Three men walking II (Alberto Giacometti)

UNITARITY CONSTRAINTS ON (IN)FINITE VOLUME THREE-BODY SCATTERING

>CERN-SEMINAR 03/23/2018<

Maxim Mai

The George Washington University

Deutsche Forschungsgemeinschaft

DFG

MOTIVATION

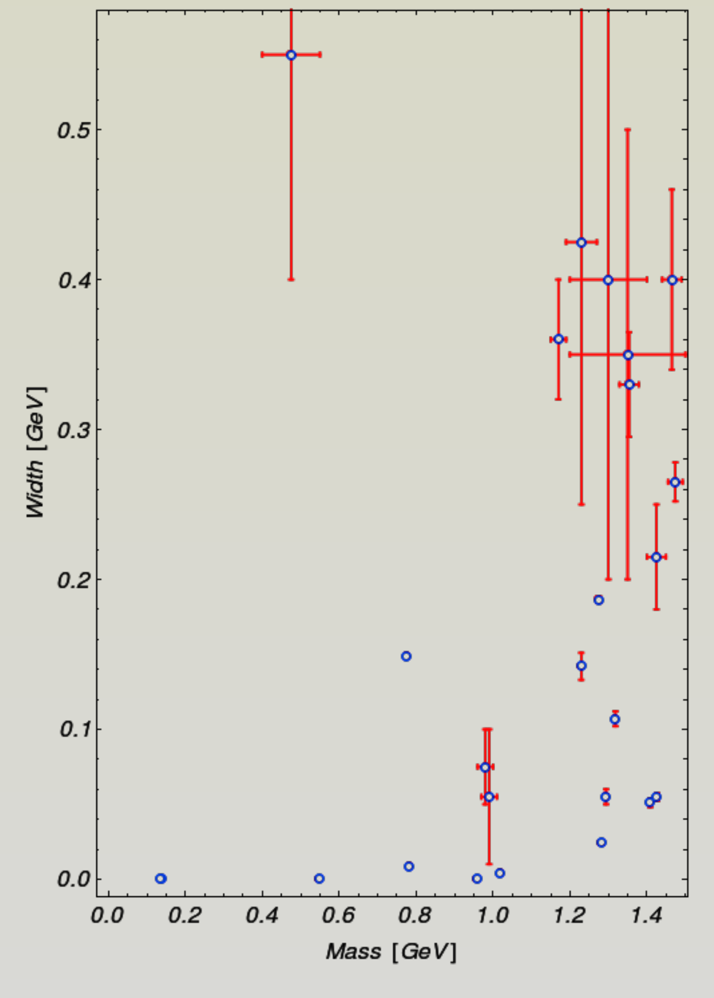
QCD at low energies → rich spectrum of excited states

Q1: how many are there?

– missing resonance problem

Q2: production mechanism?

– quark-antiquark
– gluenballs
– hadron-hadron dynamics



PDG(2018)

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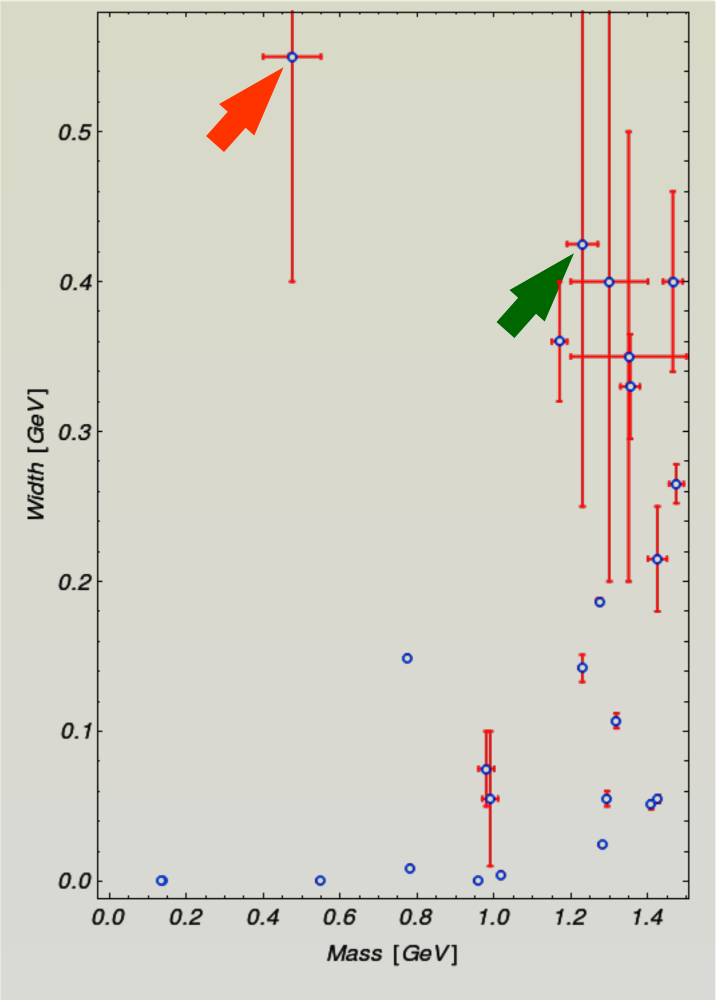

PDG(2018)

EXAMPLES:

– ***σ(500)***

couple strongly to 2π

– ***a1(1260)***

couple strongly to 3π

σ(500)

- named by Schwinger in 1957, but debated for decades → **Review Pelaez (2015)**
- dispersive techniques give the most precise results

Colangelo/Gasser/Leutwyler(2001) Caprini/Colangelo/Leutwyler(2006)
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- new source of information: **ππ phase-shifts from Lattice QCD**

HadSpec(2016) Guo et al.[GWU](2018)

→ 3 and 2 flavor calculations

→ unphysical pion masses: ***(236,391)*** and ***(227,315) MeV***

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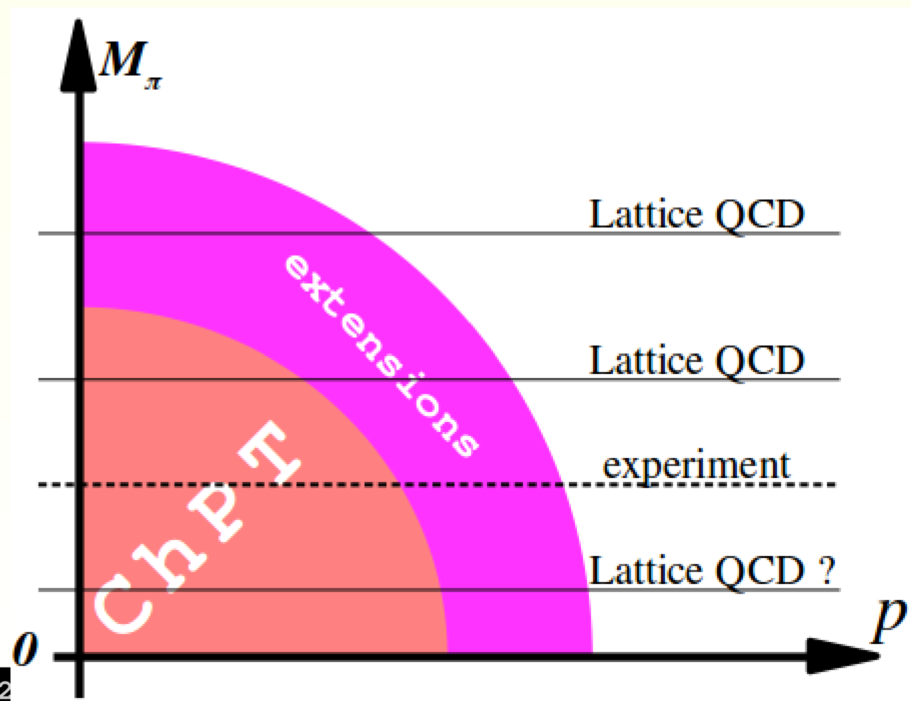

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→ for ***unphysically small*** quark masses

Li, Pagels (1971)
Bruns, MM (2017)

$$m_H(M) = \mathring{m}_H + c^H_{1m} M^2 + \cancel{d^H_m M^2 \log M^2} + O(M^3)\,,$$

$$\Gamma_H(M) = \mathring{\Gamma}_H + c^H_{1\Gamma} M^2 + \cancel{d^H_\Gamma M^2 \log M^2} + O(M^3)$$

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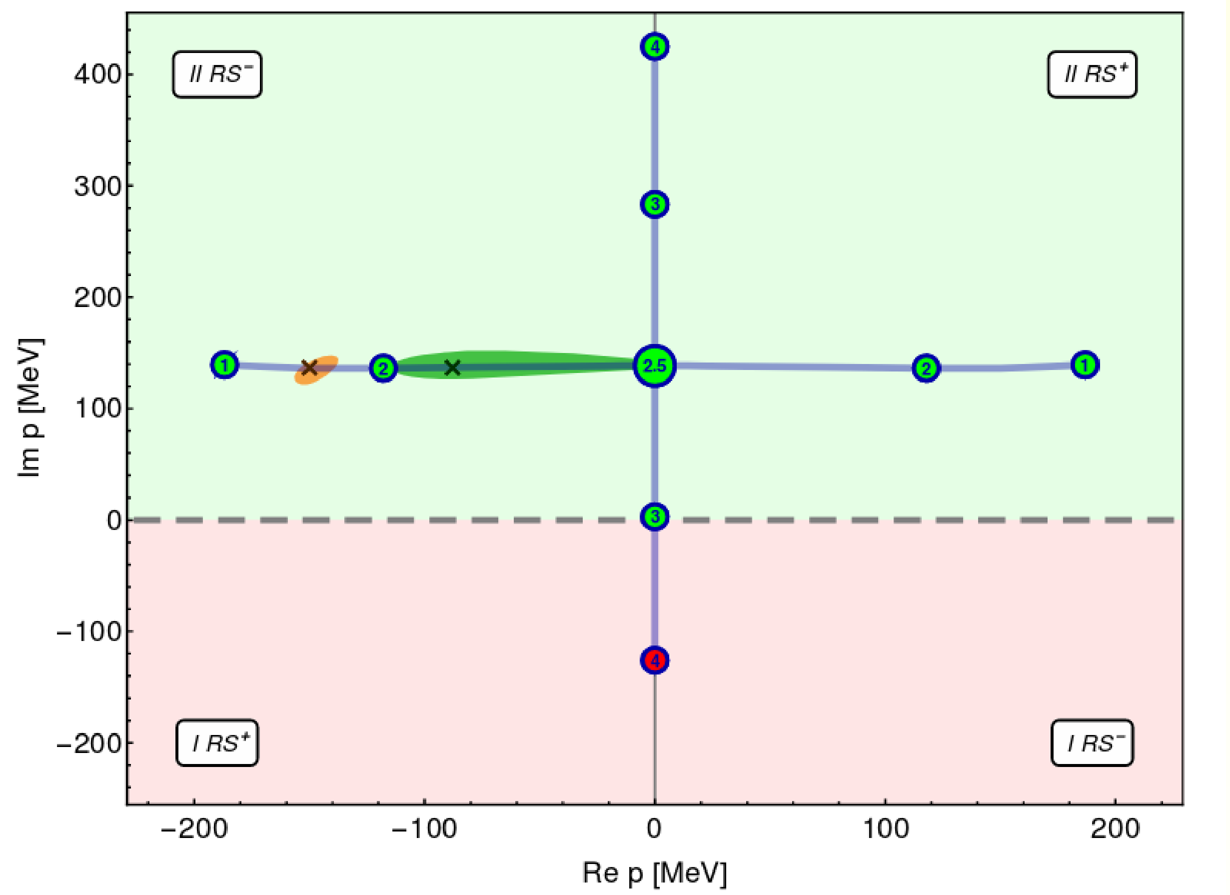
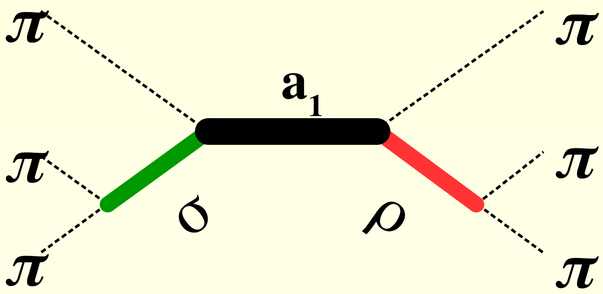


