Opportunities and Challenges with Jets at LHC and beyond, CCNU, Wuhan

Heavy quarkonium dissociation by thermal gluons in the QGP

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Contents

1. Introduction:

- Heavy quarkonium probes of hot & dense matter

2. QCD multipole expansion

- QQbar coupling to external gluons

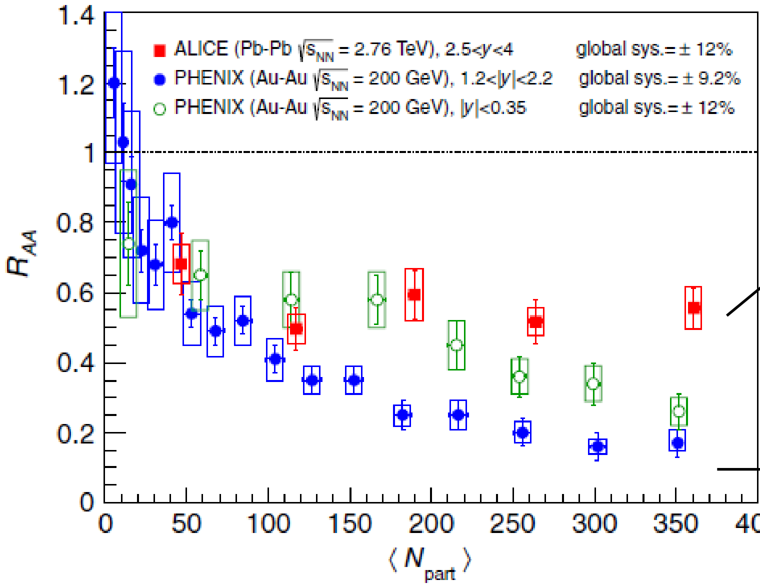
3. LO: g + Ψ → c + cbar

- Cross section: E1 vs M1
- Dissociation rate

4. NLO: g + Ψ → g + c + cbar

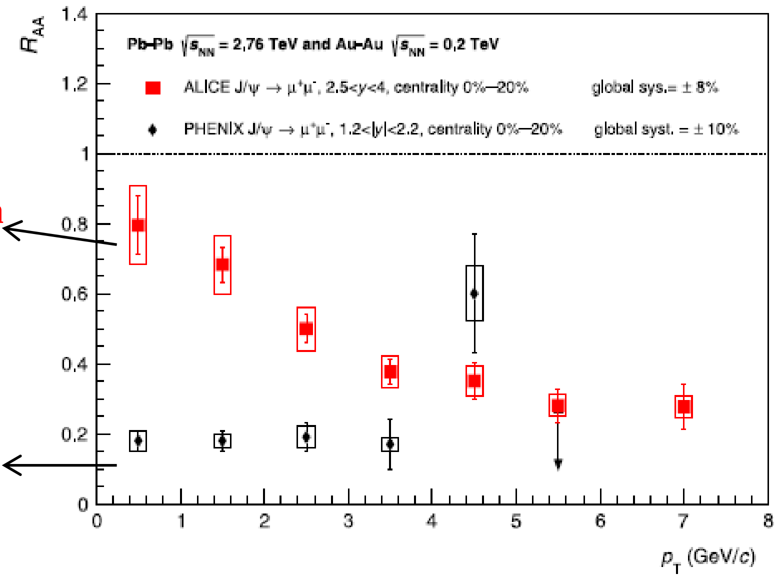
- Cross section
- Dissociation rate

Production of heavy quarkonium in QGP



Regeneration /coalescence

Suppression /dissociation



The complex story of quarkonium production Rapp, Zhuang, Ko, Strickland ...

--- static color screening Matsui & Satz 86

➔ suppression of yields

--- dynamical dissociation

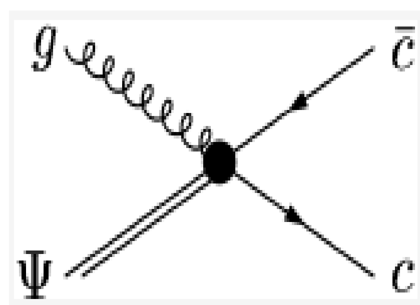
g + Ψ ➔ (g +) c + cbar

--- regeneration/coalescence

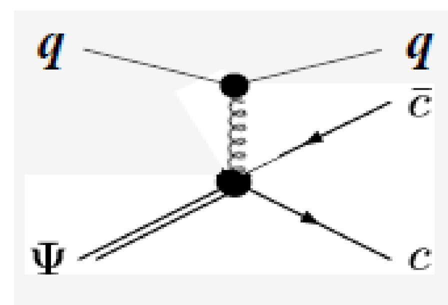
Thews, B.-Munzinger, Rapp

➔ enhance the low p_T yields

--- ImV_{QQbar} , Landau damping



LO: gluo-dissociation



NLO: “quasi-free” disso.

