Top pair production in hadron-hadron collisions at NNLO

Sebastian Sapeta

IFJ PAN Kraków

In collaboration with René Ángeles-Martinez and Michał Czakon

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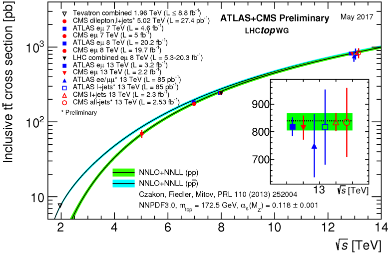
Top pair production: the status of QCD calculations

- A single *complete* NNLO result for total and differential cross section obtained with STRIPPER methodology [Czakon, Mitov Fiedler, Heymes, Mitov '13, '16]



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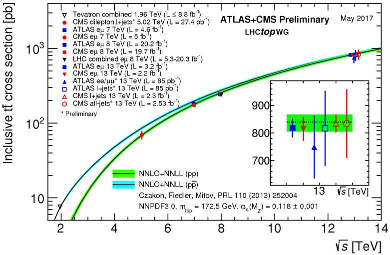

- Flavour off-diagonal channels at NNLO from q_T subtraction [Bonciani, Catani, Grazzini, Sargsyan, Torre ‘15]
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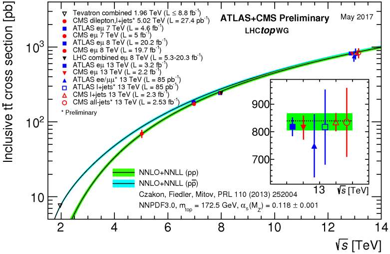

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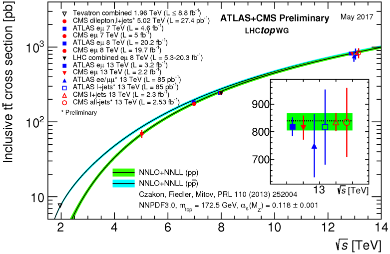

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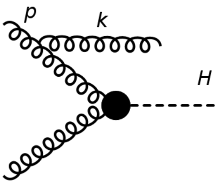
- Given the complexity of the calculation, a second result obtained with an *independent* method is highly desirable
- The framework developed to perform the above would be of direct use for N^{3}LO calculations for a range of processes relevant for the LHC

Anatomy of perturbative QCD calculations

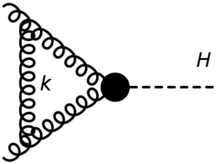
Next-to-Leading Order (NLO), UV-renormalized

Real $\int d^4 k$



$\Rightarrow$ divergent in the limits $k \to 0$ or $k \parallel p$

Virtual $\int d^{4-2\epsilon} k$



$= \frac{1}{\epsilon^2} + \frac{1}{\epsilon} + \text{finite}$

Anatomy of perturbative QCD calculations

$$\sigma_{\text{NLO}} = R + V$$

- R and V are separately divergent in the soft and collinear limits *(IR divergences)*
- Kinoshita-Lee-Nauenberg theorem guarantees that σ_{NLO} is finite
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How to carry out this cancellation in practice, given that R is integrated in 4 while V in d dimensions?

The q_T slicing method

[Catani, Grazzini '07, '15]

$$p + p \to F\,(q_T) + X$$

$$\sigma^F_{\mathrm{N}^m\mathrm{LO}} = \int_0^{q_{T,\mathrm{cut}}} dq_T \, \frac{d\sigma^F_{\mathrm{N}^m\mathrm{LO}}}{dq_T} + \int_{q_{T,\mathrm{cut}}}^{\infty} dq_T \, \frac{d\sigma^F_{\mathrm{N}^m\mathrm{LO}}}{dq_T}$$

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enough to know in small-q_T approximation

known

Soft Collinear Effective Theory (SCET)

$$\text{SCET} \simeq \text{QCD}\Big|_{\text{IR limit}}$$

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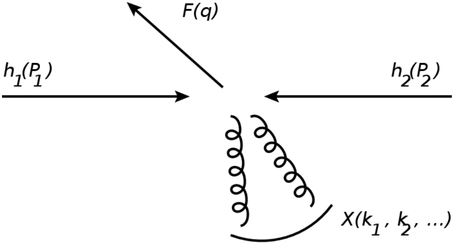
The new fields decouple in the Lagrangian

$$\mathcal{L}_{\text{SCET}} = \mathcal{L}_c + \mathcal{L}_{\bar{c}} + \mathcal{L}_s$$