σ becomes a (virtual) bound state @ $M_\pi = (345)\ 415\ MeV$

- **Dynamics is important! BUT many states have dominant 3-body content**

  e.g. $a_1(1260)$

  – important channel in GlueX @ Jlab



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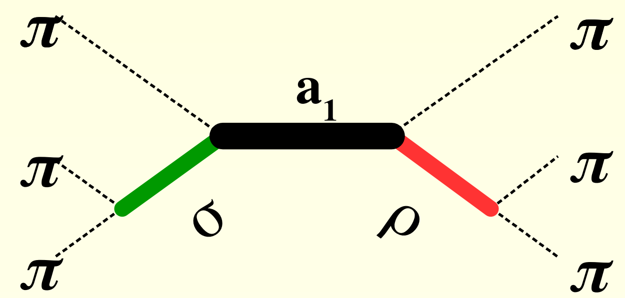
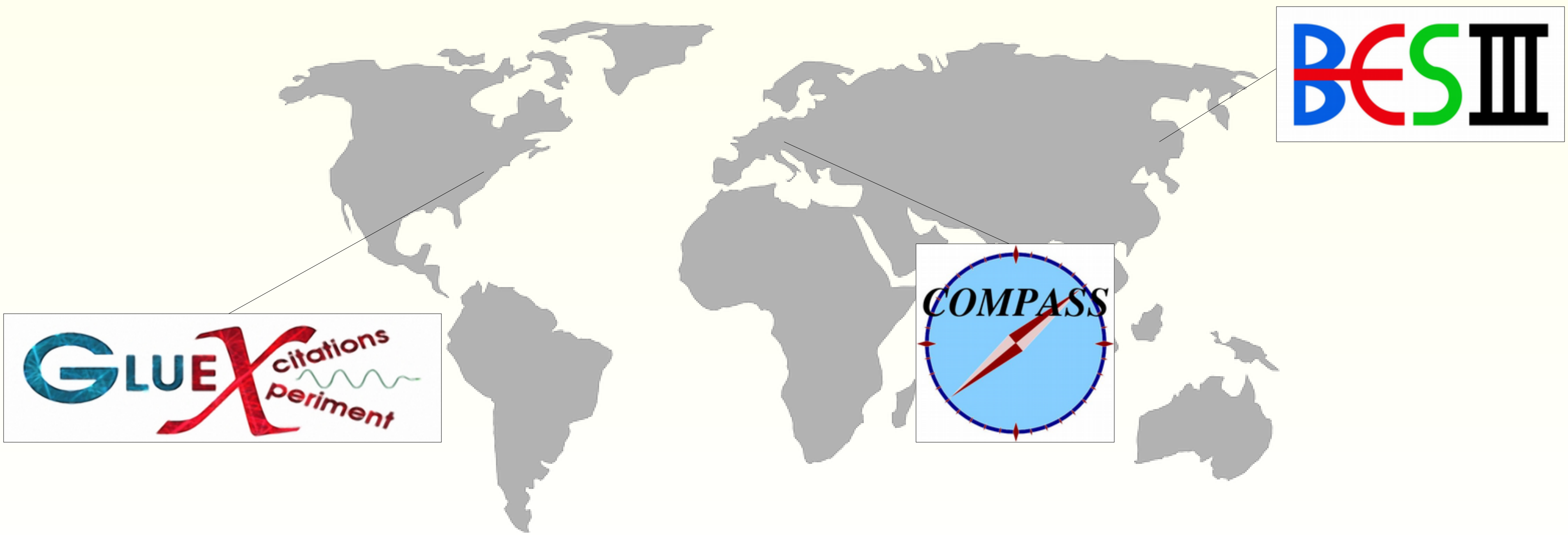


- **Exotic states (*constituent quark model*)**

  – gluonic degrees of freedom

  – cannot decay into 2 mesons but into 3 mesons

  – worldwide experimental effort



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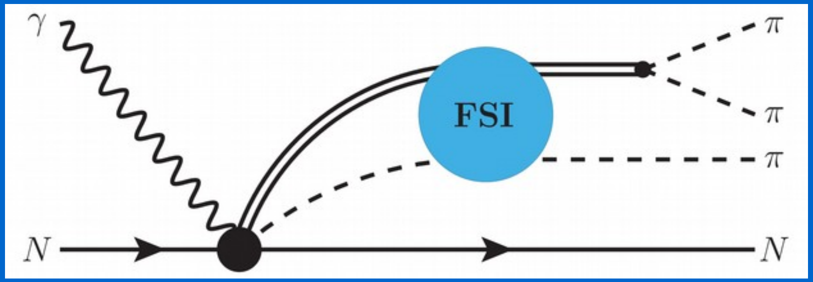

– Further applications

  * Roper puzzle ($\pi\pi N$)

  * X(3872)

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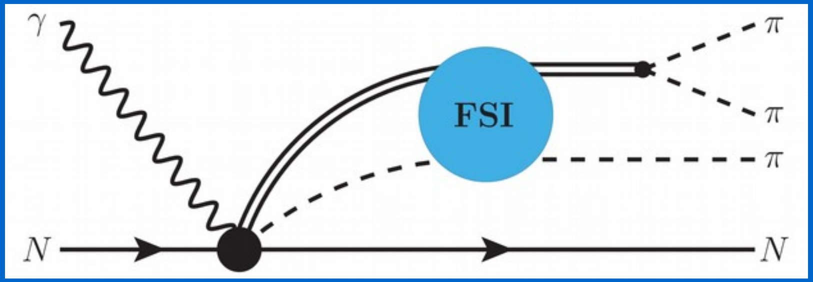

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Lattice QCD

Ab-initio numerical calculations

- first results:
  * a1(1260) Lang et al.(2014)
  * **no $\pi\pi N$ *operators* → *no* Roper** Lang et al.(2017)
  * ***$\pi\rho$(I=2)*** scattering Woss et al.(2018)
- in progress:
  * ***$\pi\rho$(I=1)*** [GWU] (2018)

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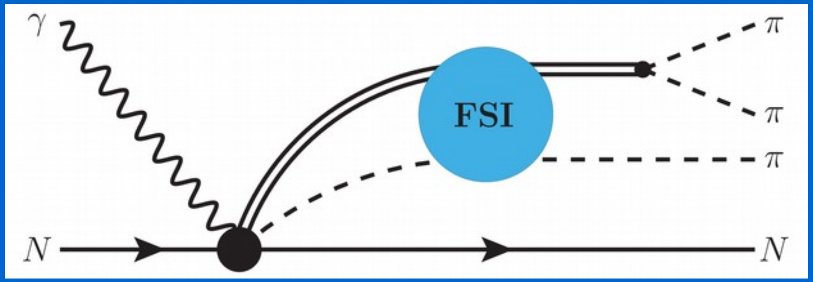

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3-BODY SCATTERING AMPLITUDE IN (UNITARY) ISOBAR-FORMULATION

UNITARITY OF S-MATRIX

IMAGINARY PARTS (INF. VOL.)

POWER LAW FIN. VOL. EFFECTS

UNITARITY OF S-MATRIX

I. PART

IMAGINARY PARTS (INF. VOL.)

II. PART

POWER LAW FIN. VOL. EFFECTS

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3→3 SCATTERING AMPLITUDE IN INFINITE VOLUME

[MM, Hu, Doring, Pilloni, Szczepaniak (2017)]

[Sadasivan, MM, Doring, Pilloni, Szczepaniak (in progress)]

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- **3 asymptotic states (scalar particles of equal mass (m))**

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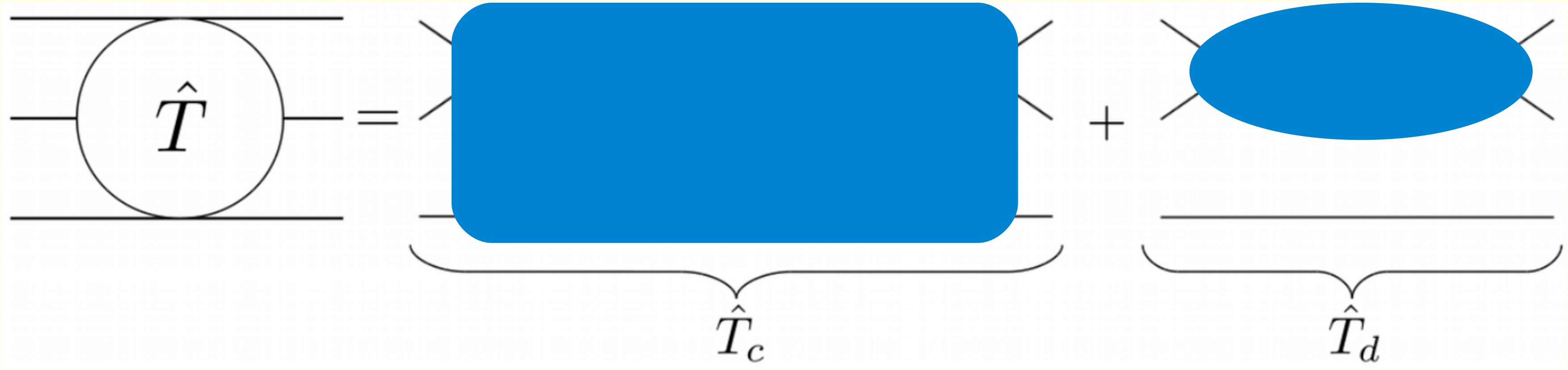

(all permutations of asympt. states are considered)

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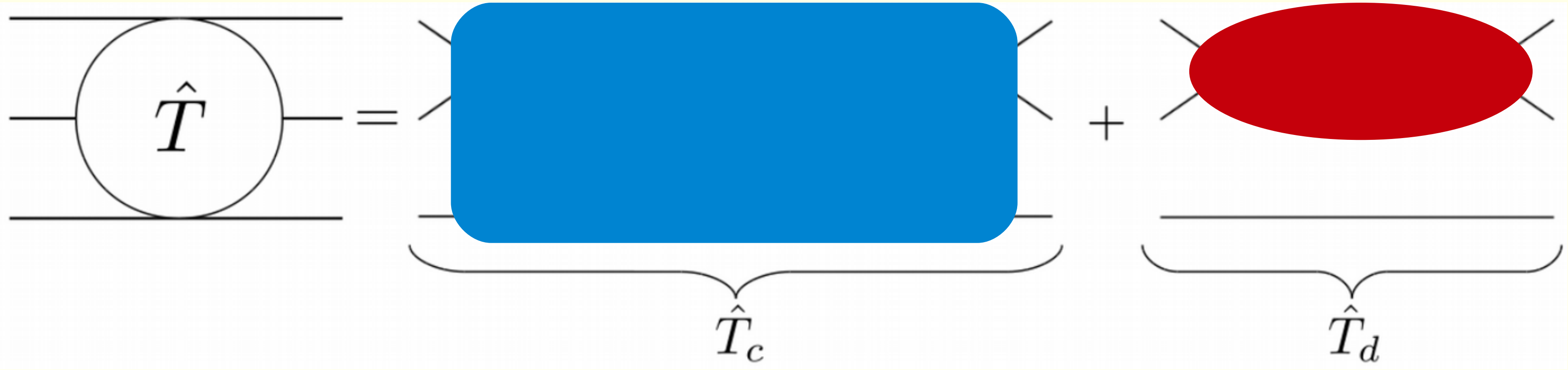

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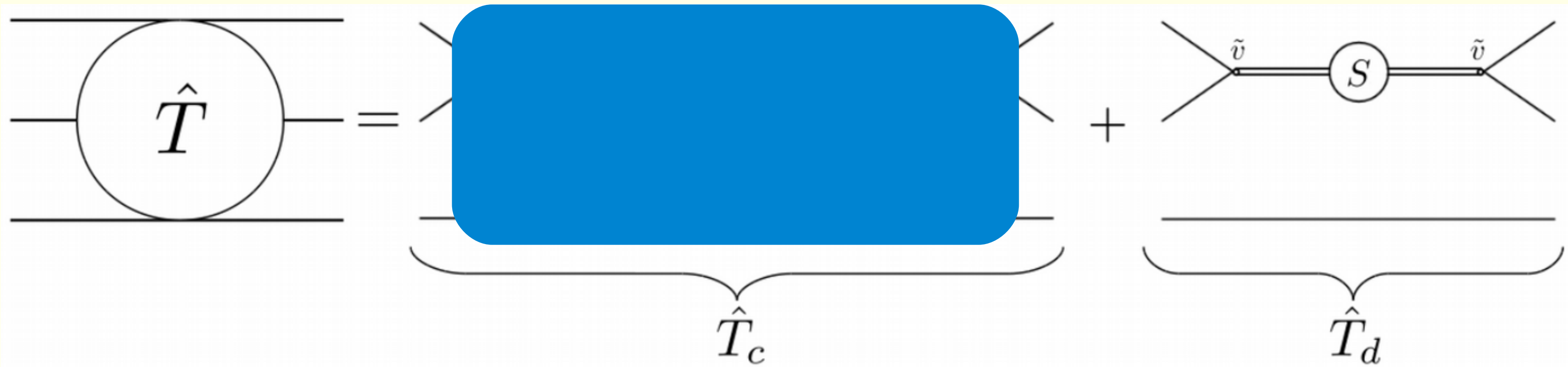

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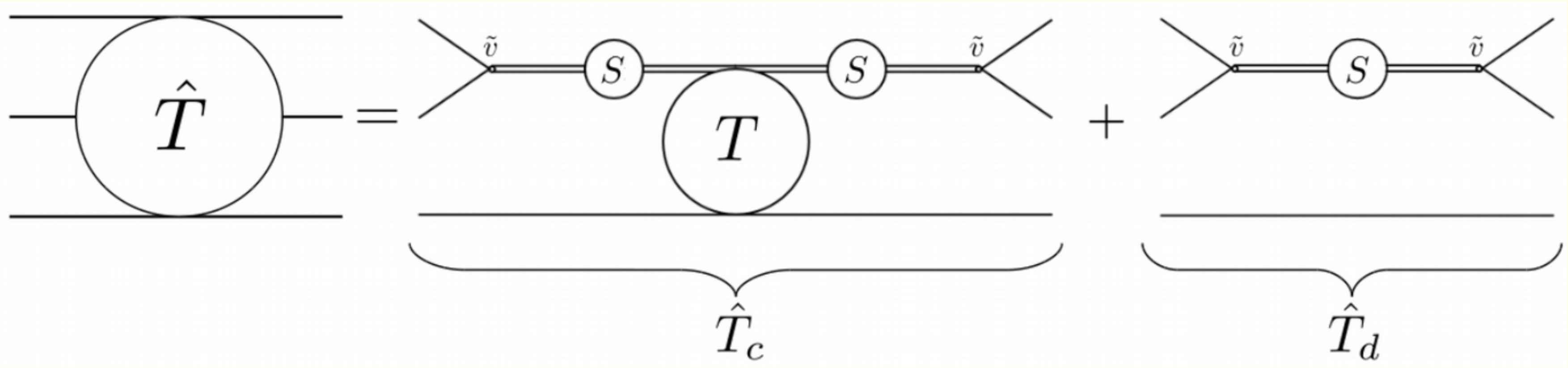

