Interplay of nonextensivity and dynamics in description of QCD matter

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Content:

(1) Why to use the nonextensive thermodynamics in description of ***dense matter*** *(hadronic or QCD)*

(2) Nonextensive Nambu – Jona-Lasinio Model - ***q-NJL***

(3) Nonextensive quasiparticle description - ***QPM***

Content:

(1) Why to use the nonextensive thermodynamics in description of dense matter (hadronic or QCD).

(2) Nonextensive Nambu – Jona-Lasinio Model

(3) Nonextensive quasiparticle description

→ *the tool for investigating the interplay of the nonextensivity and dynamics in description of dense hadronic or QCD matter*

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(*)

$$\exp(X/T) \rightarrow \exp\_q(X/T) = [1 + (q-1)(X/T)]^{1/(q-1)}$$

Nonextensive environment

[J.Rożynek, Physica A 446(2015)27]

For q=1 (extensive case): extremize entropy

$$\tilde{S} = \left[n_i \ln n_i + (1-n_i)\ln(1-n_i)\right] + \left[n_i \to 1-\bar{n}_i\right],$$

under conditions:

$$\sum_i (n_i - \bar{n}_i) = \hat{N} \quad \text{and} \quad \sum_i (n_i + \bar{n}_i) E_i = \hat{E},$$

and obtain

($e(x) = \exp(x)$, $x=\beta(E-\mu)$, $\beta=1/T$,
$\xi=+1$ for quarks, $\xi=-1$ for gluons,
μ is chemical potential):

$$n_i = \frac{1}{e(x_i)+\xi}, \quad \bar{n}_i = \frac{1}{e(\bar{x}_i)+\xi},$$

$$x_i = \beta(E_i - \mu), \quad \bar{x}_i = \beta(E_i + \mu).$$

For q > 1 (case (a)) : extremize entropy

$$\boxed{\ln_q x = \frac{x^{1-q}-1}{1-q}}$$

$$\tilde{S}_q^{(a)} = \sum_i \left[n_{qi}^q \ln_q n_{qi} + (1-n_{qi})^q \ln_q(1-n_{qi})\right] + \left[n_{qi} \to 1-\bar{n}_{qi}\right],$$

under conditions:

$$\sum_i \left(n_{qi}^q - \bar{n}_{qi}^q\right) = \hat{N}, \quad \sum_i \left(n_{qi}^q + \bar{n}_{qi}^q\right) E_{qi} = \hat{E}.$$

and obtain:

$$n_{qi} = \frac{1}{e_q(x_{qi})+\xi}, \quad \bar{n}_{qi} = \frac{1}{e_q(\bar{x}_{qi})+\xi},$$

$$x_{qi} = \beta(E_{qi} - \mu), \quad \bar{x}_{qi} = \beta(E_{qi} + \mu)$$

$$\boxed{e_q(x) = [1+(q-1)x]^{\frac{1}{q-1}}}$$

Nonextensive environment

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under conditions: $\sum_i (n_i - \bar{n}_i) = \hat{N}$ and $\sum_i (n_i + \bar{n}_i) E_i = \hat{E},$

an ... $+\xi$,

ξ= ... $\mu).$

Such form of constraints - **[n(x)]^q** instead of **n(x)** - is necessary to fulfil the basic requirements of thermodynamical consistency, for example that

$$\left(\frac{\partial S}{\partial E}\right)_{\rho=N/V} = \frac{1}{T}.$$

Fo... extremize entropy ... $\ln_q(1-n_{qi})\big]$

$$\boxed{\ln_q x = \frac{x^{1-q}-1}{1-q}}$$

$+\left[n_{qi} \to 1-\bar{n}_{qi}\right],$

under conditions: $\sum_i \left(n_{qi}^q - \bar{n}_{qi}^q\right) = \hat{N}, \quad \sum_i \left(n_{qi}^q + \bar{n}_{qi}^q\right) E_{qi} = \hat{E}.$

and obtain: $n_{qi} = \frac{1}{e_q(x_{qi}) + \xi}, \quad \bar{n}_{qi} = \frac{1}{e_q(\bar{x}_{qi}) + \xi},$

$$x_{qi} = \beta(E_{qi} - \mu), \quad \bar{x}_{qi} = \beta(E_{qi} + \mu) \qquad \boxed{e_q(x) = \left[1 + (q-1)x\right]^{\frac{1}{q-1}}}$$

Nonextensive environment

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under conditions: $\sum_i (n_i - \bar{n}_i) = \hat{N} \quad \text{and} \quad \sum_i (n_i + \bar{n}_i) E_i = \hat{E},$

and obtain (e(x) = exp(x), β=1/T, ξ=+1 for quarks, $n_i = \frac{1}{e(x_i)+\xi}, \quad \bar{n}_i = \frac{1}{e(\bar{x}_i)+\xi},$

ξ= -1 f

Definition $$e_q(x) = [1+(q-1)x]^{\frac{1}{q-1}}$$

must be supplemented by a condition ensuring that the function $\mathbf{e_q(x)}$ is always nonnegative real valued (Tsallis' cut-off prescription):

$$\begin{aligned} e_q(x) = 0, &\quad \text{for} \quad [1+(q-1)x] \le 0, \\ \text{or} \quad n_{qi} = 1, &\quad \text{for} \quad E_{qi} < \mu - \frac{1}{\beta(q-1)}. \end{aligned}$$

There are no limitations on the distribution of antiparticles, note that always $\boldsymbol{n_{q>1}(x) > n_{q=1}(x)}$.

For q

under

and ob

$x_{qi} = \beta(E_{qi} - \mu), \quad \bar{x}_{qi} = \beta(E_{qi} + \mu)$ $$e_q(x) = [1+(q-1)x]^{\frac{1}{q-1}}$$

q-NJL

Nonextensive Nambu – Jona-Lasinio Model *[RW- EJPA52(2016)13]:*

- based on effective Lagrangian
- suitable for the bosonization procedure
- only four-quark interaction

$$\mathcal{L}_{\rm eff} = \bar{q}\left(i\gamma^\mu\partial_\mu - \hat{m}\right)q + S_{ab}\left[\left(\bar{q}\lambda^a q\right)\left(\bar{q}\lambda^b q\right)\right] + P_{ab}\left[\left(\bar{q}i\gamma_5\lambda^a q\right)\left(\bar{q}i\gamma_5\lambda^b q\right)\right],$$

$$S_{ab} = g_S\delta_{ab} + g_D D_{abc}\left\langle\bar{q}\lambda^c q\right\rangle, \qquad P_{ab} = g_S\delta_{ab} - g_D D_{abc}\left\langle\bar{q}\lambda^c q\right\rangle.$$

Integrating over the momenta of quark fields in the L_{eff} one obtains an effective action expressed in terms of σ and ϕ, the natural degrees of freedom of low-energy QCD in the mesonic sector :

$$W_{\rm eff}[\varphi,\sigma] = -\frac{1}{2}\left(\sigma^a S^{-1}_{ab}\sigma^b\right) - \frac{1}{2}\left(\varphi^a P^{-1}_{ab}\varphi^b\right) - i\,{\rm Tr}\ln\left[i\gamma^\mu\partial_\mu - \hat{m} + \sigma_a\lambda^a + (i\gamma_5)(\varphi_a\lambda^a)\right].$$