- The separation of fields in the Lagrangian into collinear, anti-collinear and soft sectors, facilitates proofs of factorization theorems

Small-q_T factorization in SCET



where $F = H, Z, W, ZZ, WW, t\bar{t}, \ldots$

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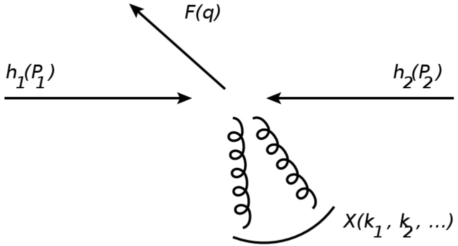

where $F = H, Z, W, ZZ, WW, t\bar{t}, \ldots$

$$\frac{d\sigma^F}{d\Phi} = \mathcal{B}_1 \otimes \mathcal{B}_2 \otimes \mathcal{H} \otimes \mathcal{S} + \mathcal{O}\left(\frac{q_T^2}{q^2}\right)$$

Small-q_T factorization in SCET

Gluons' momenta in light-cone coordinates

$$k_i^\mu = \left(k_i^+, k_i^-, \boldsymbol{k}_i^\perp\right) \qquad \text{where} \qquad k^\pm = k^0 \pm k^3$$

Expansion parameter

$$\lambda = \frac{q_T^2}{q^2} \ll 1$$

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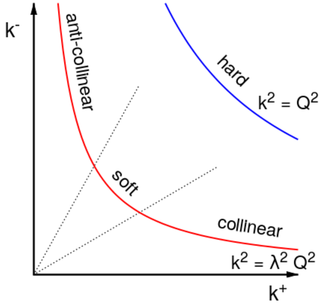
Regions

collinear	$k_i^\mu \sim (1, \lambda^2, \lambda)\, Q^2$	$\mathcal{B}_1$
anti-collinear	$k_i^\mu \sim (\lambda^2, 1, \lambda)\, Q^2$	$\mathcal{B}_2$
hard	$k_i^\mu \sim (1, 1, 1)\, Q^2$	$\mathcal{H}$
soft	$k_i^\mu \sim (\lambda, \lambda, \lambda)\, Q^2$	$\mathcal{S}$



Top pair production at small-q_T through NNLO

$$\frac{d\sigma^{\rm NNLO}}{dq_T\, dy\, dM\, d\cos\theta} = \sum_{i,\bar{i}} \mathcal{B}_{i/h_1} \otimes \mathcal{B}_{\bar{i}/h_2} \otimes {\rm Tr}\left[\mathcal{H}_{i\bar{i}} \otimes \mathcal{S}_{i\bar{i}}\right]$$

where

q_T, y, M	:	transverse momentum, rapidity, mass of top quark pair
θ	:	scattering angle of the top quark in $t\bar{t}$ rest frame

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- $\mathcal{B}$ - known up to NNLO [Gehrmann, Lübbert, Yang '12, '14]
- $\mathcal{H}$ - known up to NNLO [Czakon '08; Baernreuther, Czakon, Fiedler '13]
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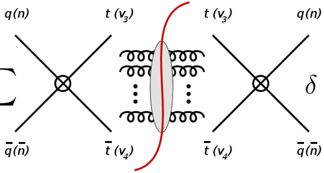
Calculating the missing NNLO correction to the soft function in the small-q_T limit, $\mathcal{S}$, is the aim of this phase of our work.

Soft function

- Represents corrections coming from exchanges of real, soft gluons, whose transverse momenta sum up to a fixed value q_T.

$$S_{\mathrm{bare}}(q_T, \beta_t, \theta) \propto \sum$$



$$\delta\left(q_T - \left|\sum_i k_{i\perp}\right|\right) \prod_i \delta^+(k_i^2)$$

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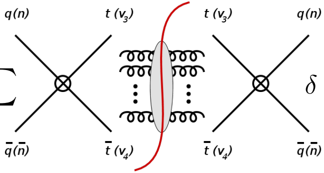

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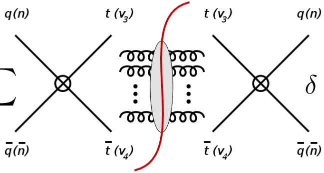