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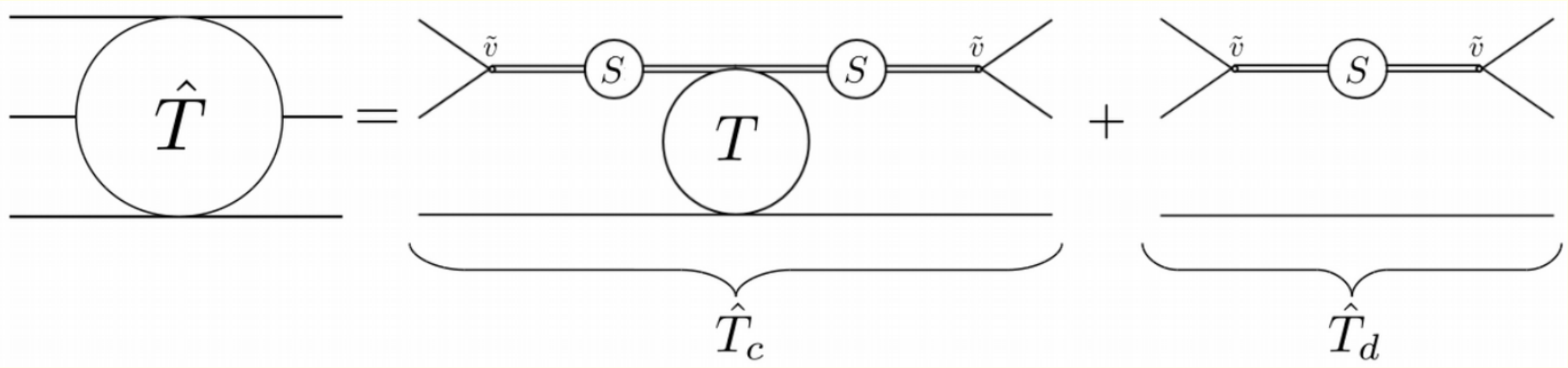

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3 unknown functions & 8 kinematic variables

3-body Unitarity (phase space integral)

$$\langle q_1, q_2, q_3 | (\hat{T} - \hat{T}^\dagger) | p_1, p_2, p_3 \rangle \quad = \quad i \int_P \langle q_1, q_2, q_3 | \hat{T}^\dagger | k_1, k_2, k_3 \rangle \langle k_1, k_2, k_3 | \hat{T} | p_1, p_2, p_3 \rangle$$

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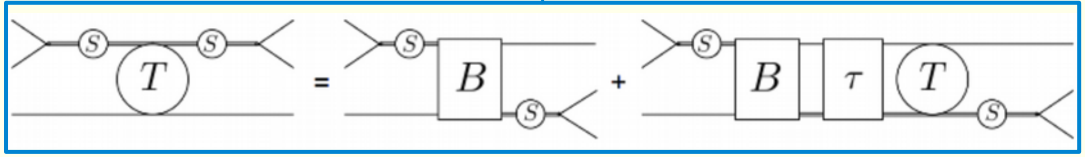

General Ansatz for the isobar-spectator interaction

→ **B & τ** are **new** unknown functions

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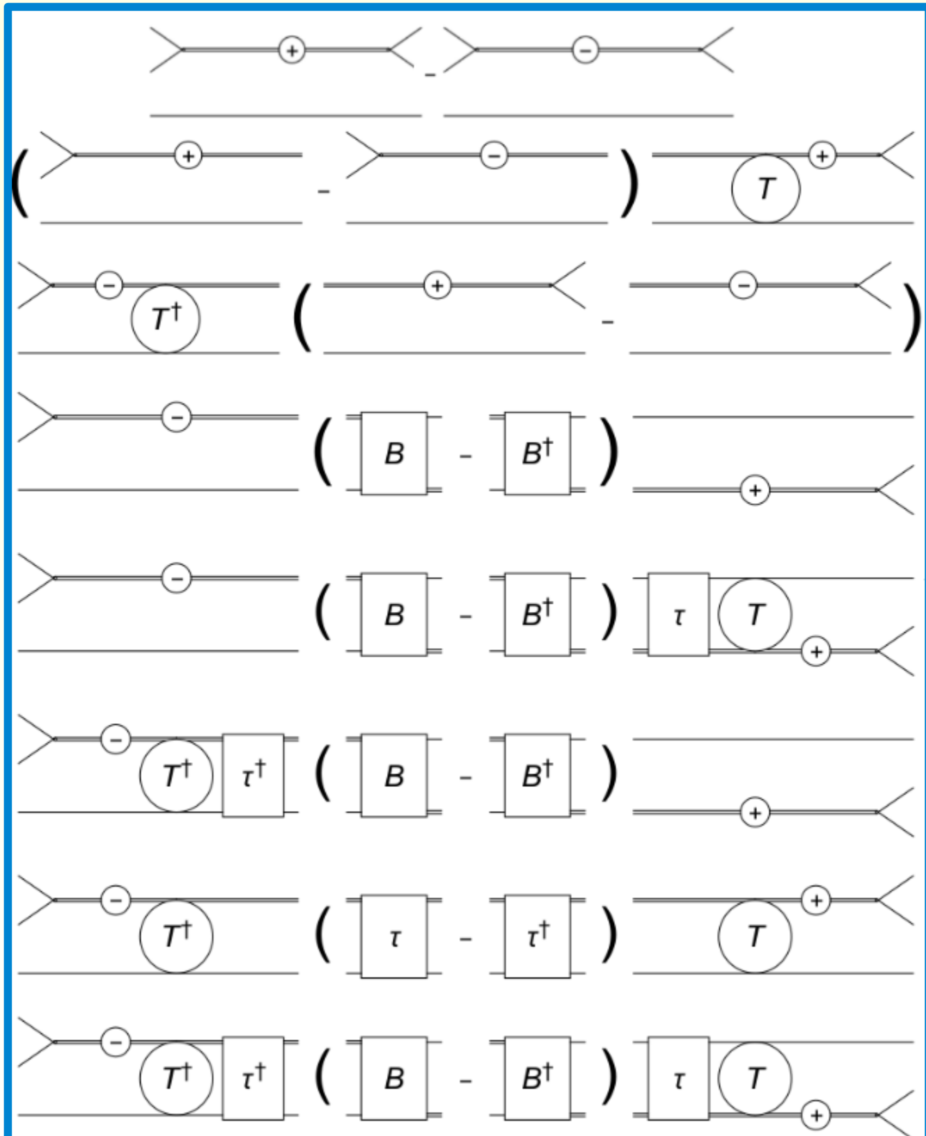

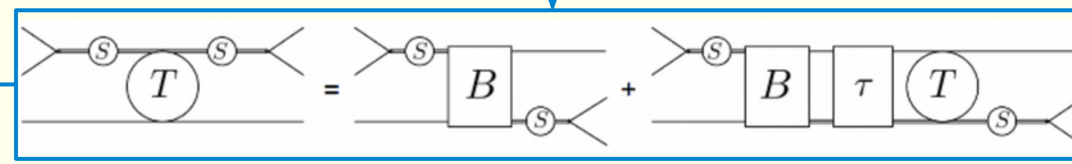

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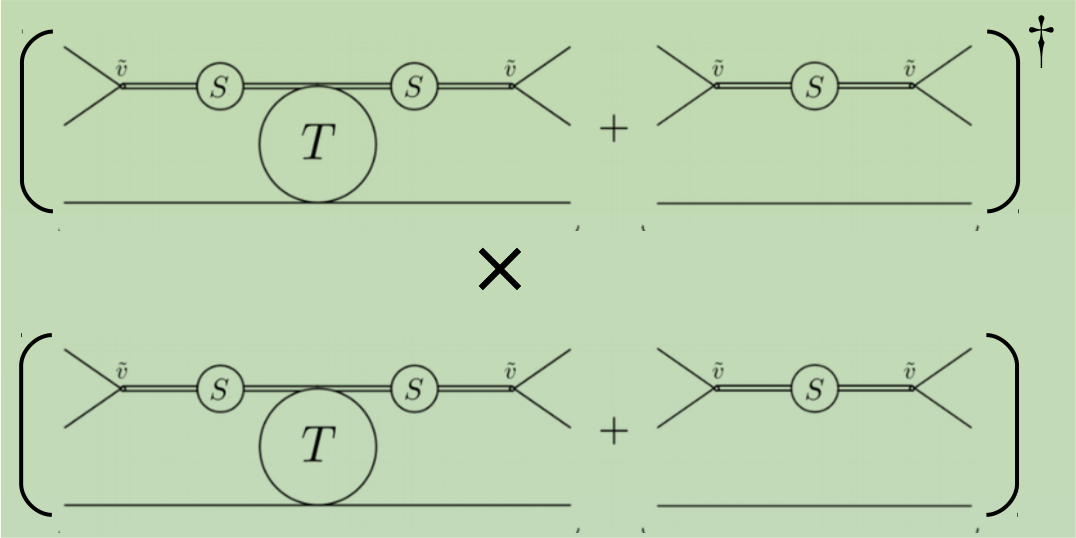

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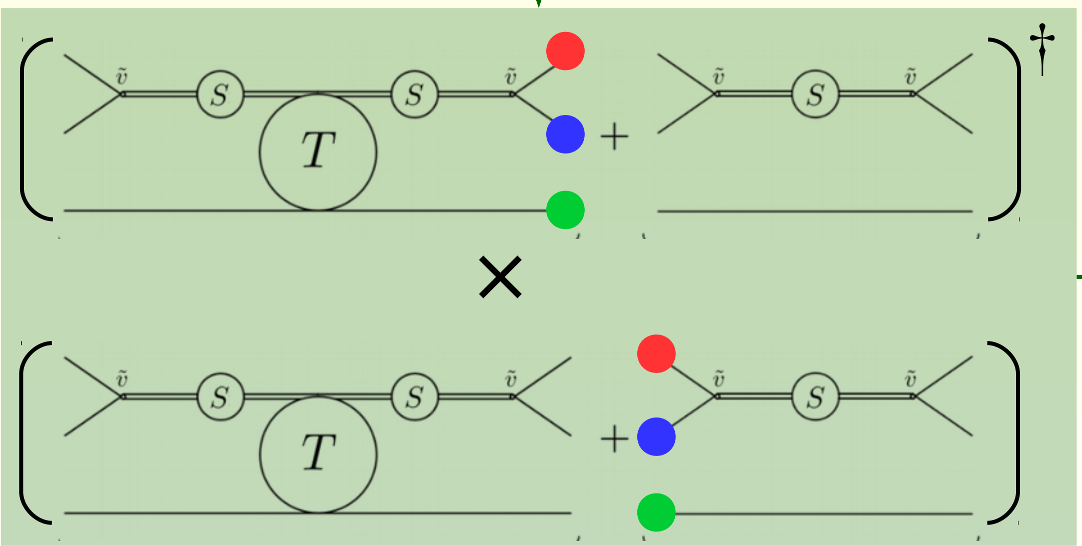

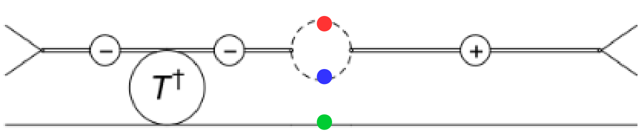

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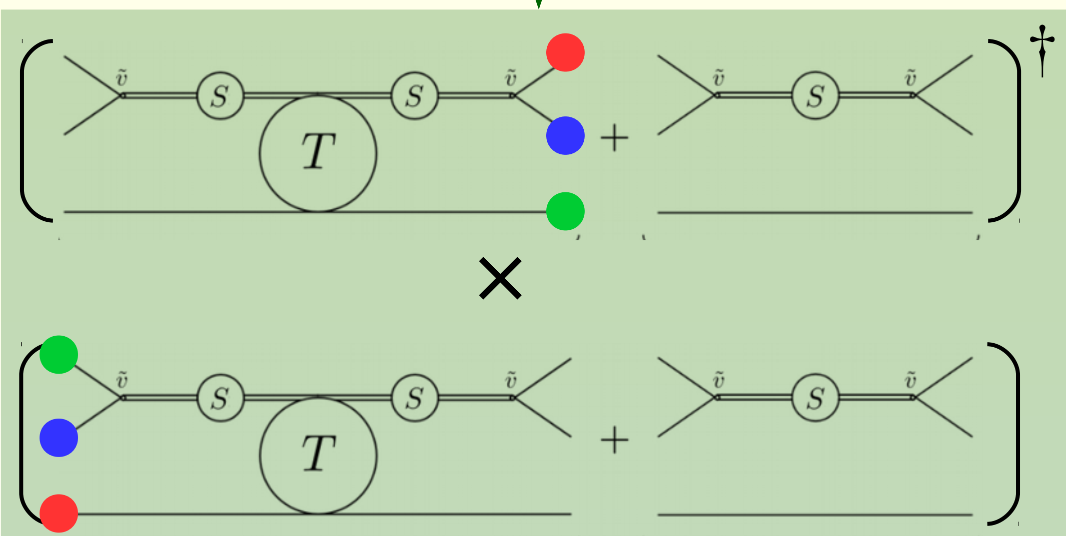

