Stony Brook University | The State University of New York

Center for Frontiers in Nuclear Science

Opportunities and Challenges with Jets at LHC and beyond

Centrality fluctuation in heavy-ion collisions

Based on arXiv:1803.01812

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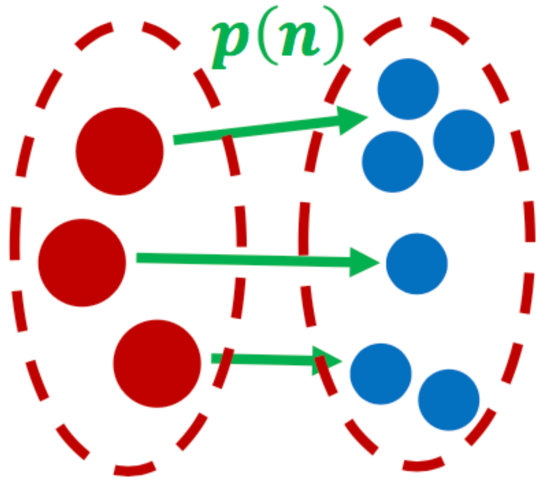
June 9-12, 2018

BROOKHAVEN NATIONAL LABORATORY

Centrality in A+A collisions

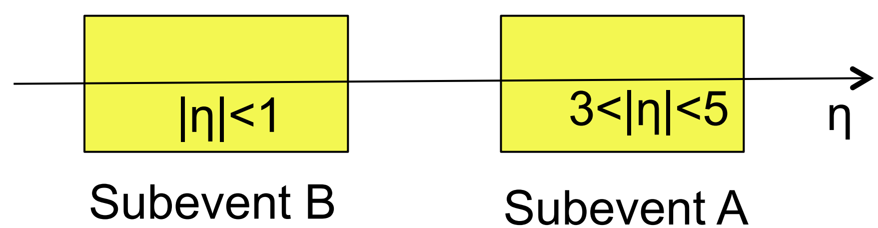
- Many variables to quantify centrality/volume.
  - At initial state: b, N_{part} , $xN_{part}+(1-x)\,N_{coll}$, N_{qp} , …
  - At final state: N_{ch}, E_T, $N_{neutron}$, …

Initial state sources



Final state particles

- Centrality def. in subevent A
- Measurement in subevent B



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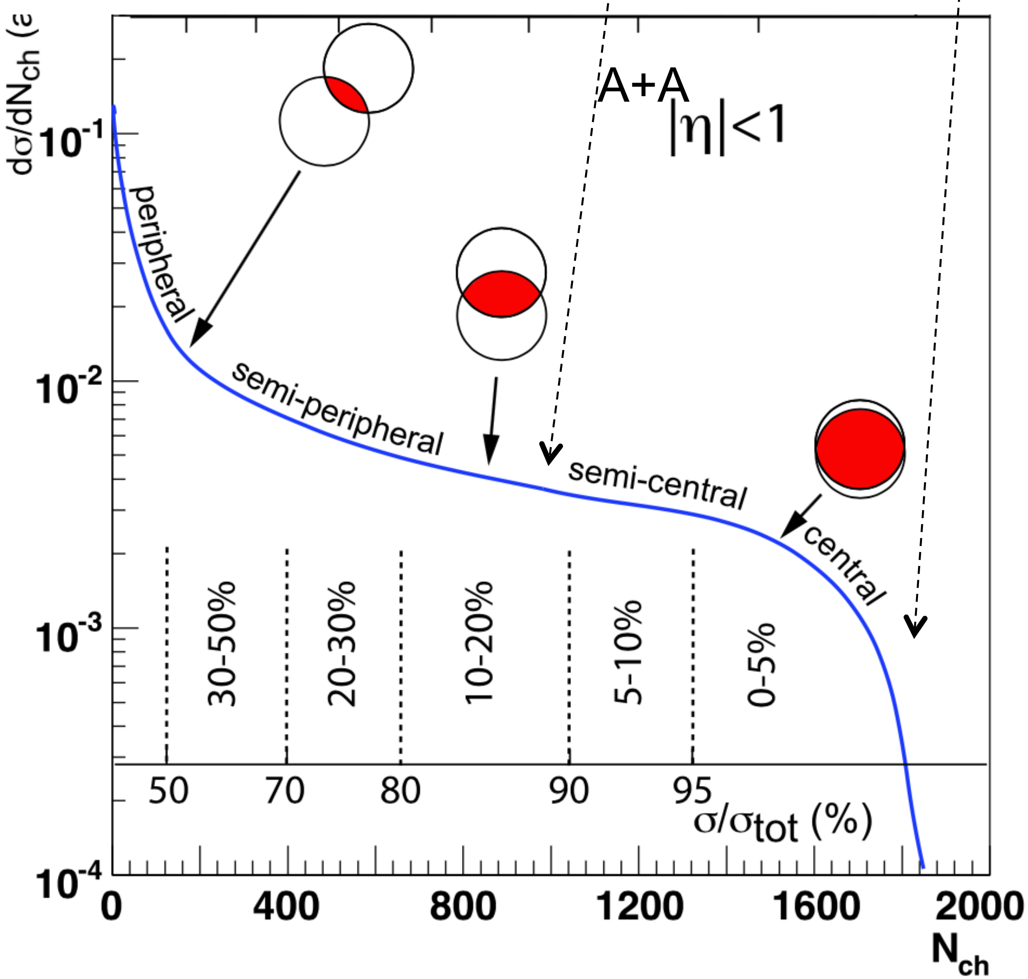
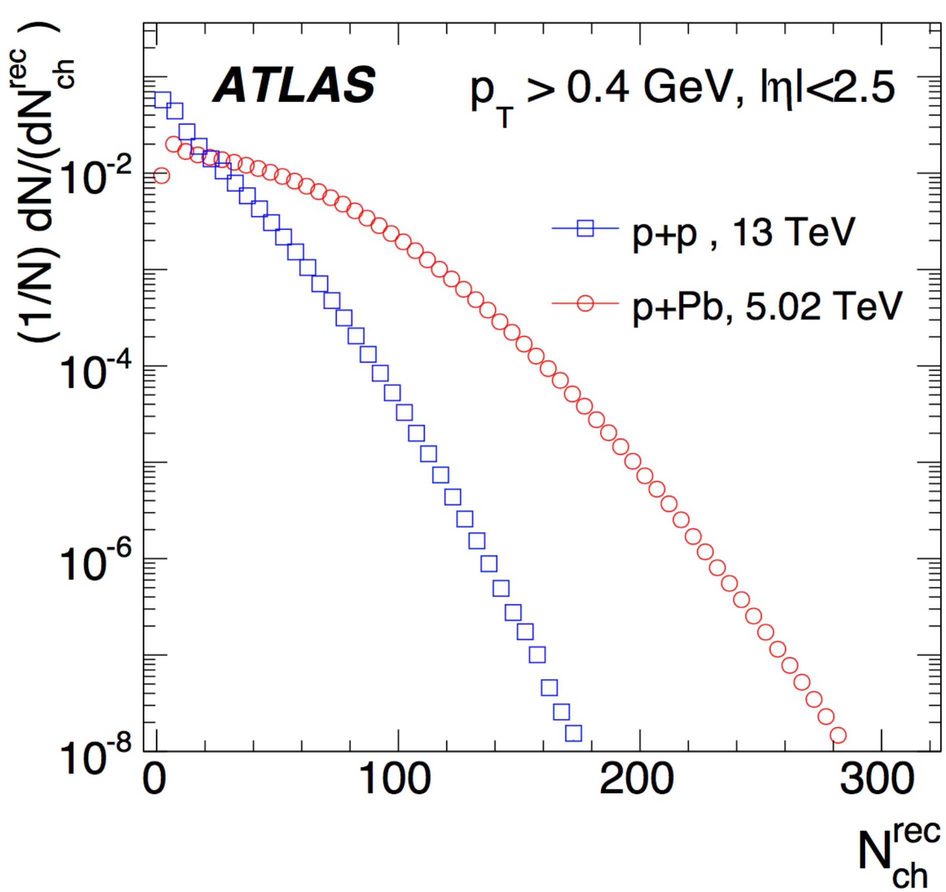
Main feature: Shoulder & Knee.

~Absent in pp, pPb

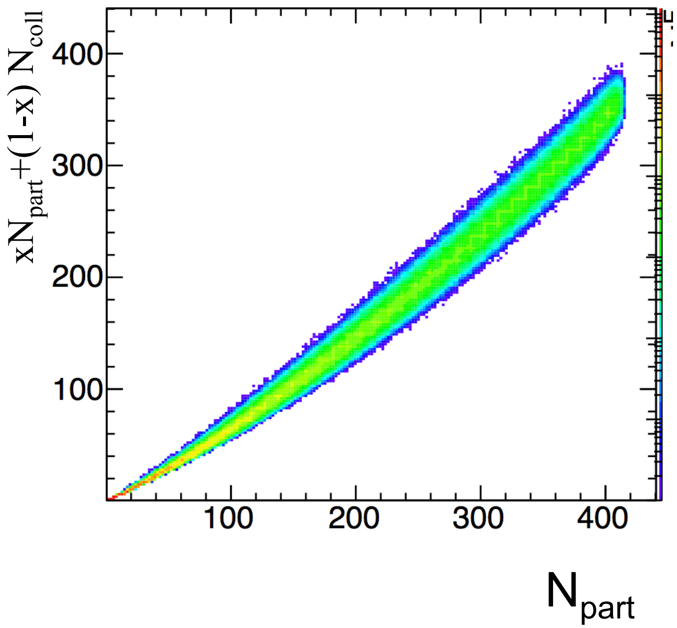




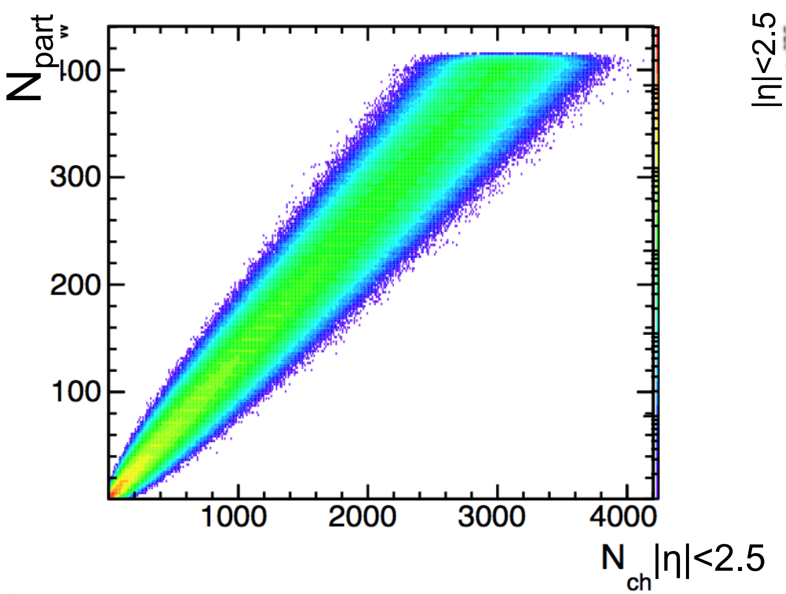
Centrality in A+A collisions

- Many variables to quantify centrality/volume.
  - At initial state: b, N_{part} , $xN_{part}+(1-x)\ N_{coll}$, N_{qp} , …
  - At final state: N_{ch}, E_T, $N_{neutron}$, …
- In absence of fluctuation→ all centrality measures are equivalent.
  - Because one-to-one mapping between different measures
  - In reality, fluc.. exist between different initial or final state variables.