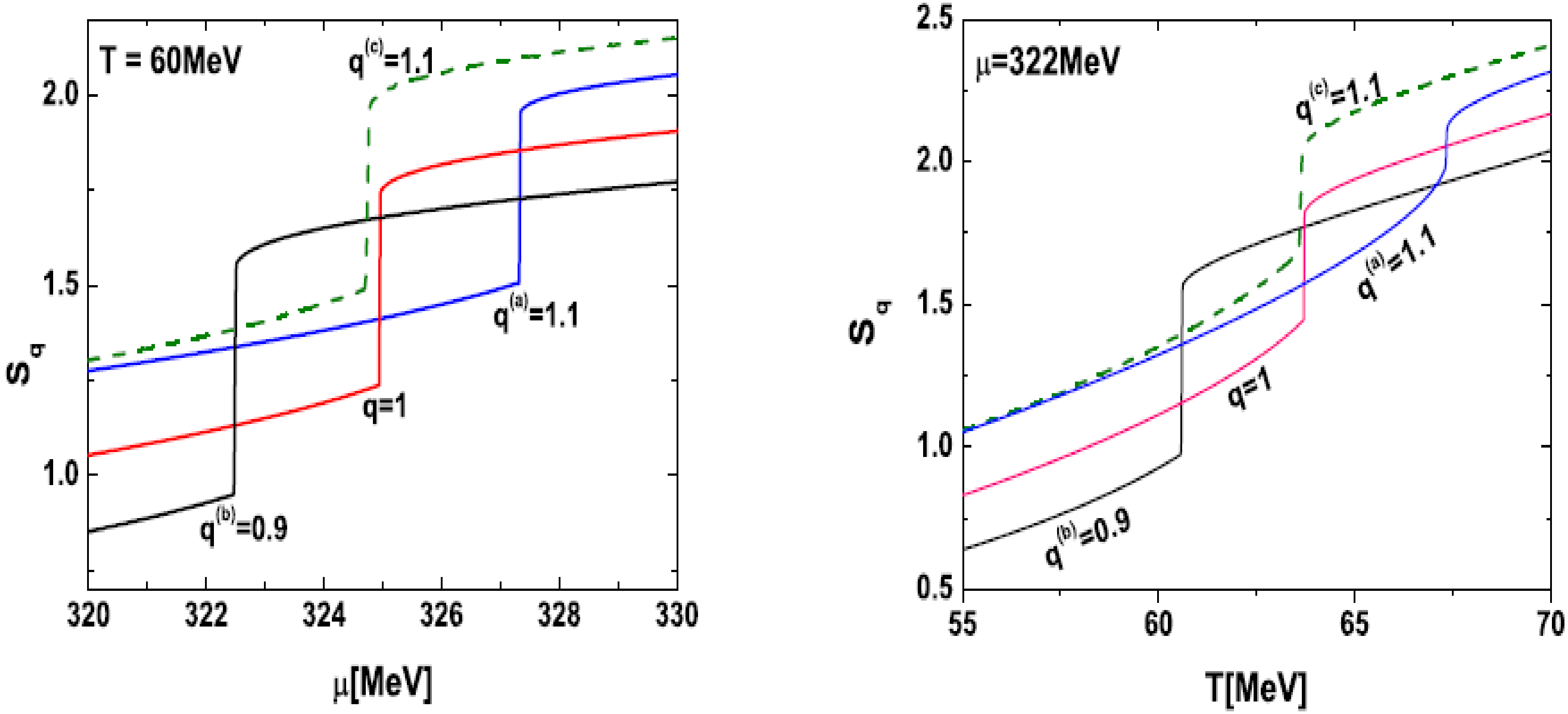
⇒ gap equations for the constituent quark masses M_i :

$$M_i = m_i - 2g_S\langle\bar{q}_i q_i\rangle - 2g_D\langle\bar{q}_j q_j\rangle\langle\bar{q}_k q_k\rangle$$

and with the quark condensates: $\langle\bar{q}_i q_i\rangle = -i\,{\rm Tr}[S_i(p)]$

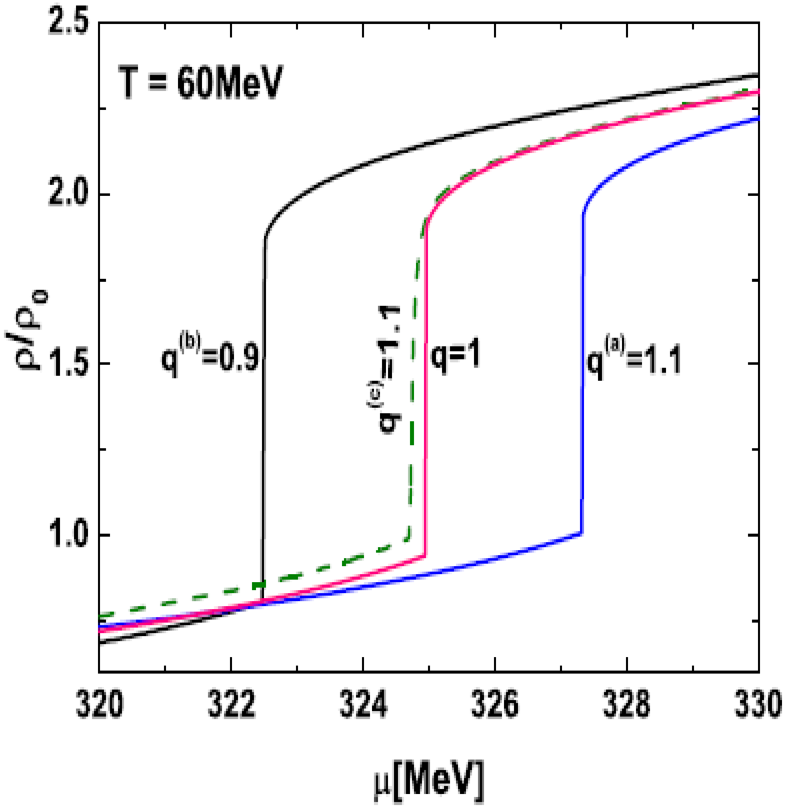
$$\boxed{\Omega_q(T,V,\mu_i) = E_q - TS_q - \sum_{i=u,d,s}\mu_i N_{qi}.}$$ q-grand canonical potential

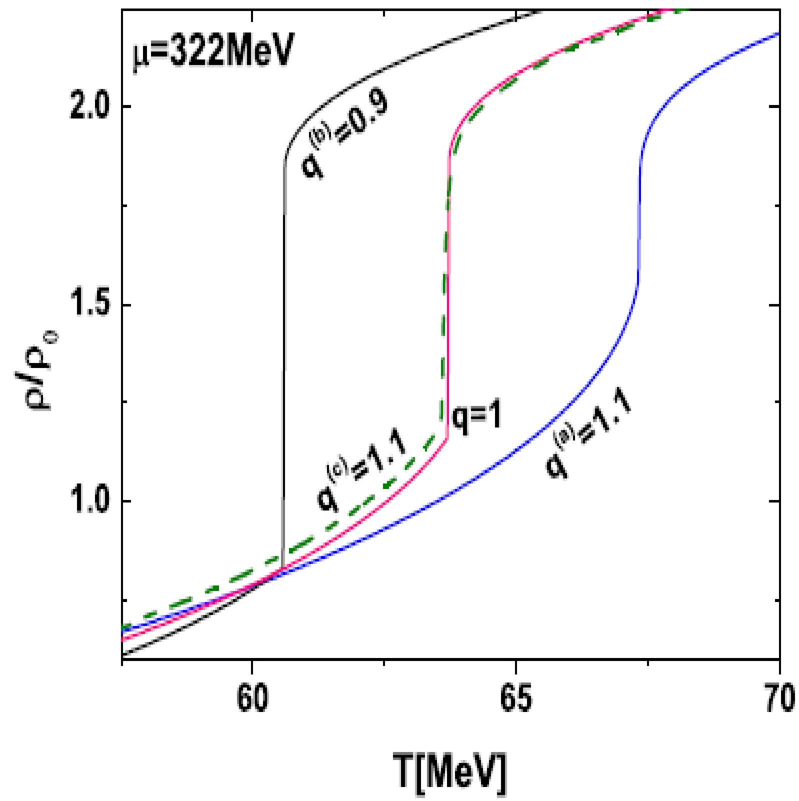
q-NJL Fig. 3



Entropy as a function of the chemical potential (left panel) and temperature (right panel) calculated for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q =1 (all M_q are running now).

