On the TMD gluon distributions for multiparton processes

Piotr Kotko

IFJ PAN

based primarily on:

M. Bury, PK, K. Kutak, arXiv:1809.08968

in connection to other works, in collab. with:

A. van Hameren, K. Kutak,
C. Marquet, E. Petreska,
S. Sapeta, A. Stasto, M. Strikman

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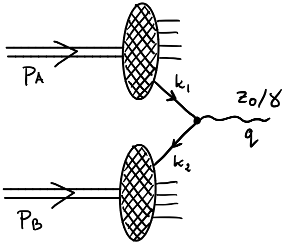
Plan

1. Very short introduction to the TMD factorization
   - Drell-Yan process
2. Nonuniversality of TMD gluon distributions
   - Wilson lines and process dependence
   - Color flow and color-ordered amplitudes
   - Operator basis for any hard process
   - Examples for 4,5,6-parton processes
3. A glimpse of applications in small x physics at LHC

Introduction

Introduction

The Drell-Yan process



Suppose we want to calculate $d\sigma/dq_T$ spectrum of the Z_0/γ system.

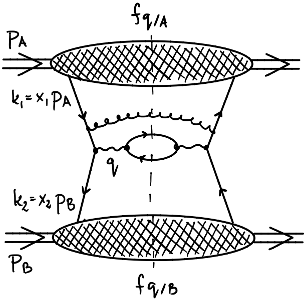
Two different regimes:

- high q_T (collinear factorization)
- low q_T (TMD factorization $\equiv$ recoil resummation by Collins-Soper-Sterman)

Introduction

Collinear factorization

$$d\sigma_{AB} \sim \int dx_1 dx_2 \; f_{q/A}\,(x_1,\mu)\, d\hat{\sigma}_{qq}(x_1,x_2,\mu)\, f_{q/B}\,(x_2,\mu)$$

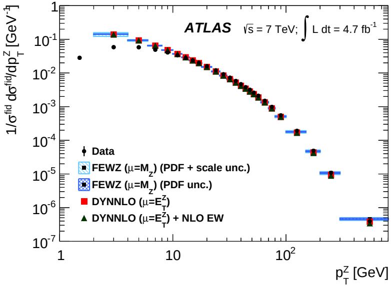


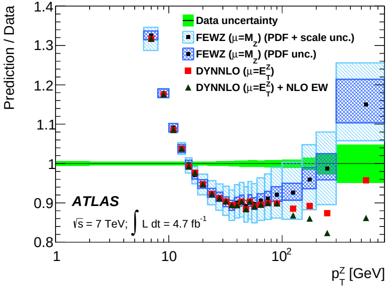
$+\ \ldots$

The domain: $\mu \sim M \sim q_T \gg \Lambda_{QCD}$

Introduction

Collinear factorization





[ATLAS, JHEP09(2014)145]

Introduction

Region $q_T \ll \mu \sim M$

Collinear factorization fails – large logs $\log(q_T/\mu)$

Resummation provided by the TMD factorization formula

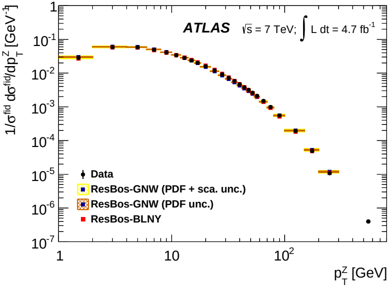
$$\frac{d\sigma}{dq_T^2} \sim \int dx_1 dx_2 \; d\hat{\sigma}_{qq} \int d^2k_T \;\; \mathcal{F}_{q/A}(x_1, k_T)\, \mathcal{F}_{q/B}(x_2, |\vec{k}_T - \vec{q}_T|)$$