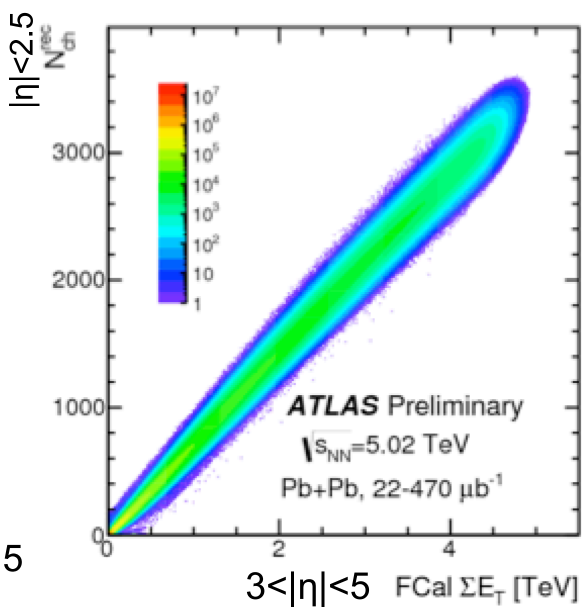
Between initial states variables



Between initial & final states



Between final states variables

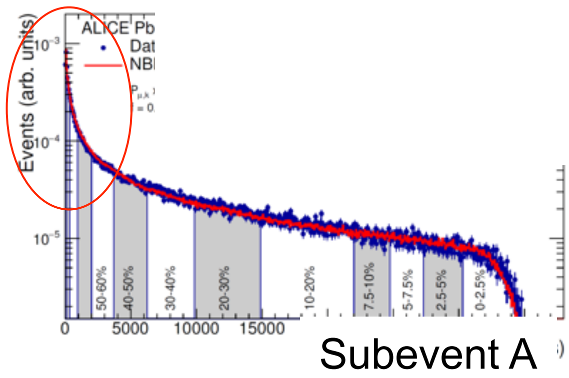


Centrality/volume fluctuation: Main uncertainty in many measurements

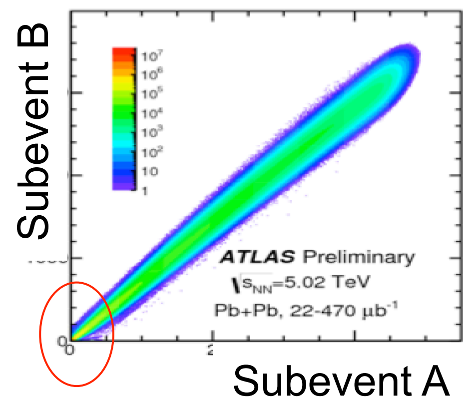
Example of centrality bias

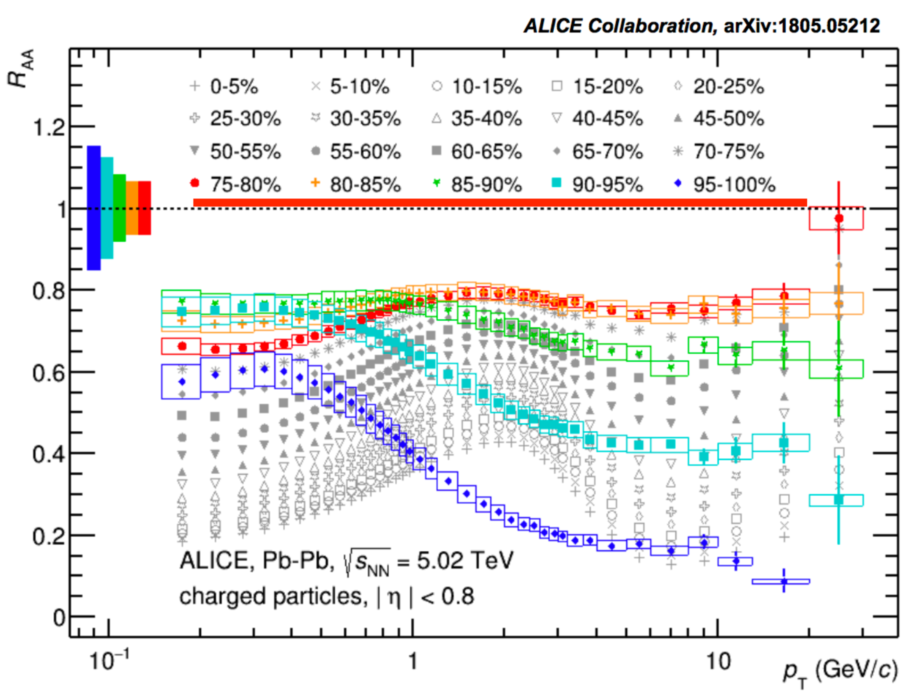
rising centrality distribution + large relative multiplicity fluctuation

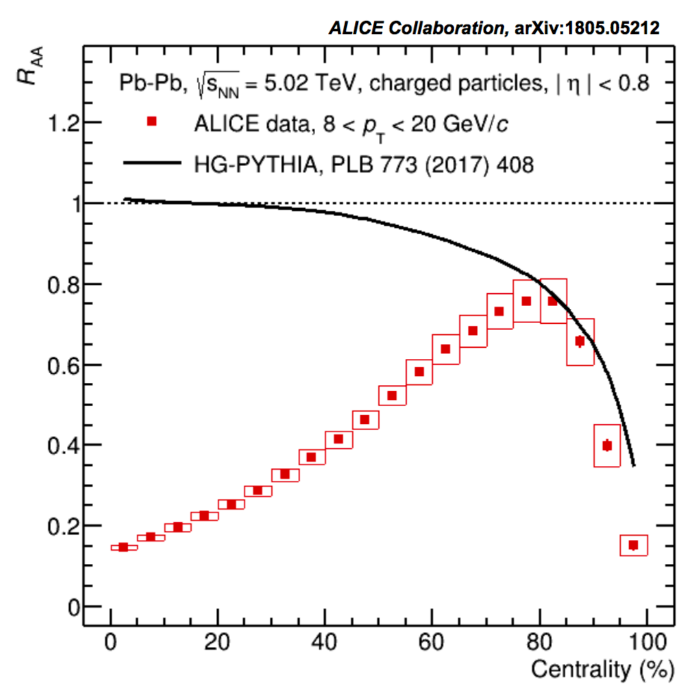
Subevent A



Subevent-B at fixed subevent A







How to quantify centrality fluctuation

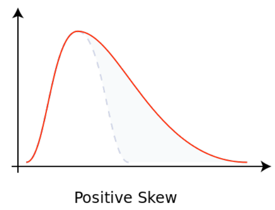
- Multiplicity cumulants:

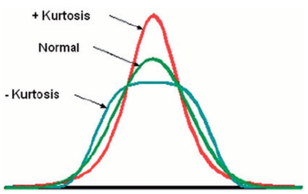
$$K_2 = \frac{\langle(\delta N)^2\rangle}{\bar{N}}, K_3 = \frac{\langle(\delta N)^3\rangle}{\bar{N}} \quad \delta N = N - \bar{N}$$

$$K_4 = \frac{\langle(\delta N)^4\rangle - 3\langle(\delta N)^2\rangle^2}{\bar{N}}$$

Quantifies the shape of p(N)

K_2 variance, K_3 Skewness, K_4 Kurtosis





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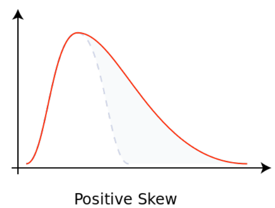

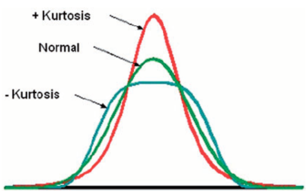

- Contains fluctuation for each source and fluctuation of number of sources

Cumulant in each source $\quad k_2 = \frac{\langle (\delta n)^2 \rangle}{\bar{n}}, \quad k_3 = \frac{\langle (\delta n)^3 \rangle}{\bar{n}}, \quad k_4 = \frac{\langle (\delta n)^4 \rangle - 3\langle (\delta n)^2 \rangle^2}{\bar{n}}, \quad \delta n = n - \bar{n}$

Cumulant for number of sources $\quad k_2^{\mathrm{v}} = \frac{\langle (\delta N_{\mathrm{s}})^2 \rangle}{\bar{N}_{\mathrm{s}}}, k_3^{\mathrm{v}} = \frac{\langle (\delta N_{\mathrm{s}})^3 \rangle}{\bar{N}_{\mathrm{s}}}, k_4^{\mathrm{v}} = \frac{\langle (\delta N_{\mathrm{s}})^4 \rangle - 3\langle (\delta N_{\mathrm{s}})^2 \rangle^2}{\bar{N}_{\mathrm{s}}}, \delta N_{\mathrm{s}} = N_{\mathrm{s}} - \bar{N}_{\mathrm{s}}$

How to quantify centrality fluctuation

- Multiplicity cumulants:

Quantifies the shape of p(N)

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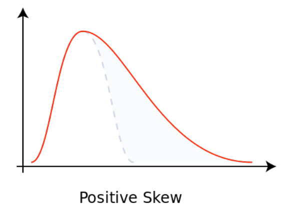

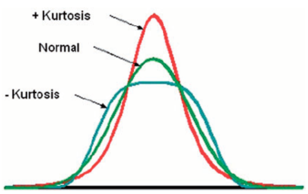

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- Practicality: centrality selection in subevent A, cumulant calc. in subevent B

$$\begin{aligned}
K_{2,B|A} &= k_{2,B} + \bar{n}_B k_{2,A}^{\mathrm{v}}, \\
K_{3,B|A} &= k_{3,B} + 3k_{2,B}\bar{n}_B k_{2,A}^{\mathrm{v}} + \bar{n}_B^2 k_{3,A}^{\mathrm{v}}, \\
K_{4,B|A} &= k_{4,B} + (4k_{3,B} + 3k_{2,B}^2)\bar{n}_B k_{2,A}^{\mathrm{v}} + 6k_{2,B}\bar{n}_B^2 k_{3,A}^{\mathrm{v}} + \bar{n}_B^3 k_{4,A}^{\mathrm{v}};
\end{aligned}$$

valid for independent source model

1205.4756, 1612.00702

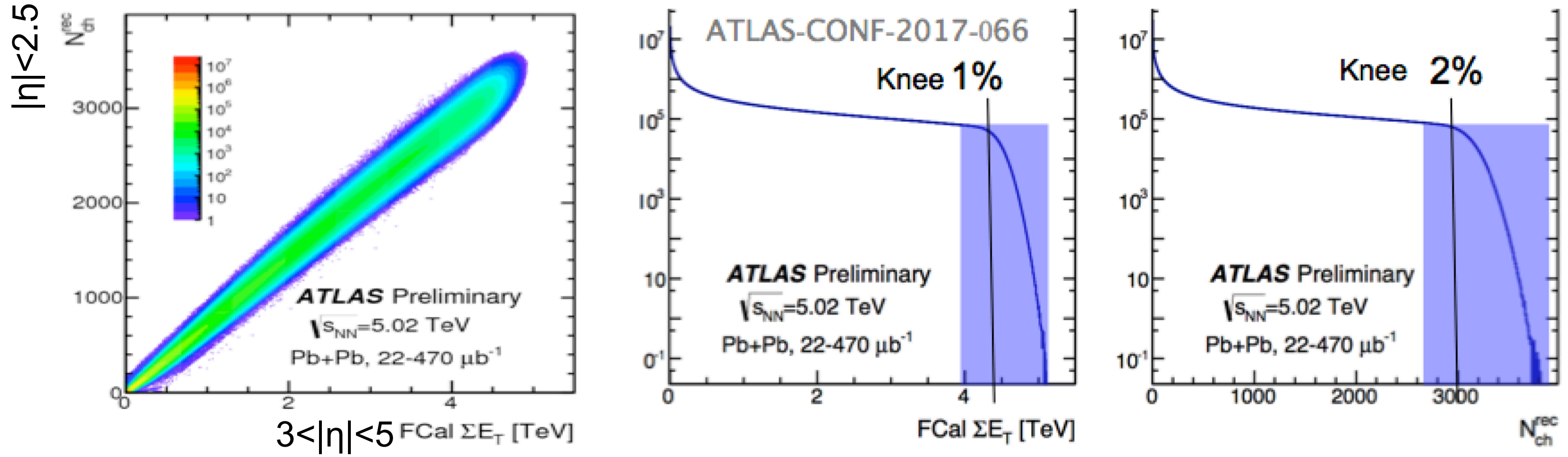
K_m is cumulant for total multiplicity in subevent B.
k_m is cumulant for one source in subevent B
k_m^{v} is cumulant for volume fluctuations in subevent A.