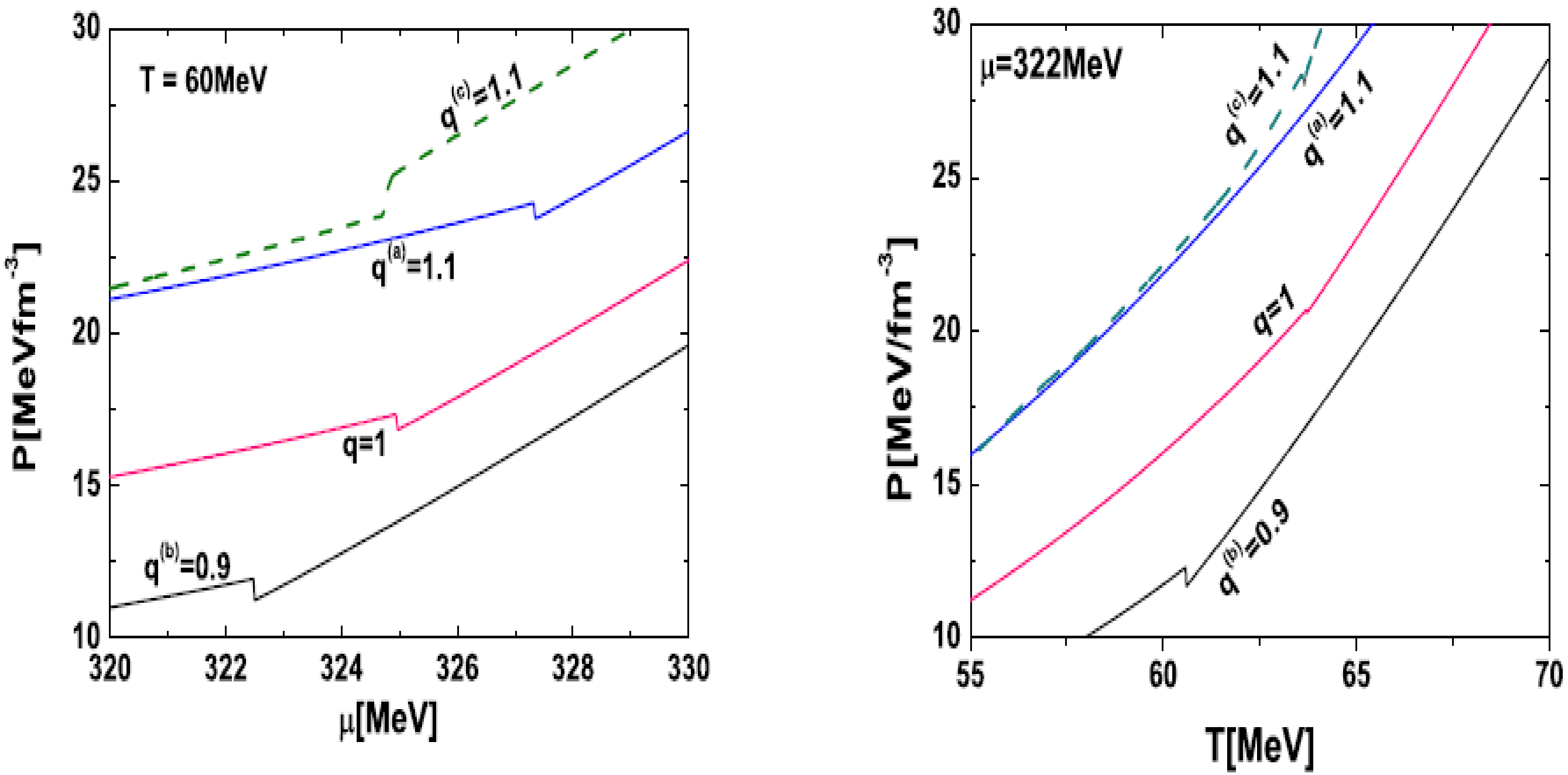
q-NJL Fig. 4





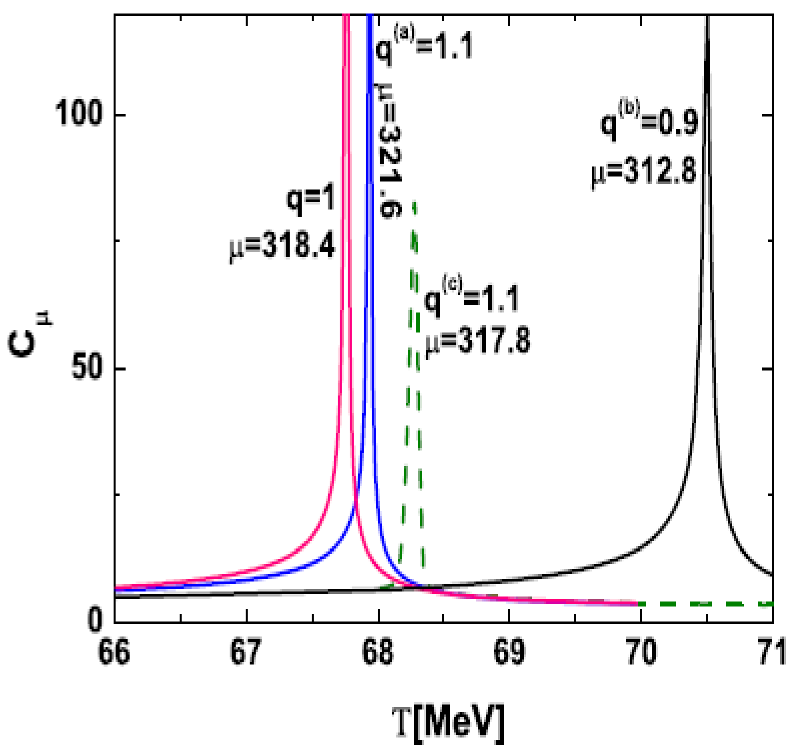
Compression ρ/ρ_0 as a function of the chemical potential (left panel) and the pressure (right panel) for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q =1.

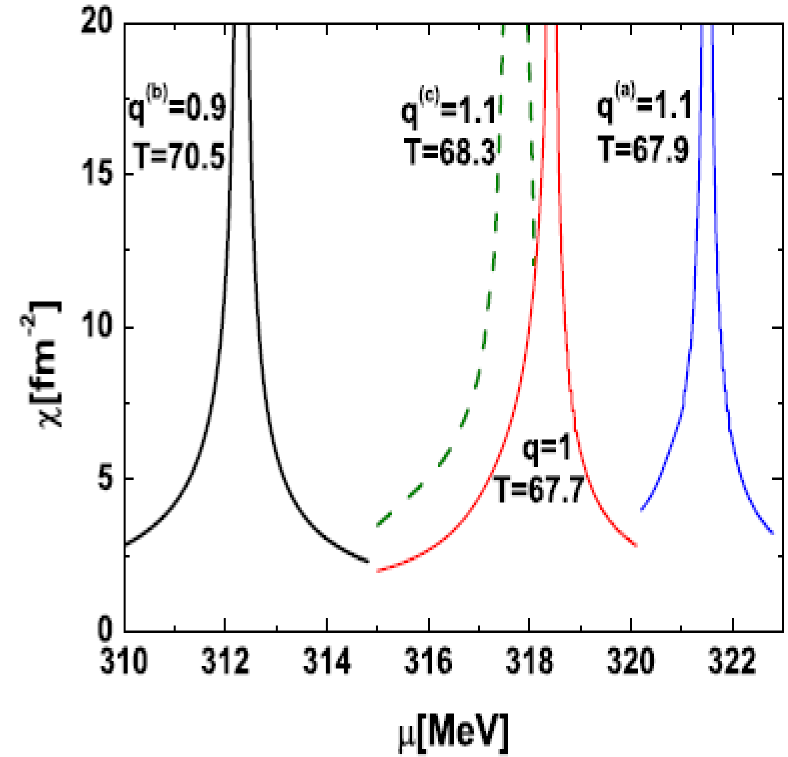
q-NJL Fig. 5



Pressure as a function of the chemical potential (left panel) and temperature (right panel) for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q =1.

q-NJL Fig. 8





Left panel: Heat capacity as a function of temperature calculated in the vicinity of the phase transition point for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q =1. The respective values of chemical potential used are indicated.

Right panel: Susceptibility as a function of chemical potential calculated in the vicinity of the phase transition point, and for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q = 1. The respective values of the temperatures are indicated.

q-NJL

Summarizing:

(1) The nonextensive description accounts (in a phenomenological way) for all situations in which one expects some dynamical correlations in the quark-antiquark system considered in our q-NJL model.

It is not a substitute for any part of the interaction described by the Lagrangian of the NJL model, but provides a different environment which can have some dynamical effects, so far undisclosed but simply parameterized by nonextensivity q.

q-NJL

Summarizing:

(2) From this perspective:

- **case (b) with $q<1$**, which has lower entropy, seems to be more suitable to describe the nonextensive mechanism in the Equation of State (EoS) of dense hadronic matter in a restricted phase space (like in (proto)neutron stars).
- **case (a), with $q>1$** and with higher entropy, is more suitable for situations in which one expects dynamical fluctuations, for example to describe heavy ion collision where dense nuclear matter is created out of equilibrium and quickly decays into hadrons.

q-NJL

Summarizing:

(3) Our work presents arguments that:

The critical properties of nuclear matter in two different environments can be different, although the phase transition occurs at the same density or compression.

The critical temperature is higher for nuclear matter created in (proto)neutron stars but the critical value of the chemical potential will be bigger for nuclear matter created in heavy ion collisions.

q-zQPM

Nonextensive quasiparticle description - q-QPM
An example of the interplay of nonextensivity and dynamics

[Rożynek&Wilk, EPJA52(2016)294].

(*) Note: in q-NJL the interplay between the nonextensivity parameter q and dynamics is quite involved.

(*) It is desirable to invent a way to demonstrated this interplay in a more straightforward way.

(*) To this end one has to simplyfy the dynamical part by describing it phenomenologically by a small number of parameters, for example by a single (T-dependent) parameter, and to look how this parameter changes in the nonextensive environment described by q.

q-zQPM

Nonextensive quasiparticle description - q-QPM
An example of the interplay of nonextensivity and dynamics

[Rożynek&Wilk, EPJA52(2016)294].