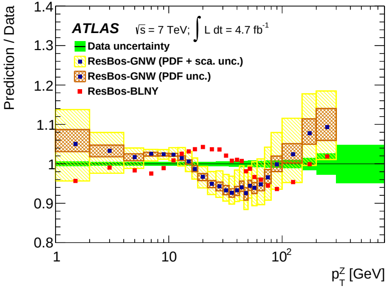
Comments:

- the factorization is proven to all orders, to the leading power $\mathcal{O}(q_T/\mu)$
- the TMD PDFs have operator definitions
- there is no all-order factorization theorem for processes involving more than two colored (tagged) partons

Introduction

Region $q_T \ll \mu \sim M$





[ATLAS, JHEP09(2014)145]

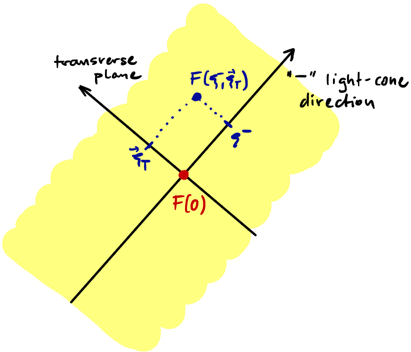
TMD gluon distributions

TMD gluon distributions

Operator definitions

Naive gluon distribution:

$$\mathcal{F}(x, k_T) \sim \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3} e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T} \langle P| F_a^{i+}(0) F_a^{i+}\left(\xi^+ = 0, \xi^-, \vec{\xi}_T\right)|P\rangle$$



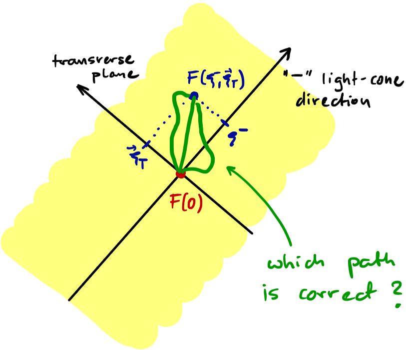
Not gauge invariant – we need a Wilson line: $U_{ab} = \mathcal{P}\exp\left\{ig\int_C dz_\mu A_c^\mu(z) f^{abc}\right\}$

TMD gluon distributions

Operator definitions

Gauge invariant definition:

$$\mathcal{F}\,(x, k_T) \sim \int \frac{d\xi^- d^2\xi_T}{(2\pi)^3}\, e^{ixP^+\xi^- - i\vec{k}_T\cdot\vec{\xi}_T}\, \langle P|\, F_a^{i+}\,(0)\, U_{ab}(0,\xi) F_b^{i+}\,(\xi)\,|P\rangle$$



TMD gluon distributions

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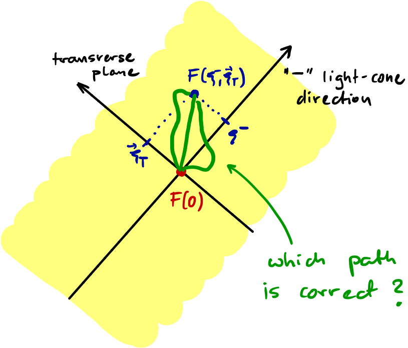
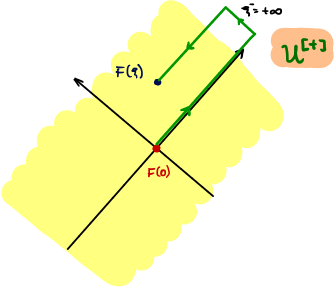


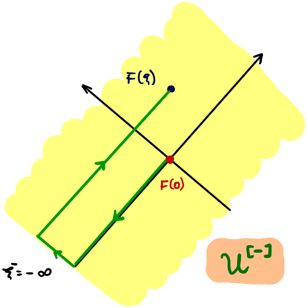
The path follows from factorization of collinear gluons.