We want k_m^{v} !

Flow fluctuation to probe centrality fluctuation

The sources determining centrality also determines ε_n ➔

Centrality fluctuation also influences flow fluctuation.

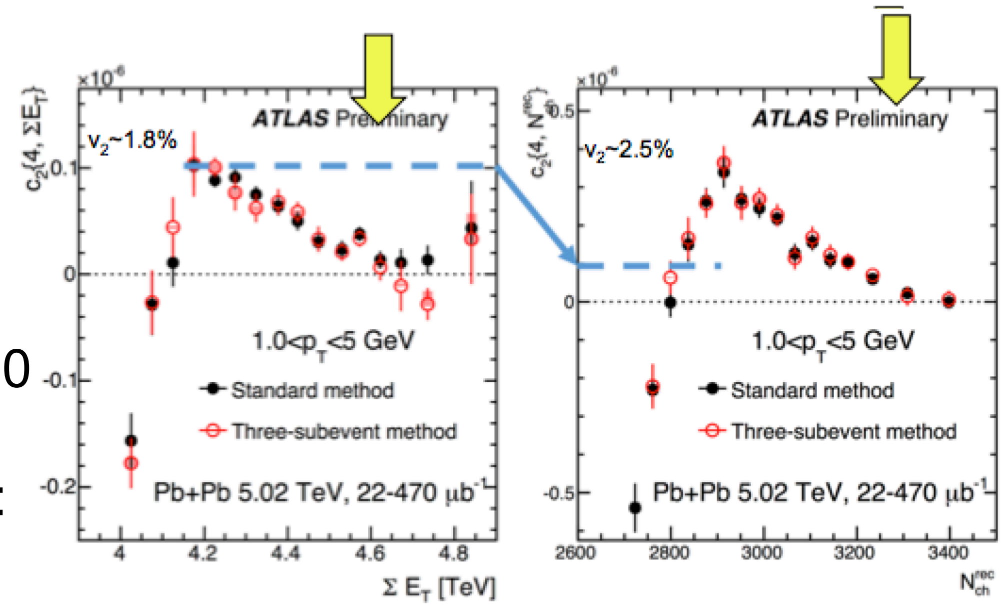


$$c_n\{4\} = \left\langle v_n^4 \right\rangle - 2\left\langle v_n^2 \right\rangle^2$$

ATLAS see $c_2\{4\}>0$ in ultra-central.

v_2 or ε_2 vary with centrality➔ mix events with different $p(\varepsilon_2)$ and combined $p(\varepsilon_2)$ has $c_2\{4\} >0$

The sign-change depends on centrality reso.: poor reso. lead to larger sign-change



Apply to all bulk observables that depend on centrality, e.g.$\langle p_T \rangle$, a_n.

Toy model for particle production: Glauber+NBD

- Independent source model: $N \equiv \sum_{i=1}^{N_s} n_i$, n_i from NBD

$$p(n;m,p) = \frac{(n+m-1)!}{(m-1)!n!} p^n (1-p)^m \quad p = \frac{\bar{n}}{\bar{n}+m}$$

p→0 Poisson p→1 Gamma

$$\hat{\sigma}^2 = \frac{\left\langle (n-\bar{n})^2 \right\rangle}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{m}$$

controls the amount of smearing

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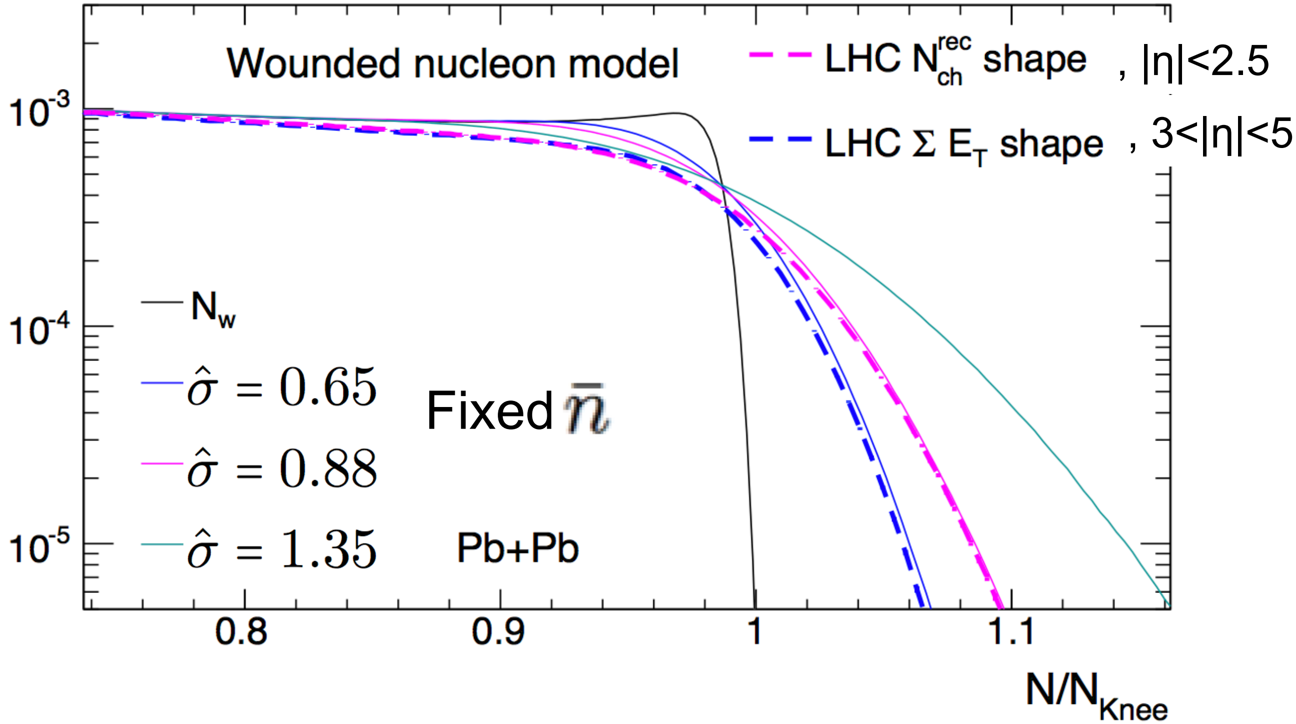
p→0 Poisson p→1 Gamma

Fixed $\bar{n}$ $$\hat{\sigma}^2 = \frac{\langle (n-\bar{n})^2 \rangle}{\bar{n}^2} = \frac{1}{\bar{n}} + \frac{1}{m}$$ controls the amount of smearing

- p(N) distribution fit to the ATLAS data.

Knee defined as n_{knee}= <n>*2A

$N_w = N_{part}$



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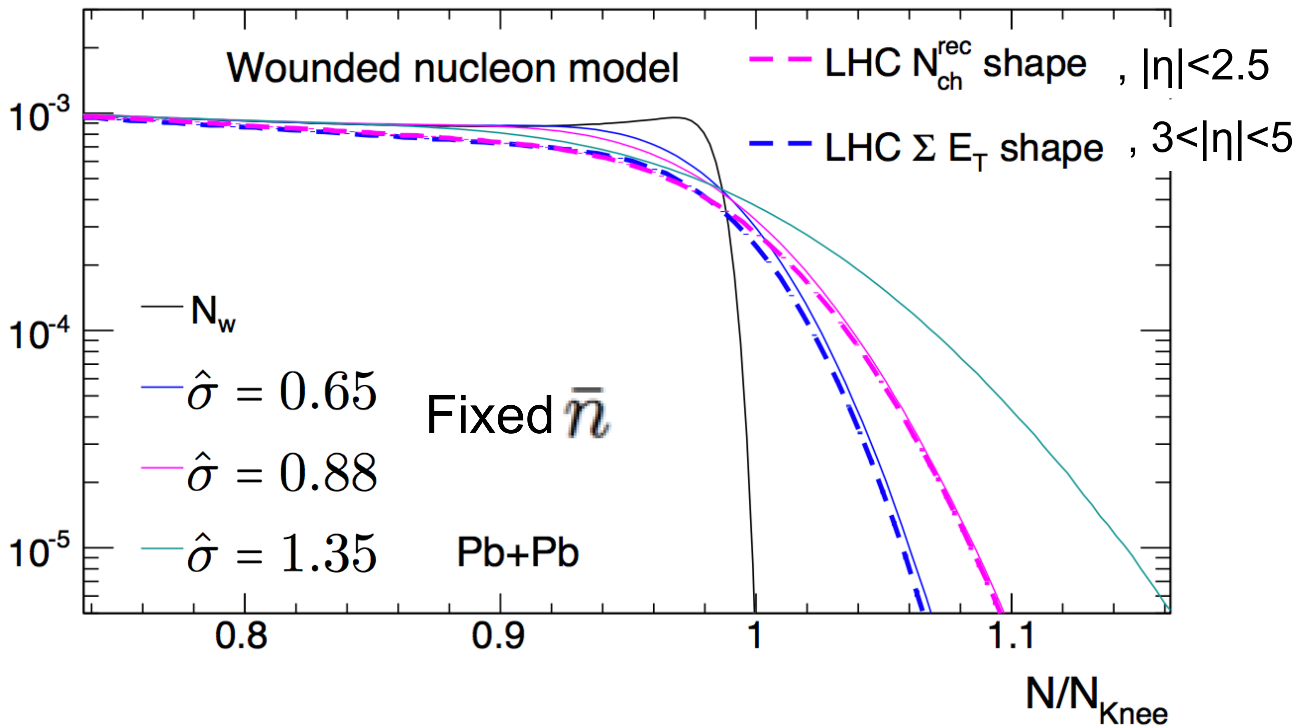
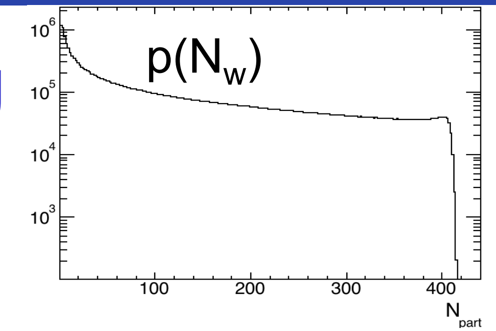


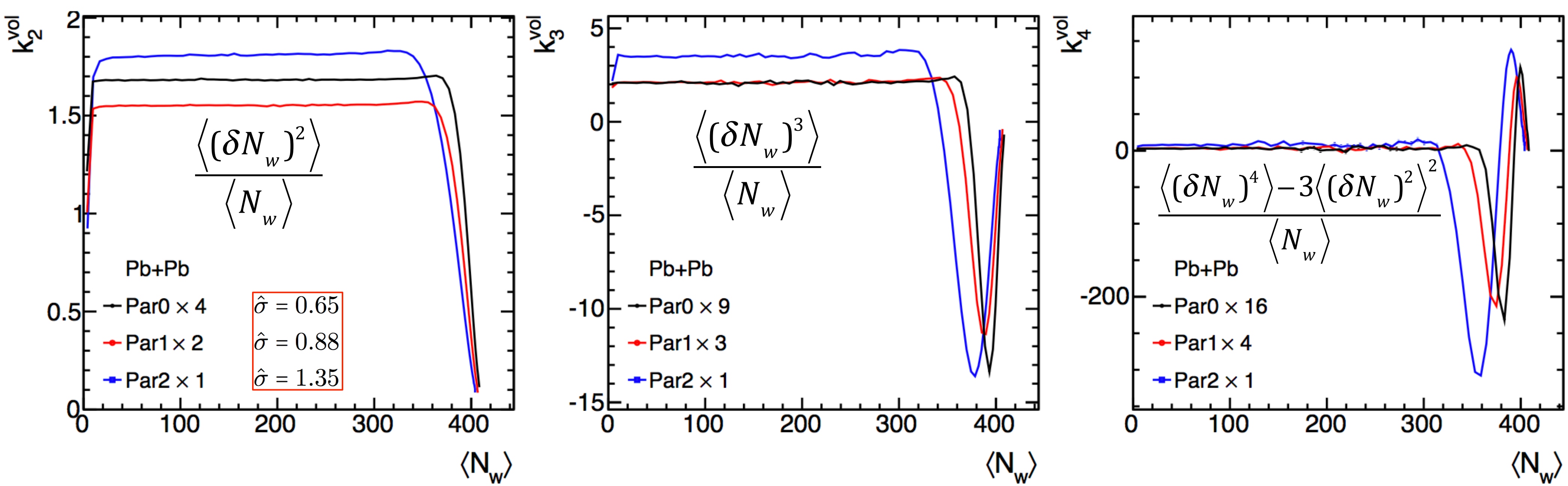
- Calculate multiplicity cumulants and eccentricity cumulants

