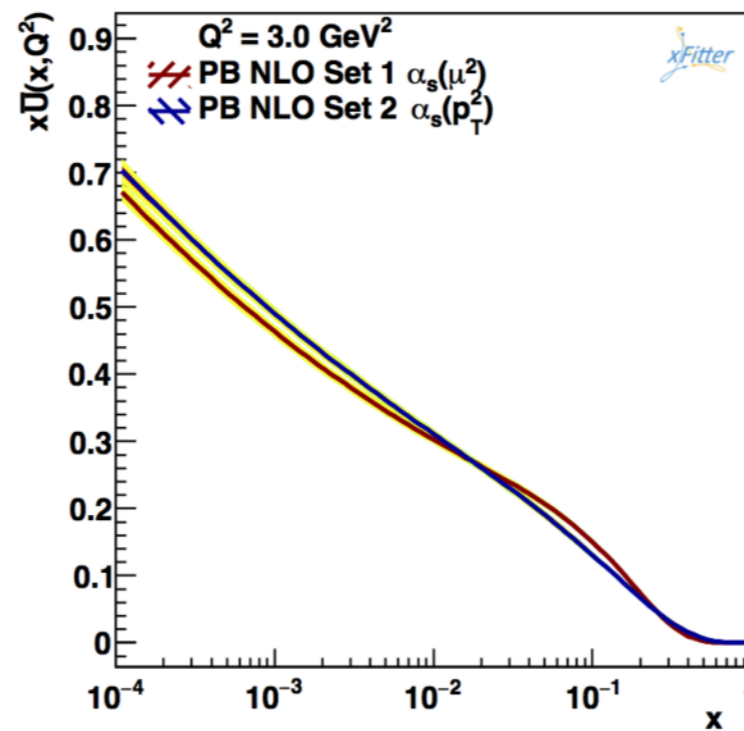
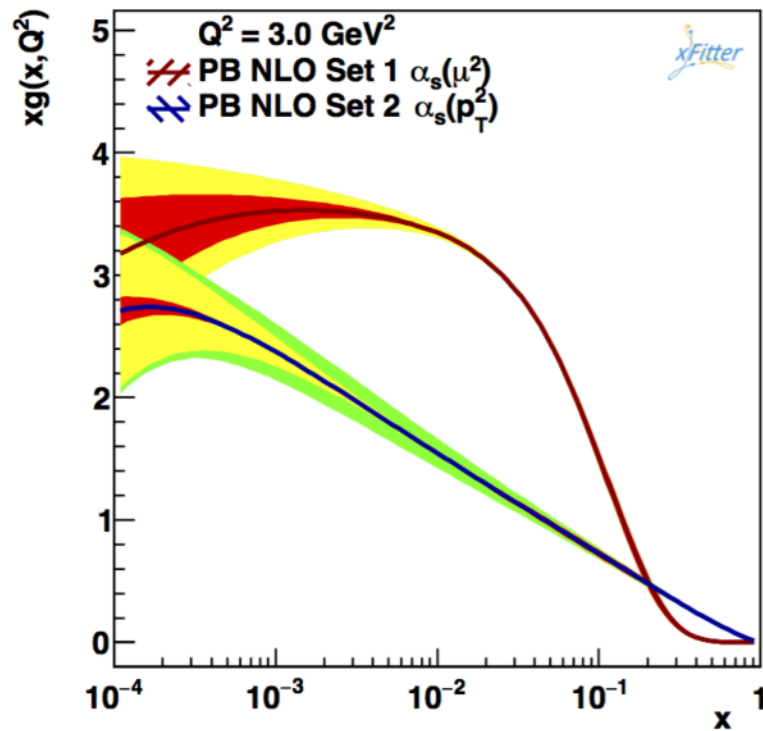


## Inclusive Charm DIS



1145 inclusive DIS data points fitted with  $\chi^2/ndf=1.2$

# Fit with different scale in $\alpha_s$

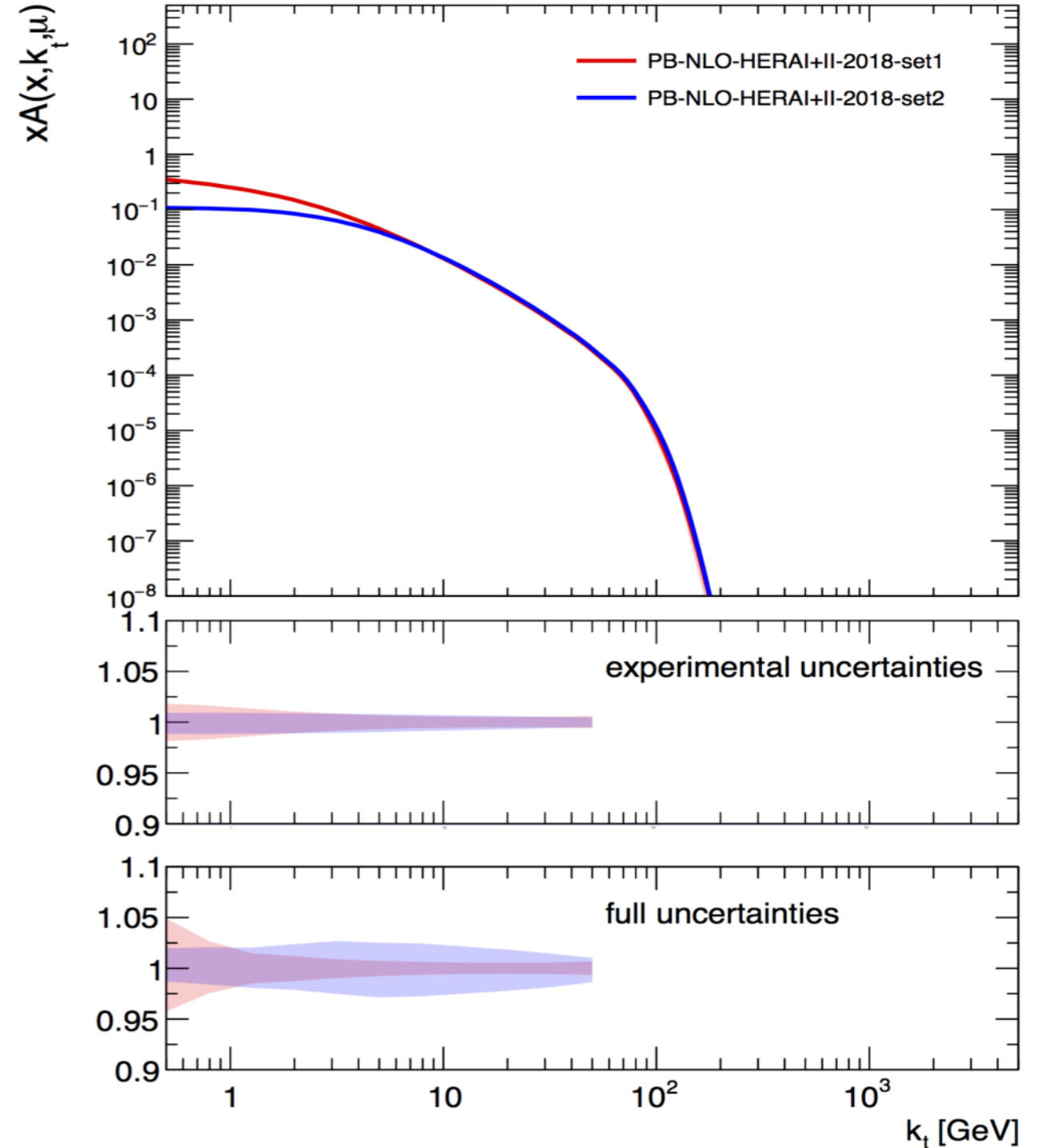
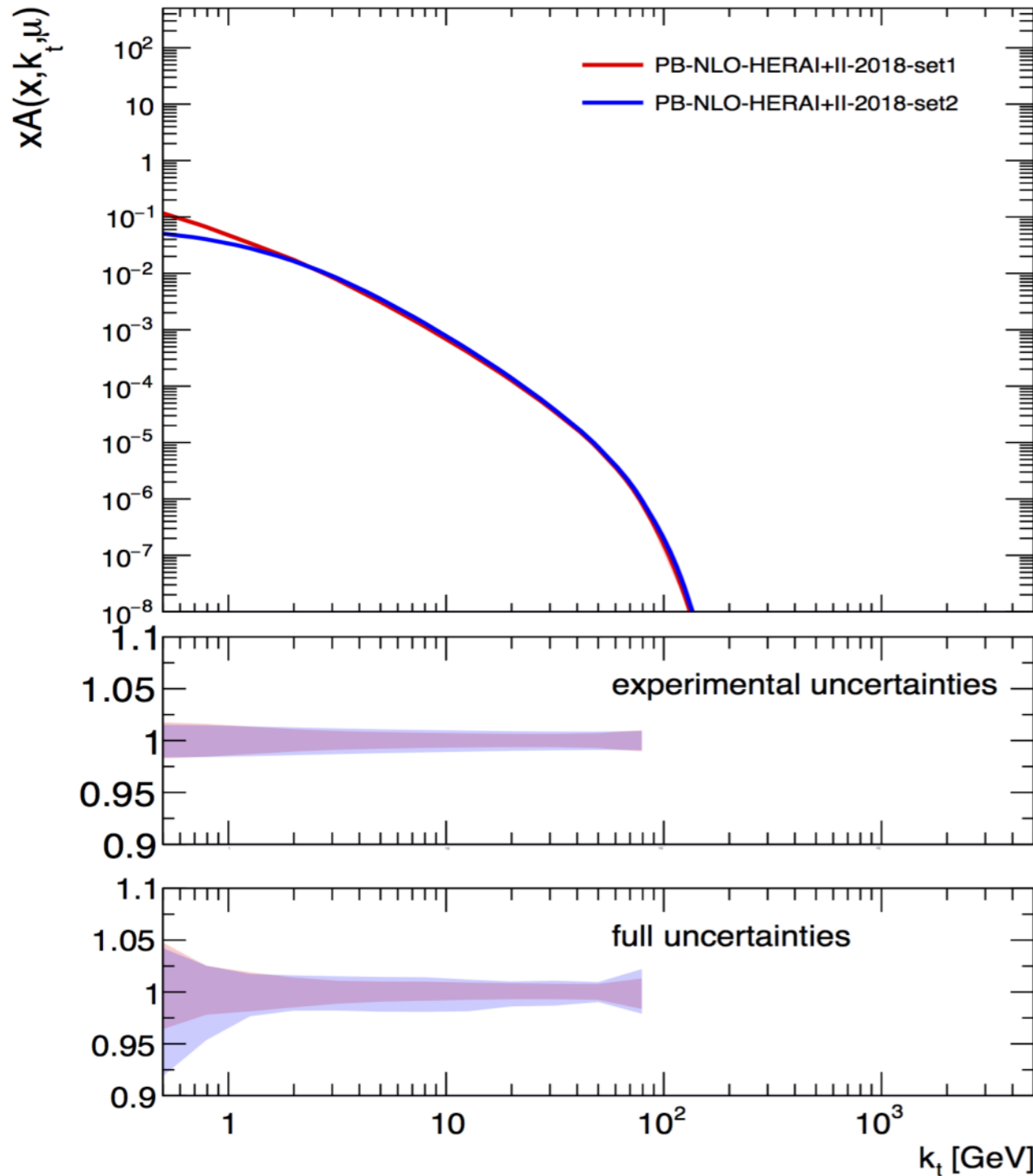


- fit 1 with  $\alpha_s(q)$ 
  - as good as HERAPDF2.0  $\chi^2/ndf = 1.2$
- fit 2 with  $\alpha_s(q(1-z))$ 
  - $\chi^2/ndf = 1.21$
- uncertainties:
  - experimental
  - starting scale
  - heavy quark masses
- gluon distribution at small  $Q^2$  depend on scale choice in  $\alpha_s$