TMD gluon distributions

Building blocks of any path (relevant to factorization)



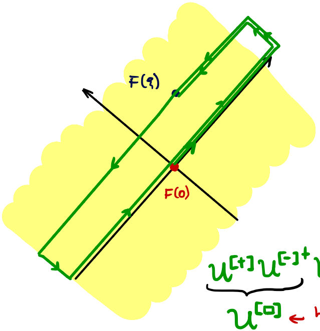
final-state interactions



initial-state interactions

TMD gluon distributions

Example of a more complicated path

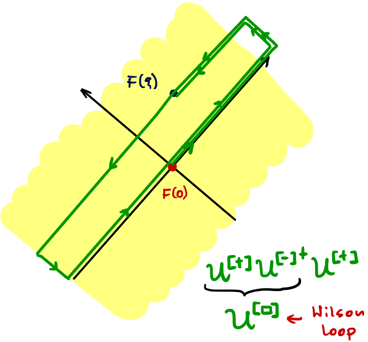


$$\underbrace{\mathcal{U}^{[+]}\mathcal{U}^{[-]\dagger}}_{\mathcal{U}^{[\square]}}\mathcal{U}^{[+]}$$

← Wilson loop

TMD gluon distributions

Example of a more complicated path



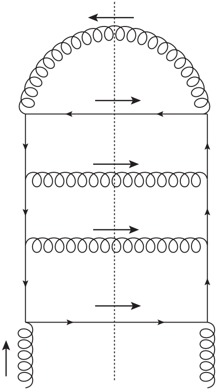
The path depends on the color structure of the hard process.
The universality of the TMD parton distributions is lost.

[e.g. C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]

TMD gluon distributions

Example

Suppose we want to determine the operator structure for the process:

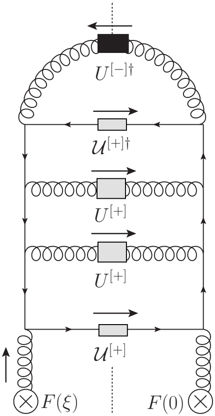


TMD gluon distributions

Example

We have to insert proper Wilson lines:
future-pointing for outgoing partons and past-pointing for incoming parton

[C.J. Bomhof, P.J. Mulders, F. Pijlman, Eur.Phys.J.C. 47, 147 (2006)]



$U^{[\pm]}$ – adjoint representation, $\mathcal{U}^{[\pm]}$ – fundamental representation

TMD gluon distributions

The procedure is straight forward, but...

- for multiparton processes there are plenty of diagrams
- gauge invariance of subsets of diagrams corresponding to the same TMD operators is not evident
- calculation simple but tedious

TMD gluon distributions

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Improvements

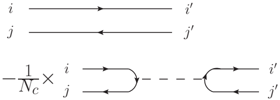
[M. Bury, PK, K. Kutak, arXiv:1809.08968]

- use color-flow representation: $A_a^\mu \rightarrow (\hat{A}^\mu)^i_j = A_a^\mu (t^a)^i_j$
- use decomposition of amplitudes into color-ordered (or dual) amplitudes

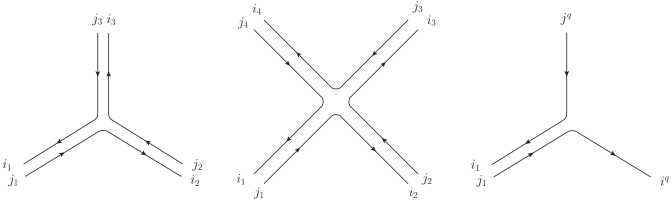
Color-flow Feynman rules

[F. Maltoni, K. Paul, T. Stelzer, S. Willenbrock, Phys.Rev. D67 (2003) 014026]

gluon propagator:



vertices:



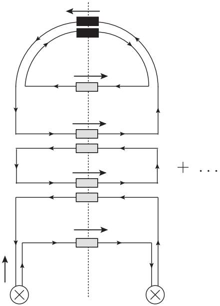
*We show only planar contributions for brevity.