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- dimensional regularization insufficient; necessity of additional regulator of "rapidity" divergencies α

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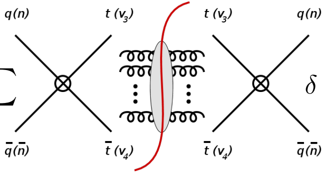

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$$\boldsymbol{S}_{i\bar{i}} = \sum_{n=0}^{\infty} \boldsymbol{S}_{i\bar{i}}^{(n)} \left(\frac{\alpha_s}{4\pi}\right)^n \qquad \boldsymbol{S}_{i\bar{i}}^{(n)} = \sum_{\{j\}} \boldsymbol{w}_{\{j\}}^{i\bar{i}} I_{\{j\}}$$

colour matrices $\longrightarrow$ ($\boldsymbol{w}$) ; (I) $\longleftarrow$ phase space integrals

Renormalization

separately divergent

$$\frac{d\sigma}{d\Phi} = \mathcal{B}_1^{(\text{bare})} \otimes \mathcal{B}_2^{(\text{bare})} \otimes \text{Tr}\left[\mathcal{H}^{(\text{bare})} \otimes \mathcal{S}^{(\text{bare})}\right]$$

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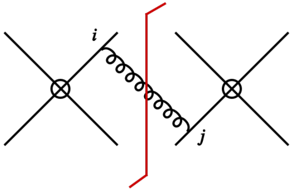
separately finite

$$\frac{d}{d\mu}\frac{d\sigma}{d\Phi} = 0 \quad \rightarrow \quad \text{Renormalization Group Equations for } \mathcal{B}, \mathcal{H} \text{ and } \mathcal{S}$$

Soft function at NLO



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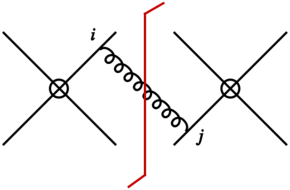


- Known in analytic form
  [Li, Li, Shao, Yan, Zhu '13; Catani, Grazzini, Torre '13]

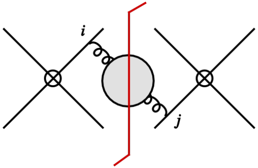
$$L_\perp = \ln \frac{x_T^2 \mu^2}{4e^{-2\gamma_E}}$$

$$\begin{aligned}
\boldsymbol{S}^{(1)}_{i\bar{i}} = {} & 4L_\perp \left(2\boldsymbol{w}^{13}_{i\bar{i}} \ln \frac{-t_1}{m_t M} + 2\boldsymbol{w}^{23}_{i\bar{i}} \ln \frac{-u_1}{m_t M} + \boldsymbol{w}^{33}_{i\bar{i}} \right) \\
& - 4\left(\boldsymbol{w}^{13}_{i\bar{i}} + \boldsymbol{w}^{23}_{i\bar{i}}\right) \mathrm{Li}_2\left(1 - \frac{t_1 u_1}{m_t^2 M^2}\right) + 4\boldsymbol{w}^{33}_{i\bar{i}} \ln \frac{t_1 u_1}{m_t^2 M^2} \\
& - 2\boldsymbol{w}^{34}_{i\bar{i}} \frac{1+\beta_t^2}{\beta_t} \left[L_\perp \ln x_s - \mathrm{Li}_2\left(-x_s \, \mathrm{tg}^2 \frac{\theta}{2}\right) + \mathrm{Li}_2\left(-\frac{1}{x_s} \mathrm{tg}^2 \frac{\theta}{2}\right) \right. \\
& \left. + 4 \ln x_s \ln \cos \frac{\theta}{2} \right] + \mathcal{O}(\epsilon)
\end{aligned}$$

Soft function at NNLO

Three distinct groups of diagrams:

- Bubble



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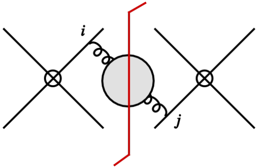
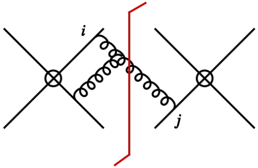


- Single-cut



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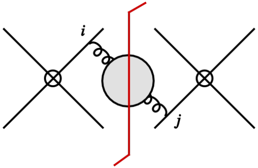

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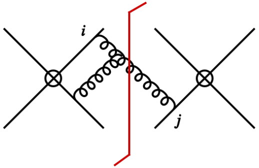
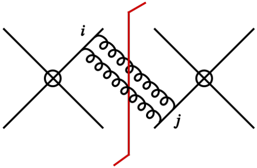
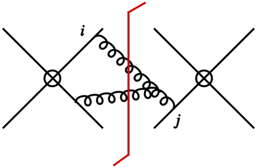
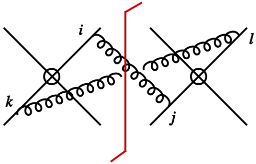


- Double-cut







+ . . .