Dissociation rates used in phenomenology

- **Semi-classical transport of heavy quarkonium in QGP**

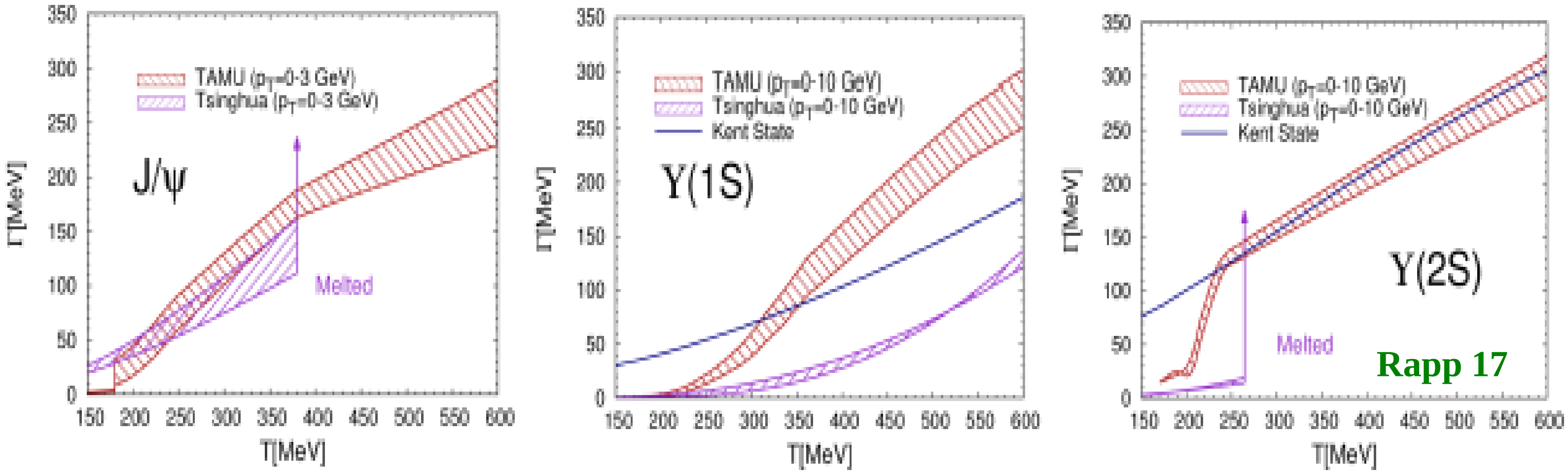
  --- Boltzmann eq.

$$\partial f_\Psi/\partial t + \vec{v}_\Psi \cdot \vec{\nabla} f_\Psi = -\alpha_\Psi f_\Psi + \beta_\Psi$$

  --- upon assuming full equilibrium charm quark distributions ➔ rate eq.

$$\frac{\mathrm{d}N_\Psi(t)}{\mathrm{d}t} = -\Gamma_\Psi(t)[N_\Psi(t) - N_\Psi^{\mathrm{eq}}(t)]$$

- **Transport coefficient: dissociation rate**



--- TAMU: quasi-free VS Tsinghua: gluo-diss. VS Kent: ImV$_{QQbar}$

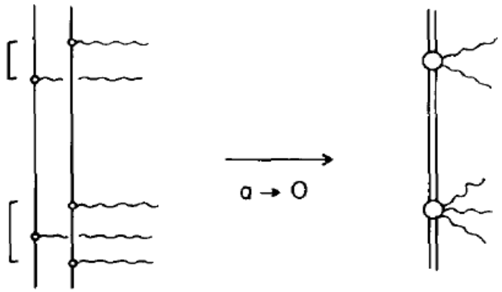
--- rates used differ considerably between different groups