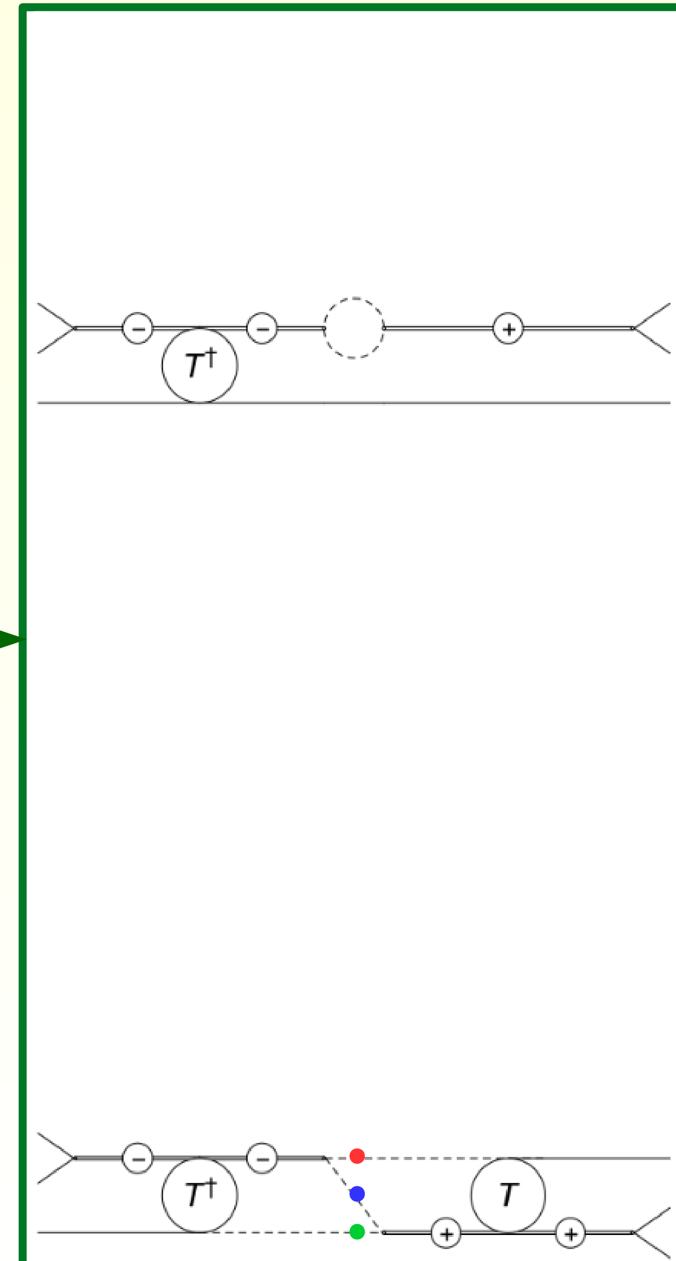

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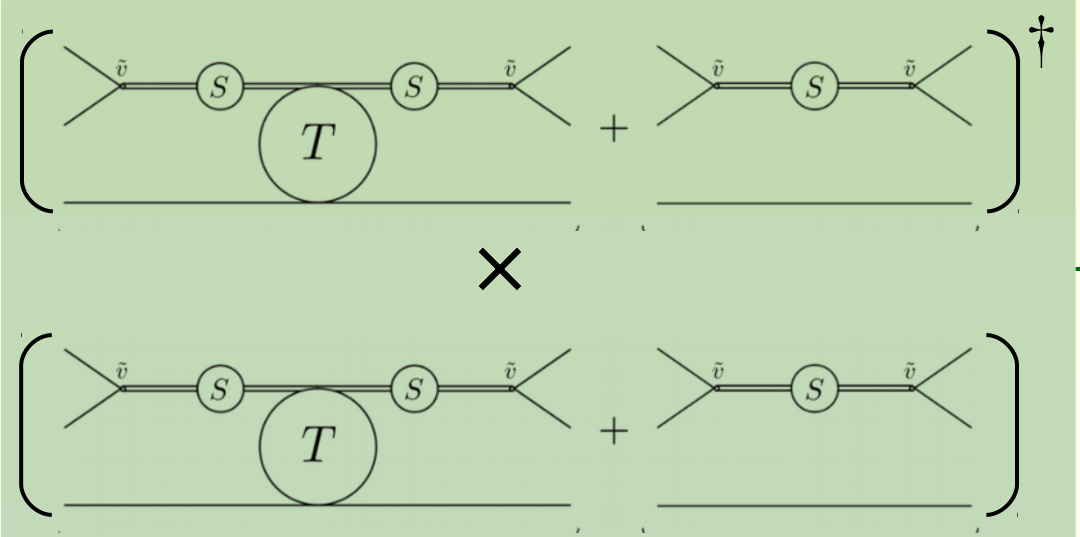
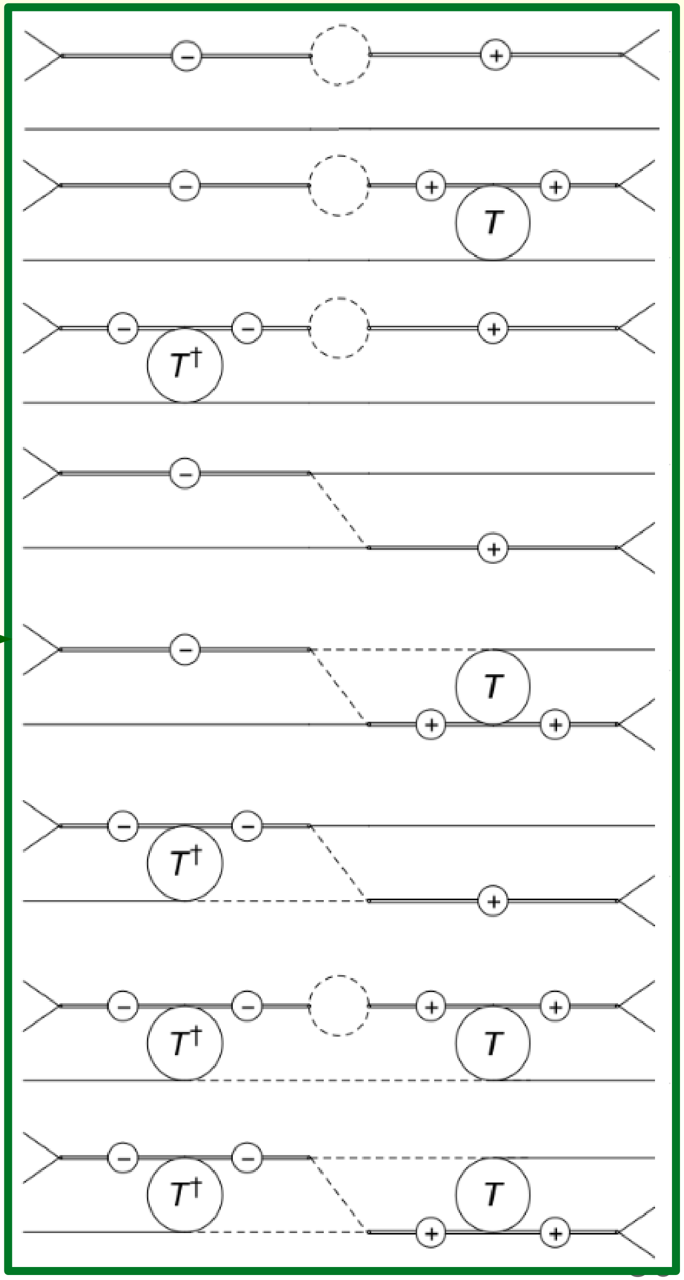


8 top.



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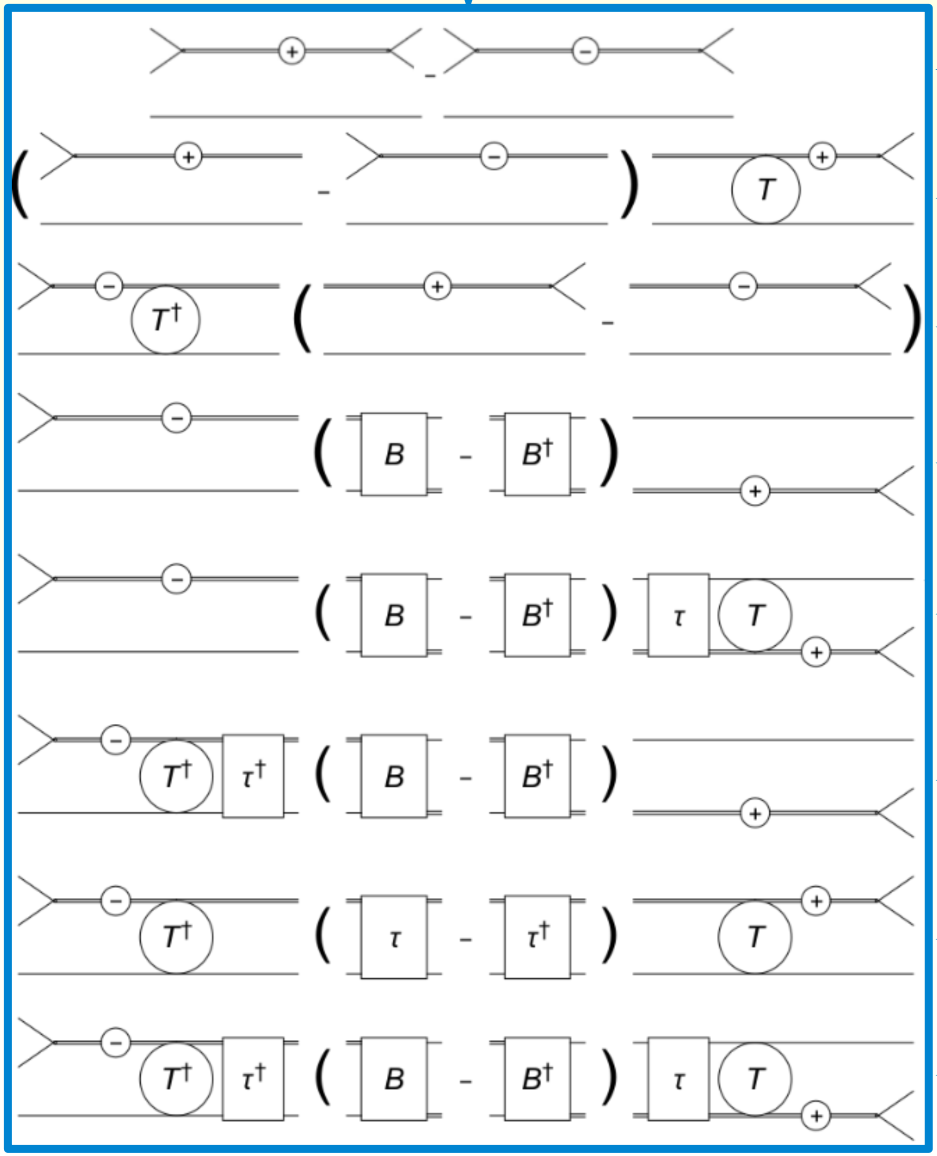

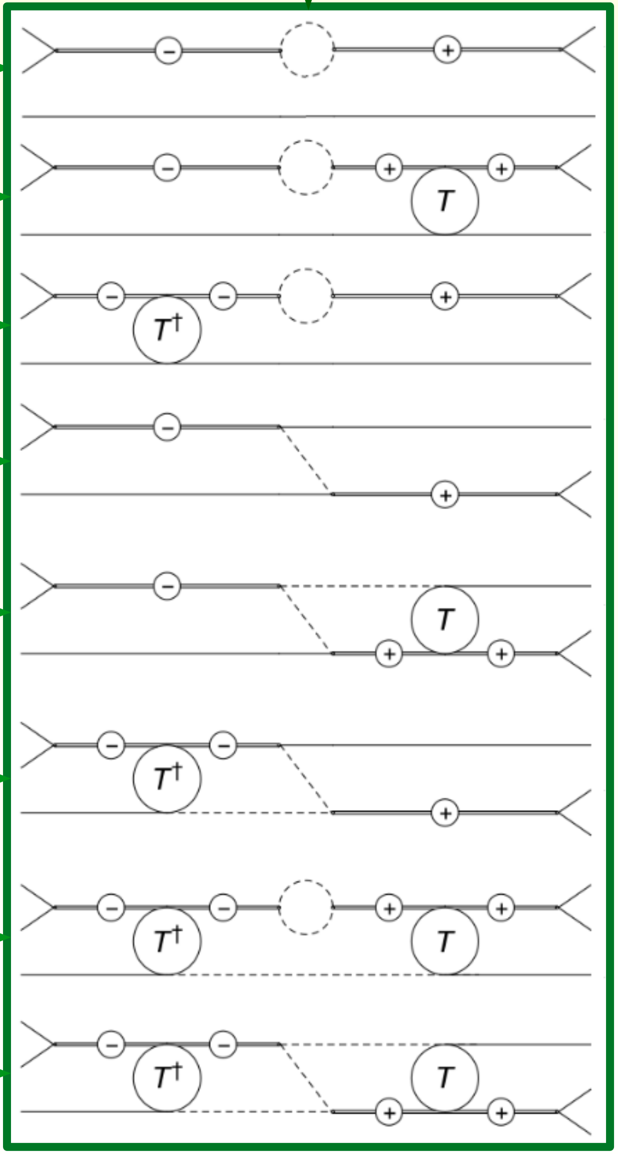

Unitarity/matching

Dispersion relation

$$\text{Disc}\, B(u) = 2\pi i\, \frac{\delta\left(E_Q - \sqrt{m^2 + \mathbf{Q}^2}\right)}{2\sqrt{m^2 + \mathbf{Q}^2}} v^2$$