# TMD distributions from fit to HERA data

anti-up,  $x = 0.01$ ,  $\mu = 100$  GeV

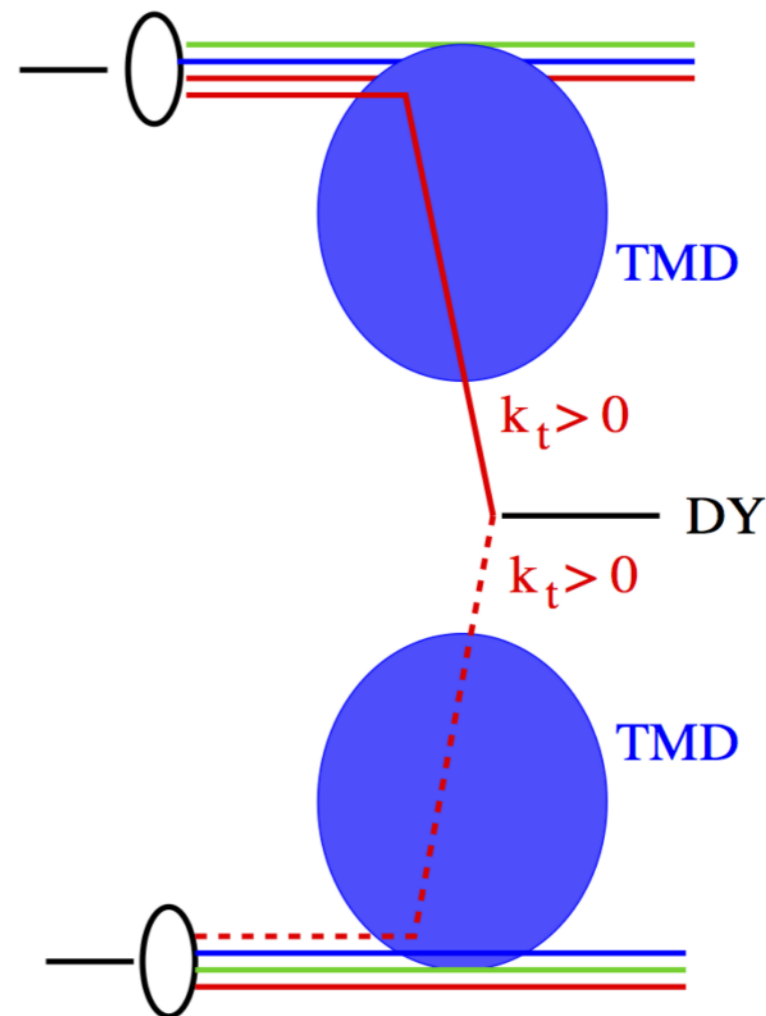
gluon,  $x = 0.01$ ,  $\mu = 100$  GeV



- model dependence larger than experimental uncertainties

# Application to DY $q_T$ - spectrum

- Use LO DY production
  - $q\bar{q} \rightarrow Z_0$
  - add  $k_t$  for each parton as function of  $x$  and  $\mu$  according to TMD
- keep final state mass fixed:
  - $x_1$  and  $x_2$  (light-cone fraction) are different after adding  $k_t$



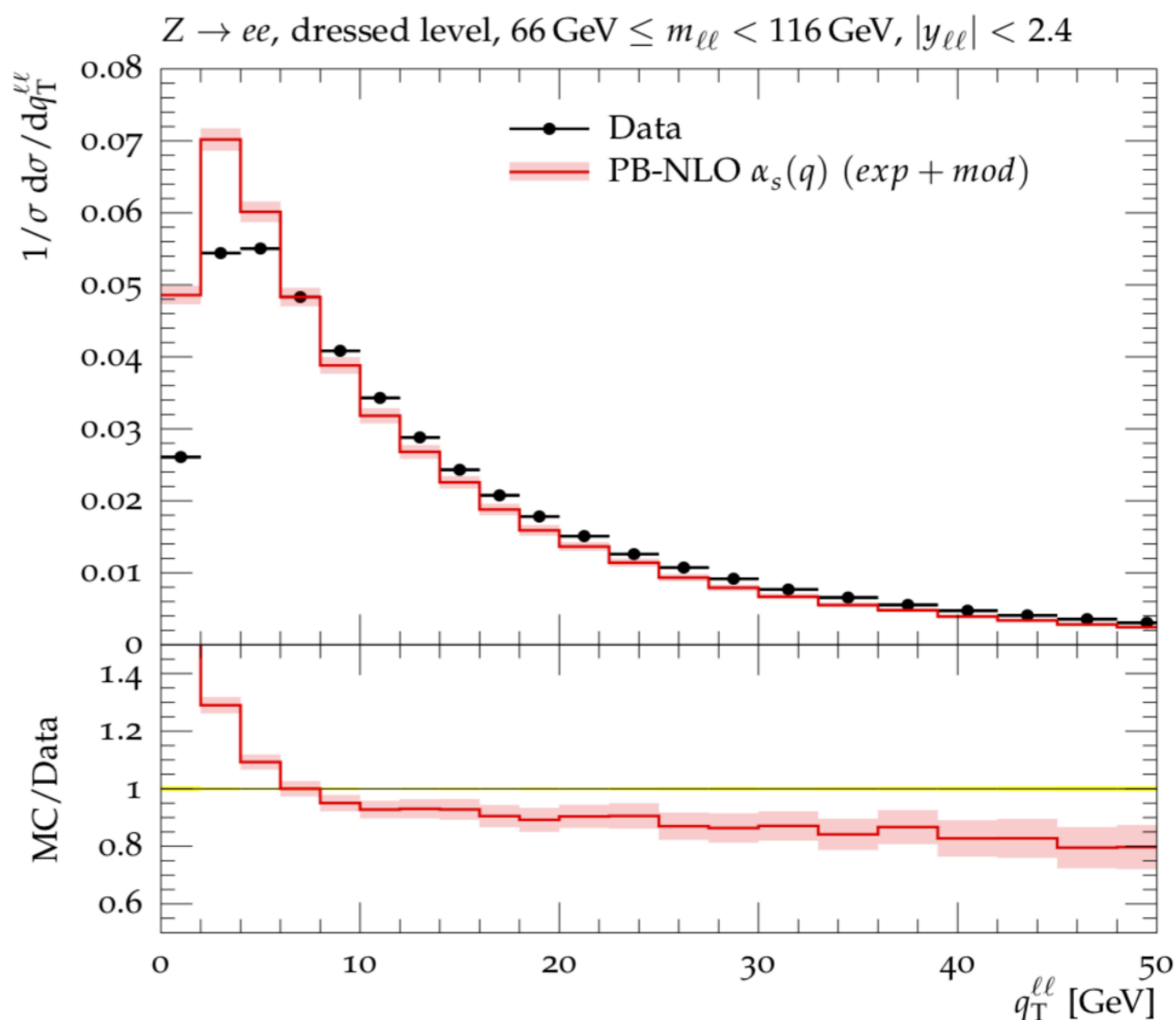
# Application to DY $q_T$ - spectrum

ATLAS Collaboration Eur. Phys. J. C76 (2016), 291  
[arXiv:1512.02192]

- Use LO DY production

$$q\bar{q} \rightarrow Z_0$$

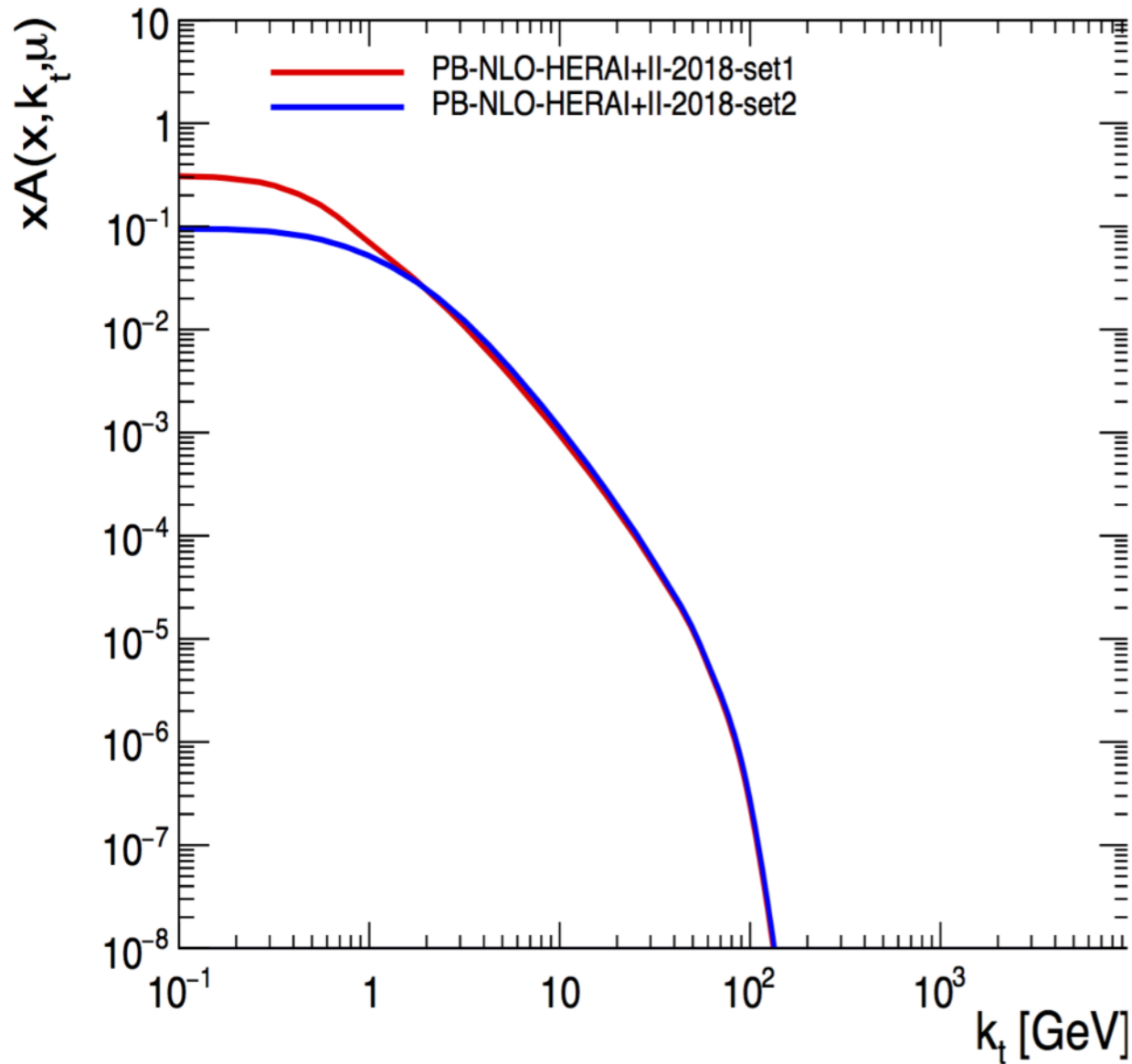
- TMD with angular ordering including  $\alpha_s(q)$