Peskin: Gluon – quarkonium coupling

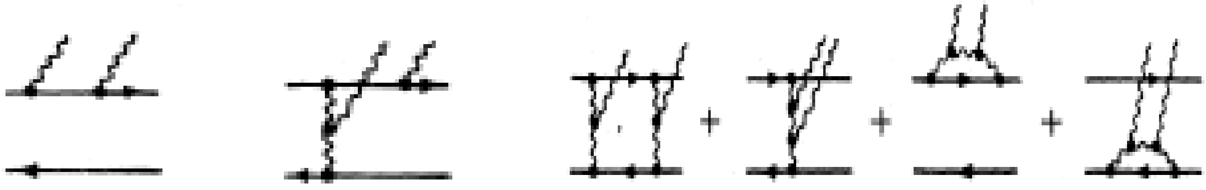
- **Peskin's OPE analysis** Peskin 79
  color-octet QQbar can only persist over short space-time range

$$\Delta t \sim \frac{1}{V_8 - V_1} \sim \frac{a}{g_s^2} \sim \frac{1}{\epsilon_B},$$

➔gluon emissions assemble into small singlet clusters: OPE local operators



- **E.g. summing up all 2-gluon emissions, in particular including gluon self-coupling diagrams**

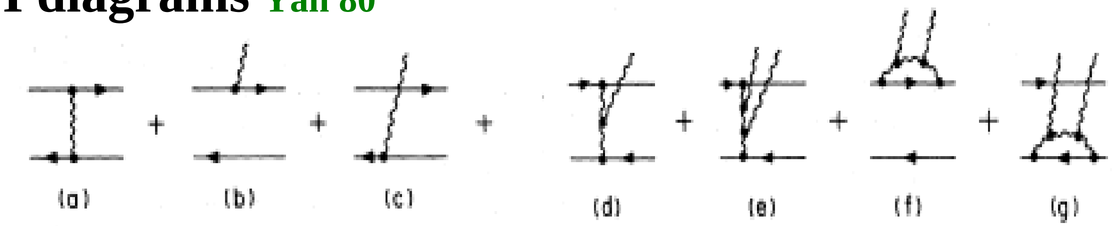


➔arrive at a gauge invariant 2nd order color-electric dipole transition:

$$-\frac{g^2}{2N}\langle\phi|\boldsymbol{r}\cdot\boldsymbol{E}\frac{1}{H_a+\epsilon+iD^0}\boldsymbol{r}\cdot\boldsymbol{E}|\phi\rangle$$

QCD multipole expansion: gluon-quarkonium coupling

- **2-PI diagrams** Yan 80



--- one iteration ➔ Peskin's result
--- full summation ➔ effective Lagrangian

$$L_Q = \int d^3x\, \bar{\psi}[\gamma^\mu(i\partial_\mu - gA_\mu) - m]\psi$$

$$-\tfrac{1}{2}\int d^3x\, d^3y \sum_{a=0}^{8} \bar{\Psi}(\vec{x},t)\gamma^0 \tfrac{1}{2}\lambda_a \Psi(\vec{x},t)[\delta_{a0}V_1(|\vec{x}-\vec{y}|) + (1-\delta_{a0})V_2(|\vec{x}-\vec{y}|)]\bar{\Psi}(\vec{y},t)\gamma^0\tfrac{1}{2}\lambda_a\Psi(\vec{y},t) + \cdots ,$$

- **Transcription into a NR Hamiltonian via multipole expansion**

$$H_0 = \frac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \frac{\lambda^a}{2}\frac{\bar{\lambda}^a}{2} V_2(|\vec{r}|),$$

$$H_I = Q^a A_0^a(t,\vec{0}) - \vec{d}^a \cdot \vec{E}^a(t,\vec{0}) - \vec{m}^a \cdot \vec{B}^a(t,\vec{0}) + \ldots,$$

with

$$\vec{d}^a = \frac{1}{2} g_s \vec{r} \left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right),$$

$$\vec{m}^a = \frac{g_s}{2m_Q}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\left(\frac{\vec{\sigma}}{2} - \frac{\vec{\sigma}\,'}{2}\right);$$

---- **similar to QED dipoles, albeit with color indices**
--- inclusion of gluon self-coupling diagrams essential

Deriving the gluo-dissociation cross section: g + ψ → c + cbar

- **Color E1 transition**

--- Weyl gauge

$$H_{E1} = -\vec{d}^a \cdot \vec{E}^a(t,\vec{0}) = \frac{g_s}{2}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\vec{r} \cdot \frac{\partial \vec{A}^a}{\partial t}$$

with

$$\vec{A}^a(t,\vec{x}) = \sum_{\vec{k},\lambda} N_{\vec{k}} \vec{\epsilon}_{\vec{k}\lambda}\left[a^a_{\vec{k}\lambda} e^{i\vec{k}\cdot\vec{x} - i\omega_{\vec{k}} t} + h.c.\right]$$