We can also derive color-flow representation for Wilson lines
(effectively, this amounts to replacing U by a pair $\mathcal{U}$, $\mathcal{U}^\dagger$, up to $1/N_c$)

TMD gluon distributions

Example (continued)

In color-flow representation we get (the leading N_c contribution):



$$N_c \mathrm{Tr}\left\{\hat{F}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}(0)\,\mathcal{U}^{[+]}\right\}\,\mathrm{Tr}\mathcal{U}^{[\square]}\,\mathrm{Tr}\mathcal{U}^{[\square]\dagger}$$

Above the fundamental representation for gluon field is used: $\hat{F}^{\mu\nu} = F_a^{\mu\nu} t^a$.

Color decomposition of amplitudes

Pure gluon amplitude

- decomposition in fundamental basis

$$\mathcal{M}^{a_1 \dots a_n}(k_1, \dots, k_n) = \sum_{\pi \in S_n/Z_n} \mathrm{Tr}\left(t^{a_{\pi(1)}} \dots t^{a_{\pi(n)}}\right) \mathcal{A}(\pi(1), \dots, \pi(n))$$

  $\mathcal{A}$ – gauge invariant color-ordered amplitudes
  (only planar diagrams, with fixed order of external legs, contribute)

- decomposition in color-flow basis

$$\mathcal{M}^{i_1 \dots i_n}_{j_1 \dots j_n}(k_1, \dots, k_n) = 2^{-n/2} \sum_{\pi \in S_{n-1}} \delta^{i_1}_{j_{\pi(2)}} \delta^{i_{\pi(2)}}_{j_{\pi(3)}} \delta^{i_{\pi(3)}}_{j_{\pi(4)}} \dots \delta^{i_{\pi(n)}}_{j_1} \mathcal{A}(1, \pi(2), \dots, \pi(n))$$

Similar decompositions for amplitudes with quarks.

The hard ME can be expressed as

$$|\mathcal{M}|^2 = \vec{\mathcal{A}}\,\mathbf{C}\,\vec{\mathcal{A}}^{\dagger}, \qquad \text{where } \mathbf{C} \text{ is the color matrix}$$

To calculate TMD structure for a given process, it is enough to consider all color flows – no need to consider all diagrams.

TMD gluon distributions

Summary of new results (using the discussed techniques)

[M. Bury, PK, K. Kutak, arXiv:1809.08968]

- The gluon TMD for *any process* can be build from 10 'basis' TMD distributions.
- The operator structures were calculated explicitly for hard processes with five and six colored partons.
- In the large N_c limit, the TMD for pure gluonic hard process with n legs was calculated for any n (in the small x limit though)

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Although formally there is no TMD factorization for processes with more than two colored partons, these operators are very useful at small x (relation to Color Glass Condensate).

TMD gluon distributions

'Basis' TMD gluon distributions

[M. Bury, PK, K. Kutak, arXiv:1809.08968]

$$\mathcal{F}_{qg}^{(1)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(2)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{qg}^{(3)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(1)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(2)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\right]\mathrm{Tr}\left[\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(3)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(4)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[-]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[-]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(5)} \sim \left\langle \mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[\square]}\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(6)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]\dagger}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

$$\mathcal{F}_{gg}^{(7)} \sim \left\langle \frac{\mathrm{Tr}\left[\mathcal{U}^{[\square]}\right]}{N_c}\mathrm{Tr}\left[\hat{F}^{i+}(\xi)\,\mathcal{U}^{[\square]\dagger}\mathcal{U}^{[+]\dagger}\hat{F}^{i+}(0)\,\mathcal{U}^{[+]}\right]\right\rangle$$