(*) Such dynamical approaches are known and are called quasi-particle models (QPM) of the QCD matter.

Their common feature is that the interacting particles (quarks and gluons) are replaced by some free, noninteracting quasi-particles.

(*) Known QPM:

(i) model with interactions encoded in the

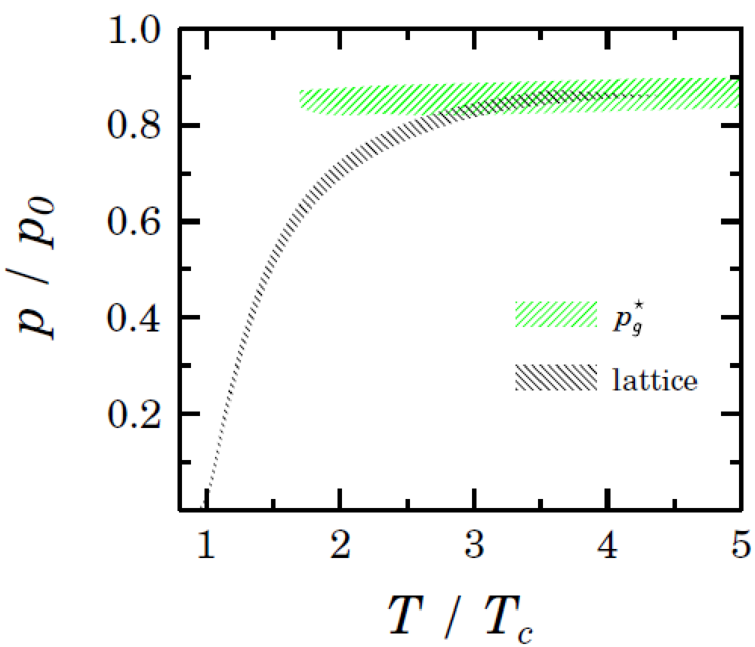
(ii) model using extensively the Polyakov loop concept,

(iii) model inspired by the Landau theory of Fermi liquids with dynamics described by ***fugacities z***

Effective masses,

Phenomenological quasiparticle model

$$\omega_i^2 = k^2 + m_i^2, \text{ where } m_i \sim gT$$

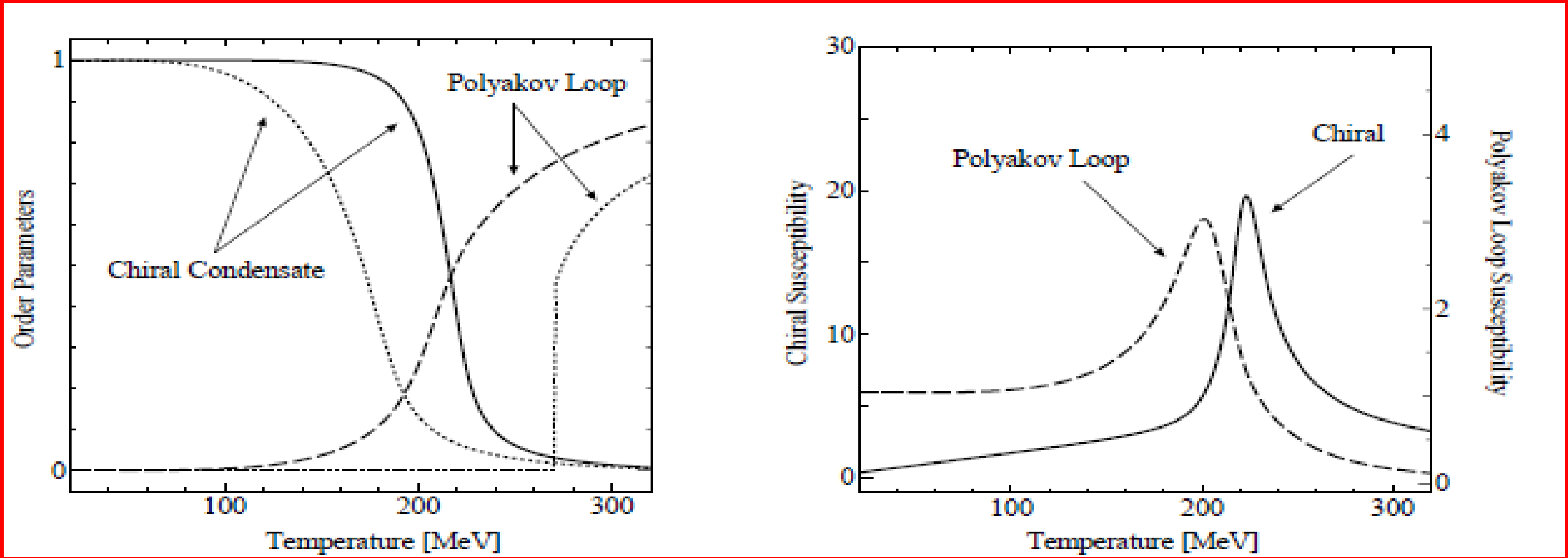


Lattice results vs. to SU(3) plasma from the HTL qusiparticle approximation.

A. Peshier, B. Kampfer, G. Soff, Phys.Rev. C 61, 045203

The model using extensively the Polyakov loop concept

$$\Omega/V = V_{\mathrm{glue}}[L] + \frac{1}{2G}(M - m_q)^2 - 2N_c N_f \int \frac{d^3p}{(2\pi)^3}\left\{E_p + T\frac{1}{N_c}\mathrm{Tr_c}\ln\left[1 + L e^{-(E_p-\mu)/T}\right] + T\frac{1}{N_c}\mathrm{Tr_c}\ln\left[1 + L^\dagger e^{-(E_p+\mu)/T}\right]\right\}$$



Kenji Fukushima PLB(2004)

Chiral effective model with the Polyakov loop

q-zQPM

The effects of interaction are now modelled by some special, T-dependent factors, effective fugacities $z^{(i)}$, the form of which is obtained from fits to the lattice QCD results.

The corresponding equilibrium distribution function is assumed as:

$$f_{eq}^{(i)}(x) = \frac{z^{(i)} e(-x_i)}{1 - \xi \cdot z^{(i)} e(-x_i)} = \frac{1}{\frac{1}{z^{(i)}} e(x_i) - \xi}$$

$e(x) = exp(x)$, $x_i = \beta E$, $\xi = +1$ for bosons and -1 for fermions.
There is no chemical potential, only massless u and d quarks and gluons and massive s-quarks

$$f_{eq}^{(i)}(\tilde{x}) = \frac{1}{e(\tilde{x}^{(i)}) - \xi},$$

$$\tilde{x}^{(i)} = \beta E_i - \mu^{(i)}(T)$$

$$\mu^{(i)}(T) = \ln z^{(i)}(T)$$

effective chemical potential, $\mu^{(i)}$, which depends on temperature T and replaces the action of the fugacities $z^{(i)}$. **$f^{(i)}(x) \to f_q^{(i)}(x)$**

q-zQPM

(*) We use, as our input, results for the scaled temperature dependence of the fugacities, $z^{(i)} = z^{(i)}(\tau)$ (where $\tau = T/T_c$ and T_c is the critical temperature), obtained in *PRD84(2011)074013* in the usual extensive environment from their fits to the lattice QCD results.

(*) Because in the q-thermodynamics all thermodynamic relations are preserved, we can compare the pressures in extensive and nonextensive environments using the usual thermodynamic relation,

$$\boxed{P_q \beta V = \ln_q\left(\Xi_q\right)}$$

calculated, respectively, for the q = 1 and for q ≠ 1 cases,

$$\boxed{P_{q=1}\left(\tilde{x}_i\right) = P_q\left[x_q^{(i)}\right]}$$

q-zQPM

$$\ln_q\left(\Xi_q\right) = -\int \frac{d^3p}{(2\pi)^3}\sum_i \xi L_q\left[x_q^{(i)}\right] = \frac{1}{3}\int \frac{d^3p}{(2\pi)^3}\sum_i p\left[n_q\left(x_q^{(i)}\right)\right]^q \frac{\partial x_q^{(i)}}{\partial p},$$

$$L_q(x) = \ln_{2-q}\left[1-\xi e_{2-q}(-x)\right].$$

$$\frac{\partial x_q^{(g,q)}}{\partial p} = \beta \quad \text{and} \quad \frac{\partial x_q^{(s)}}{\partial p} = \beta\frac{p}{\sqrt{p^2+m^2}}$$