Peskin'Coulomb reproduced

---- Fermi's golden rule: 1st order perturbation

$$\sigma_{E_1}^{g+J/\psi\to c+\bar{c}}(E_g) = \frac{g_s^2\pi}{2\cdot 9} E_g \frac{V}{(2\pi)^3}\int d^3\vec{p} \times \left| < (c\bar{c})_8, \vec{p}|\vec{r}|J/\psi > \right|^2 \delta\left(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}\right),$$

Coulomb appro.

$$\sigma_{E_1,\mathrm{Coulomb}}^{g+J/\psi\to c+\bar{c}}(E_g) = \frac{2^7}{9} g_s^2 \frac{\epsilon_B^{5/2}}{m_Q} \frac{(E_g - \epsilon_B)^{3/2}}{E_g^5}$$

- **Color M1 transition**

A novel contribution

$$H_{M1} = -\vec{m}^a \cdot \vec{B}^a(t,\vec{0}) = -\frac{g_s}{2m_Q}\left(\frac{\lambda^a}{2} - \frac{\bar{\lambda}^a}{2}\right)\left(\frac{\vec{\sigma}}{2} - \frac{\vec{\sigma}\,'}{2}\right) \cdot \nabla \times \vec{A}^a(t,\vec{0})$$

--- 1st order perturbation

$$\sigma_{M_1}^{g+J/\psi\to c+\bar{c}}(E_g) = \frac{g_s^2\pi}{2\cdot 3} \frac{E_g}{m_Q^2} \frac{V}{(2\pi)^3}\int d^3\vec{p} \times \left| < (c\bar{c})_8 | J/\psi > \right|^2 \delta\left(E_g - \epsilon_B - \frac{\vec{p}^2}{m_Q}\right).$$

Coulomb appro.

$$\sigma_{M_1,\mathrm{Coulomb}}^{g+J/\psi\to c+\bar{c}}(E_g) = \frac{2^3}{3} g_s^2 \frac{\epsilon_B^{5/2}}{m_Q^2} \frac{(E_g - \epsilon_B)^{1/2}}{E_g^3}$$