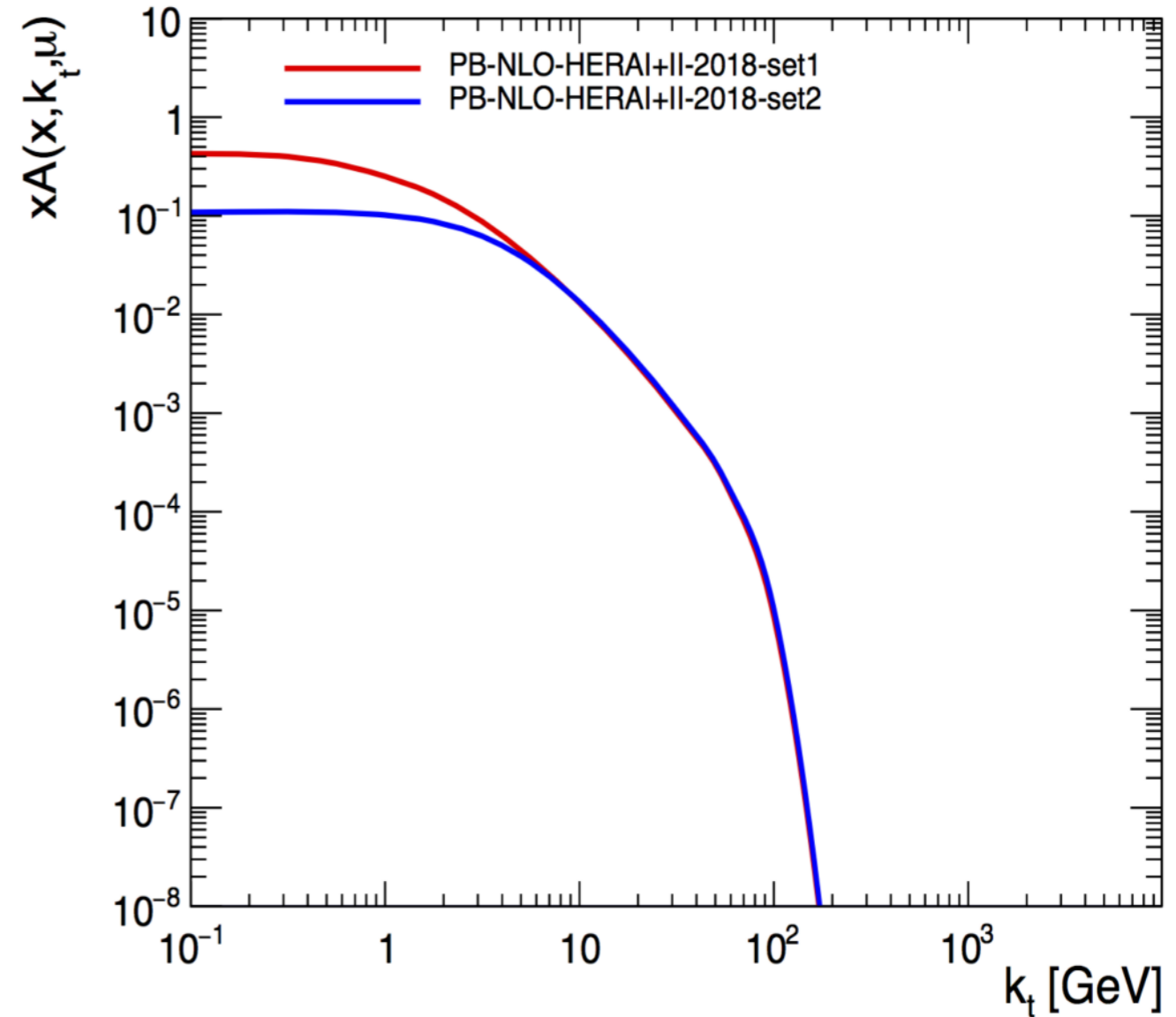


# TMD distributions

up,  $x = 0.01$ ,  $\mu = 100$  GeV



gluon,  $x = 0.01$ ,  $\mu = 100$  GeV



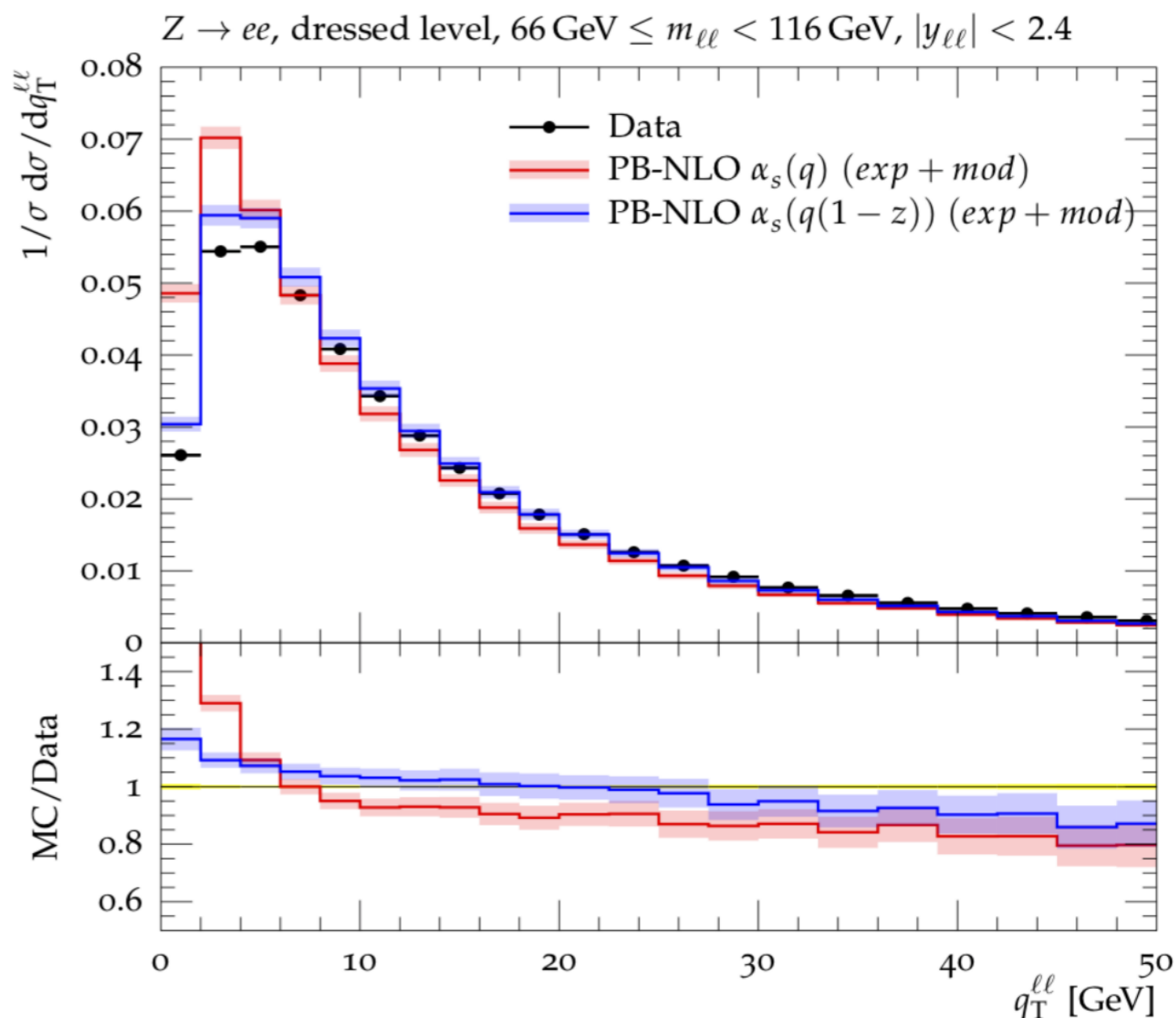
TMDplotter 2.2.0

- Differences essentially in low  $k_T$  region
  - introducing  $q_T$  instead of  $q$ , suppresses further soft gluons at low  $k_T$  !

# Application to DY $q_T$ - spectrum

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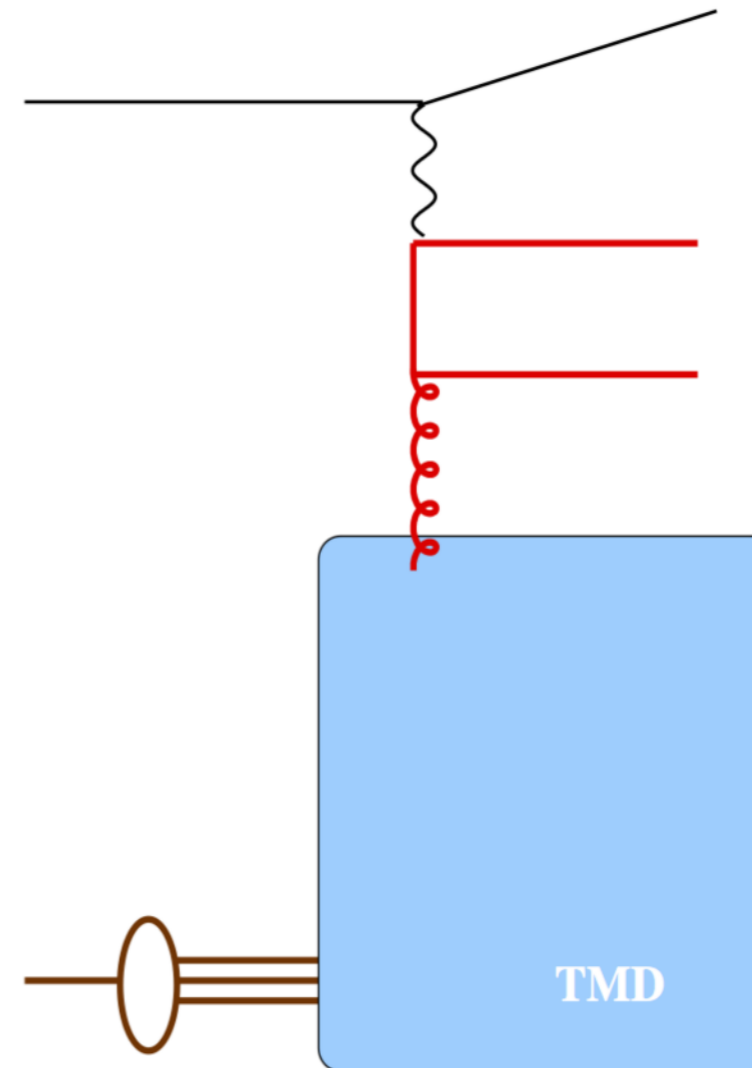
- Use LO DY production
- TMD with angular ordering including  $\alpha_s(q)$
- TMD with angular ordering including  $\alpha_s(p_T)$  much better !
- Additional issues:
  - resolvable branching
  - freeze  $\alpha_s$
  - intrinsic  $k_T$



# MCEG: TMDs and parton shower

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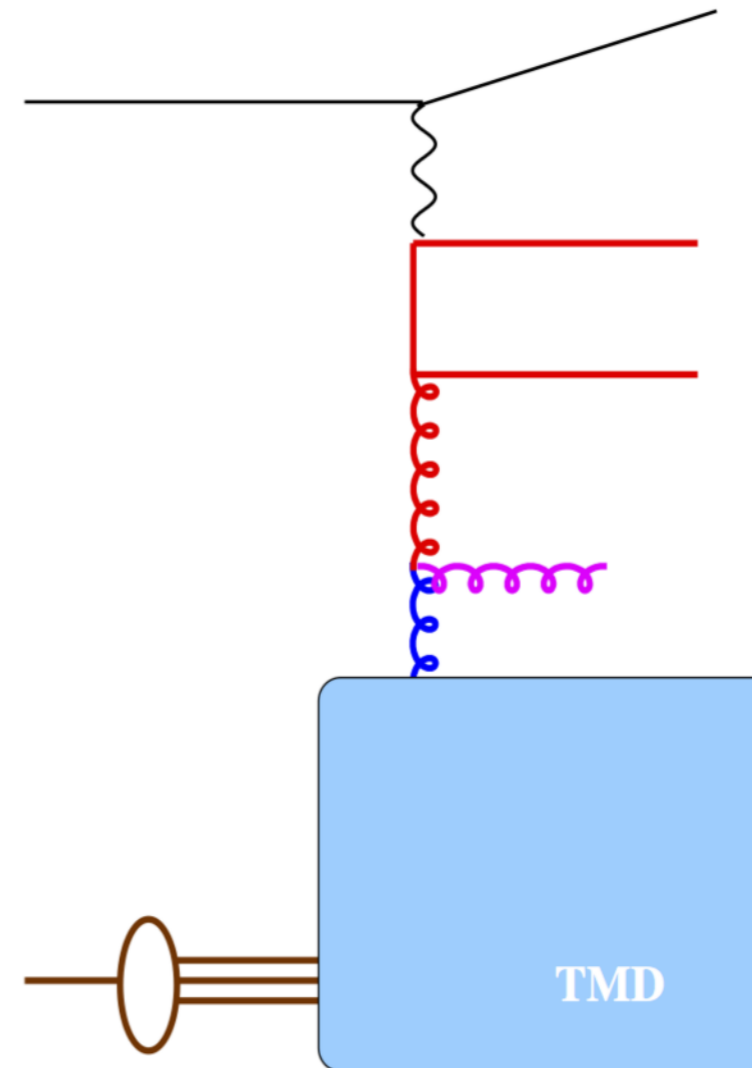
- basic elements are:
  - Matrix Elements:
    - on shell/off shell
  - PDFs
    - TMDs