$$P_q\beta V = \ln_q\left(\Xi_q\right)$$

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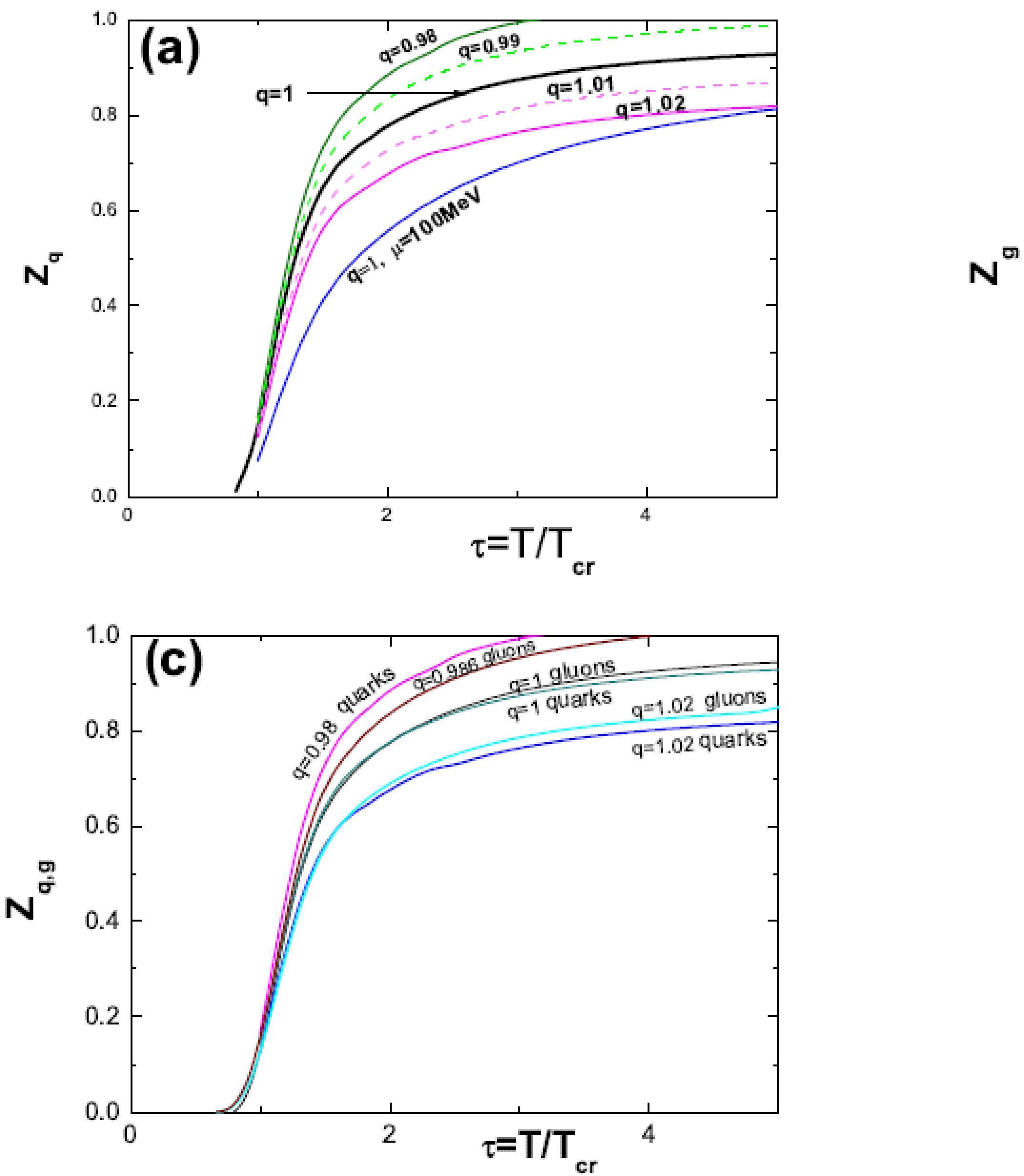
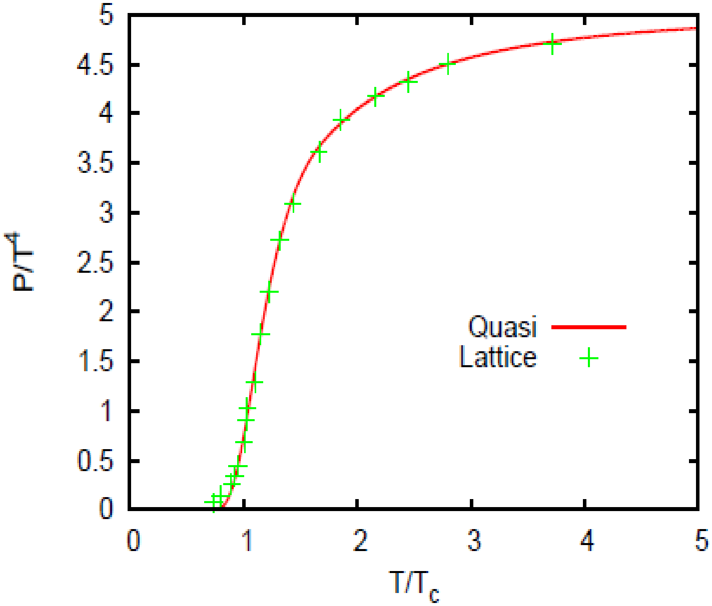
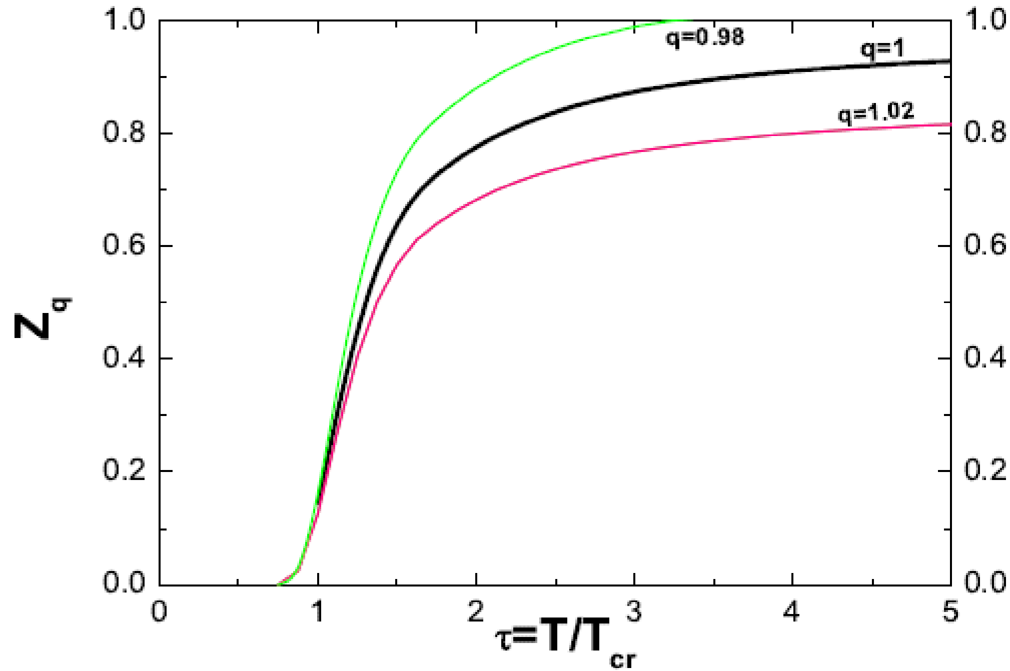
q-zQPM