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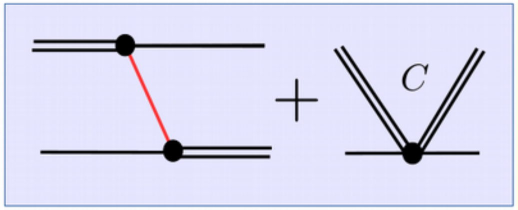

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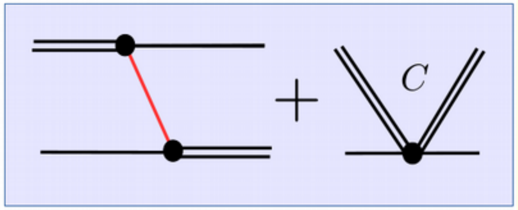

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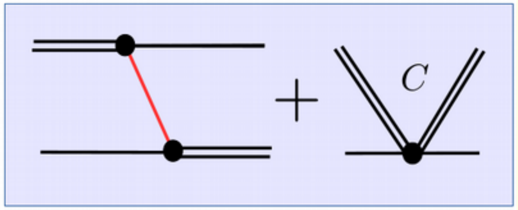

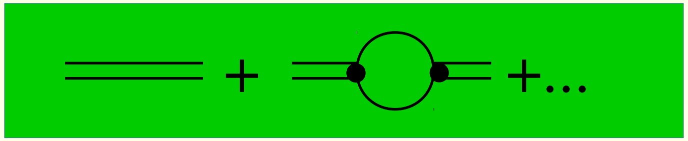

THE ONLY IMAGINARY PARTS REQUIRED BY 3b UNITARITY

Unitarity/matching | **Dispersion relation**

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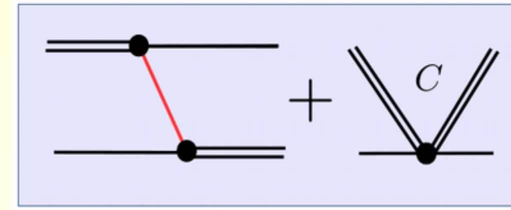

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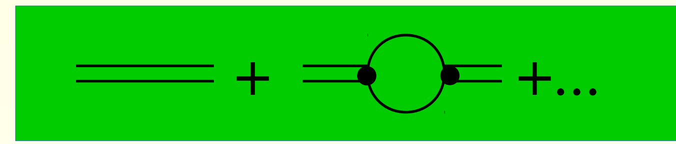

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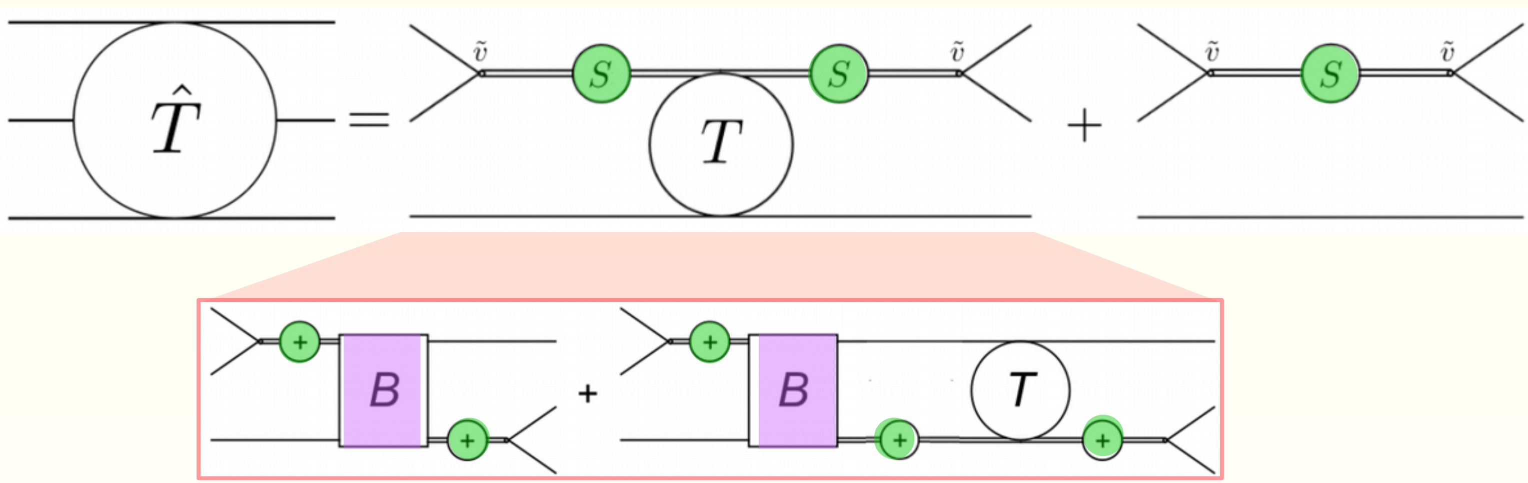


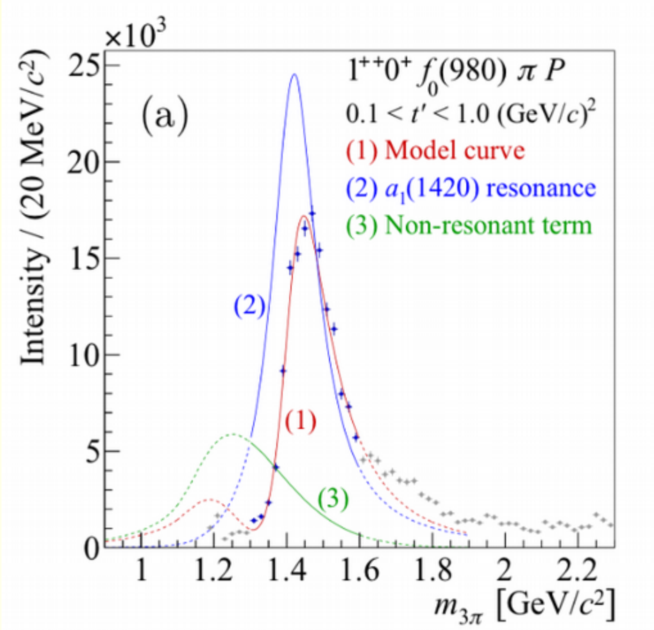
- 3-dim integral equation
- Unknown: ***C,v, parameters of the isobar (subtraction constant)***

Interesting application: a1(1420)

– observed in COMPASS

– in f0(980)π final state

– one explanation:



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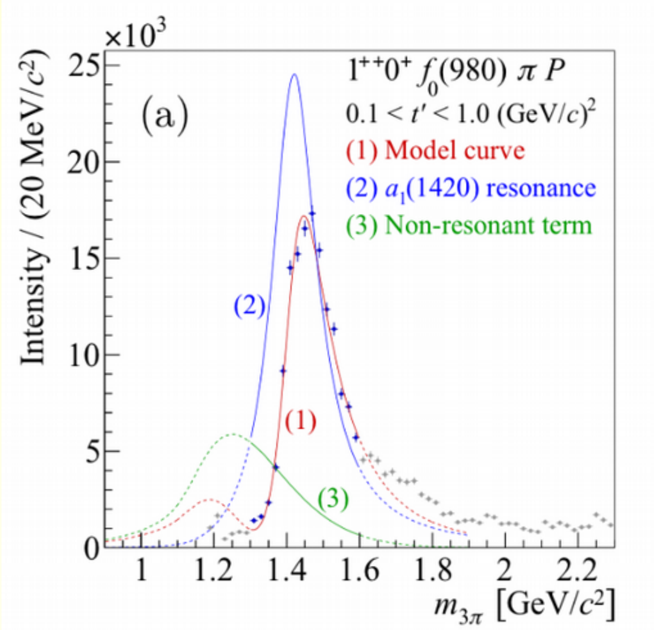

Log-like behavior of the "triangle-diagram"

Mikhasenko/Ketzer/Sarantsev(2015)

Aceti/Dai/Oset(2016)

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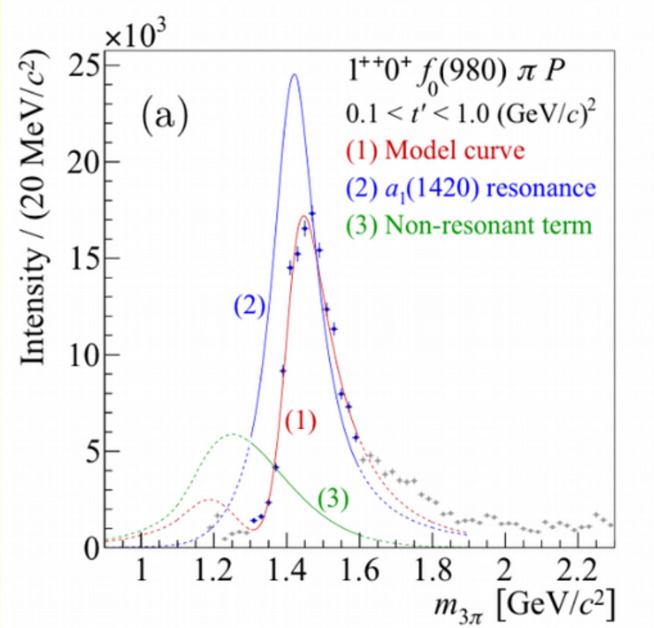
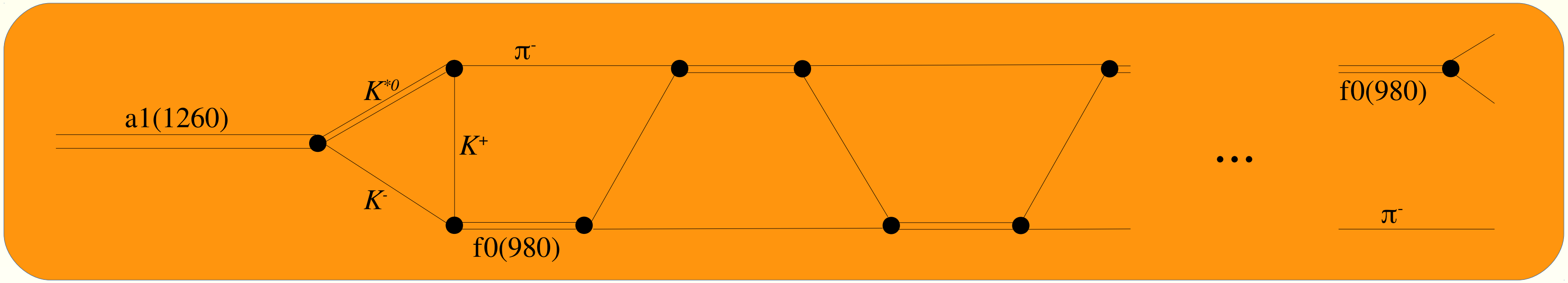


Log-like behavior of the "triangle-diagram"

Mikhasenko/Ketzer/Sarantsev(2015)
Aceti/Dai/Oset(2016)



– **Q:** Does such a feature exist in full 3b-unitary FSI?

Sadasivan in progress...

UNITARITY OF S-MATRIX

I. PART

IMAGINARY PARTS (INF. VOL.)

II. PART

POWER LAW FIN. VOL. EFFECTS

UNITARITY OF S-MATRIX

I. PART

3→3 SCATTERING AMPLITUDE IN FINITE VOLUME

IMAGINARY PARTS (INF. VOL.)