TMD gluon distributions

Example result: $gg \to q\bar{q}g$

The hard ME convoluted with the TMD PDFs can be expressed as

$$|\mathcal{M}|^2 \otimes \mathrm{TMDs} = \vec{\mathcal{A}}\, \boldsymbol{\Phi}_{gg\to q\bar{q}g}\, \vec{\mathcal{A}}^\dagger$$

where the TMD matrix reads

$$\boldsymbol{\Phi}_{gg\to q\bar{q}g} = \begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_2 & \Phi_3 & \Phi_3 & \Phi_4 \\ \Phi_2 & \Phi_2 & \Phi_5 & \Phi_6 & \Phi_3 & \Phi_3 \\ \Phi_2 & \Phi_5 & \Phi_2 & \Phi_3 & \Phi_6 & \Phi_3 \\ \Phi_3 & \Phi_6 & \Phi_3 & \Phi_2 & \Phi_5 & \Phi_2 \\ \Phi_3 & \Phi_3 & \Phi_6 & \Phi_5 & \Phi_2 & \Phi_2 \\ \Phi_4 & \Phi_3 & \Phi_3 & \Phi_2 & \Phi_2 & \Phi_1 \end{pmatrix}$$

with

$$\Phi_1 = \frac{-N_c^2(\mathcal{F}_{gg}^{(1)} + \mathcal{F}_{gg}^{(3)}) + \mathcal{F}_{gg}^{(6)} N_c^4 + \mathcal{F}_{gg}^{(3)}}{(N_c^2 - 1)^2}, \qquad \Phi_2 = \frac{\mathcal{F}_{gg}^{(1)} N_c^2 - \mathcal{F}_{gg}^{(3)}}{N_c^2 - 1},$$

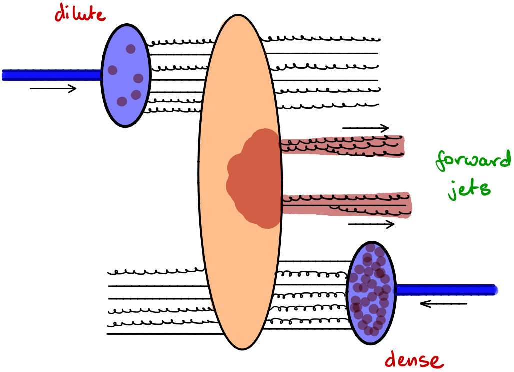
and so on...

Applications

Small x TMD factorization

Forward jets

Basic idea



Framework: Small-x Improved TMD factorization (ITMD)

Factorization formula for forward dijets in pA

[PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, A. van Hameren, JHEP 1509 (2015) 106]

$$\frac{d\sigma_{AB}}{dy_1 d^2p_{T1} dy_2 d^2p_{T1}} \sim \sum_{a,c,d} f_{a/B}\left(x_B,\mu^2\right) \sum_{i=1,2} \Phi^{(i)}_{ag\to cd}\left(x_A, k_T^2, \mu^2\right) K^{(i)}_{ag\to cd}(k_T,\mu^2)$$

$f_{a/B}$ – collinear PDFs in proton

$\Phi^{(i)}_{ag\to cd}$ – TMD gluon distributions in nucleus for $ag \to cd$

$K^{(i)}_{ag\to cd}$ – off-shell gauge invariant hard factors

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$K^{(i)}_{ag\to cd}$ – off-shell gauge invariant hard factors

$$\Phi^{(1)}_{qg\to gq} = \mathcal{F}^{(1)}_{qg}, \qquad \Phi^{(2)}_{qg\to gq} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(2)}_{qg} - \mathcal{F}^{(1)}_{qg}\right),$$

$$\Phi^{(1)}_{gg\to q\bar{q}} = \frac{1}{N_c^2-1}\left(N_c^2\mathcal{F}^{(1)}_{gg} - \mathcal{F}^{(3)}_{gg}\right), \qquad \Phi^{(2)}_{gg\to q\bar{q}} = \mathcal{F}^{(3)}_{gg} - N_c^2\mathcal{F}^{(2)}_{gg},$$