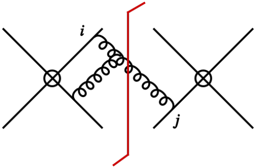
Soft function at NNLO

Three distinct groups of diagrams:

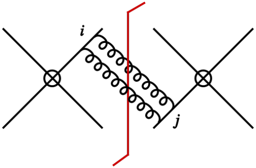
- Bubble

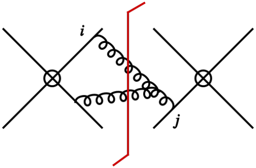
DIFFERENTIAL EQUATIONS

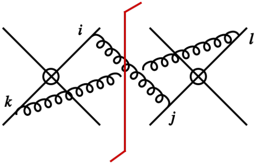
- Single-cut



- Double-cut







+ ...

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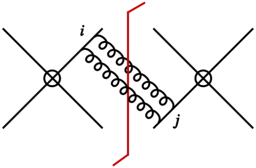

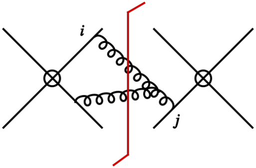

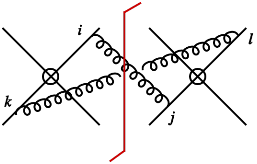

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SECTOR DECOMPOSITION

Double-cut NNLO integrals

Example:

$$\tilde{I}_{3gv,ij} = \int \frac{d^d k_1 \, d^d k_2 \; \delta^+(k_1^2) \, \delta^+(k_2^2) \, \delta((k_1+k_2)_T^2 - q_T^2)}{(n\cdot k_1)^\alpha \; (n \cdot k_2)^\alpha \; (n_i \cdot k_1) \; (n_j \cdot (k_1+k_2)) \; (k_1+k_2)^2}$$

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- divergent in the limits $\epsilon \to 0$ and $\alpha \to 0$
- a range of overlapping singularities
- complication introduced by $\delta((k_1+k_2)_T^2 - q_T^2)$ which additionally couples gluon's momenta

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To disentangle overlapping singularities and calculate regularized integrals we use the method of sector decomposition [Binoth, Heinrich, '00; Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, Zirke '17].

Sector decomposition

$$\int_0^1 dx\,dy\,\frac{\mathcal{W}(x,y)}{(x+y)^{2+\epsilon}}$$

Sector decomposition

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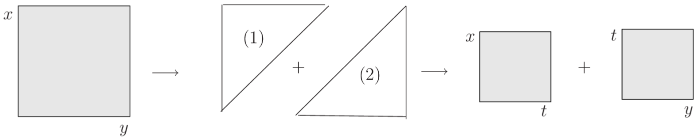

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In general, each integral can be expressed as

$$\mathcal{I} = \sum_{i \in \text{sectors}} \int_0^1 \frac{dx_1}{x_1^{1+a_1\epsilon}} \frac{dx_2}{x_2^{1+a_2\epsilon}} \cdots \frac{dx_n}{x_n^{1+a_n\epsilon}} \mathcal{W}_i(x_1, x_2, \ldots, x_n)$$

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After the above procedure is performed, all divergences become explicit and are turned in to ϵ poles

Sector decomposition

Two types of singularities

- Endpoint, *e.g.* soft:

$$\left(k_1^+, k_1^-, k_1^\perp\right) \to 0$$

Sector decomposition

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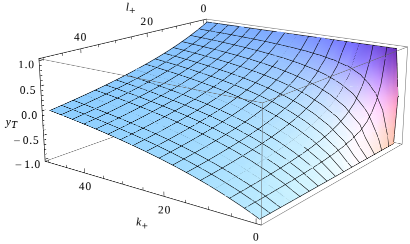
- Endpoint, *e.g.* soft:

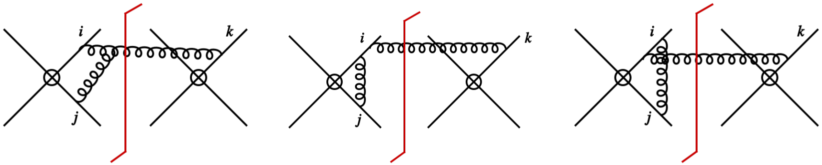
$$\left(k_1^+, k_1^-, k_1^\perp\right) \to 0$$

- Manifold, *e.g.* collinear

$$k_1 \cdot k_2 \to 0$$



Single-cut (real-virtual)



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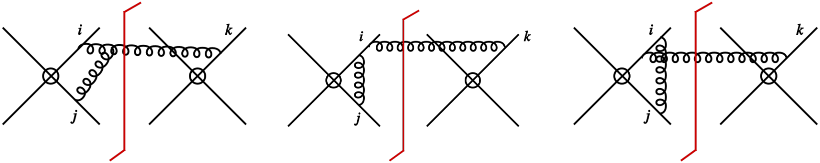

$$S^{(2)}_{\text{1-cut}} = \sum_{ijk} \int d^d l \, \frac{\delta^+(l^2)\, \delta(l_T - q_T)}{l_+^{\alpha}\, n_k \cdot l} \, n_k^{\mu}\, T_k^a J^{\mu}_{ij,a}(l)$$

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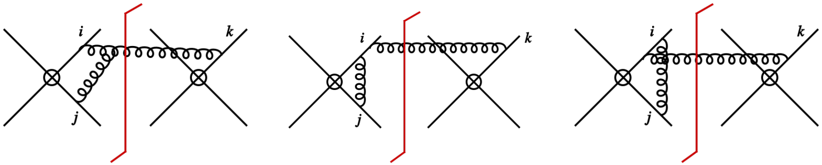

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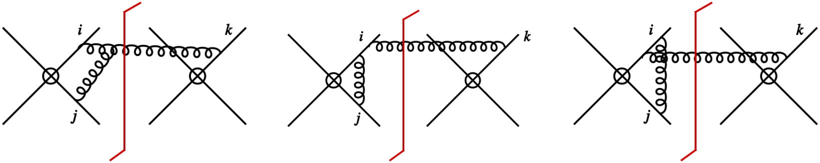

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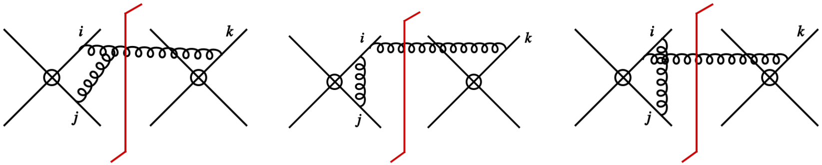

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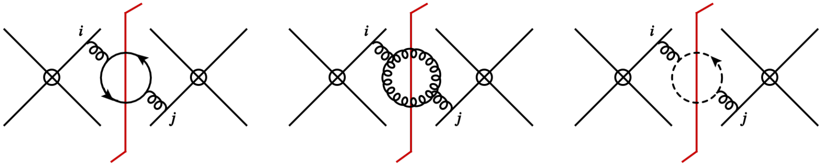
- $S^{(2)}_{\text{1-cut}}$ can be obtained by a relatively simple integration over l^μ.
- Single-cut piece of the soft function exhibits both real and imaginary part. The latter when $i \neq j \neq k$, the former, otherwise.

Bubble



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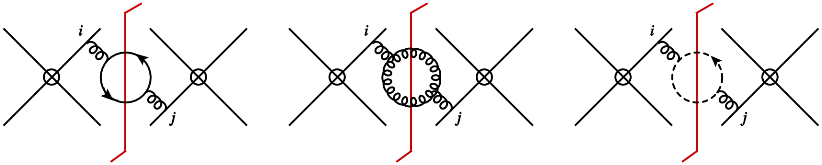
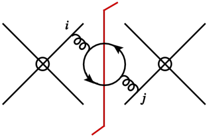
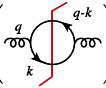


- Solvable analytically: direct cross check of our sector decomposition-based implementation
- Non-trivial tensor structure → challenging numerators
- Laboratory to stress-test sector decomposition-based methodology