II. PART

POWER LAW FIN. VOL. EFFECTS

MM,Doring (2017)

Sadasivan, MM, Doring (in progress)

LATTICE QCD

Ab-initio numerical calculations of QCD Greens functions

- **in Euclidean space-time** → Osterwalder/Schrader(1973,1975)

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- in Euclidean space-time $\rightarrow$ `Osterwalder/Schrader(1973,1975)`
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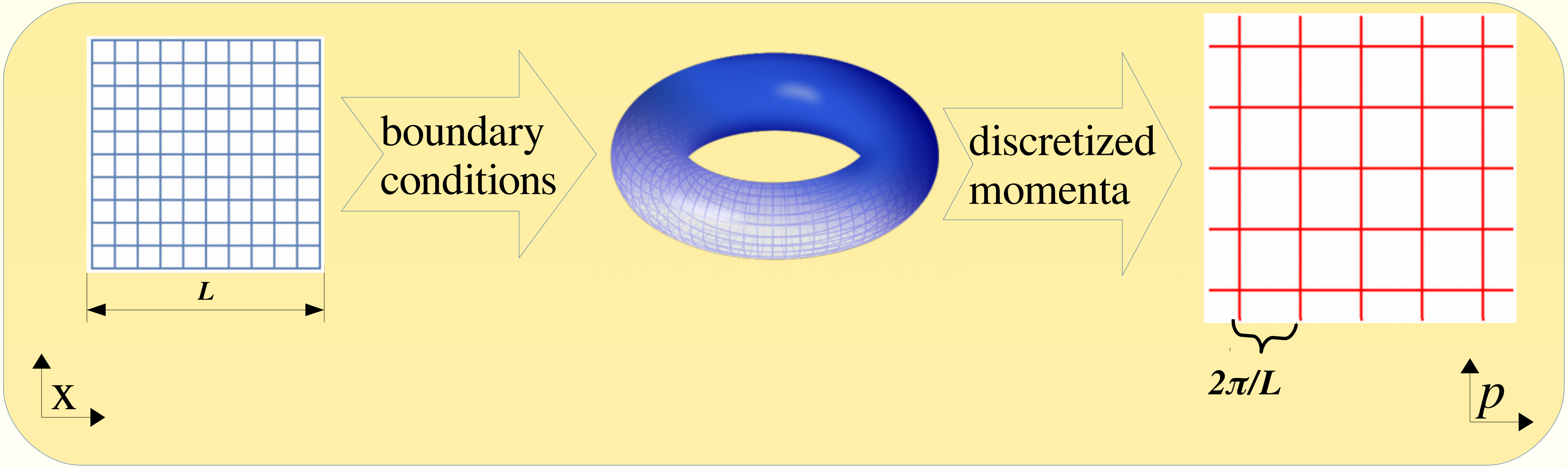
- **in finite volume**



→ momenta & spectra are discretized

FINITE VOLUME EFFECTS

2-body case

– well understood

Lüscher (1986)

– multi-channels, spin, ...

Gottlieb,Rummukainen,Feng,Li,Liu,
Doring,Briceno,Rusetsky,Bernard...

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One way of thinking:

Unitarity

$$T(E) = \frac{1}{K^{-1}(E) + i\Phi(E)}$$

→

Discrete Momenta

$$T^{FV}(E) = \frac{1}{K^{-1}(E) + \frac{2}{\sqrt{\pi}L} Z_{00}(E, L)}$$

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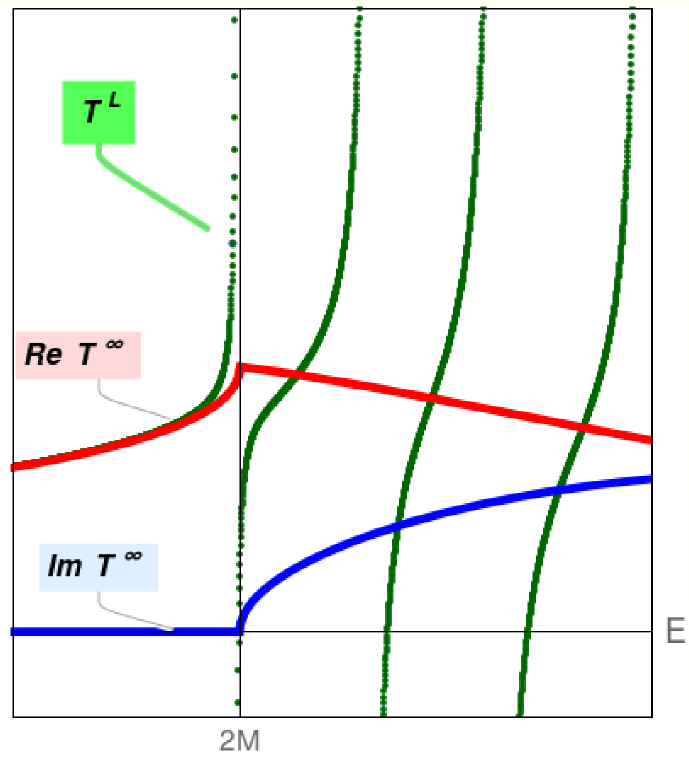
(pole-pos.) ↔ (energy eigenvalues)



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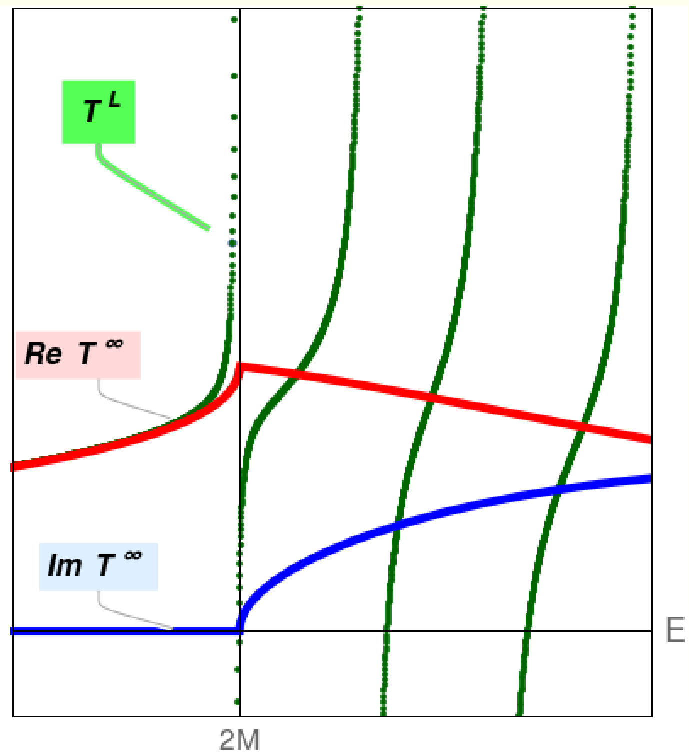
$$Det[K^{-1}(E^*) + \frac{2}{\sqrt{\pi}L} Z_{00}(E^*, L)] = 0$$



FINITE VOLUME EFFECTS

3-body case

– important formal developments

Sharpe, Rusetsky, Hansen, Polejaeva, Briceno, Davoudi, Guo, MM, Doring...

– pilot numerical investigations

Pang/Hammer/Rusetsky/Wu(2017)

MM/Doring(2017)

Hansen/Briceno/Sharpe(2018)

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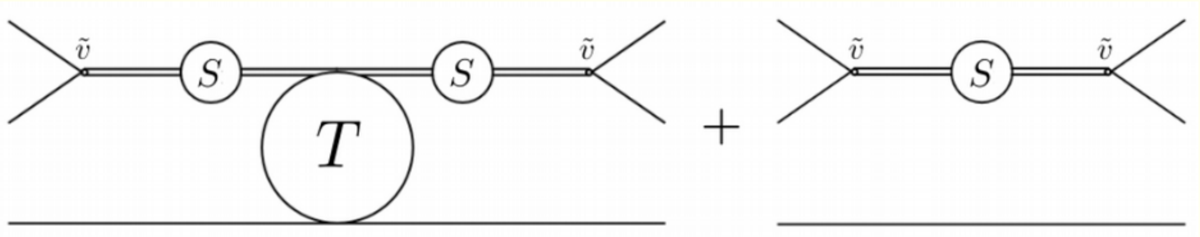
This work: **Discretize infinite volume scattering amplitude** MM/Doring(2017)



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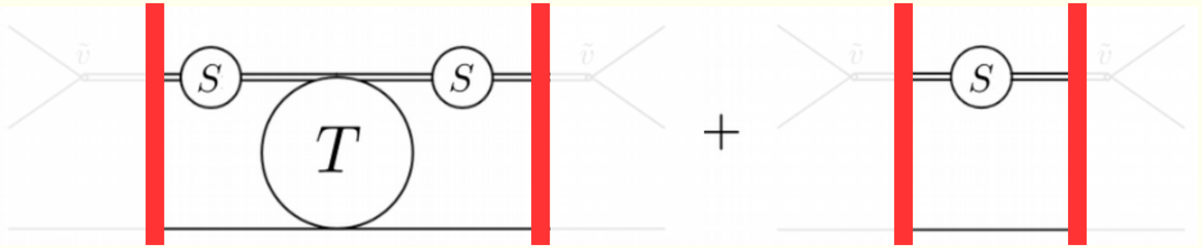

→ v **is cut-free**

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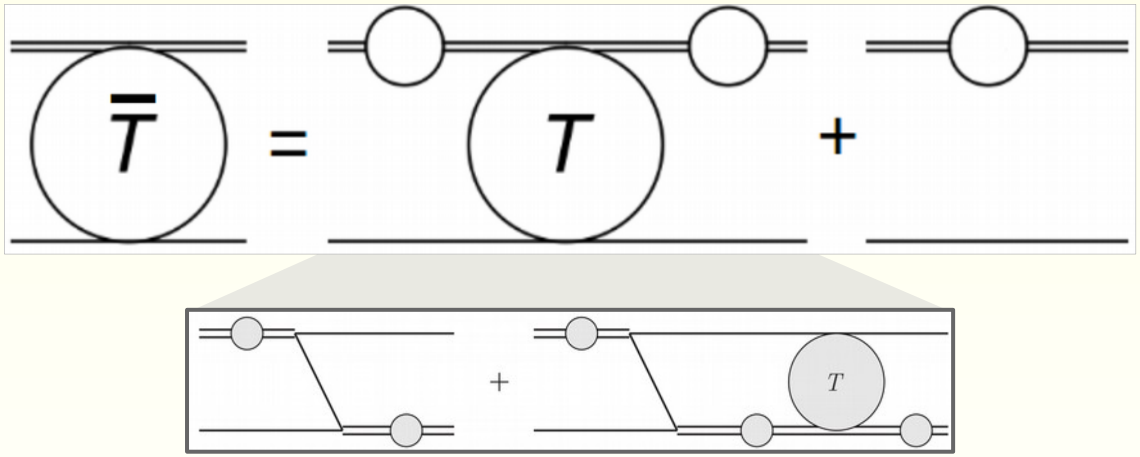

→ $\boldsymbol{v}$ **is cut-free**

FINITE VOLUME EFFECTS

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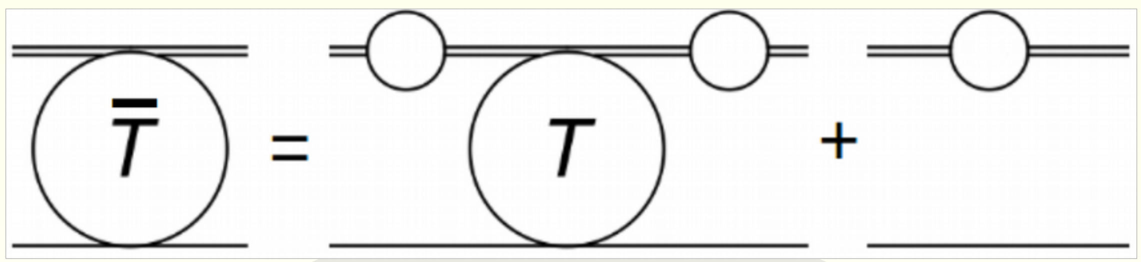
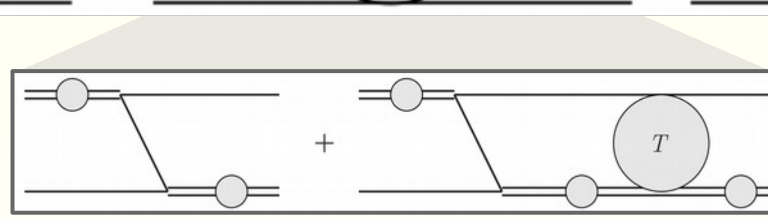




→ v **is cut-free**

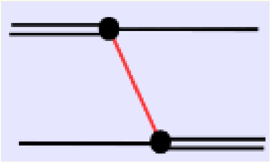
→ generic 3b-Quantization Condition

$$\mathrm{Det}\left[\mathbf{B(E)}+\tau(\mathbf{E})^{-1}\right]=\mathbf{0}$$

PROJECTION TO IRREPS

– High-dimensional problem

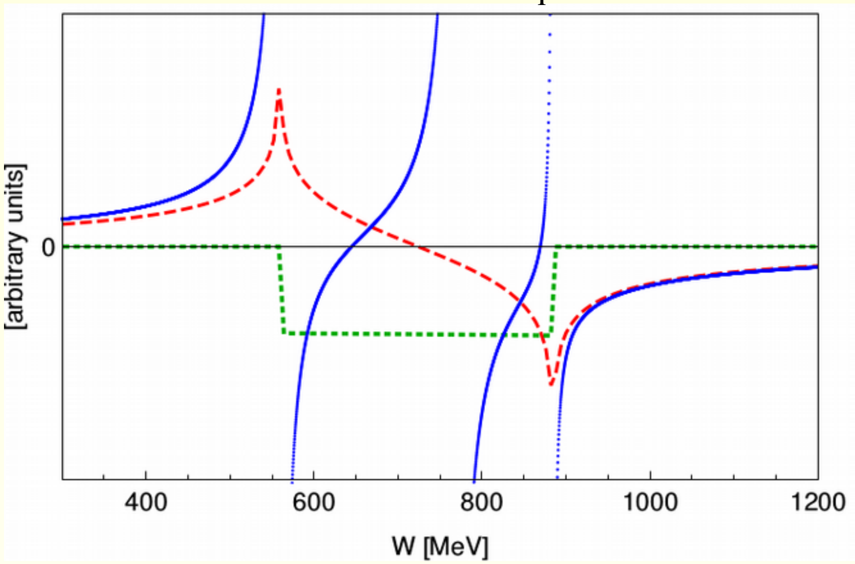
– B (OPE potential) is singular!