Centrality fluctuation via multiplicity cumulants

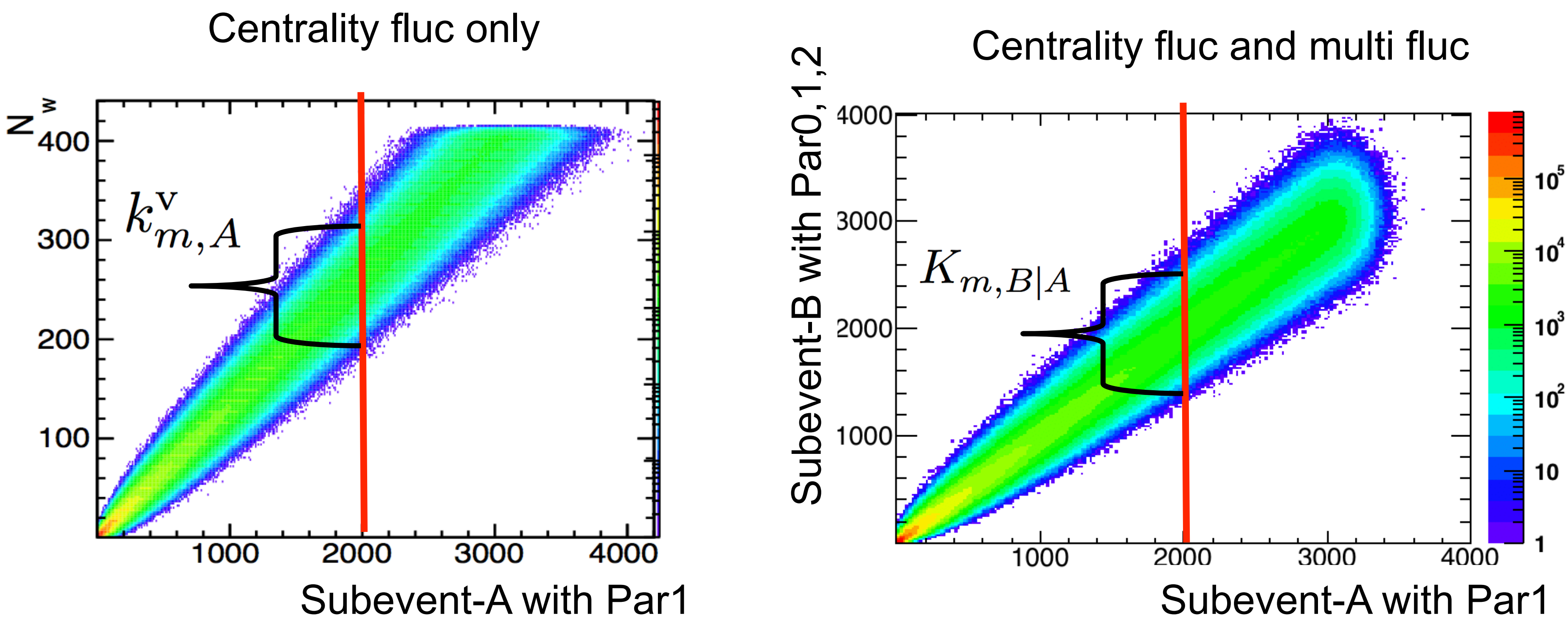
Centrality fluctuation



- Flat in $30<N_w<300$, where $p(N_w)$ is slowly varying
- Strong variation in UCC region $N_w>350$
  - Strong suppression for k=2. Large oscillation for k=3,4
  - All return to 0 at largest N_w, where centrality fluc. is constrained!
- More smearing (blue) leads to larger centrality fluctuation.



Include both multiplicity and centrality fluctuations



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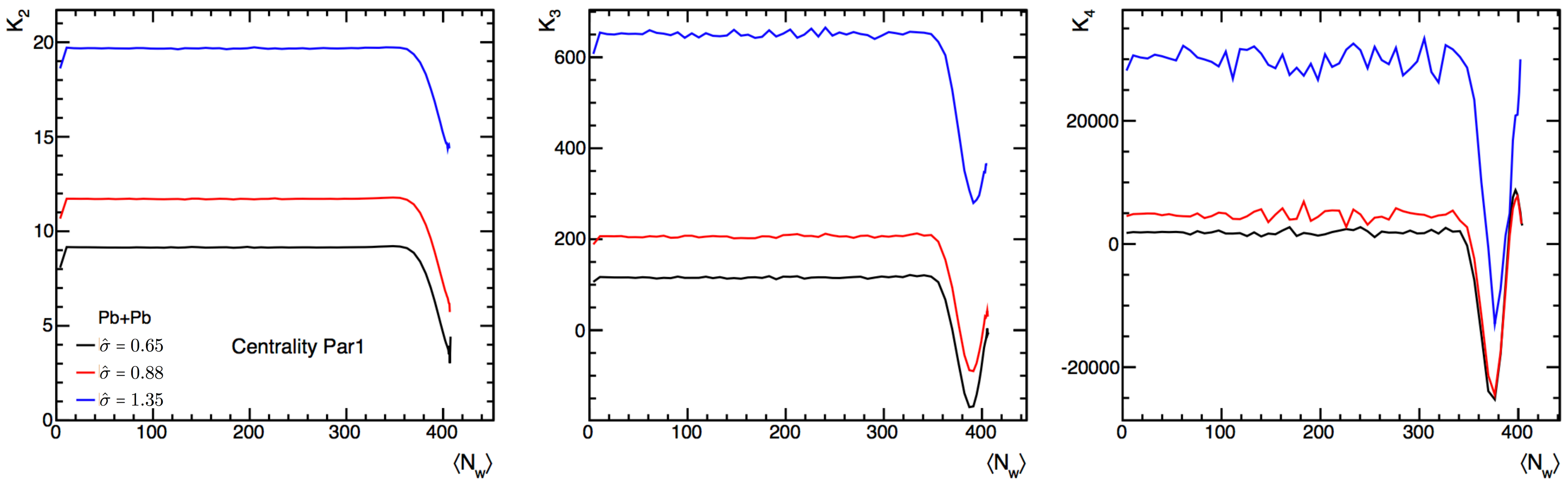
\end{aligned}
$$

Fixed $\bar{n}$

Fix centrality fluc, $k_{n,A}{}^{v}$ but increase multi fluc in subevent B, $k_{n,B}$

Include both multiplicity and centrality fluctuations



Observe: same centrality fluctuation + different offset

$$
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Fixed $\bar{n}$

Centrality fluctuation via flow cumulants

Flow observables via eccentricities

- Eccentricity ε_n has linear response with v_n, n=2,3,4 $\quad v_n = k_n \epsilon_n$

- Define the cumulants for ε_n e.g $\quad c_{n,\epsilon}\{4\} = \langle \epsilon_n^4 \rangle - 2 \langle \epsilon_n^2 \rangle^2$

- Then the cumulant ratios for v_n and ε_n should be equal

$$\frac{\langle v_n^4 \rangle - 2 \langle v_n^2 \rangle^2}{\langle v_n^2 \rangle^2} = \frac{\langle \epsilon_n^4 \rangle - 2 \langle \epsilon_n^2 \rangle^2}{\langle \epsilon_n^2 \rangle^2}$$

$$\boxed{\hat{c}_{n,\epsilon}\{2k\} \equiv \frac{c_{n,\epsilon}\{2k\}}{c_{n,\epsilon}\{2\}^k}}$$

- Similarly for symmetric cumulants

$$\boxed{\mathrm{nsc}_\epsilon(2,3) = \frac{\langle \epsilon_2^2 \epsilon_3^2 \rangle}{\langle \epsilon_2^2 \rangle \langle \epsilon_3^2 \rangle} - 1}$$

Related to the counterpart for flow:

$$\mathrm{nsc}(2,3) = \frac{\langle v_2^2 v_3^2 \rangle}{\langle v_2^2 \rangle \langle v_3^2 \rangle} - 1$$

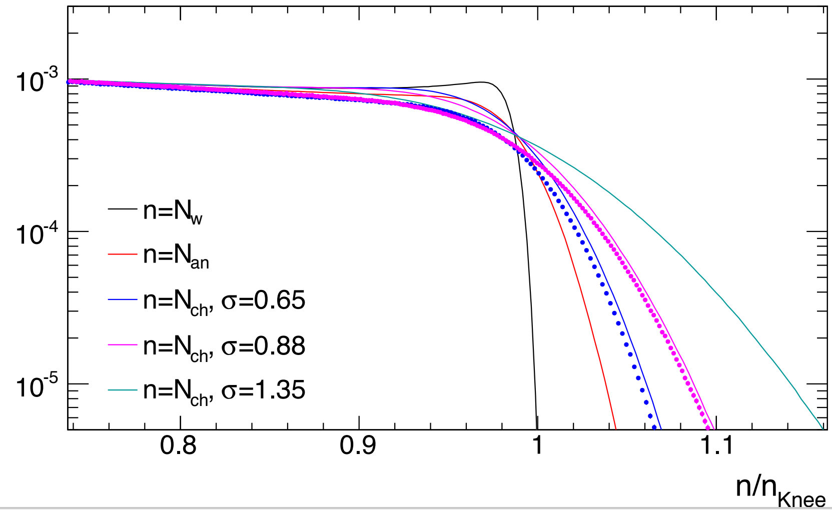
Centrality fluctuation effects on flow cumulants

- Calculate the cumulant ratios from ε_2.

Knee defined as n_{knee}= <n>*2A

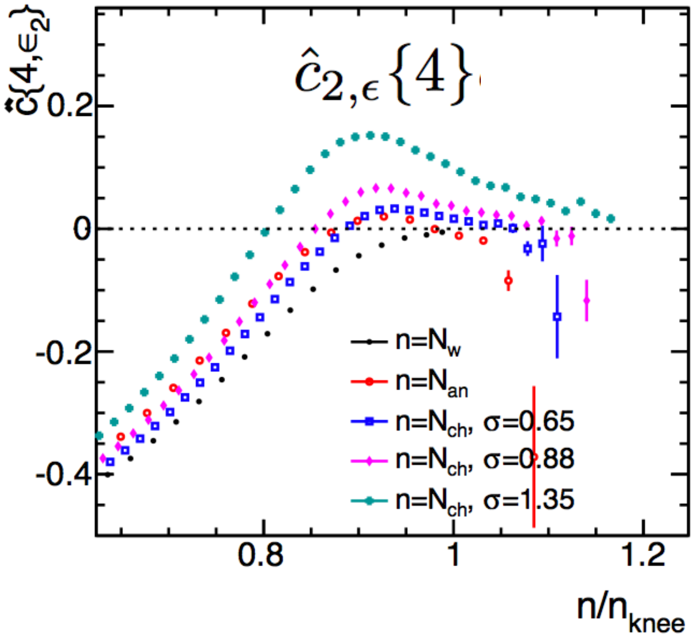
$$\hat{c}_{n,\epsilon}\{4\} = \frac{\langle \epsilon_n^4 \rangle - 2\langle \epsilon_n^2 \rangle^2}{\langle \epsilon_n^2 \rangle^2}$$