$$\Phi^{(1)}_{gg\to gg} = \frac{1}{2N_c^2}\left(N_c^2\mathcal{F}^{(1)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right),$$

$$\Phi^{(2)}_{gg\to gg} = \frac{1}{N_c^2}\left(N_c^2\mathcal{F}^{(2)}_{gg} - 2\mathcal{F}^{(3)}_{gg} + \mathcal{F}^{(4)}_{gg} + \mathcal{F}^{(5)}_{gg} + N_c^2\mathcal{F}^{(6)}_{gg}\right)$$

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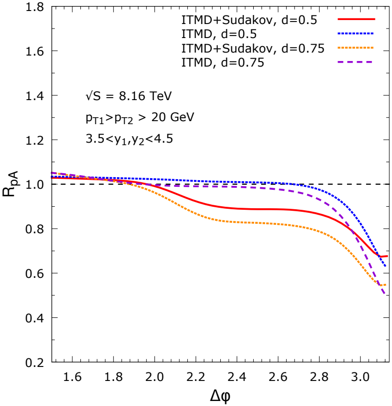
$K^{(i)}_{ag\to cd}$ – off-shell gauge invariant hard factors

- The formula can be justified within the Color Glass Condensate (CGC) theory
- Various TMD gluon distribution can be calculated with the help of CGC
- Despite the lack of universality, we have the predictive power at small-x

Results for dijet production in *pPb* at LHC

Azimuthal decorrelations for nuclear modification ratio R_{pA}

[A. van Hameren, PK, K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]



Kinematic cuts:

- CM energy: $\sqrt{S} = 8.16\,\mathrm{TeV}$
- require two jets with $(\Delta\phi)^2 + (\Delta\eta)^2 > R^2, R = 0.5$
- transverse momenta cuts: $p_{T1} > p_{T2} > 20\,\mathrm{GeV}$
- rapidity cuts: $3.5 < y_1, y_2 < 4.5$

Summary & Outlook

Summary

We have studied the gauge invariant operators defining the TMD gluon distributions for multiparton processes.

- We find 10 different structures that build up any TMD gluon distribution
- We calculate explicitly the strucures appearing for 5,6 parton processes using the color flow method
- We obtain the result for arbitrary-multiplicty pure gluonic process at large N_c in the small-x regime

On phenomenology ground, the generalized factorization formula allows to make calculations for particle production in dilute-dense collisions. So far we obtained predictions for dijets in pA and UPC.

BACKUP

Off-shell gauge invariant amplitudes

Color decomposition of gluon amplitudes:

$$\mathcal{M}^{a_1\ldots a_N}\left(\varepsilon_1^{\lambda_1},\ldots,\varepsilon_N^{\lambda_N}\right)=\sum_{\sigma\in S_N}\mathrm{Tr}\left(t^{a_{\sigma_1}}t^{a_{\sigma_2}}\ldots t^{a_{\sigma_N}}\right)\mathcal{M}\left(\sigma_1^{\lambda_{\sigma_1}},\sigma_2^{\lambda_{\sigma_2}}\ldots,\sigma_N^{\lambda_{\sigma_N}}\right)$$

a_i- color indices, $\varepsilon_i^{\lambda_i}$ - polarization vectors with helicity λ_i, S_N - set of noncyclic permutations.