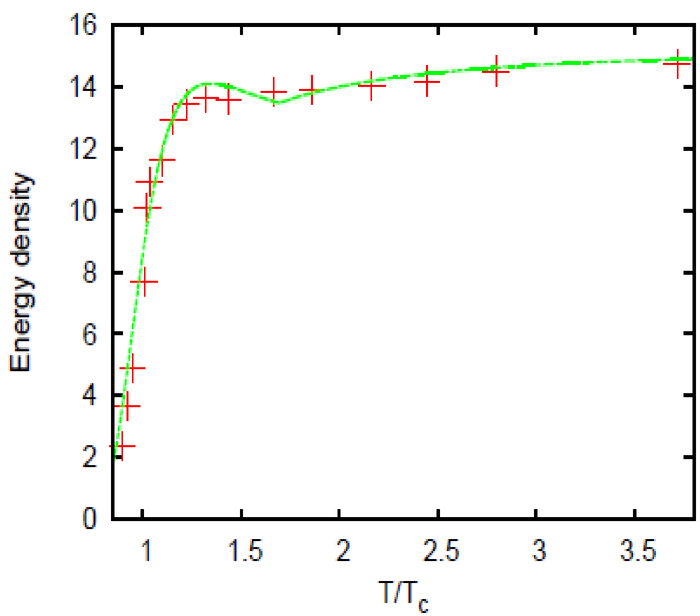


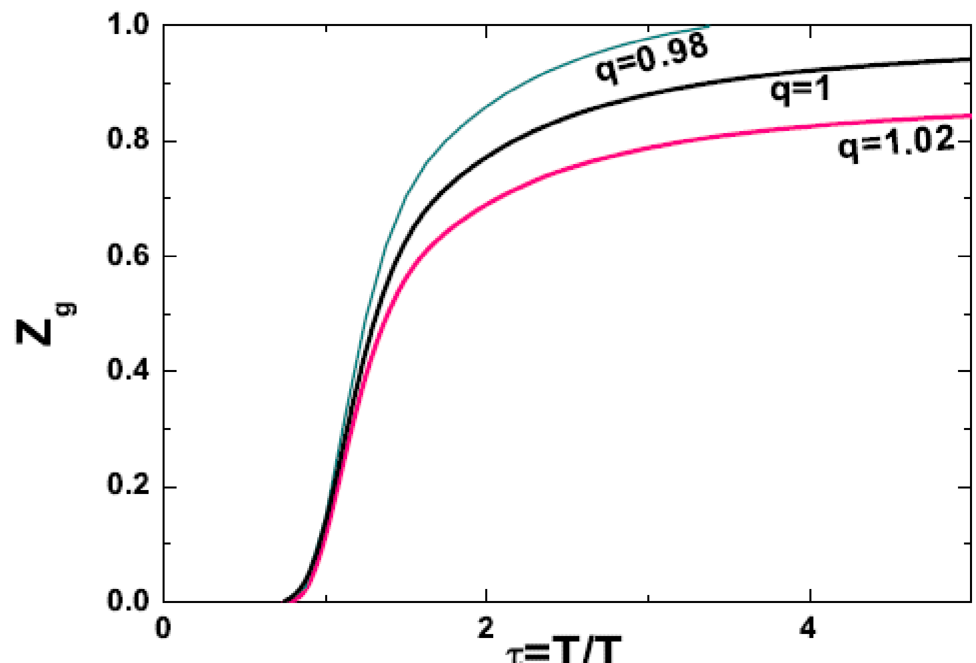
Nonextensive effective fugacities z_q for quarks (a) and gluons (b) . In (a) for comparison curve with $q = 1$ but $\mu = 100$ MeV was also shown. Quarks and gluons are compared in (c).