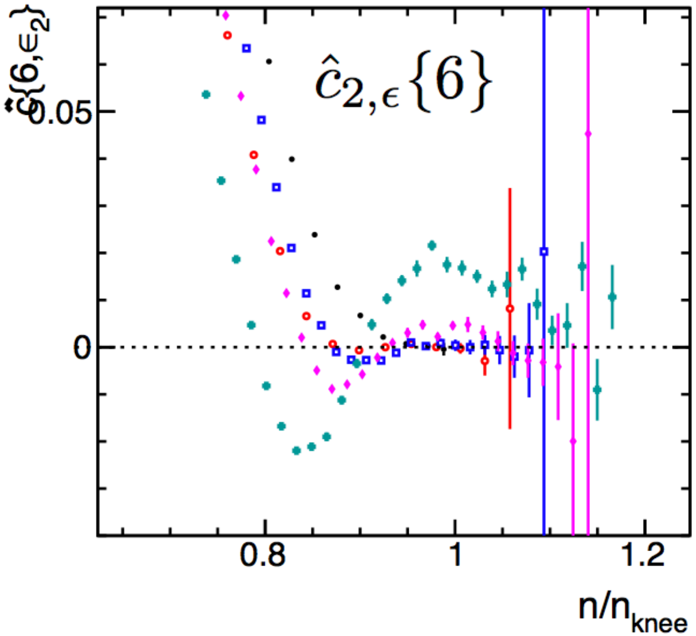
$$\hat{c}_{n,\epsilon}\{6\} = \frac{\langle \epsilon_n^6 \rangle - 9\langle \epsilon_n^4 \rangle \langle \epsilon_n^2 \rangle + 12\langle \epsilon_n^2 \rangle^3}{4\langle \epsilon_n^2 \rangle^3}$$



No sign-change when binned in N_w

sign-change increase for stronger smearing from particle production





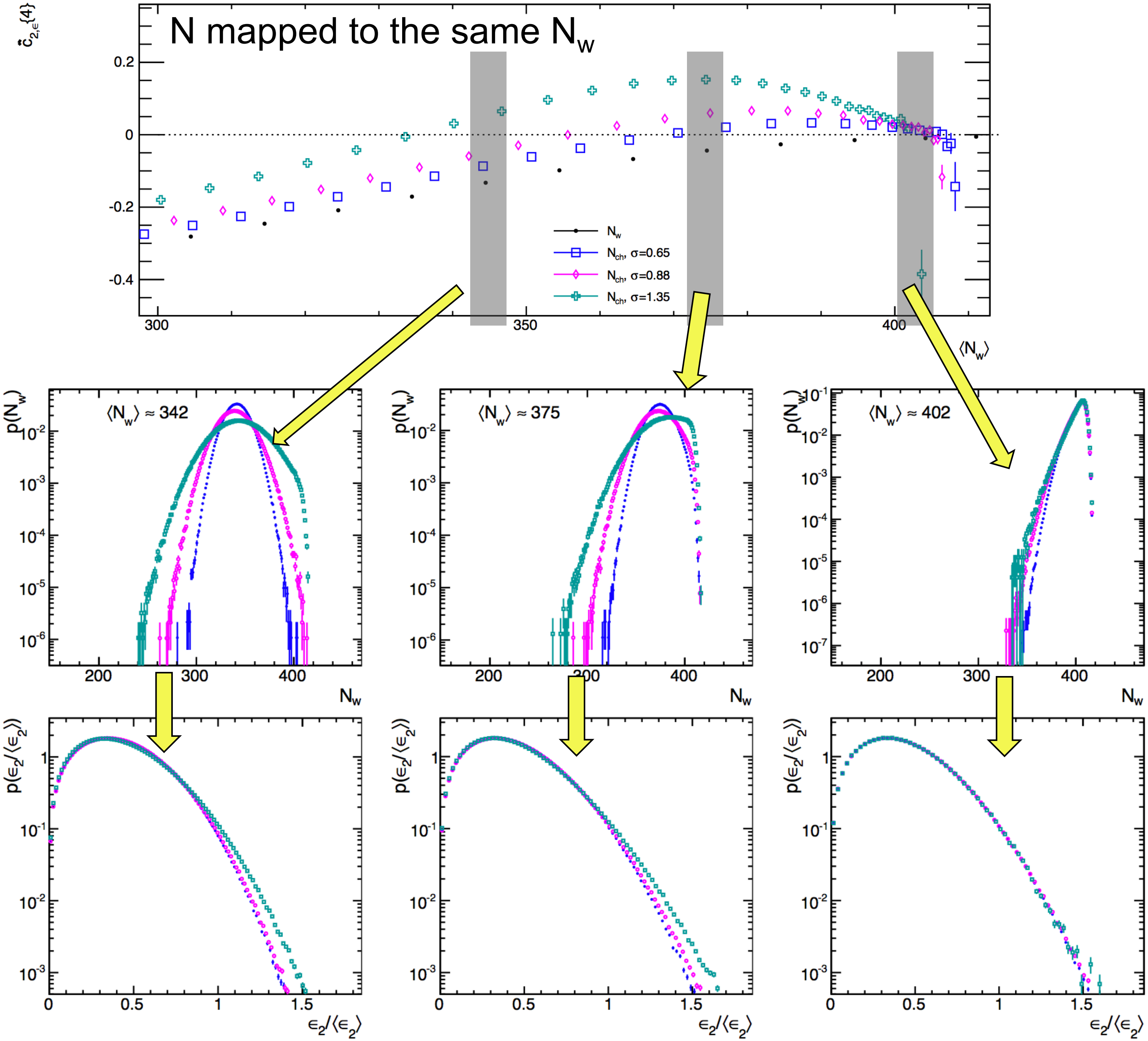
Smearing of $p(N_w)$ and $p(\varepsilon_2)$

Mixing events with different N_w changes the $p(\varepsilon_2)$, and lead to strong non-Gaussian tails

$\hat{c}_{2,\epsilon}\{4\}$

$p(N_{part})$

$p(\varepsilon_2/<\varepsilon_2>)$



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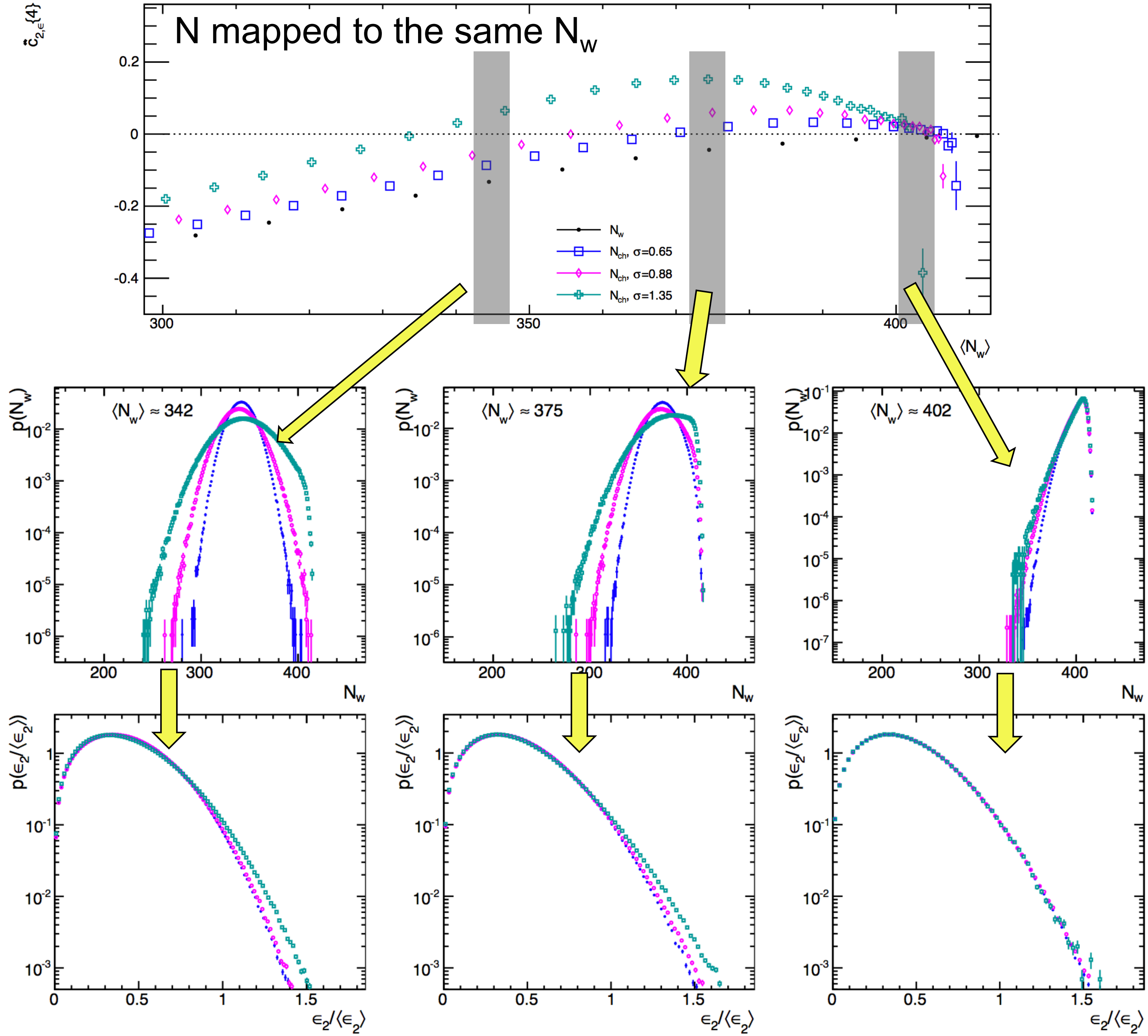

Flow cumulants are sensitive to centrality fluctuations!!

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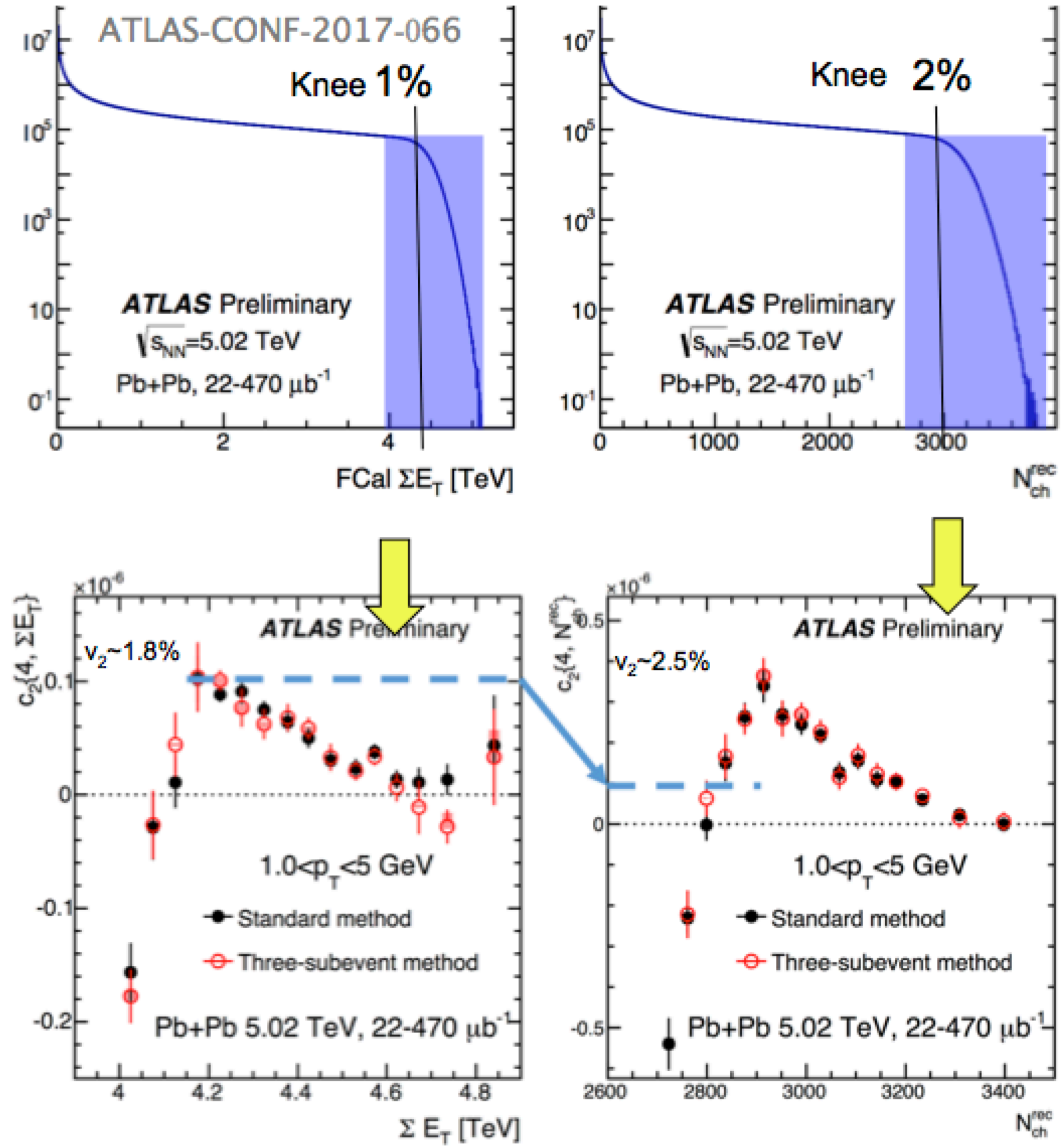