# MCEG: TMDs and parton shower

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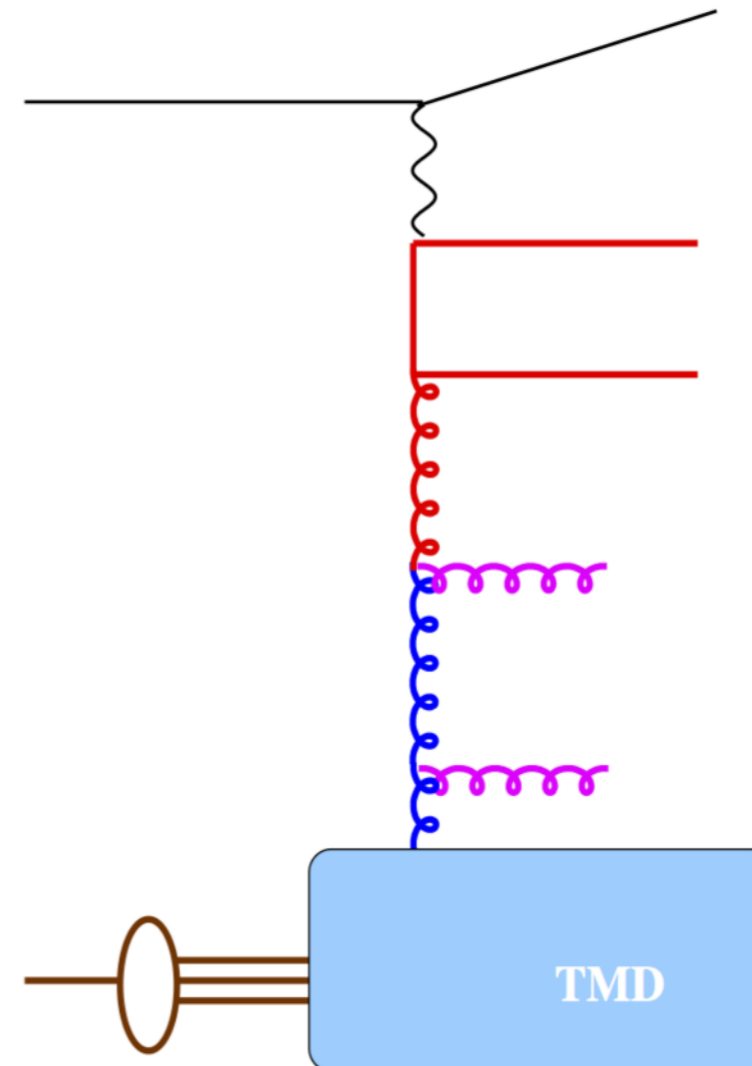
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  - Parton Shower
    - following TMDs for initial state !



# MCEG: TMDs and parton shower

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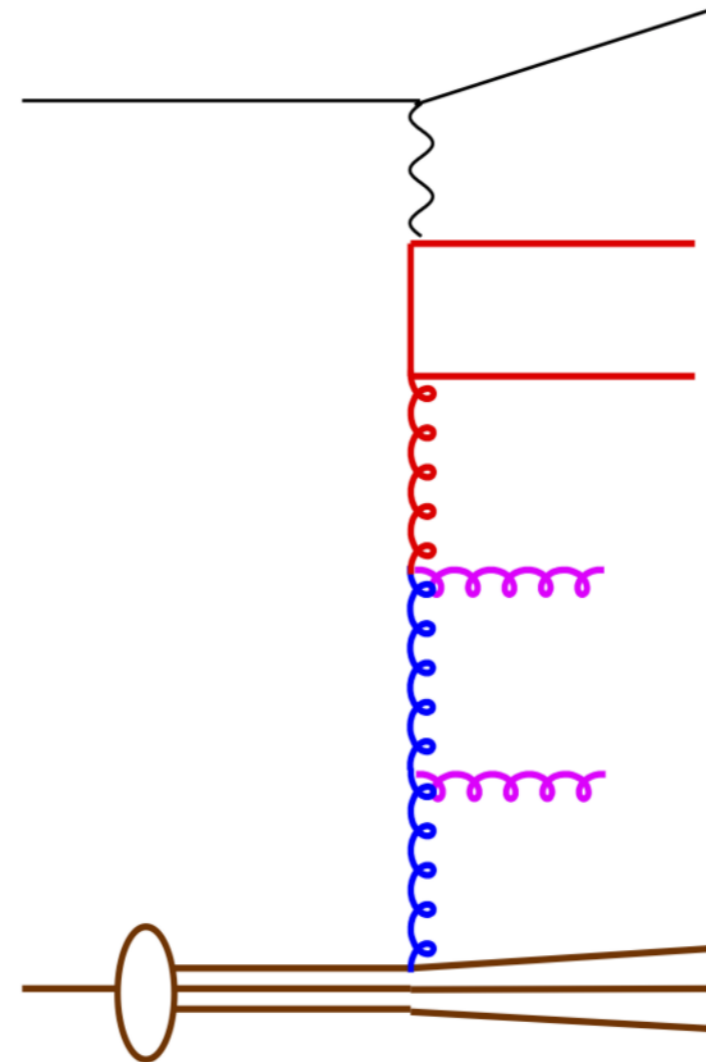
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# MCEG: TMDs and parton shower

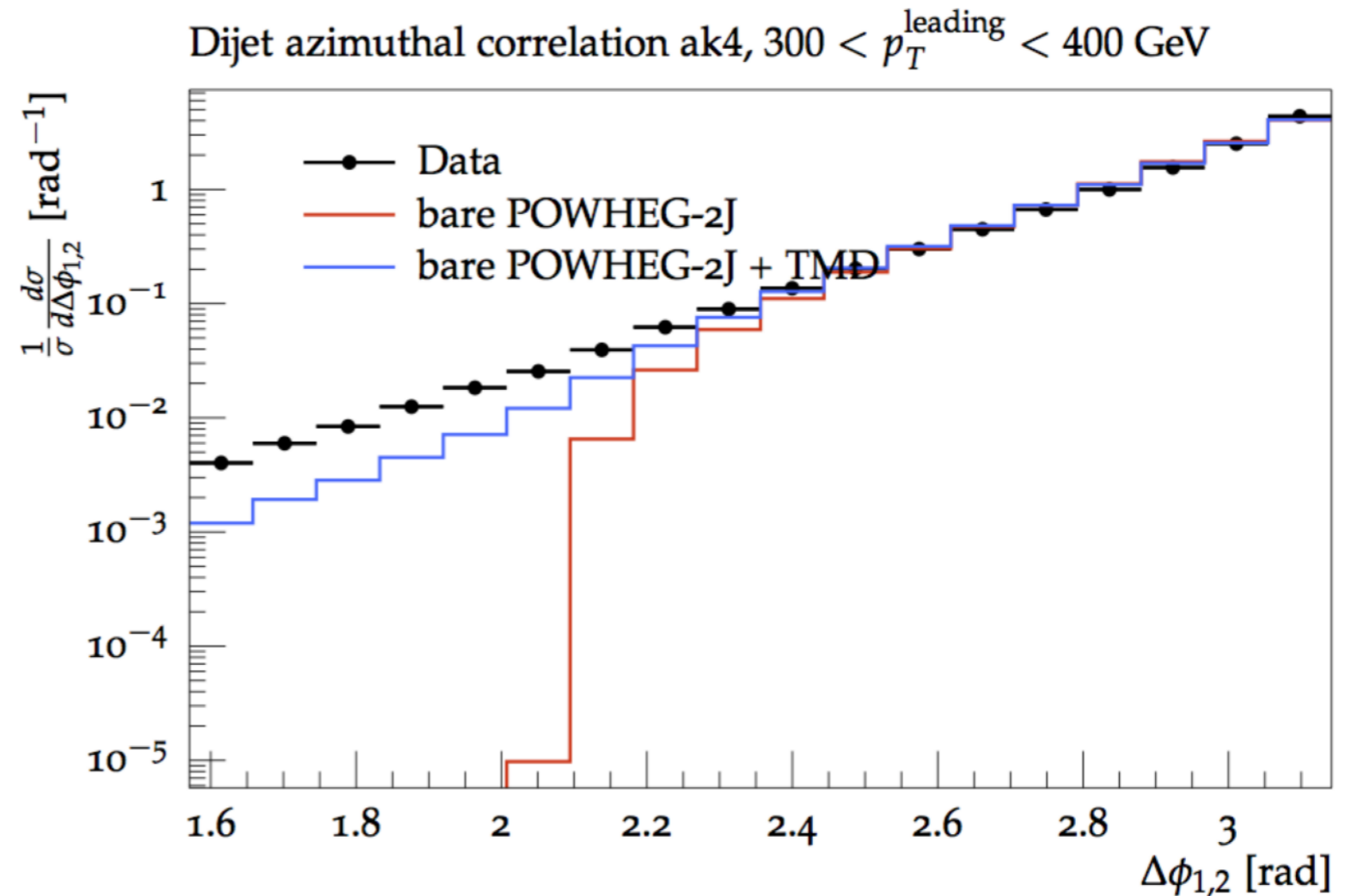
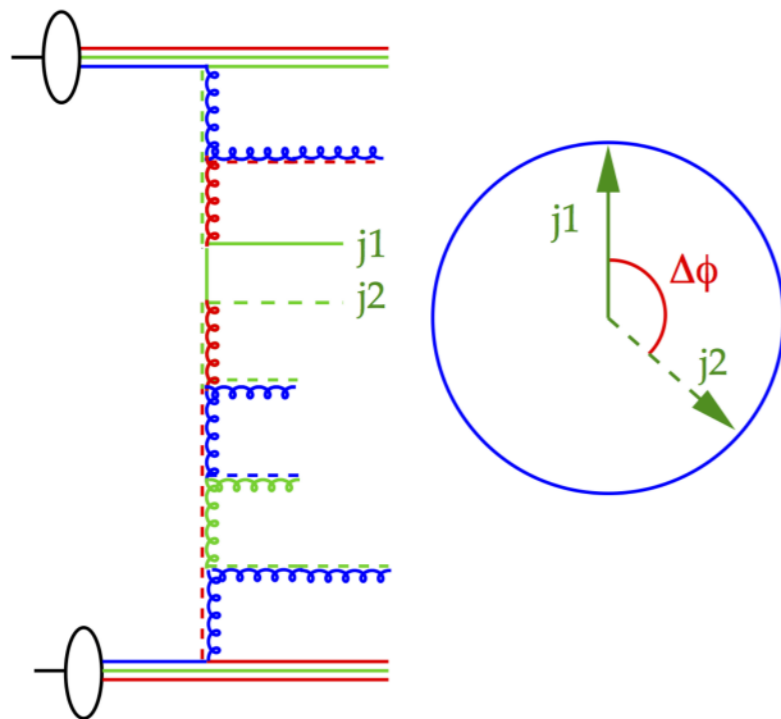
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- basic elements are:
  - **Matrix Elements:**
    - on shell/off shell
  - **PDFs**
    - TMDs
  - **Parton Shower**
    - following TMDs for initial state !
- Proton remnant and hadronization handled by standard hadronization program, e.g. PYTHIA