Fit to data

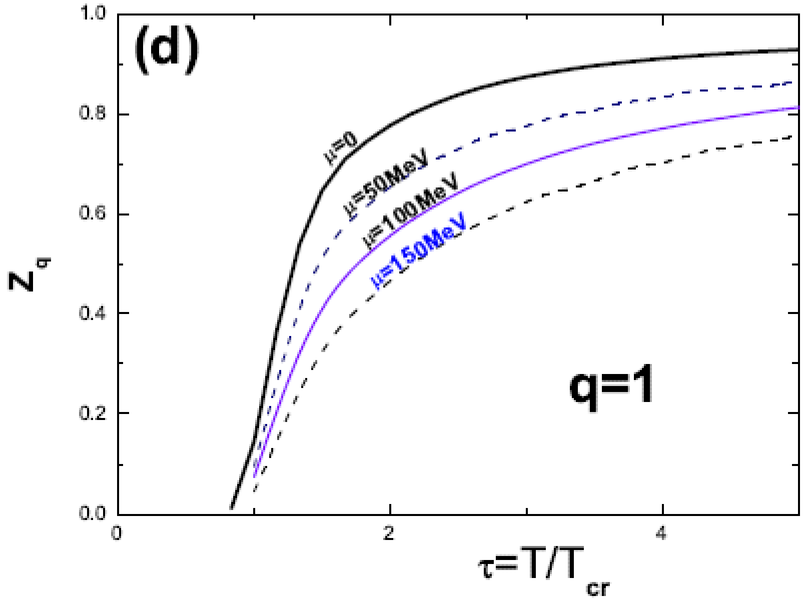

P/T^4
Quasi
Lattice
T/T_c


Z_q
q=0.98
q=1
q=1.02
τ=T/T_cr


Energy density
T/T_c

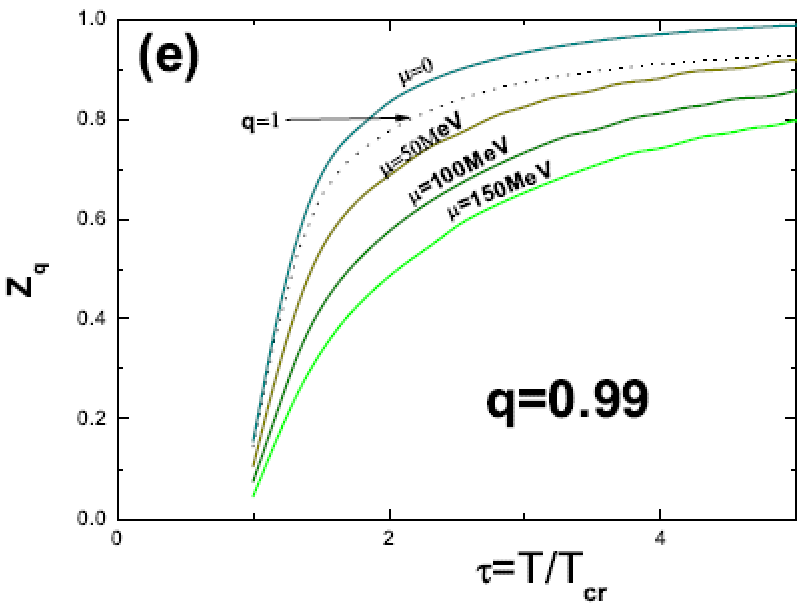

Z_g
q=0.98
q=1
q=1.02

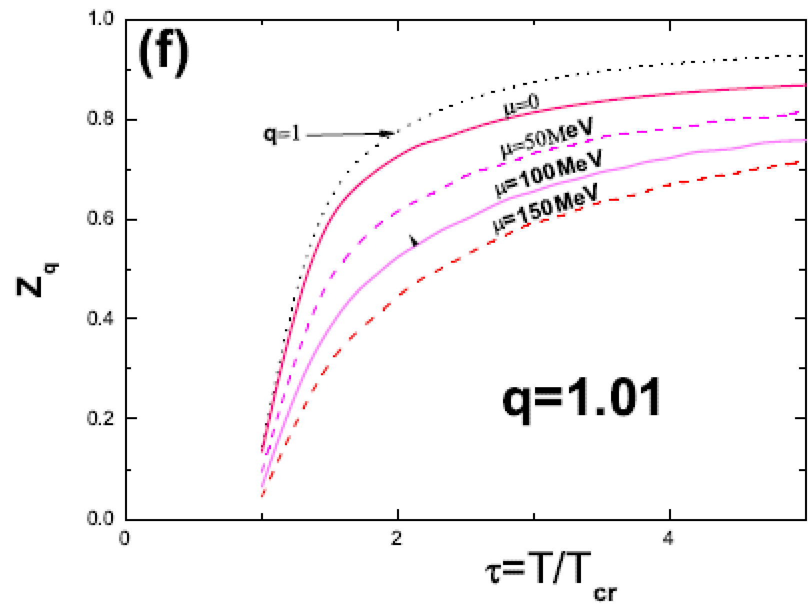
q-zQPM



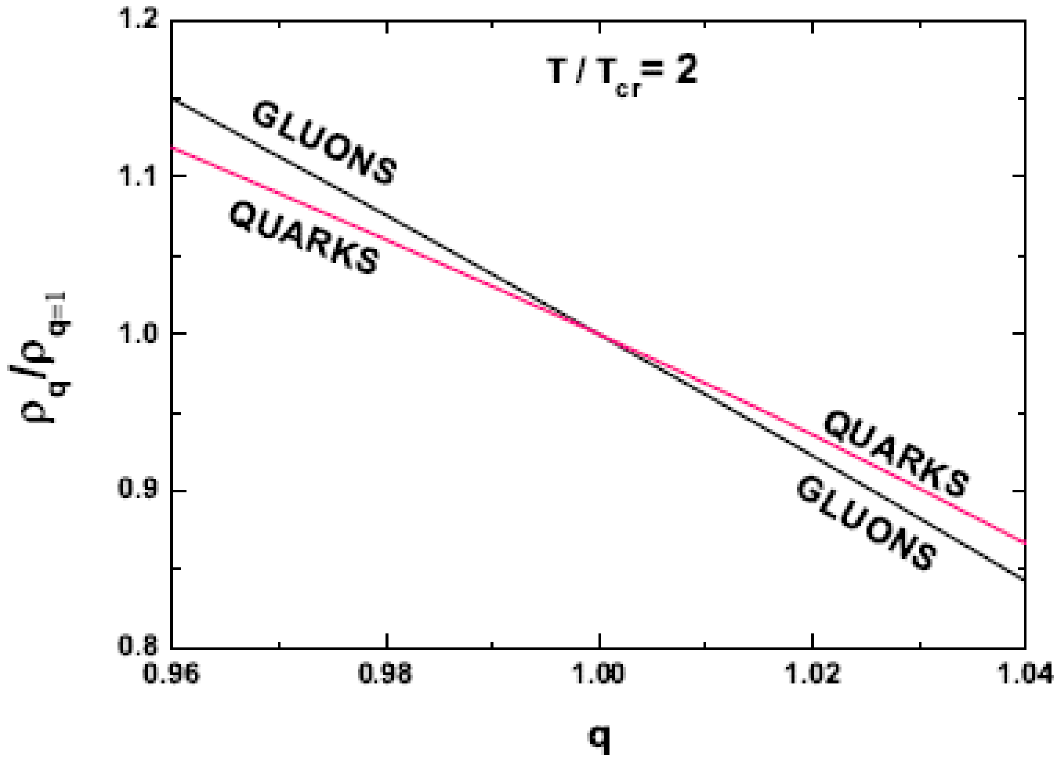
Nonextensive effective fugacities z_q for quarks . Panels show effects of introducing additional nonzero chemical potential μ for, respectively,

q = 1, q = 0.99 and q = 1.01.





Nonextensive Compressibility for T/Tcr=2



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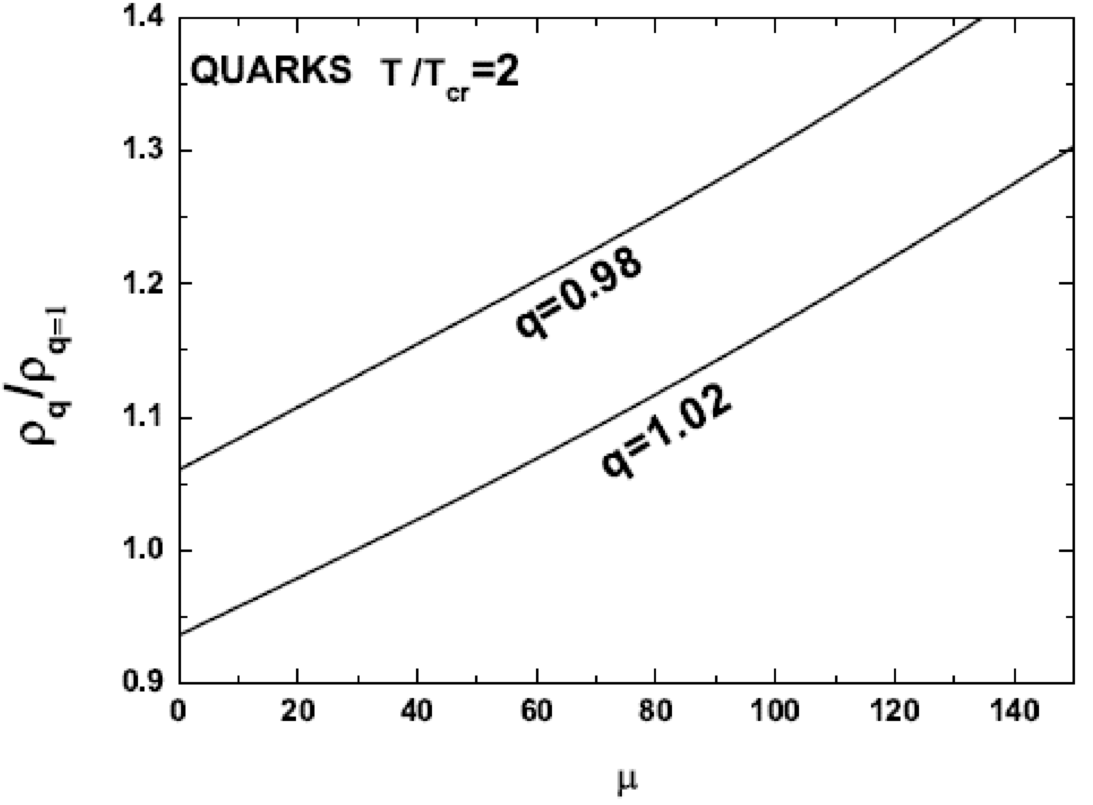
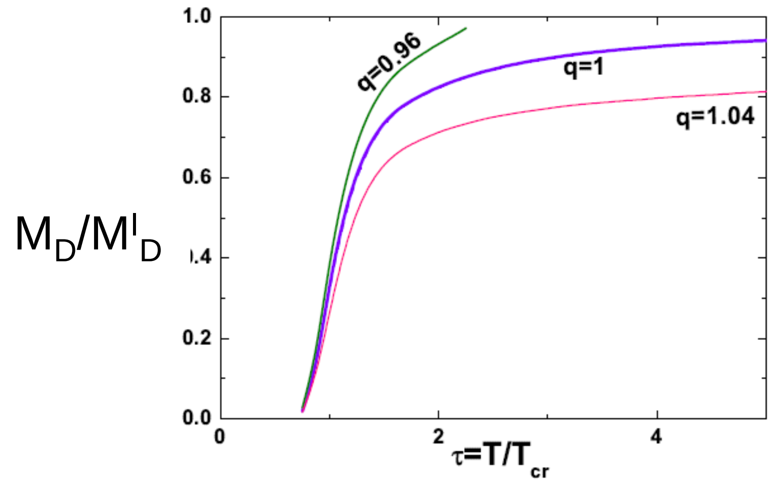
Nonextensive Compressibility for $T/T_{cr}=2$ and non-zero chemical potential

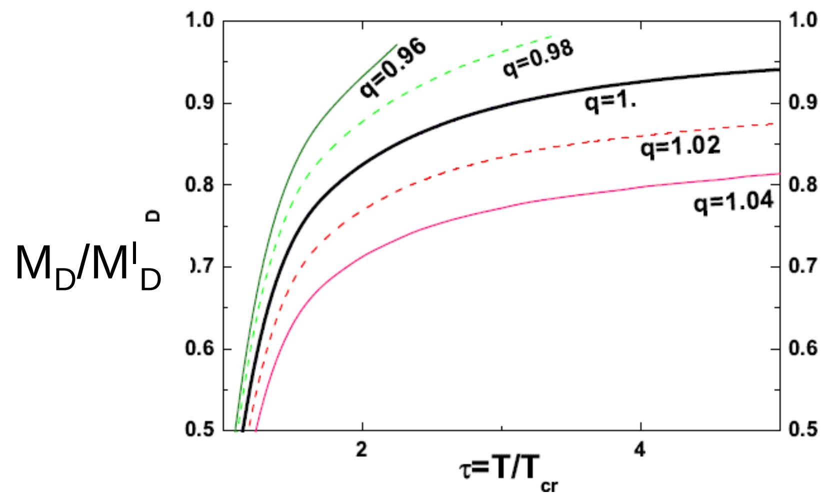


Debye Mass - from transport theory

$$M_D^2 = -2N_c Q^2 \int \frac{d^3p}{8\pi^3} \partial_p n_{eq}^{(g)} - $$
$$ -Q^2 \int \frac{d^3p}{8\pi^3} \partial_p \left(4n_{eq}^{(q)} + 2n_{eq}^{(s)}\right)$$



$$\left(M_D^I\right)^2 = Q^2 T^2 \left(\frac{N_c}{3} + \frac{1}{2} - \frac{m^2}{4\pi^2 T^2} \ln 2\right).$$



q-zQPM - Summary

(1) Note that, as shown in the qNJL model:

The $q > 1$ results in the enhancement of the growth of pressure and entropy observed in the critical region of phase transition from quark matter to hadronic matter in lattice calculations for finite temperature. In this case limit of noninteracting particles (corresponding to $z_q = 1$) is never reached. It is reached for the $q < 1$ (both for quarks and gluons).

q-zQPM - Summary

(2) The physical significance of the effective nonextensive fugacities, z_q, is best seen by looking at the corresponding dispersion:

$$\varepsilon_q = -\frac{\partial}{\partial \beta}\left(\Xi_q\right)$$

$$\varepsilon_q^{(i)} = E_i + T^2 \frac{\partial \mu_q^{(i)}}{\partial T} = E_i + T^2 \left[\frac{1}{z_q^{(i)}} \frac{\partial z_q^{(i)}}{\partial T}\right]$$

nonextensivity affects only the interaction term represented by fugacity **z** (there is no direct dependence on the nonextensivity q). It results in some additional contributions to quasiparticle energies from the collective excitations. It occurs because of the temperature dependence of the effective fugacities, z = z(T). It can be interpreted as representing the action of the gap equation in the q-NJL (but with constant energy). „Landau" quasi-particle.

q-zQPM - Summary

(3) Both the nonextensivity (**q**) and quasi-particle approach (**z**) deform the original particle number distribution, but do it in different ways:

(i) The **z-deformation is local** and **z=z(T)**.