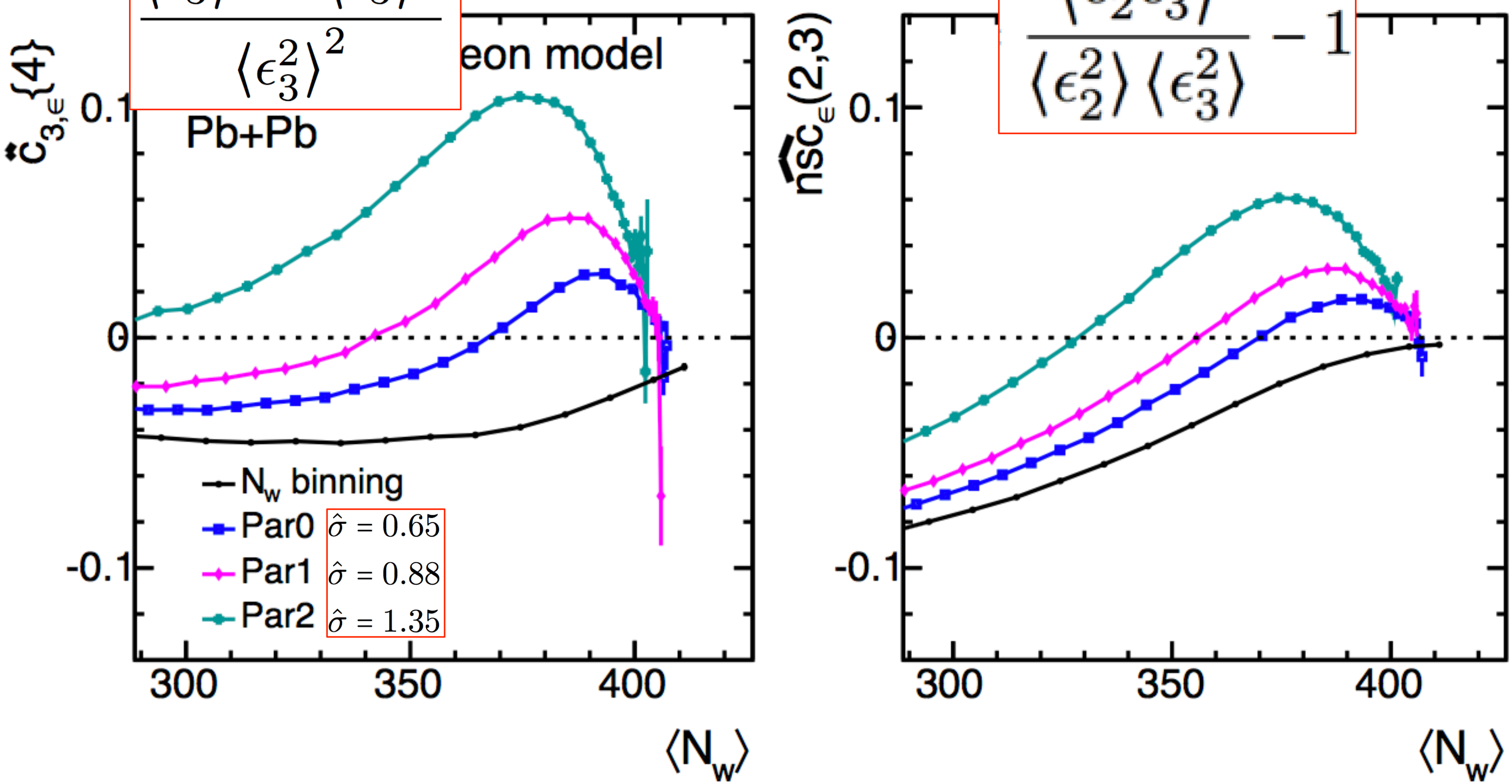
For any given model framework can tune centrality fluctuation to match observed sign-change

Influences of $p(N_w)$ on $p(\varepsilon_3)$ and $p(\varepsilon_2,\varepsilon_3)$

Sensitive to nature of sources and their fluctuations imposed by centrality selection.

$$\frac{\langle\epsilon_3^4\rangle - 2\langle\epsilon_3^2\rangle^2}{\langle\epsilon_3^2\rangle^2}$$

$$\frac{\langle\epsilon_2^2\epsilon_3^2\rangle}{\langle\epsilon_2^2\rangle\langle\epsilon_3^2\rangle} - 1$$



Centrality fluctuation and subevent cumulants

Centrality fluctuation in subevent cumulants

- Multiplicity cumulants are prone to statistical fluctuations, very dependent on efficiency and acceptance.

Consider NBD: $k_2(p) = \dfrac{1}{1-p} = 1 + \dfrac{\bar{n}}{m},\ k_3(p) = \dfrac{1+p}{(1-p)^2},\ k_4(p) = \dfrac{1}{1-p} + \dfrac{6p}{(1-p)^3}$

Efficiency/acceptance reduces p values➔ k_m=1 for p=0, Poisson limit

- Avoid statistical effects by extending the multiplicity cumulants to subevents, each from independent phase space: Charge, PID, or η.

Subevent works because centrality fluctuation is global effect !

Subevent cumulants for p(N) and p(n)

denote subevent as a, b, c…, each from a unique η.
Use normalized cumulant notation:

Total:

For 2-particle correlation: $$C_{2,ab} = \frac{\langle \delta N_a \delta N_a \rangle}{\bar{N}_a \bar{N}_b}$$

For 3-particle correlation: $$C_{3,abc} = \frac{\langle \delta N_a \delta N_b \delta N_c \rangle}{\bar{N}_a \bar{N}_b \bar{N}_c}$$

For 4-particle correlation:

$$C_{4,abcd} = \frac{\langle \delta N_a \delta N_b \delta N_c \delta N_d \rangle - \langle \delta N_a \delta N_b \rangle \langle \delta N_c \delta N_d \rangle - \langle \delta N_a \delta N_c \rangle \langle \delta N_b \delta N_d \rangle - \langle \delta N_a \delta N_d \rangle \langle \delta N_b \delta N_c \rangle}{\bar{N}_a \bar{N}_b \bar{N}_c \bar{N}_d}$$

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Use normalized cumulant notation:

Total:

For each source:

For 2-particle correlation: $C_{2,ab} = \frac{\langle \delta N_a \delta N_a \rangle}{\bar{N}_a \bar{N}_b}$ $\quad c_{2,ab} = \frac{\langle \delta n_a \delta n_b \rangle}{\bar{n}_a \bar{n}_b}$

For 3-particle correlation: $C_{3,abc} = \frac{\langle \delta N_a \delta N_b \delta N_c \rangle}{\bar{N}_a \bar{N}_b \bar{N}_c}$ $\quad c_{3,abc} = \frac{\langle \delta n_a \delta n_b \delta n_c \rangle}{\bar{n}_a \bar{n}_b \bar{n}_c}$

For 4-particle correlation:

$$C_{4,abcd} = \frac{\langle \delta N_a \delta N_b \delta N_c \delta N_d \rangle - \langle \delta N_a \delta N_b \rangle \langle \delta N_c \delta N_d \rangle - \langle \delta N_a \delta N_c \rangle \langle \delta N_b \delta N_d \rangle - \langle \delta N_a \delta N_d \rangle \langle \delta N_b \delta N_c \rangle}{\bar{N}_a \bar{N}_b \bar{N}_c \bar{N}_d}$$

$$c_{4,abcd} = \frac{\langle \delta n_a \delta n_b \delta n_c \delta n_d \rangle - \langle \delta n_a \delta n_b \rangle \langle \delta n_c \delta n_d \rangle - \langle \delta n_a \delta n_c \rangle \langle \delta n_b \delta n_d \rangle - \langle \delta n_a \delta n_d \rangle \langle \delta n_b \delta n_c \rangle}{\bar{n}_a \bar{n}_b \bar{n}_c \bar{n}_d}$$

Factorize CF and fluc. within each source

For two-particle correlation $$C_{2,ab} = \frac{1}{\langle N_{\rm part}\rangle}\frac{\langle \delta n_a \delta n_b\rangle}{\bar{n}_a \bar{n}_b} + \frac{\langle \delta N_{\rm part}^2\rangle}{\langle \delta N_{\rm part}\rangle^2} = \frac{c_{2,ab} + k_2^{\rm v}}{\langle N_{\rm part}\rangle}$$

For 3-particle correlation $$C_{3,abc} = \frac{\langle \delta N_a \delta N_b \delta N_c\rangle}{\bar{N}_a \bar{N}_b \bar{N}_c} = \frac{c_{3,abc} + \left(c_{2,ab} + c_{2,ac} + c_{2,bc}\right)k_2^{\rm v} + k_3^{\rm v}}{\langle N_{\rm part}\rangle^2}$$

For 4-particle correlation $$C_{4,abcd} = \frac{c_{4,abcd} + \left(c_{2,ab}c_{2,cd} + pe. + c_{3,abc} + pe.\right)k_2^{\rm v} + \left(c_{2,ab} + pe.\right)k_3^{\rm v} + k_4^{\rm v}}{\langle N_{\rm part}\rangle^3}$$

Expect 2nd-order or FB correlation scales as $1/N_{\rm part}$, 3rd-order $1/N_{\rm part}^2$…

Factorize CF and fluc. within each source

For two-particle correlation $$C_{2,ab}=\frac{1}{\langle N_{\rm part}\rangle}\frac{\langle \delta n_a \delta n_b\rangle}{\bar{n}_a\bar{n}_b}+\frac{\langle \delta N_{\rm part}^2\rangle}{\langle \delta N_{\rm part}\rangle^2}=\frac{c_{2,ab}+k_2^{\rm v}}{\langle N_{\rm part}\rangle}$$

For 3-particle correlation $$C_{3,abc}=\frac{\langle \delta N_a \delta N_b \delta N_c\rangle}{\bar{N}_a\bar{N}_b\bar{N}_c}=\frac{c_{3,abc}+\left(c_{2,ab}+c_{2,ac}+c_{2,bc}\right)k_2^{\rm v}+k_3^{\rm v}}{\langle N_{\rm part}\rangle^2}$$

For 4-particle correlation $$C_{4,abcd}=\frac{c_{4,abcd}+\left(c_{2,ab}c_{2,cd}+pe.+c_{3,abc}+pe.\right)k_2^{\rm v}+\left(c_{2,ab}+pe.\right)k_3^{\rm v}+k_4^{\rm v}}{\langle N_{\rm part}\rangle^3}$$