→ **Project to irreps of cubic group:**

$\{A_1|A_2|E|T_1|T_2\}$

S-wave infinite volume vs. A_1^+ finite volume



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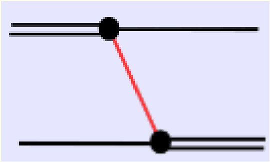

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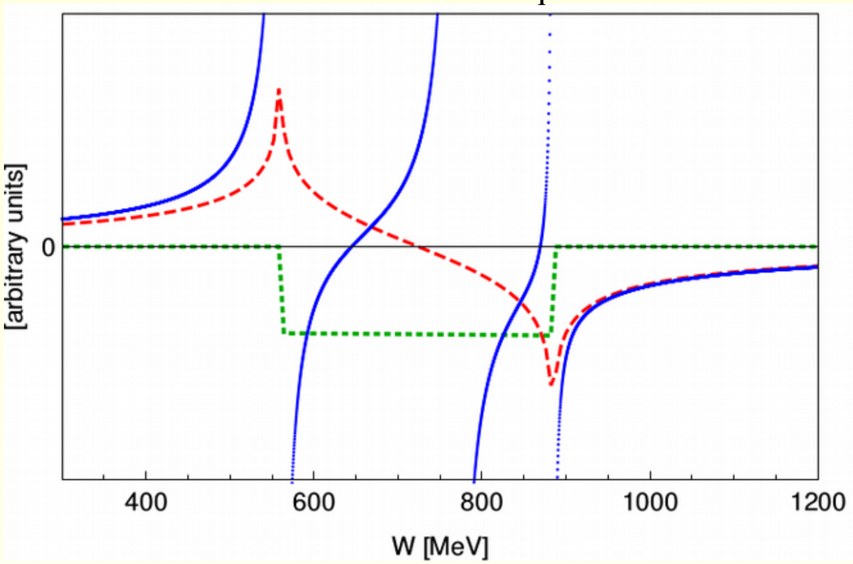


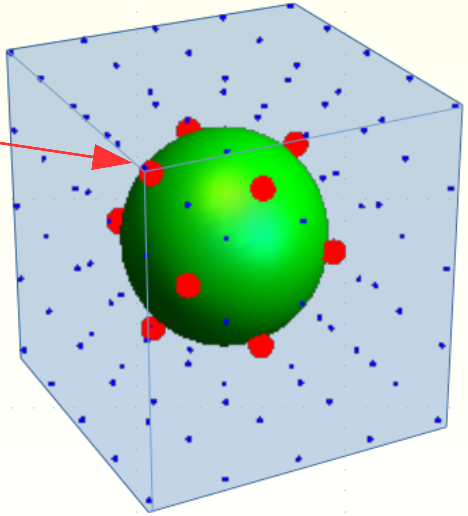
1) Separation of variables

– shells = sets of points related by $\boldsymbol{O_h}$

2) Find the ONB of functions on each shell

– $f^s(\hat{\mathbf{p}}_j) = \sqrt{4\pi}\sum_{\Gamma\alpha}\sum_u f_u^{\Gamma\alpha s}\chi_u^{\Gamma\alpha s}(\hat{\mathbf{p}}_j)$

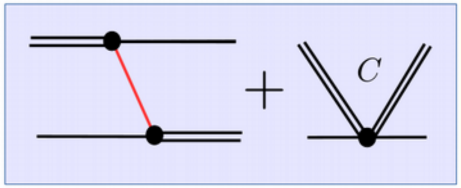
– *inf. vol. analog:* PWA



[Döring,Hammer,MM,Pang,Rusetsky,Wu(2018)]

QUANTIZATION CONDITION

$$\mathrm{Det}\left(\mathbf{B}^{\Gamma \mathbf{ss}'}_{\mathbf{uu}'}(\mathbf{W}^2)+\frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_{\mathbf{s}}(\mathbf{W}^2)^{-1}\delta_{\mathbf{ss}'}\delta_{\mathbf{uu}'}\right)=\mathbf{0}$$



W – total energy

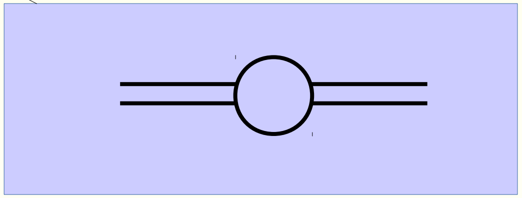
s/s' - shell index

u/u' - basis index

ϑ – multiplicity

L – lattice volume

Es – 1p. energy



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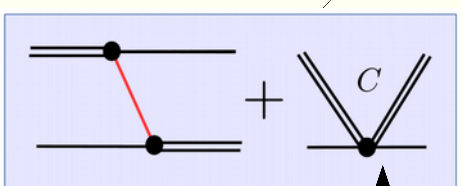

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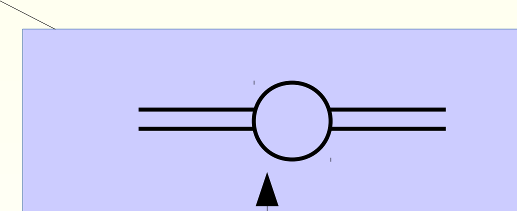

Fix to 3→3 data

Fix to 2→2 data:

$T_{22} = v\ \tau\ v$

QUANTIZATION CONDITION

$$\mathrm{Det}\left(\mathbf{B}^{\Gamma\mathbf{s}\mathbf{s}'}_{\mathbf{u}\mathbf{u}'}(\mathbf{W}^2) + \frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_\mathbf{s}(\mathbf{W}^2)^{-1}\delta_{\mathbf{s}\mathbf{s}'}\delta_{\mathbf{u}\mathbf{u}'}\right) = \mathbf{0}$$

- **3 particles in finite volume:** ***m=138 MeV, L=3 fm***

QUANTIZATION CONDITION

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- 3 particles in finite volume: ***m=138 MeV, L=3 fm***
- **one S-wave isobar → two unknowns:**
  - vertex(Isobar→2 stable particles)
  - subtraction constant (~mass)

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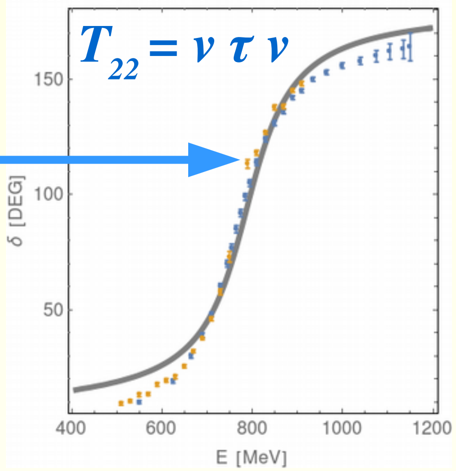

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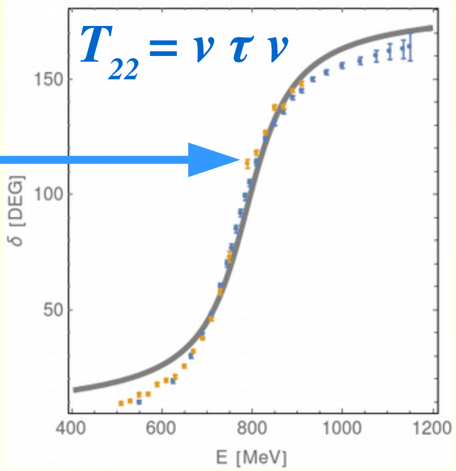
- **Project to *Γ = A¹⁺***
  - **→ prediction of 3body energy-eigenlevels**



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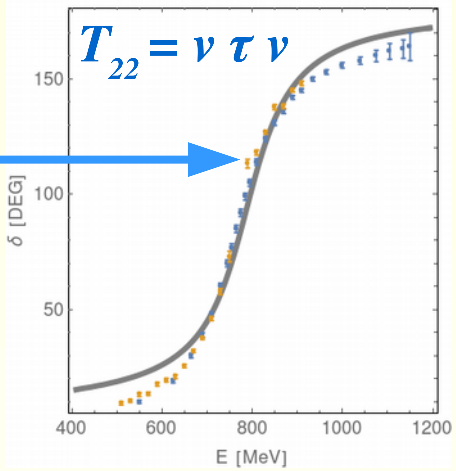
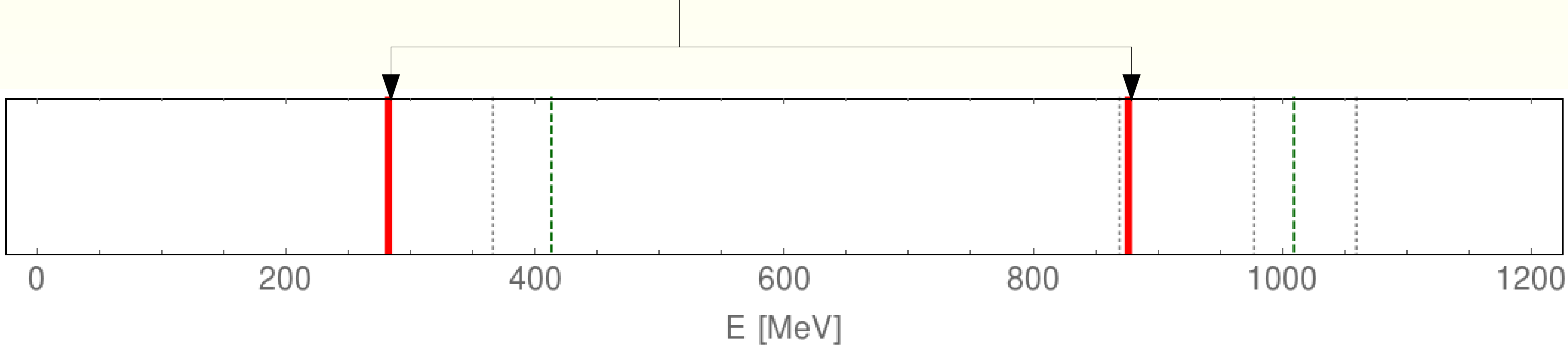


- **Project to $\Gamma = A^{1+}$**

  → prediction of 3body energy-eigenlevels



QUANTIZATION CONDITION

$$\mathrm{Det}\left(\mathbf{B}^{\Gamma\mathbf{ss'}}_{\mathbf{uu'}}(\mathbf{W^2})+\frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_{\mathbf{s}}(\mathbf{W^2})^{-\mathbf{1}}\delta_{\mathbf{ss'}}\delta_{\mathbf{uu'}}\right)=\mathbf{0}$$