# Application to high $p_T$ dijets in pp

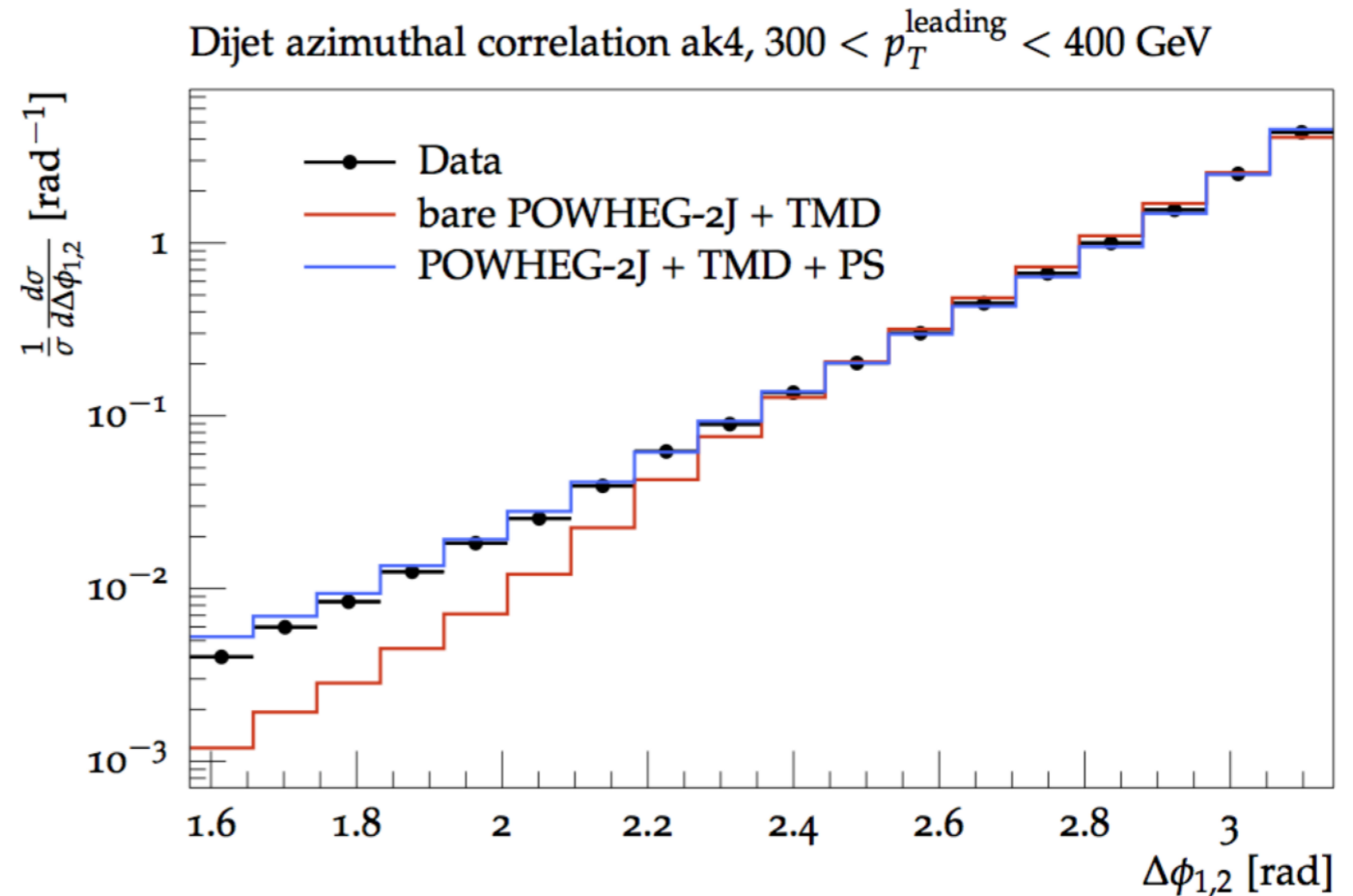
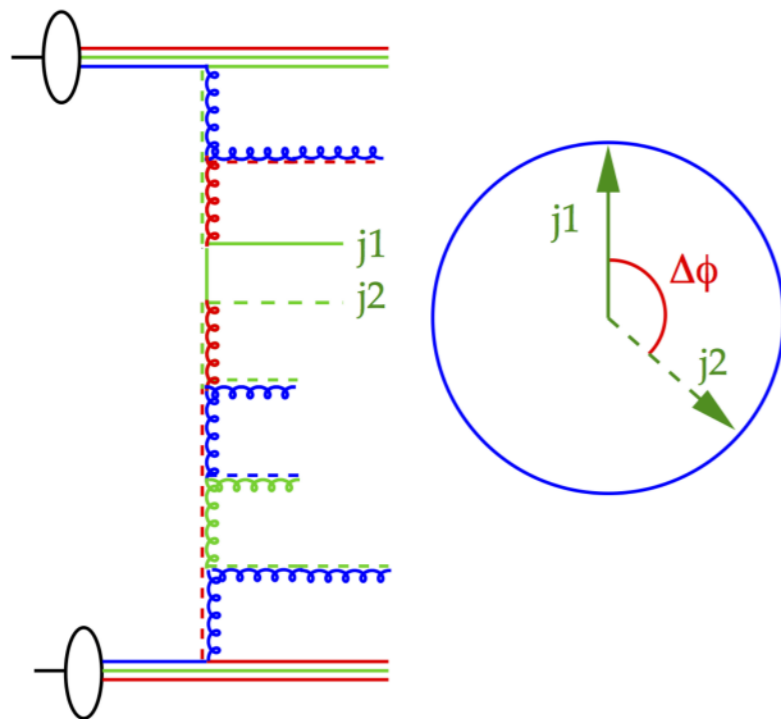
- Dijet production at in pp, a test for TMDs and PS :



- TMDs with NLO dijets get closer to data !

# Application to high $p_T$ dijets in pp

- Dijet production at in pp, a test for TMDs and PS :

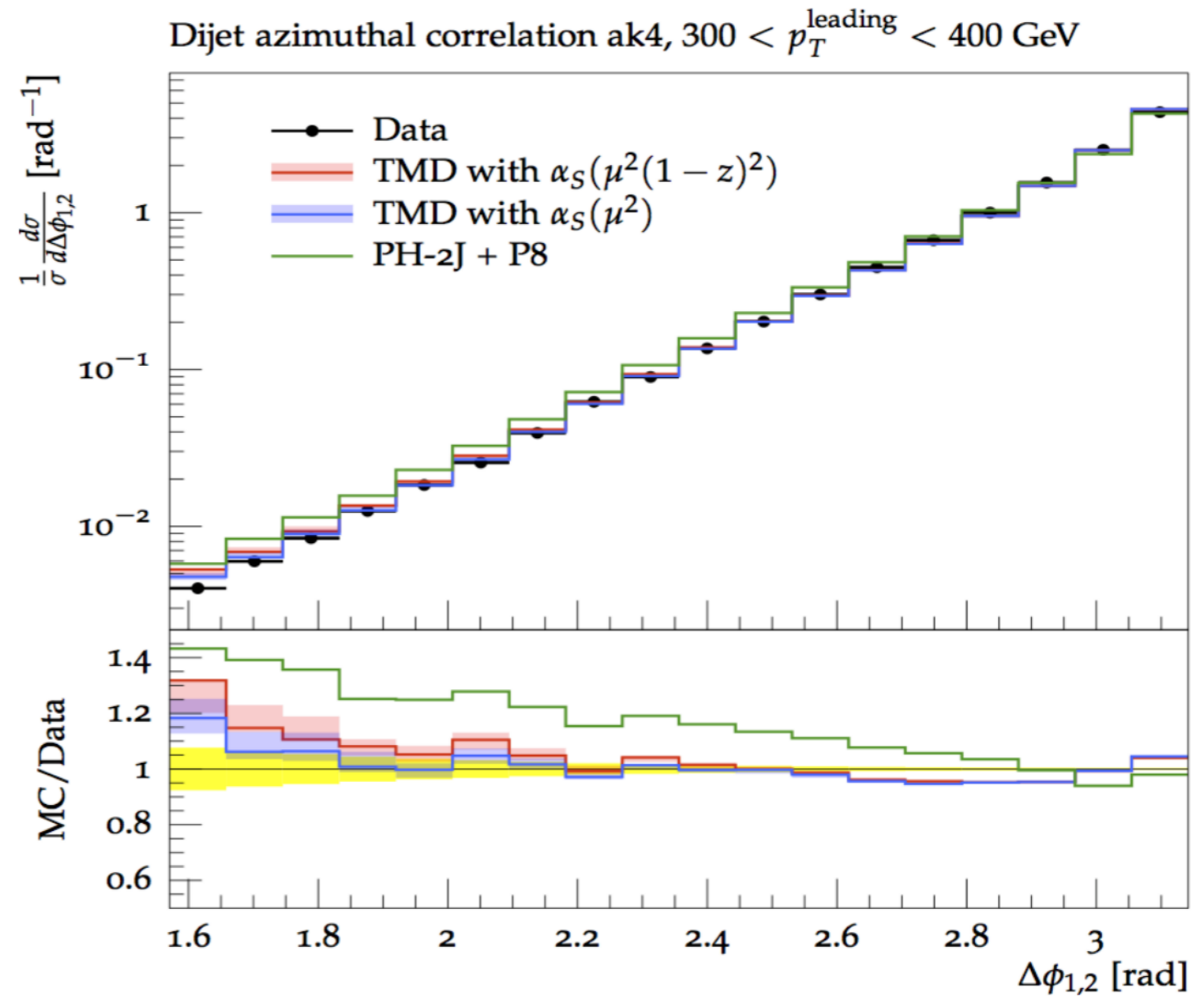
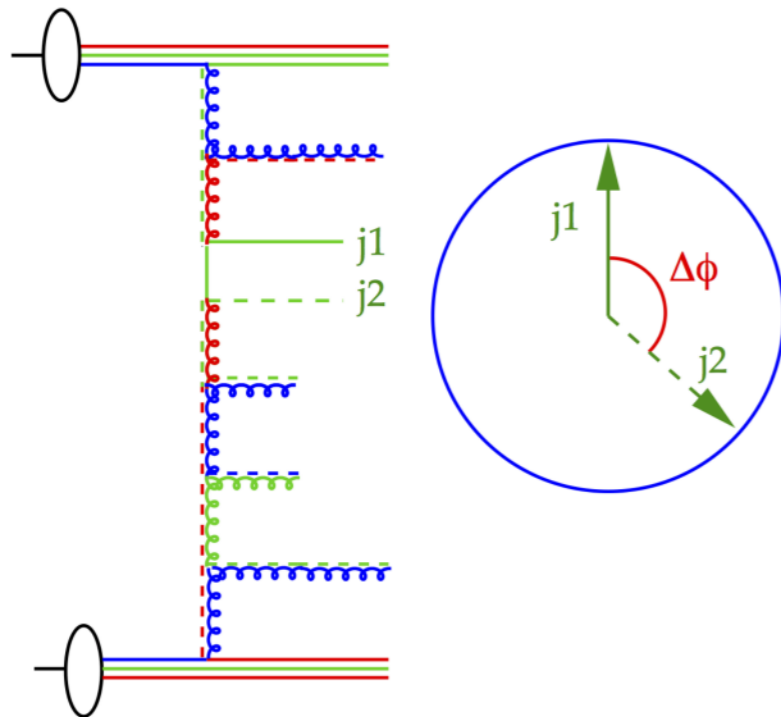


- TMDs with NLO dijets + parton shower (following TMD) describes data!



# Application to high $p_T$ dijets in pp

- Dijet production at in pp, a test for TMDs and PS :



- TMDs with NLO dijets + parton shower (following TMD) describes data!
- different TMD sets are very similar

# Conclusion

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- Parton Branching method to solve DGLAP equation at LO, NLO and NNLO
  - ➔ consistence for collinear (integrated) PDFs shown
  - ➔ advantages of Parton Branching method !
- method directly applicable to determine  $k_t$  distribution (as would be done in PS)
  - ➔ TMD distributions for all flavors determined at LO and NLO, without free parameters
  - ➔ TMD evolution implemented in xFitter – fits to DIS processes at the moment
- Application to pp processes: DY, jets:
  - ➔ DY  $q_T$  - spectrum without new parameters
  - ➔ TMD initial parton shower:
    - ➔ backward evolution following exactly the TMD density
    - ➔ dijet  $\Delta\phi$  very well described with NLO dijets + TMD + TMD shower

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# Appendix

# Evolution equation and parton branching method

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- use momentum weighted PDFs:  $x f(x, t)$

$$x f_a(x, \mu^2) = \Delta_a(\mu^2) x f_a(x, \mu_0^2) + \sum_b \int_{\mu_0}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mu'^2)} \int_x^{z_M} dz P_{ab}^{(R)}(\alpha_s, z) \frac{x}{z} f_b\left(\frac{x}{z}, \mu'^2\right)$$

- with  $P_{ab}^{(R)}(\alpha_s(t'), z)$  real emission probability (without virtual terms)
  - $z_M$  introduced to separate real from virtual and non-emission probability
  - reproduces DGLAP up to  $\mathcal{O}(1 - z_M)$
- make use of momentum sum rule to treat virtual corrections
  - use Sudakov form factor for non-resolvable and virtual corrections

$$\Delta_a(z_M, \mu^2, \mu_0^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz z P_{ba}^{(R)}(\alpha_s), z\right)$$

# Advantages of parton branching method

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- DGLAP equation:

$$\mu^2 \frac{\partial}{\partial \mu^2} f(x, \mu^2) = \int \frac{dz}{z} \frac{\alpha_s(\mu_r)}{2\pi} P_+(z) f\left(\frac{x}{z}, \mu^2\right)$$

- Advantages of parton branching method for collinear PDFs:
  - access to all kinematic variables and combinations between them
    - full freedom of choosing:
      - renormalisation scale:  $\alpha_s(\mu_r)$
      - evolution scale:  $\mu_f$
  - studies of different ordering conditions possible for the first time
    - angular ordering with  $\alpha_s(q)$
    - but angular ordering suggests that renormalization scale is  $p_T$  and not angle
      - angular ordering with  $\alpha_s(p_T) \rightarrow \alpha_s(q(1-z))$
      - repeat fits with changed renormalisation scale in pdf (but not yet in coefficient fct)