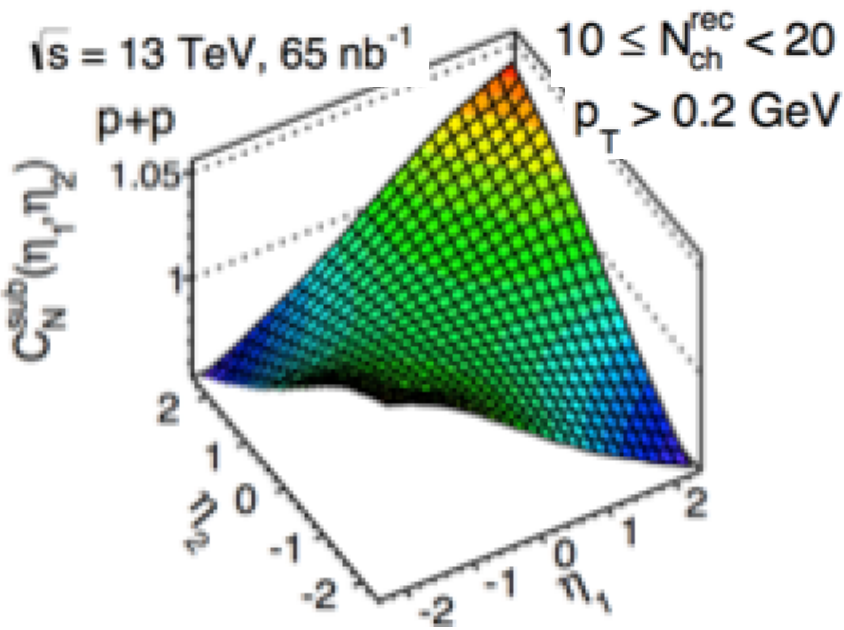
Expect 2nd-order or FB correlation scales as $1/N_{\rm part}$, 3rd-order $1/N_{\rm part}^2$…

Example two-particle correlation:

Long-range component scales as:

$$C_{2,ab}\approx a\eta_a\eta_b$$

a ~ 0.015, maximum signal <5%



Summary

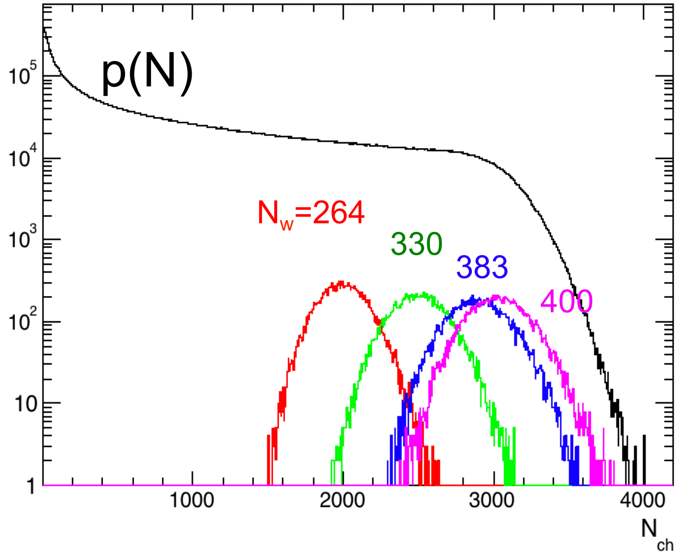
- Centrality fluctuation (CF) is one of the main uncertainties for interpreting the centrality dependence of experimental results.

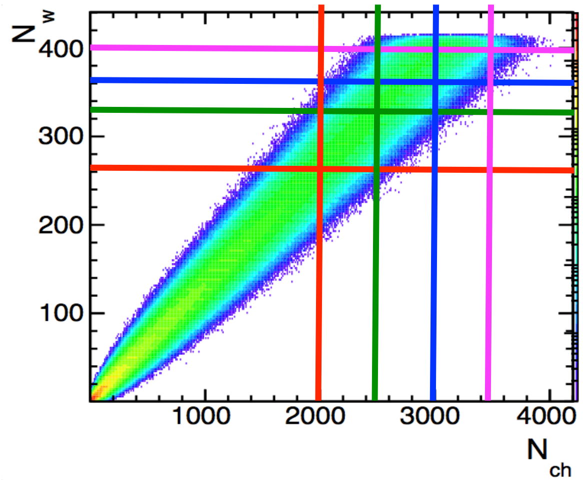
- The origin & influence of CF investigated using simple Glauber+NBD model
  - Centrality is represented by number of particle production sources, $N_{s.}$
  - CF influence the multiplicity and flow fluctuations and can be studied using multiplicity cumulants and flow cumulants
  - These cumulants are valuable especially in ultra-central events (also peripheral), where CF is strongly distorted.

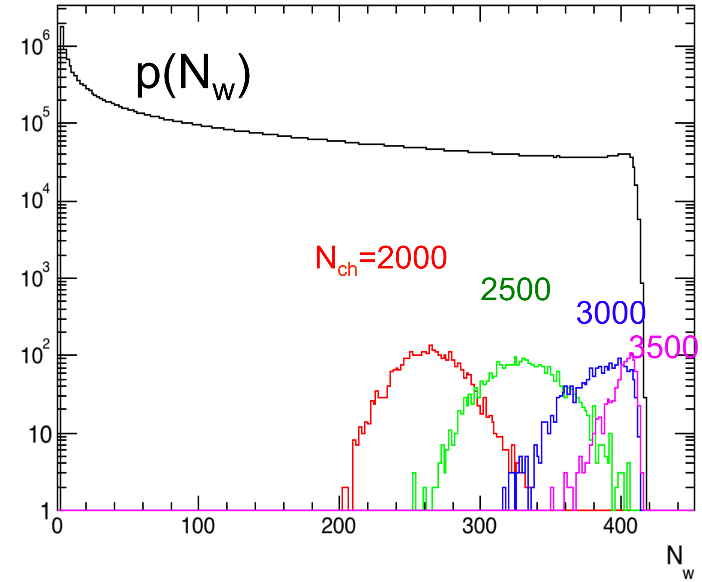
- Centrality is a long-range global effect, so one can investigate its fluctuation in the longitudinal direction via subevent method.

- Extending these studies to small systems is a promising direction.

Expected behavior of multiplicity cumulants







$$\left\langle (N-\bar{N})^k \right\rangle_{N_\mathrm{w}} = \int (N-\bar{N})^k p(N;N_\mathrm{w})dN$$

$$\left\langle (N_\mathrm{w}-\bar{N}_\mathrm{w})^k \right\rangle_{N} = \int (N_\mathrm{w}-\bar{N}_\mathrm{w})^k p(N;N_\mathrm{w})p(N_\mathrm{w})dN_\mathrm{w}$$

$p(N_w)$ treated as constant for narrow centrality bin ➔ N_w cumulant at fixed N is related to N cumulant at fixed N_w : Valid for NBD

$$\boxed{\bar{n}^{m-1}k_m^{\mathrm{v}}(1-p) = \frac{p^{m-1}}{(1-p)^{m-1}}k_m(p)}$$

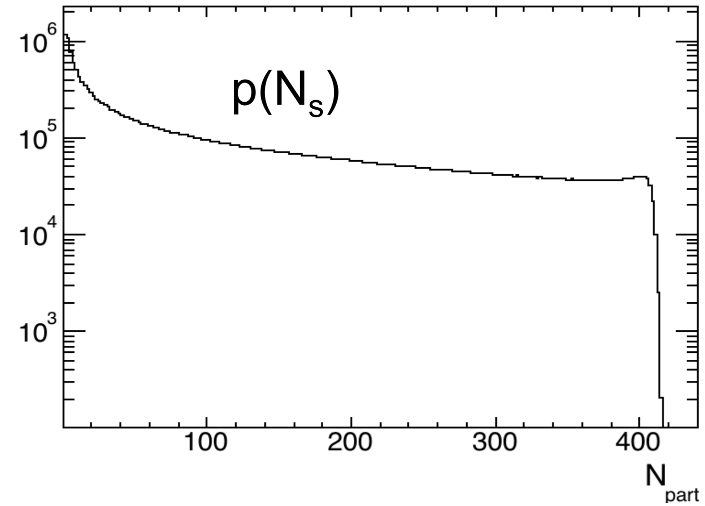
$$\frac{\bar{n}^2\left\langle(\delta N_s)^2\right\rangle}{\left\langle(\delta N)^2\right\rangle} = 1 \qquad \frac{\bar{n}^3\left\langle(\delta N_s)^3\right\rangle}{\left\langle(\delta N)^3\right\rangle} = \frac{2-p}{1+p} \qquad \frac{\bar{n}^4\left(\left\langle(\delta N_s)^4\right\rangle - 3\left\langle(\delta N_s)^2\right\rangle^2\right)}{\left\langle(\delta N)^4\right\rangle - 3\left\langle(\delta N)^2\right\rangle^2} = \frac{p^2+6(1-p)}{(1-p)^2+6p}$$

works only in mid-central collisions (centrality plateau)

Expected behavior of multiplicity cumulants

- CLT: for large N_s, fluctuation of N for fixed N_s is Gaussian:

$$p(N;N_s) \approx \frac{1}{\sqrt{2\pi\sigma^2 N_s}} e^{-\frac{(N-\bar{n}N_s)^2}{2\sigma^2 N_s}}, \bar{N} = \bar{n}N_s$$



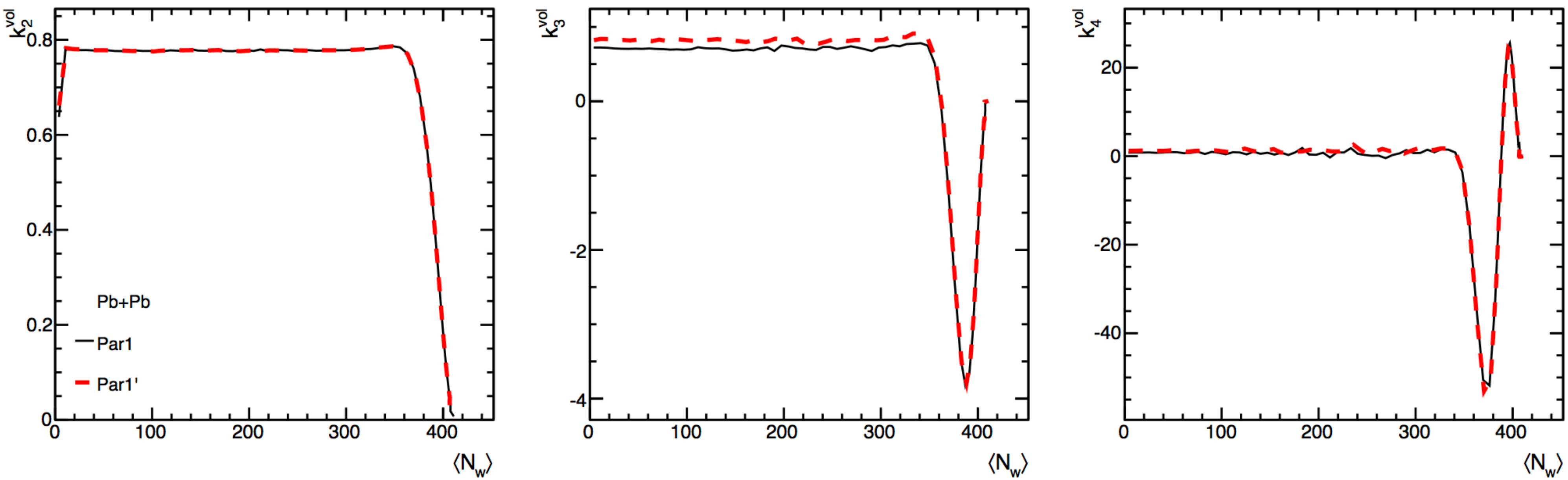
- In ultra-central collisions, cumulants are dominated by shape of $p(N_s)$, where it is a strongly varying function

$$\left\langle (\delta N_s)^k \right\rangle \approx \int (\delta N_s)^k \frac{1}{\sqrt{2\pi\hat{\sigma}^2 N_s}} e^{-\frac{(N_s-\bar{N}_s)^2}{2\hat{\sigma}^2 N_s}} p(N_s) dN_s$$

Depend only on $p(N_s)$ and σ of p(n)