Bubble part of the soft function from differential equations



$$\propto \int \frac{d^d q\,\delta(q_T - 1)\,\theta^+(q^2)\,n_i^\mu n_j^\nu}{q^4\,(n_i\cdot q)\,(n_j\cdot q)} \Bigg($$



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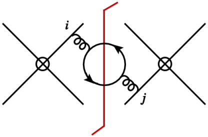

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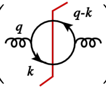
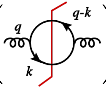


where



$$\left(\quad \right)_{\mu\nu} = \int \frac{d^d k\, N_{\mu\nu}\, \delta^+(k^2)\, \delta^+((q-k)^2)}{(n \cdot k)^\alpha\, (n \cdot (q-k))^\alpha\, k^2 (q-k)^2}$$

$$= T_{00}\, g^{\mu,\nu} + T_{qq}\, q^\mu q^\nu + T_{nn}\, n^\mu n^\nu + T_{qn}\, (n^\mu q^\nu + q^\mu n^\nu)$$

Bubble part of the soft function from differential equations

- Reverse unitarity [Anastasiou, Melnikov '02, Cutkosky '60]

$$2i\pi\delta(p^2 - m^2) \to \frac{1}{p^2 - m^2 + i\epsilon} - \frac{1}{p^2 - m^2 - i\epsilon}$$

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- Topology:

$$\int \frac{d^d k}{(n\cdot k)^{a_1+2\alpha}\,(\bar{n}\cdot k)^{a_2}\,(v_3\cdot k)^{a_3}\,(v_4\cdot k)^{a_4}\,(k^2 - m^2)^{a_5}\,((n\cdot k)\,(\bar{n}\cdot k) - m^2 - 1)^{a_6}}$$

Integration by parts and differential equations

- Identities (5 standard IBPs + 2 specific)

$$\int d^d k \, \frac{\partial}{\partial k^\mu} q^\mu I(a_1, a_2, \ldots, a_6) = 0 \,, \qquad q^\mu = n^\mu, \bar{n}^\mu, v_3^\mu, v_4^\mu, k^\mu$$

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- Final set of bubble integrals:

$$\left\{ I_{jk}(\beta_t, \theta) \right\}$$

Complete Soft Function at NNLO: structure of the result

$$S^{(2,\text{bare})}(L_\perp, \beta_t, \theta) = \left[\frac{1}{\epsilon} + L_\perp + L_\perp^2 + \ldots\right]$$

$$\times \left[S^{(2)}_{\text{bubble}}(\beta_t, \theta, \epsilon) + S^{(2)}_{\text{1-cut}}(\beta_t, \theta, \epsilon) + S^{(2)}_{\text{2-cut}}(\beta_t, \theta, \epsilon)\right]$$

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- The only term that has to be obtained through direct calculation is the $L_\perp$-independent part of $S^{(2,0)}(L_\perp)$.
- However, we calculate all terms and use the redundant ones for cross checks against Renormalization Group prediction.

Vanishing of higher order poles

Even though the NNLO Soft Function exhibits at most $\frac{1}{\epsilon^2}$ singularity, higher order poles appear in individual contributions.

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Even though the NNLO Soft Function exhibits at most $\frac{1}{\epsilon^2}$ singularity, higher order poles appear in individual contributions.

- All α poles, including $\frac{\epsilon}{\alpha}$, as well as $\frac{1}{\epsilon^4}$ pole cancel within each colour structure, for example

$$\frac{1}{\epsilon^4}\begin{pmatrix} 0.00009\,N_c{}^{-1} - 0.00009\,N_c & -0.00002\,N_c^2 - 0.00009\,N_c{}^{-2} + 0.0001 \\ -0.00002\,N_c^2 - 0.00009\,N_c{}^{-2} + 0.0001 & 0.00008\,N_c^3 - 0.00006\,N_c + 0.00007\,N_c{}^{-3} - 0.00009\,N_c{}^{-1} \end{pmatrix}$$