# The limit $z_M$

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- Investigating the large  $z$  part:

$$\begin{aligned} & \sum_b \int_x^1 dz K_{ab}(\alpha_s) \frac{1}{(1-z)_+} \tilde{f}_b(x/z, \mu^2) \\ &= \sum_b \int_x^1 dz K_{ab}(\alpha_s) \frac{1}{1-z} \tilde{f}_b(x/z, \mu^2) - \sum_b \int_0^1 dz K_{ab}(\alpha_s) \frac{1}{1-z} \tilde{f}_b(x, \mu^2) \end{aligned}$$

- in the region  $1 > z > z_M$  expand:

$$\tilde{f}_b(x/z, \mu^2) = \tilde{f}_b(x, \mu^2) + (1-z) \frac{\partial \tilde{f}_b}{\partial \ln x}(x, \mu^2) + \mathcal{O}(1-z)^2$$

- up to  $\mathcal{O}(1-z_M)$  :

$$\begin{aligned} & \sum_b \int_x^1 dz K_{ab}(\alpha_s) \frac{1}{(1-z)_+} \tilde{f}_b(x/z, \mu^2) \\ &= \sum_b \int_x^{z_M} dz K_{ab}(\alpha_s) \frac{1}{1-z} \tilde{f}_b(x/z, \mu^2) - \sum_b \int_0^{z_M} dz K_{ab}(\alpha_s) \frac{1}{1-z} \tilde{f}_b(x, \mu^2) \end{aligned}$$

# Where to find TMDs ? TMDlib and TMDplotter

- TMDlib proposed in 2014 as part of REF workshop and developed since
- combine and collect different ansaetze and approaches:

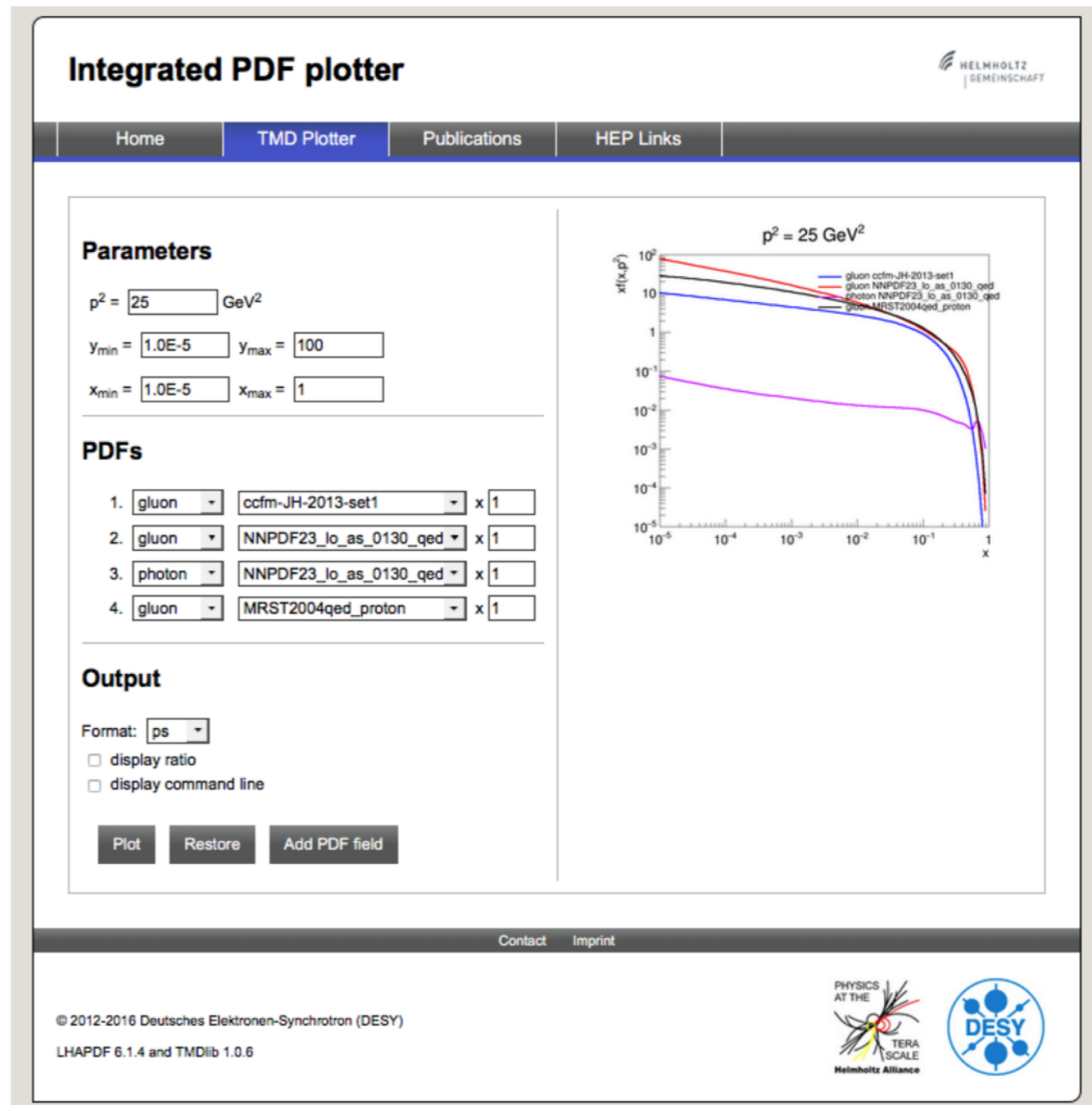
<http://tmd.hepforge.org/> and  
<http://tmdplotter.desy.de>

- TMDlib: a library of parametrization of different TMDs and uPDFs (similar to LHAPdf)

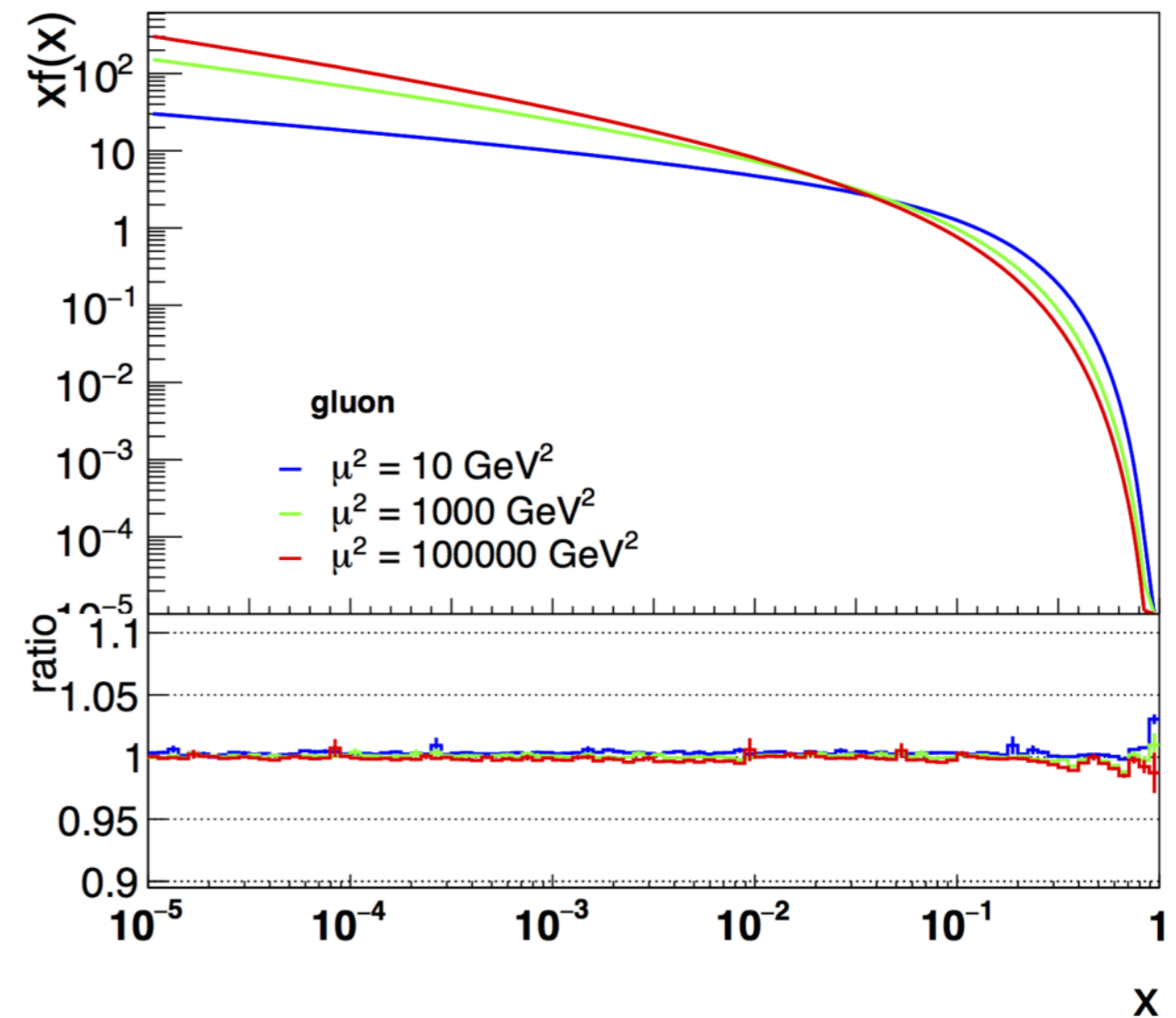
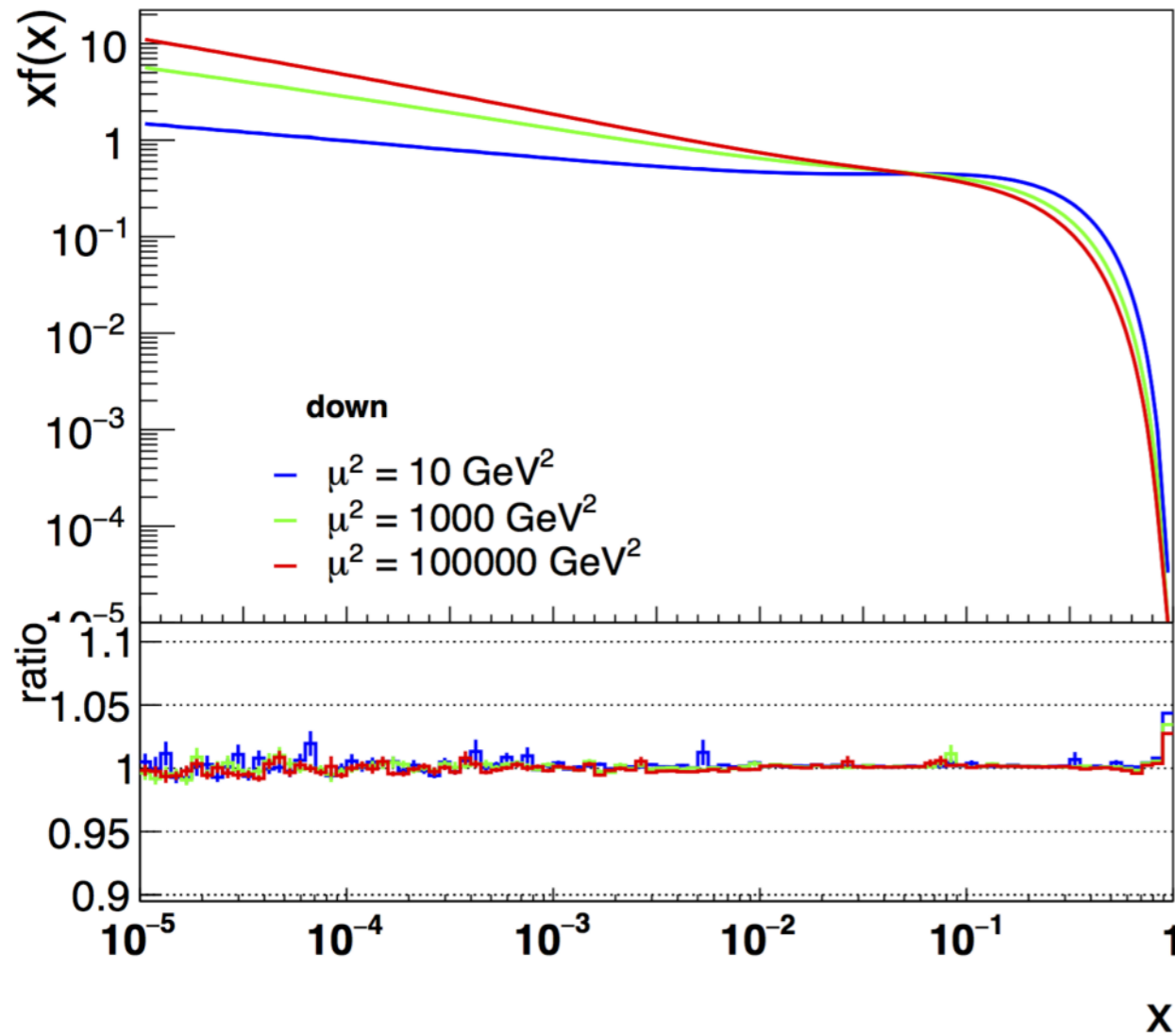
TMDlib and TMDplotter: library and plotting tools for transverse-momentum-dependent parton distributions, *F. Hautmann et al. arXiv 1408.3015, Eur. Phys. J., C 74(12):3220, 2014.*

- Also integrated pdfs (including photon pdf are available via LHAPDF)

- Feedback and comments from community is needed – just use it !