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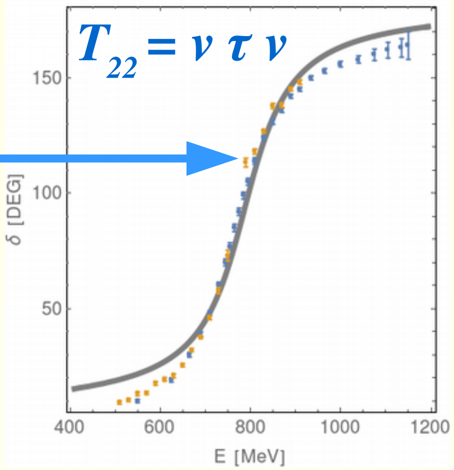
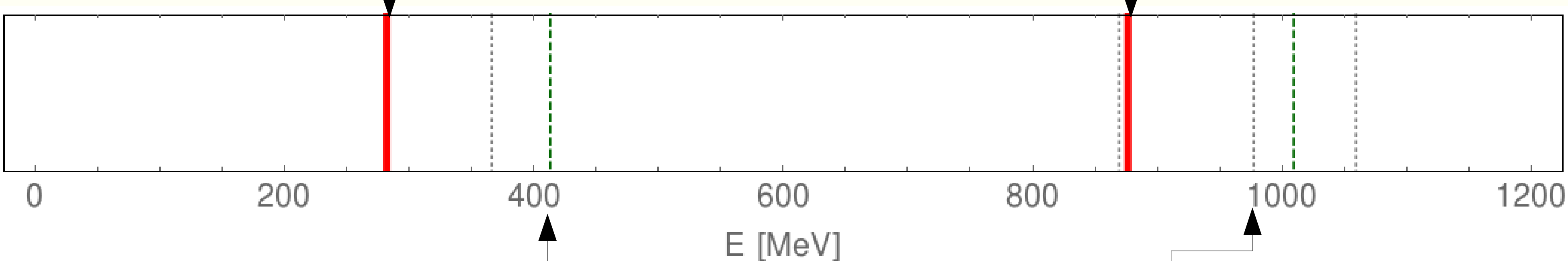
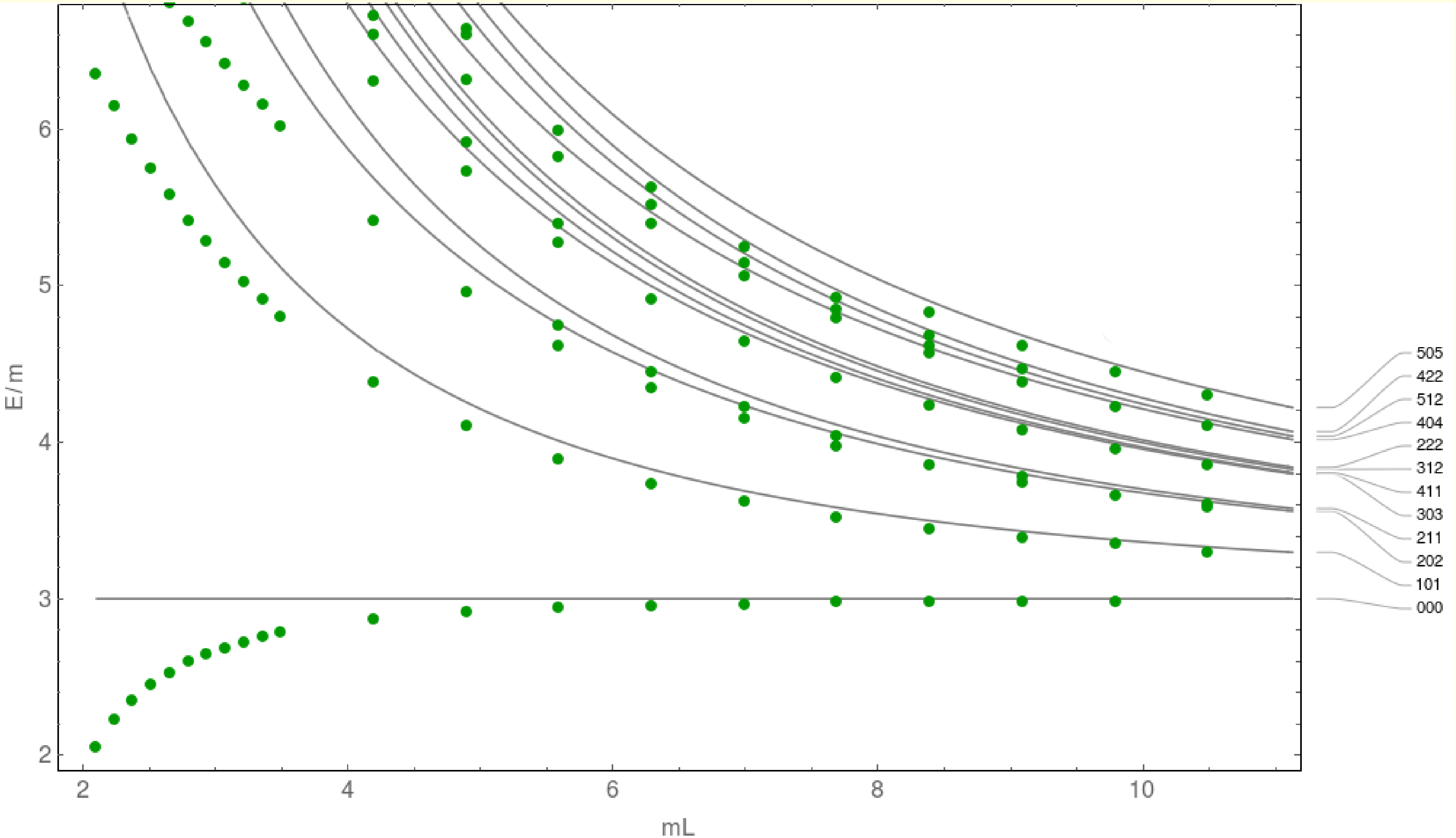


- **Project to** ***Γ = A***

→ prediction of 3body energy-eigenlevels



unphysical lvls cancel out (exact proof available)



E/m
mL
2
3
4
5
6
2
4
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8
10
505
422
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222
312
411
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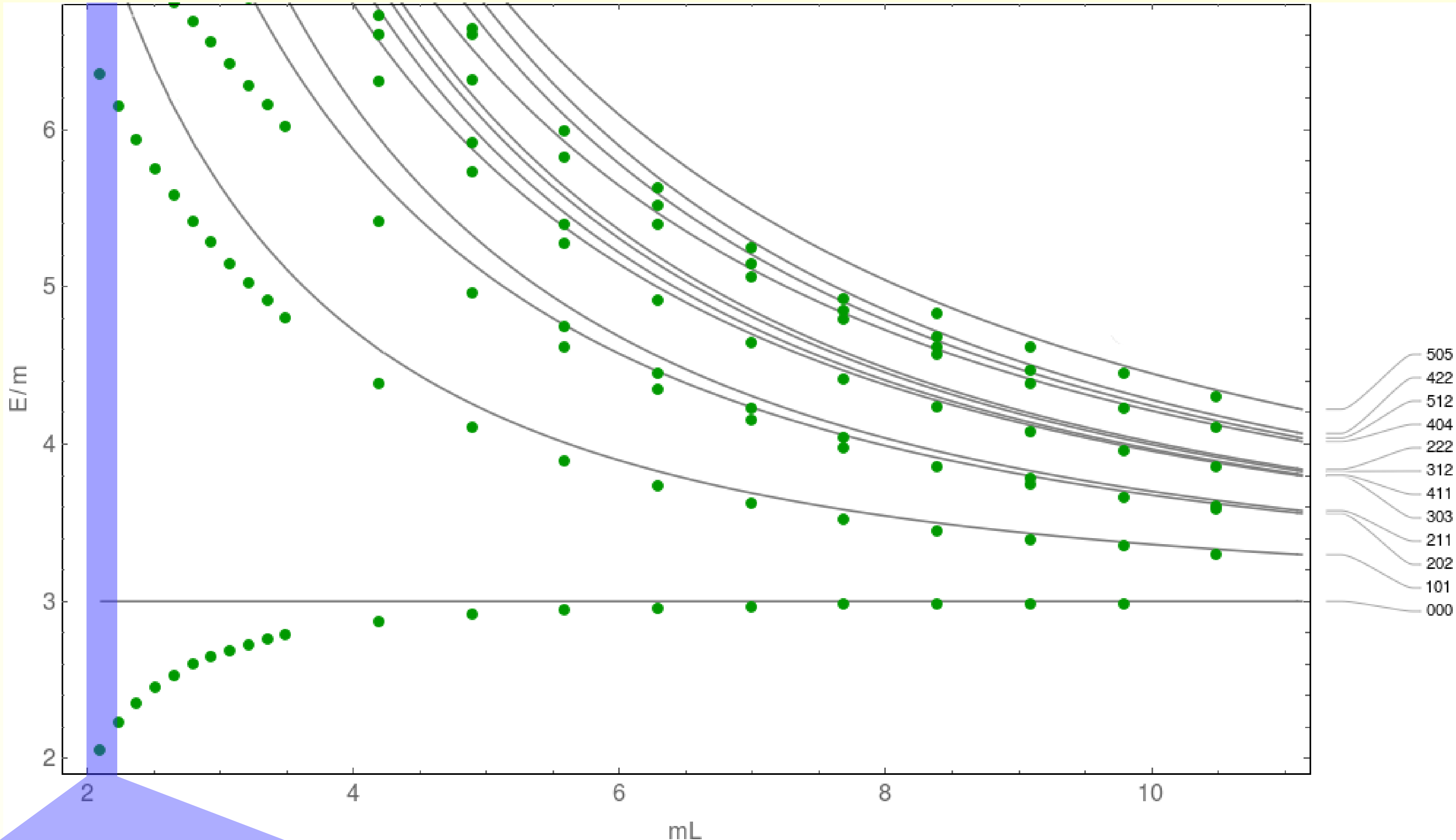
211
202
101
000


E / m
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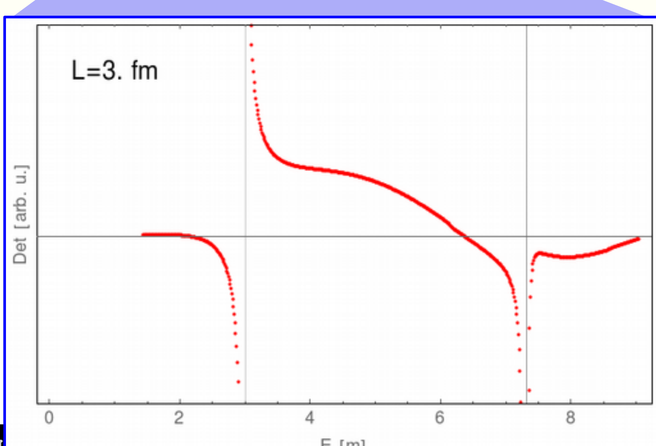
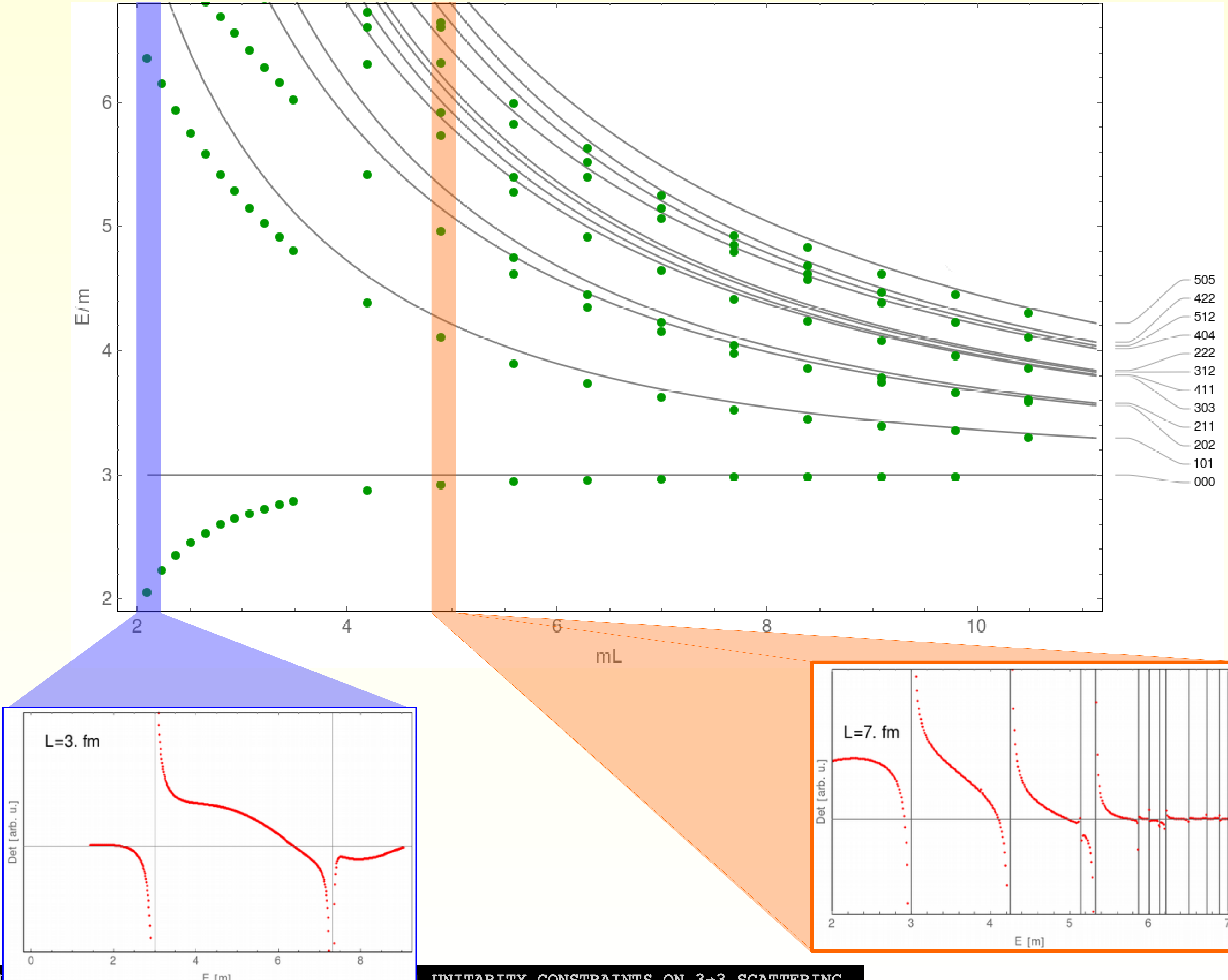

L=3. fm
Det [arb. u.]
E [m]


E / m
mL
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422
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404
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312
411
303
211
202
101
000
L=3. fm
Det [arb. u.]
E [m]
L=7. fm
Det [arb. u.]
E [m]

SUMMARY/OUTLOOK

3-body scattering amplitude from 2&3 body Unitarity

– interaction kernel = one-particle-exchange

– single approximation: number of isobars

– flexible parametrization: real contributions can be added to the kernel

3-body Quantization Condition in fin. vol. derived

$$\mathrm{Det}\left(\mathbf{B}^{\Gamma\mathbf{ss}'}_{\mathbf{uu}'}(\mathbf{W^2})+\frac{\mathbf{2E_s\,L^3}}{\vartheta(\mathbf{s})}\tau_\mathbf{s}(\mathbf{W^2})^{-1}\delta_{\mathbf{ss}'}\delta_{\mathbf{uu}'}\right)=\mathbf{0}$$

– cancellations of unphysical poles revealed analytically

– projection to irreps done

– technical feasibility on a numerical example

TBD: multiple channels

TBD: inclusion of isospin & angular momentum

Three men walking II (Alberto Giacometti)

THANK YOU !