(ii) The **q-deformation is global**, **independent on T**.

Formally such replacement is possible, but resulting fugacity becomes the energy dependent quantity. It means that we cannot replace action of nonextensivity by some reasonable (i.e., energy independent) fugacity.

Both quantities present incomparable aspects of dynamics

Summary

(*) The nonextensive description accounts (in a phenomenological way) for all situations in which one expects some dynamical correlations and/or fluctuations in the system considered (hadronic or quark-antiquark or yet another not yet disclosed) .

(*) It is not a substitute for any part of the interaction already known and described by, for example, some Lagrangian (as in the NJL model), but provides a different environment which can have some dynamical effects, so far undisclosed but simply parameterized by the nonextensivity parameter $q \neq 1$.

(*) Therefore, when dynamical description becomes more complete gradualy $q \to 1$

Back-up slides

q-NJL

Concluding remarks:

(i) Our approach should not be confounded with the similar in spirit approach based on quantum algebras (or on the so called q-deformed algebras) which was used to formulate a q-deformed NJL model [(#) S.S. Avancini et al. PlB507(2001)129].

Their common feature is the use of some suitable deformation of the mean field NJL model (based on nonextensive statistical mechanics in our case or on quantum algebras in (#)), which may account for intrinsic correlations and fluctuations that go beyond the mean field formulation and, in a certain limit, approach the more realistic lattice calculations.

(ii) There exists another, potentially very interesting, approach to nonextensivity, based on the so called Kaniadakis entropy especially suitable for relativistic systems *[G. Kaniadakis, EPJA40(2009)275, EPJB70(2009)3, EPL92(2010)35002]*. In *[A.M.Teweldeberhan et al., IJMPE12(2003)669]* it was used to study the formation of the quark-gluon plasma (and compared with nonextensive approach) whereas in *[F.I.M.Pereira, et al., NPA828(2009)136]* it was used to investigate the relativistic nuclear EoS in the context of the Walecka quantum hadrodynamics theory. However, its thermodynamic consistency still remains to be checked.

q-NJL

$$E_q = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \left[\int p^2 \mathrm{d}p \frac{p^2 + m_i M_{qi}}{E_{qi}} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right]$$

$$-g_S V \sum_{i=u,d,s} \left(\langle \bar{q}_i q_i \rangle_q \right)^2 - 2 g_D V \langle \bar{u} u \rangle_q \langle \bar{d} d \rangle_q \langle \bar{s} s \rangle_q,$$

$$S_q = -\frac{N_c}{\pi^2} V \sum_{i=u,d,s} \int p^2 \mathrm{d}p \cdot \tilde{S}_{qi}^{(R)},$$

$$N_{qi} = \frac{N_c}{\pi^2} V \int p^2 \mathrm{d}p \, (n_{qi}^q - \bar{n}_{qi}^q) \, .$$

The corresponding nonextensive energy, entropy, and number density

$$M_{qi} = m_i - 2 g_S \langle \bar{q}_i q_i \rangle_q - 2 g_D \langle \bar{q}_j q_j \rangle \langle \bar{q}_k q_k \rangle_q,$$

q-gap equations

$$\langle \bar{q}_i q_i \rangle_q = -\frac{N_c}{\pi^2} \sum_{i=u,d,s} \left[\int \frac{p^2 M_{qi}}{E_{qi}} (1 - n_{qi}^q - \bar{n}_{qi}^q) \right] \mathrm{d}p.$$

q-quark condensates:

q-NJL

Basic q-formulas for q-NJL:

$$\Omega_q(T,V,\mu_i) = E_q - TS_q - \sum_{i=u,d,s} \mu_i N_{qi}.$$

q-grand canonical potential

$$P_q(\mu,T) = -\frac{1}{V}[\Omega_q(\mu,T) - \Omega_q(0,0)],$$

$$\varepsilon_q(\mu,T) = \frac{1}{V}[E_q(\mu,T) - E_q(0,0)],$$

q-pressure and the energy densities

$$S_q = \sum_{i=u,d,s} \frac{\partial \Omega_q}{\partial T}\Big|_\mu \quad \text{and} \quad \varrho_q = \sum_{i=u,d,s} \frac{\partial \Omega_q}{\partial \mu}\Big|_T.$$

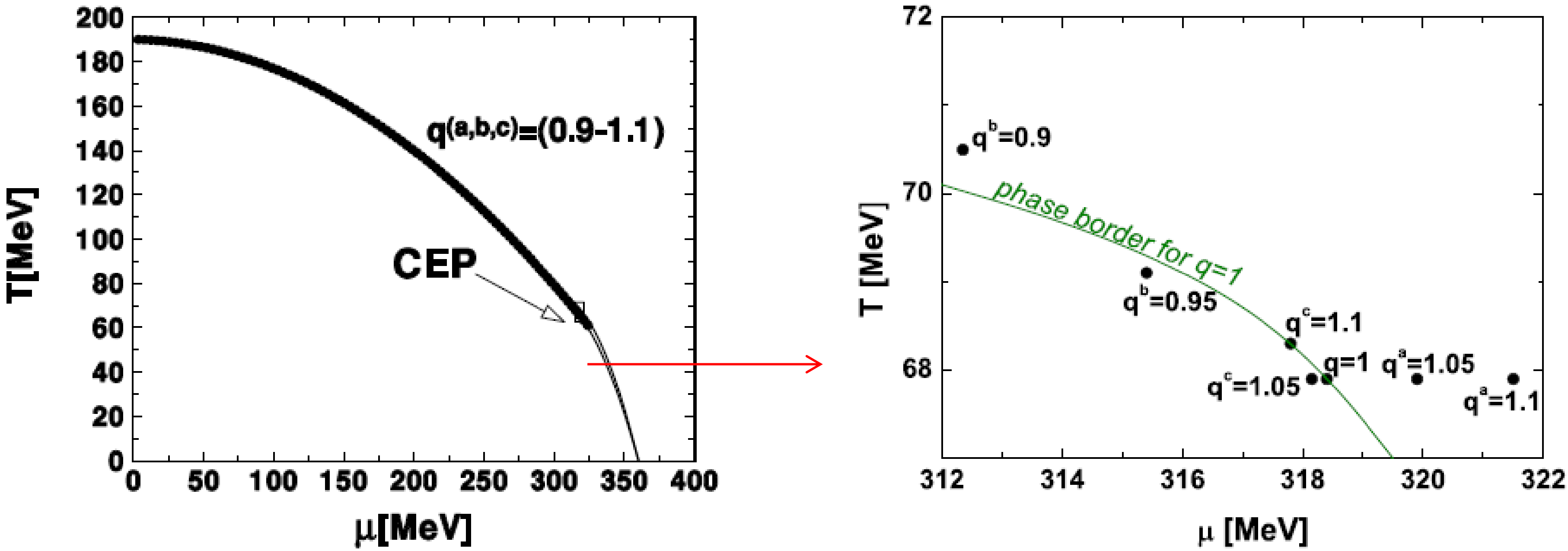
q-entropy and q-density,

$$C_\mu = \frac{\partial S_q}{\partial T}\Big|_\mu \quad \text{and} \quad \chi_B = \frac{\partial \varrho_q}{\partial \mu}\Big|_T.$$

nonextensive versions of the heat capacity and the barionic susceptibility

$$\mathrm{d}E_q = \frac{\partial E_q}{\partial S_q}\mathrm{d}S_q + \frac{\partial E_q}{\partial N_q}\mathrm{d}N_q = \frac{\partial E_q}{\partial T}\mathrm{d}T + \frac{\partial E_q}{\partial \mu}\mathrm{d}\mu, \quad \Longrightarrow \quad \frac{1}{T} = \frac{\partial S_q}{\partial E_q} \quad \text{and} \quad \frac{1}{T^2 C_\mu} = -\frac{\partial^2 S_q}{\partial E_q^2}.$$

q-NJL Fig. 9



Phase diagram in the q-NJL model in the T – μ plane for different values of q corresponding to different realizations of the q-NJL model and compared with the BG case of q = 1. The left panel shows a general view where, for the scale used, all curves essentially coincide. The right panel shows an enlarged region near the critical point (CEP).