# Validation of method with QCDnum at **NLO**



- Very good agreement with **NLO** - QCDnum over all  $x$  and  $\mu^2$ 
  - the same approach works also at NNLO !

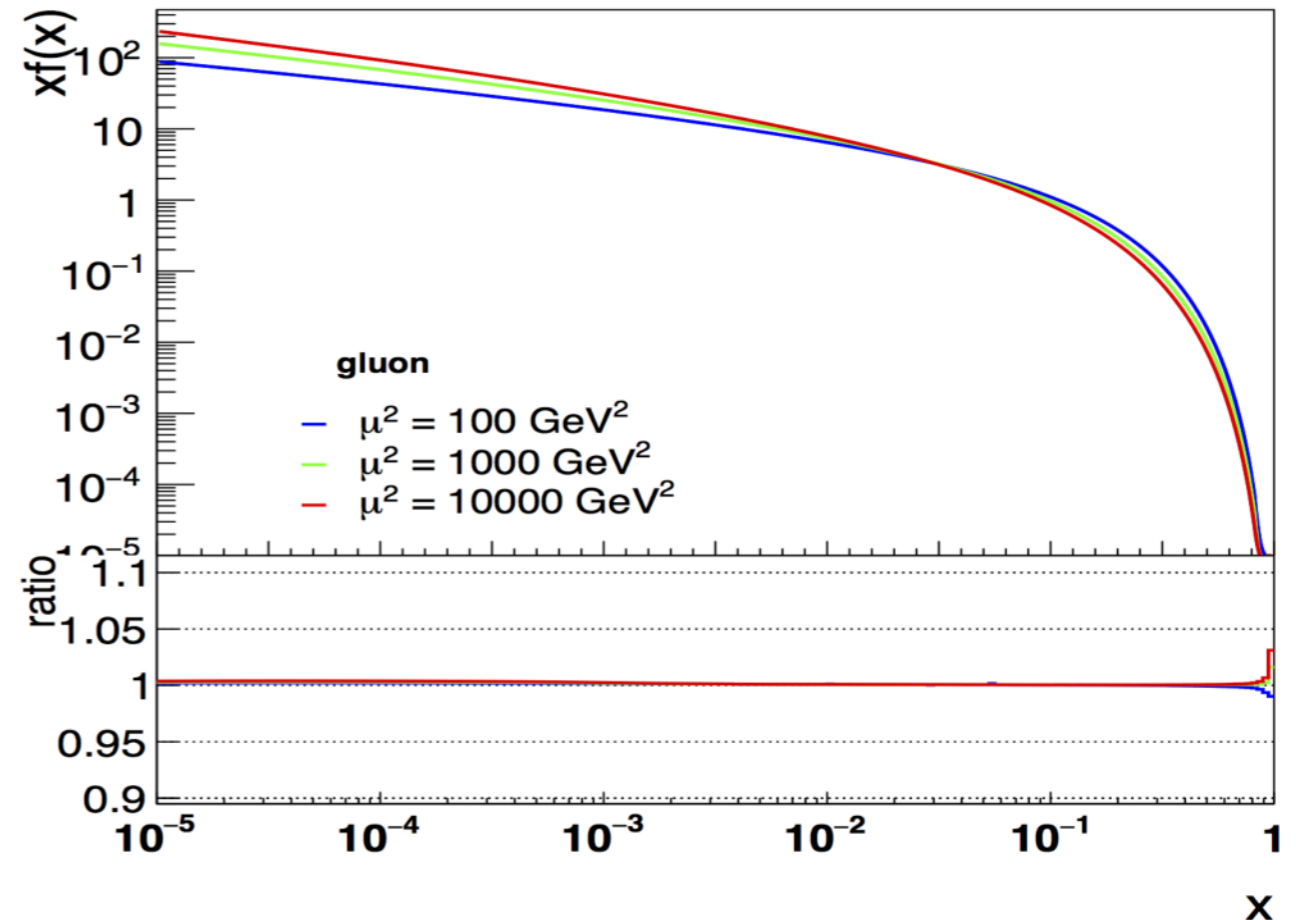
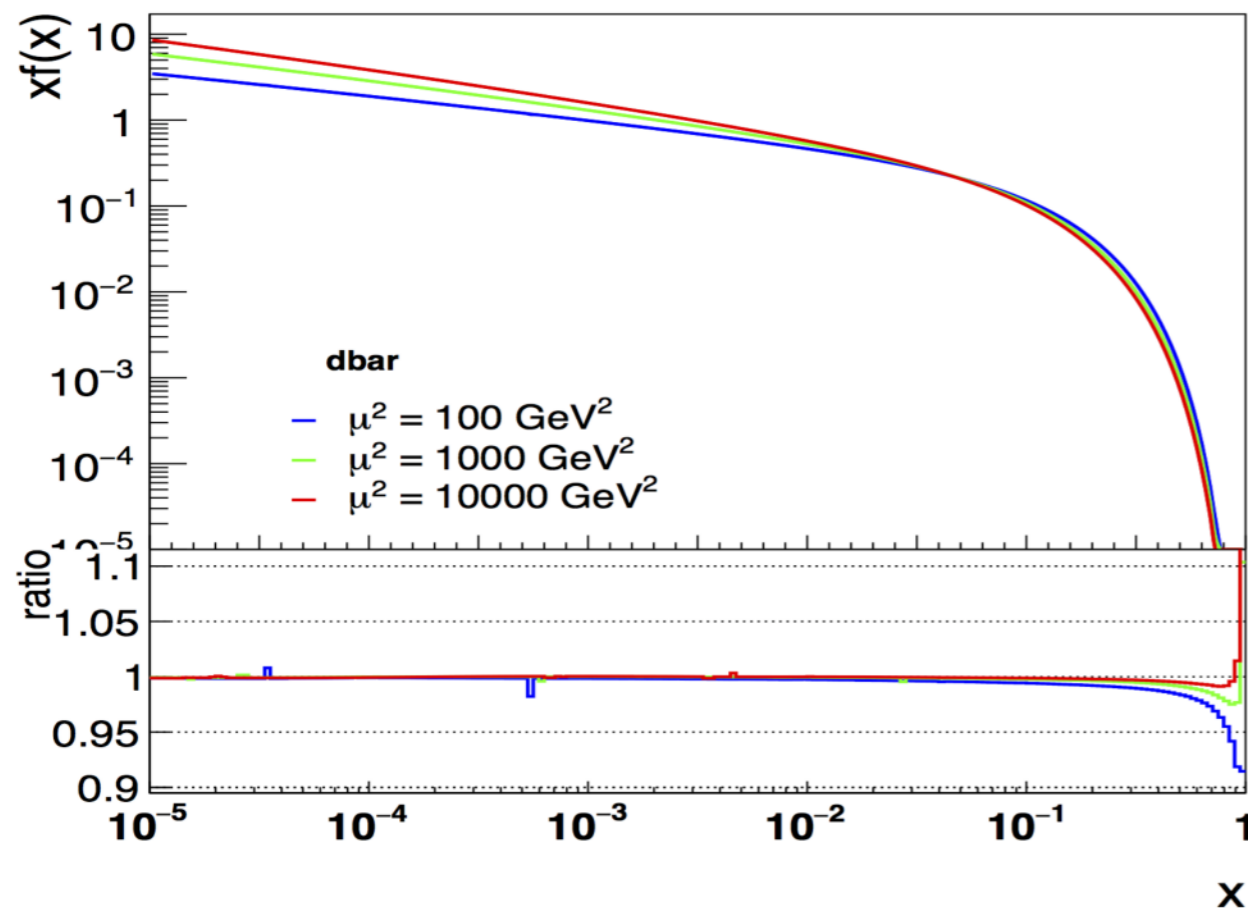


# Parton branching method in xFitter

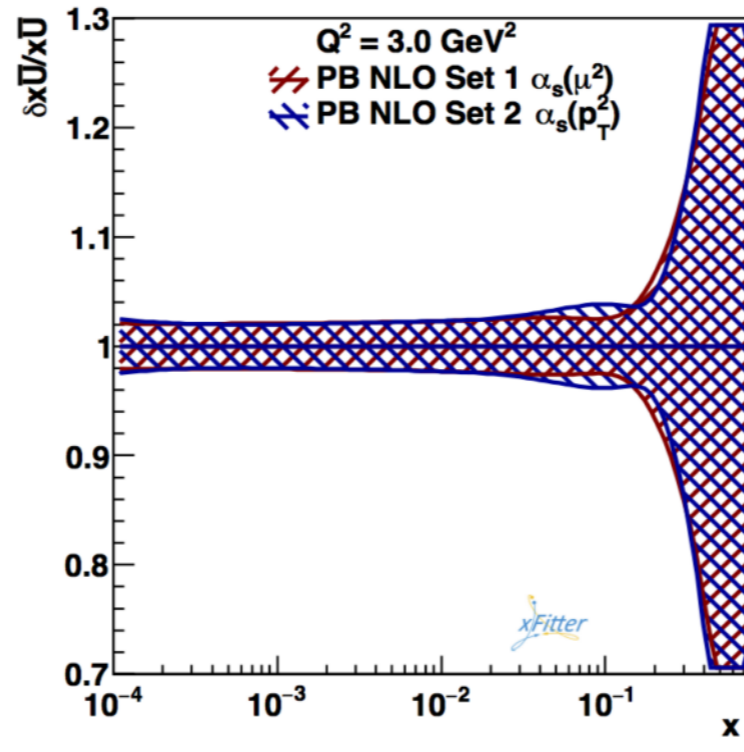
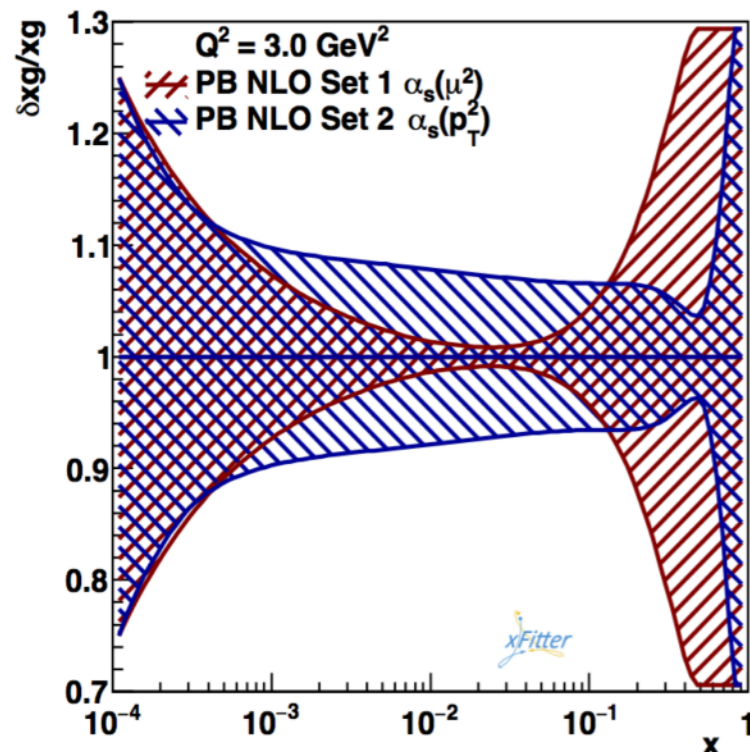
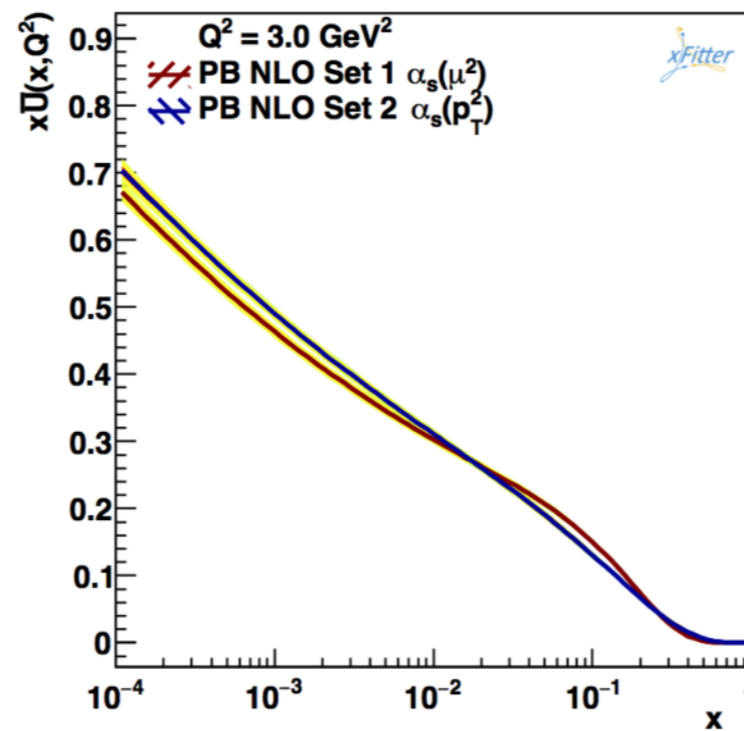
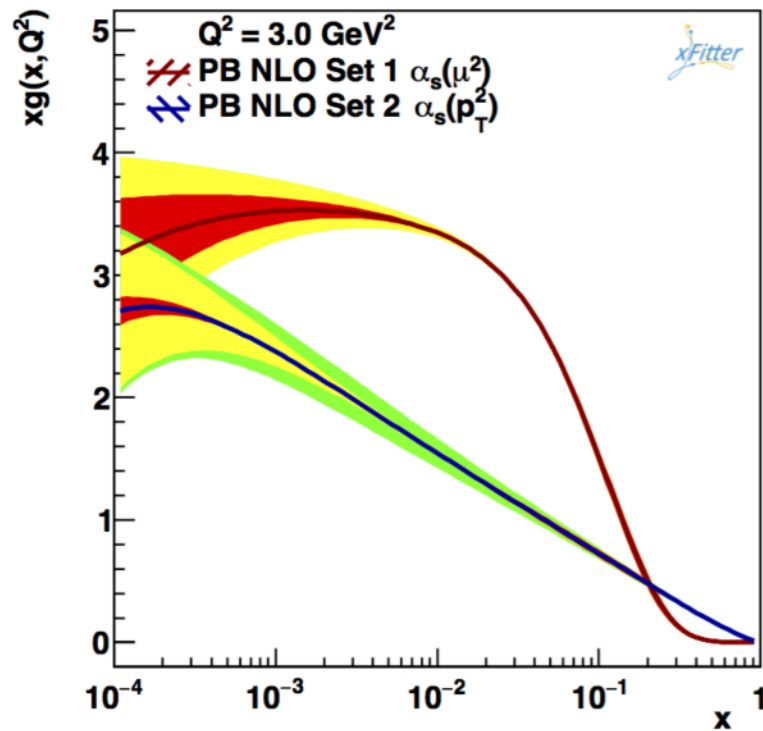
- Convolution of kernel with starting distribution