In spinor formalism, the non-zero off-shell helicity amplitudes have the form of the MHV amplitudes with certain modification of spinor products:

[A. van Hameren, PK, K. Kutak, JHEP 1212 (2012) 029]

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^-,3^+,4^+\right)=2g^2\,\rho_1\,\frac{\langle 1^*2\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^-,4^+\right)=2g^2\,\rho_1\,\frac{\langle 1^*3\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

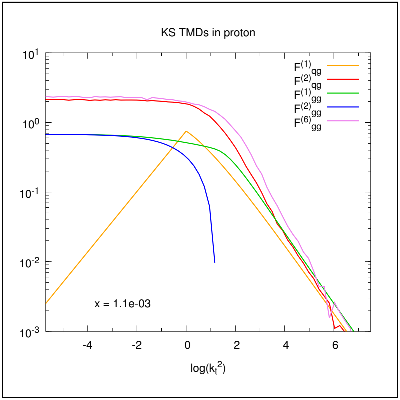
$$\mathcal{M}_{g^*g\to gg}\left(1^*,2^+,3^+,4^-\right)=2g^2\,\rho_1\,\frac{\langle 1^*4\rangle^4}{\langle 1^*2\rangle\langle 23\rangle\langle 34\rangle\langle 41^*\rangle}$$

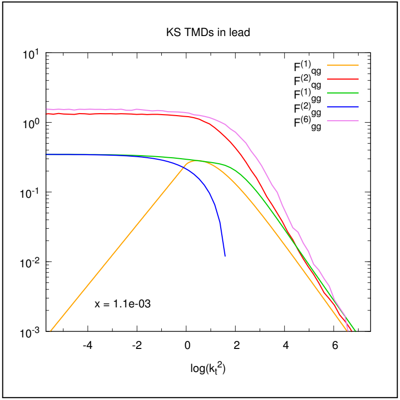
where $\langle ij\rangle=\langle k_i-|k_j+\rangle$ with spinors defined as $|k_i\pm\rangle=\frac{1}{2}\left(1\pm\gamma_5\right)u\left(k_i\right)$. Spinor products for off-shell states involve only longitudinal component of the off-shell momentum $\langle 1^*i\rangle=\langle p_1 i\rangle$, where $k_1=p_1+k_{T1}$, $k_1^2\neq 0$, $p_1^2=0$.

TMD gluon distributions

TMDs (contributing at large N_c), obtained, through the Gaussian approximation of CGC, from a single distribution fitted to HERA data by Kutak and Sapeta[1] (KS) using the nonlinear evolution equation of Kutak-Kwieciński[2].

[A. van Hameren, P.K., K. Kutak, C. Marquet, E. Petreska, S. Sapeta, JHEP 1612 (2016) 034]





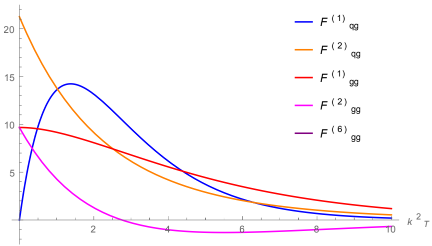
[1] K. Kutak, S. Sapeta, Phys. Rev. D 86 (2012) 094043 [2] K. Kutak, K. Kwiecinski, Eur. Phys. J. C 29 (2003) 521

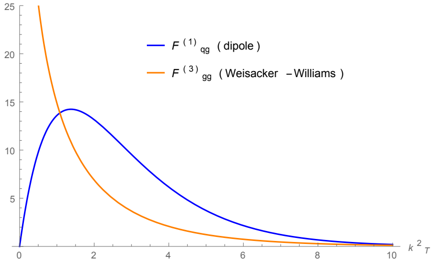
TMD gluon distributions: GBW model

The Golec-Biernat-Wusthoff (GBW) model:

$$\mathcal{F}_{qg}^{(1)}\left(x,k_T^2\right)=\frac{N_c S_\perp}{2\pi^3\alpha_s}\,\frac{k_T^2}{Q_s^2\left(x\right)}\,\exp\left(-\frac{k_T^2}{Q_s^2\left(x\right)}\right),\;\; Q_s\left(x\right)=Q_{s0}^2\left(\frac{x}{x_0}\right)^{\lambda}$$

Assuming gaussian distribution of colour sources all five gluons can be calculated analytically[1]





[1] E. Petreska, Proceedings, 7th International Workshop MPI@LHC 2015