Two p(n) with same relative width

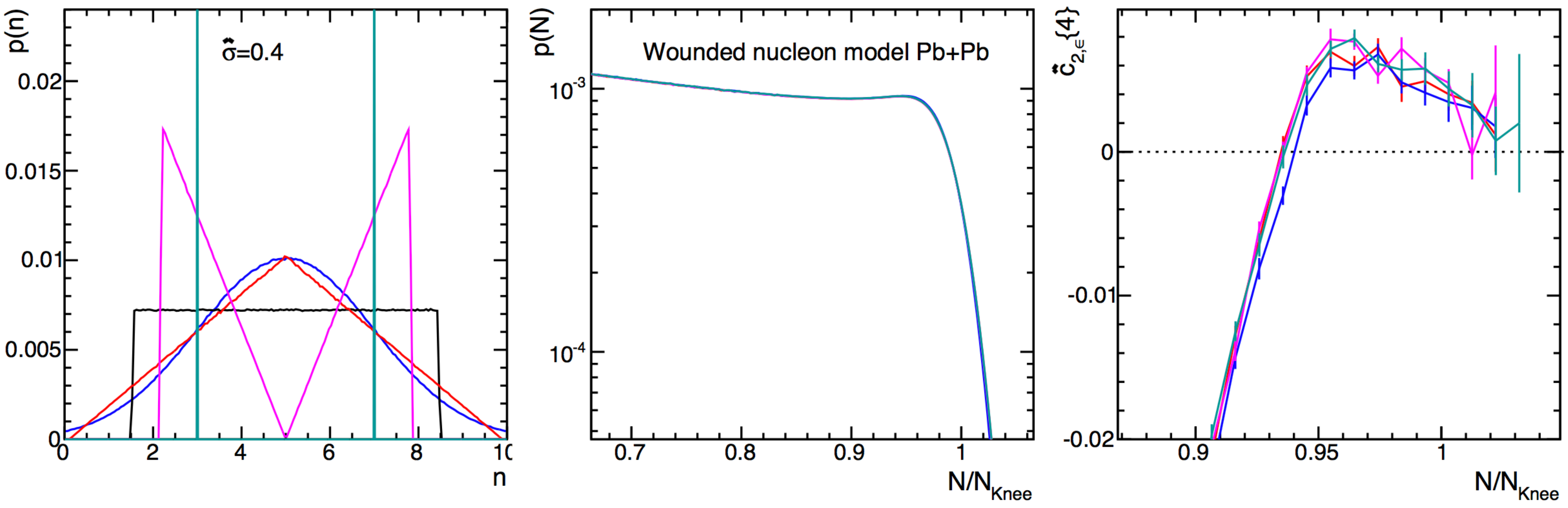
Wounded nucleon model				
	p	m	mean $\bar{n}$	RMS/mean $\hat{\sigma}$
Par1	0.831	1.55	7.58	0.88
Par1′	0.644	2.00	3.64	0.88



2nd-order agree perfectly, for higher order they agree in UCC region.

Dependence on the shape of p(n)

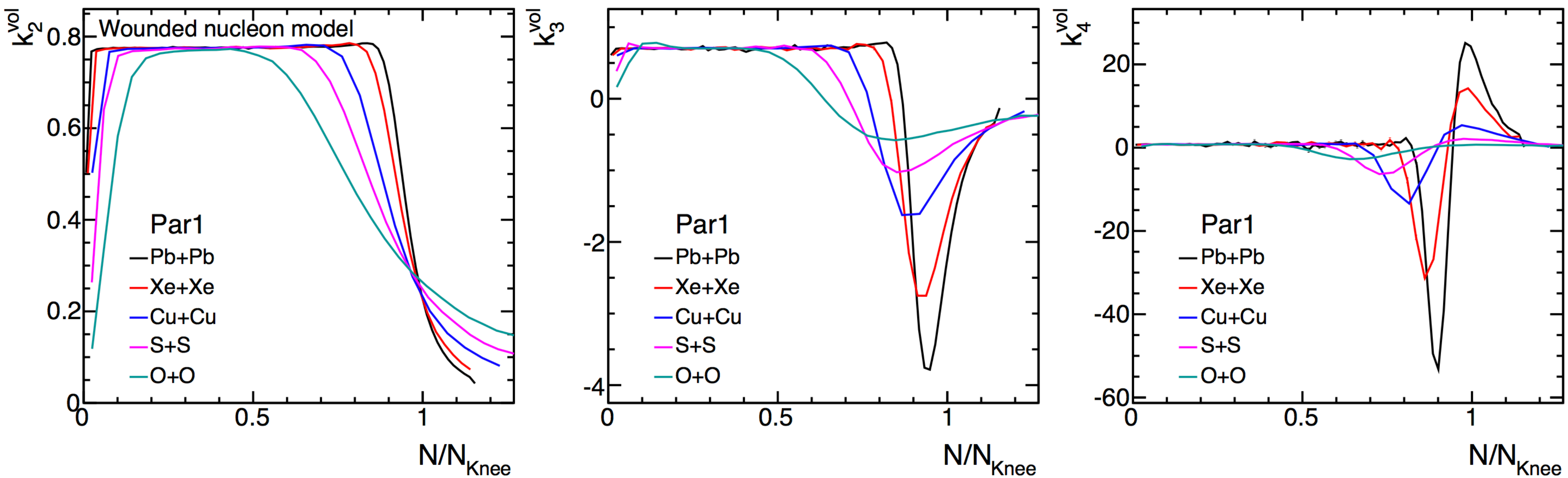
- Very different functional form for each source but same relative width
- The final p(N) and $c_2\{4\}$ are same



System dependence: centrality fluctuations

Check Xe+Xe, Cu+Cu, S+S and O+O

2A= 258, 126, 64, 32

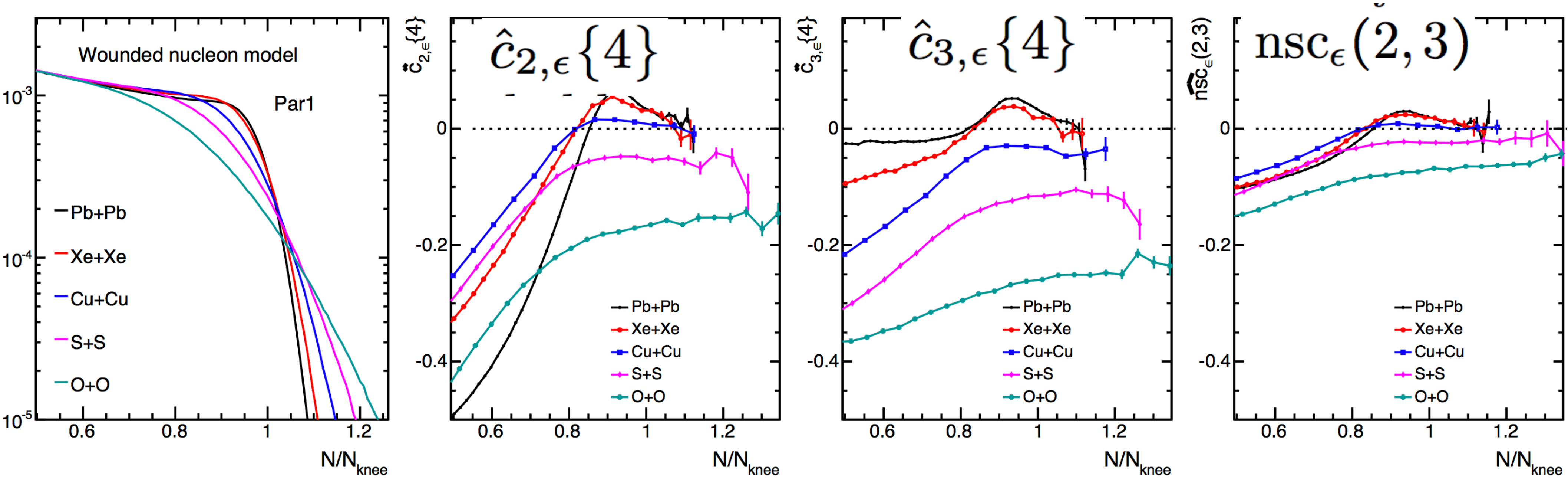


Influence broader range for smaller system

System dependence: eccentricity fluctuations

Check Xe+Xe, Cu+Cu, S+S and O+O

2A= 258, 126, 64, 32



Sign-change only for large system, but generally flattens out for other systems

$$N_{\mathrm{knee}} = 2A\bar{n}.$$

Eccentricity and centrality mixed fluctuation

Multiplicity level:

$$F(K_1,\epsilon_n)=\frac{\left\langle N\epsilon_n^2\right\rangle}{\langle N\rangle\left\langle\epsilon_n^2\right\rangle}-1,\ F(K_2,\epsilon_n)=\frac{\left\langle(\delta N)^2\epsilon_n^2\right\rangle}{\langle(\delta N)^2\rangle\left\langle\epsilon_n^2\right\rangle}-1$$

Centrality fluctuation component

$$F(k_1^{\mathrm{v}},\epsilon_n)=\frac{\left\langle N_{\mathrm{s}}\epsilon_n^2\right\rangle}{\langle N_{\mathrm{s}}\rangle\left\langle\epsilon_n^2\right\rangle}-1,\ F(k_2^{\mathrm{v}},\epsilon_n)=\frac{\left\langle(\delta N_{\mathrm{s}})^2\epsilon_n^2\right\rangle}{\langle(\delta N_{\mathrm{s}})^2\rangle\left\langle\epsilon_n^2\right\rangle}-1$$

Associate with subevent explicitly:

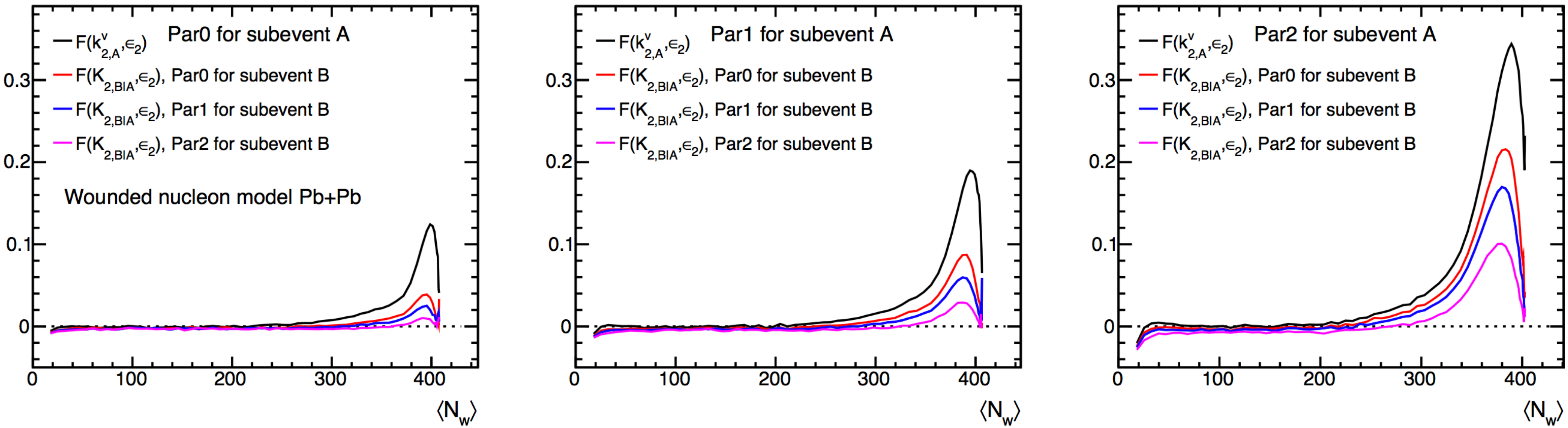
$$F(K_{1,B|A},\epsilon_n)=F(k_{1,A}^{\mathrm{v}},\epsilon_n)$$

$$F(K_{2,B|A},\epsilon_n)=\frac{k_{2,B}F(k_{1,A}^{\mathrm{v}},\epsilon_n)+\bar{n}_B k_{2,A}^{\mathrm{v}}F(k_{2,A}^{\mathrm{v}},\epsilon_n)}{k_{2,B}+\bar{n}_B k_{2,A}^{\mathrm{v}}}$$

2^{nd}-order cumulant for p(N, ε_2)

$$F(K_{2,B|A},\epsilon_2) = \frac{k_{2,B}F(k^{v}_{1,A},\epsilon_2) + \bar{n}_B k^{v}_{2,A}F(k^{v}_{2,A},\epsilon_2)}{k_{2,B} + \bar{n}_B k^{v}_{2,A}}$$

Values depends on the subevent used for centrality selection as well as on the smearing in subevent B



Large fluctuation in centrality selection➔ larger signal

Large fluctuation in measurement subevent➔ smaller signal