$$\begin{aligned}
 x f_a(x, \mu^2) &= x \int dx' \int dx'' \mathcal{A}_{0,b}(x') \tilde{\mathcal{A}}_a^b(x'', \mu^2) \delta(x'x'' - x) \\
 &= \int dx' \mathcal{A}_{0,b}(x') \cdot \frac{x}{x'} \tilde{\mathcal{A}}_a^b\left(\frac{x}{x'}, \mu^2\right)
 \end{aligned}$$

- kernel defined on grid (for integrated and TMD distribution)
- validation of method:

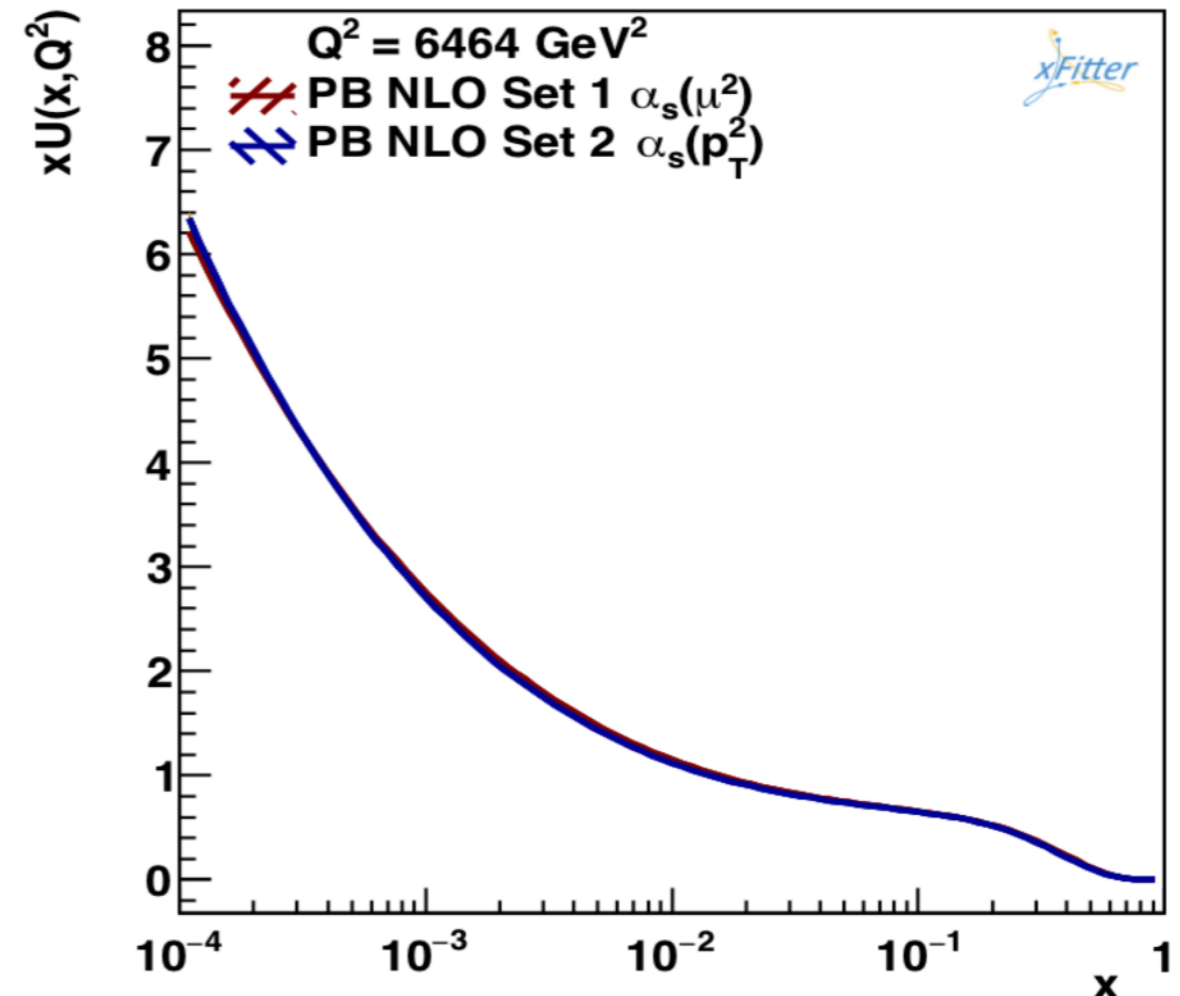
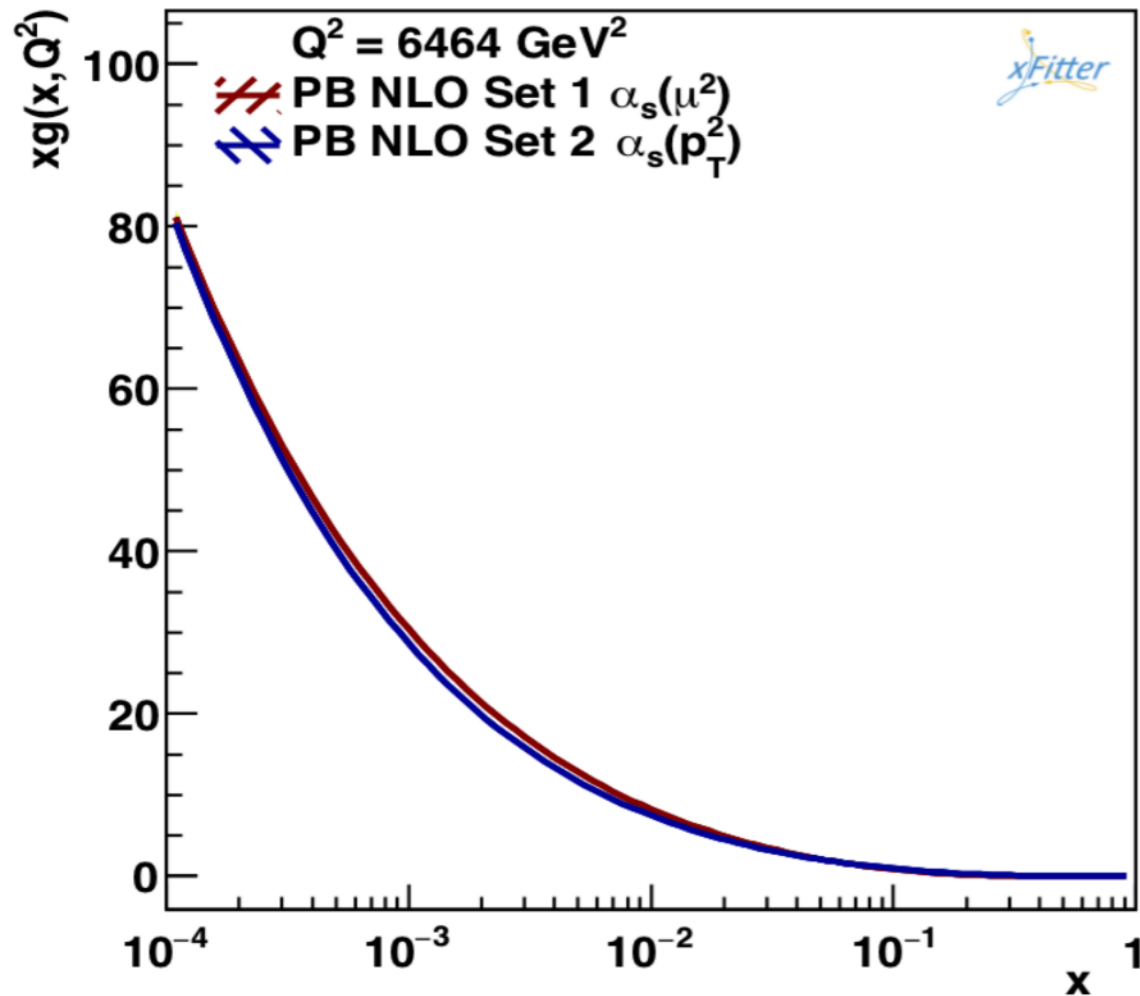


# Fit with different scale in $\alpha_s$ : at small $Q^2$



- fit 1 with  $\alpha_s(q)$ 
  - as good as HERAPDF2.0  
 $\chi^2/ndf = 1.2$
- fit 2 with  $\alpha_s(q(1-z))$ 
  - $\chi^2/ndf = 1.21$
- very different gluon distribution obtained at small  $Q^2$

# Fit with different scale in $\alpha_s$ : at large $Q^2$



# MCEG: TMDs, parton shower

- basic elements are:
  - **Matrix Elements:**
    - on shell/off shell
  - PDFs
    - TMDs
  - Parton Shower
    - following TMDs for initial state !
- Proton remnant and hadronization handled by standard hadronization program, e.g. PYTHIA
- **Parton shower with TMDs follows exactly the evolution of the TMD**
  - no (!) free parameter in shower
  - resolvable branchings and calculation of  $k_T$  defined in TMD
  - no adjustment of kinematics during/after shower