Vanishing of higher order poles

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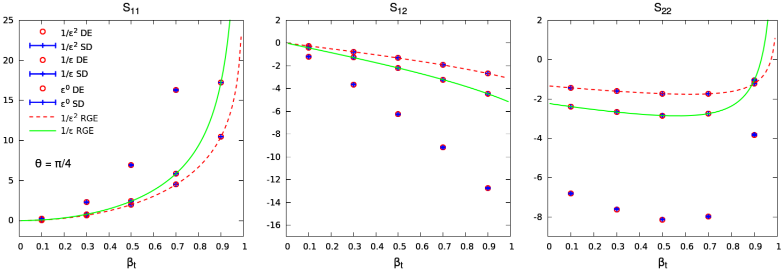
- $\frac{1}{\epsilon^3}$ pole cancels between 1-cut and 2-cut contributions

$$\frac{1}{\epsilon^3}\begin{pmatrix} 0.0004\,N_c^3 - 0.0007\,N_c + 0.0004\,N_c^{-1} & 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} \\ 0.0004\,N_c^2 - 0.0004\,N_c^{-2} - 7.\times 10^{-6} & -0.0004\,N_c^3 - 0.00001\,N_c + 0.0003\,N_c^{-3} + 0.0002\,N_c^{-1} \end{pmatrix}$$

† We used $\beta_t = 0.4$, $\theta = 0.5$.

NNLO, small-q_T soft function for top pair production

Quark bubble contribution ($q\bar{q}$ channel)

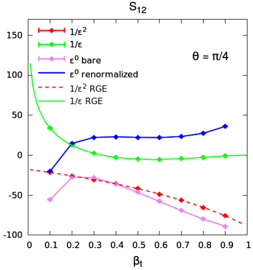


Validation of the framework

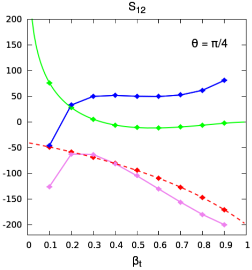
- Perfect agreement of the quark bubble results obtained from *differential equations* and *sector decomposition* for all terms in ϵ expansion
- Reproduction of the n_f part of the Renormalization Group result

Imaginary part

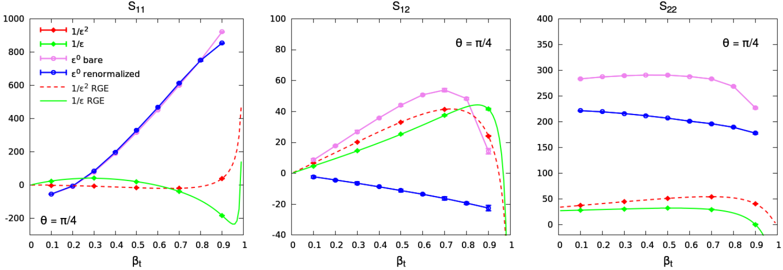
($q\bar{q}$ channel)



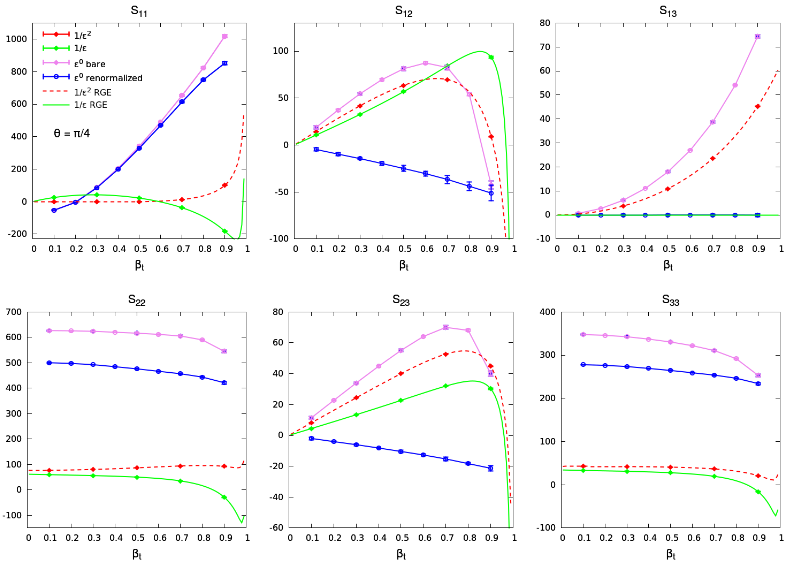
(gg channel)



Real part ($q\bar{q}$ channel)



Real part (gg channel)



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  1. Cancellation of α poles, including ϵ/α, and ϵ poles beyond $1/\epsilon^2$
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- The soft function can now be used to obtain full $t\bar{t}$ cross section at NNLO as well for resummation up to NNLL'

Acknowledgements

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