- **Selection rules**

--- E1: ΔL = 1, ΔS = 0; M1: ΔL = 0, ΔS = 1 from singlet to octet transition

--- P-wave states χ_c & χ_b also derived

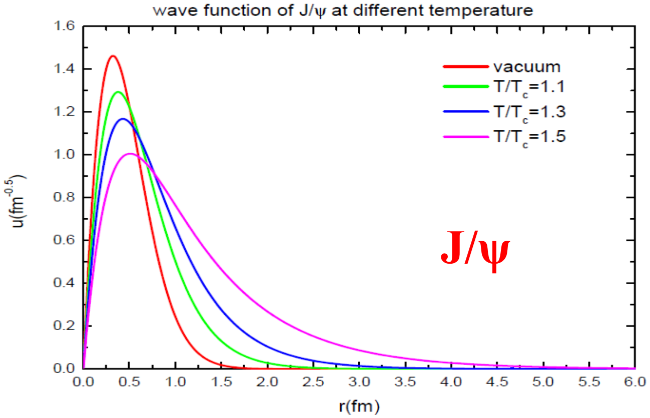
In-medium potential Schrodinger

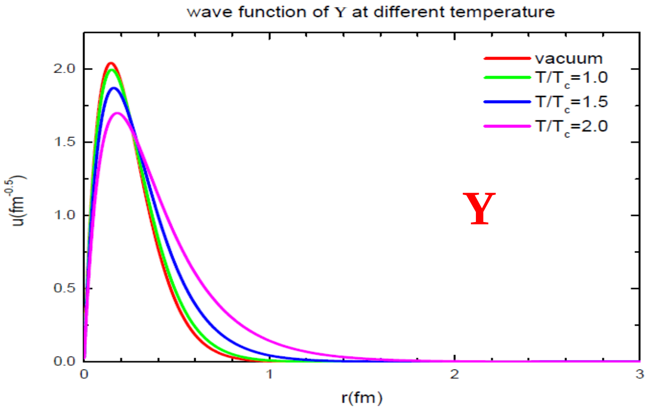
- **In-medium potential model** Satz 88

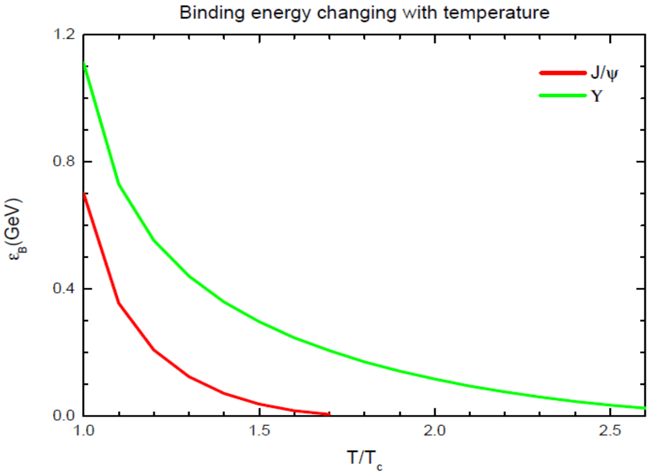
$$V(r,T) = -\frac{\alpha}{r}e^{-m_D(T)r} + \frac{\sigma}{m_D(T)}(1 - e^{-m_D(T)r})$$

$$[H(r,T) - E_{n,l}(T)]\psi_{n,l}(r,\theta,\phi) = 0$$

- **Vacuum/in-medium quarkonium wave func., binding $\varepsilon_B(T)$**



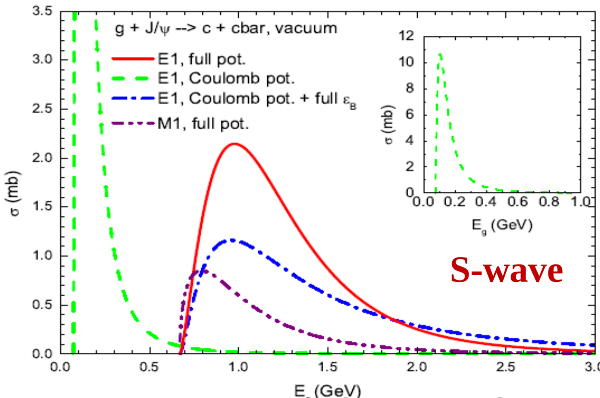


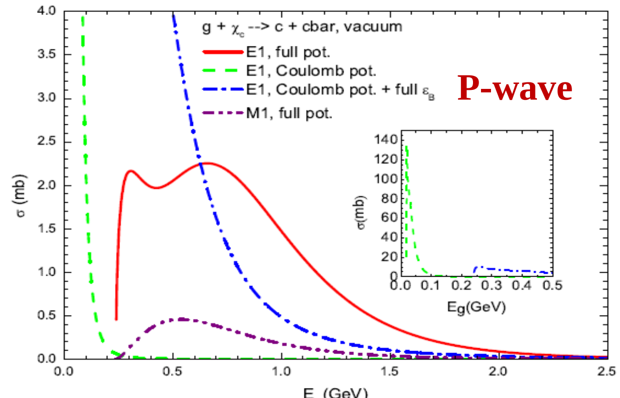


--- higher T, stronger screening
--- bound state size grows, binding energy decreases
--- $T_d(J/\psi)\sim1.7T_c$, $T_d(\Upsilon)\sim2.6T_c$

Gluo-diss. g + ψ → c + cbar cross section

- **Vacuum cross sections**

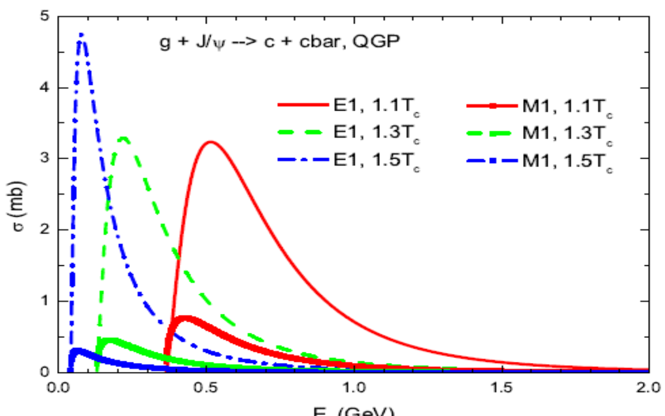


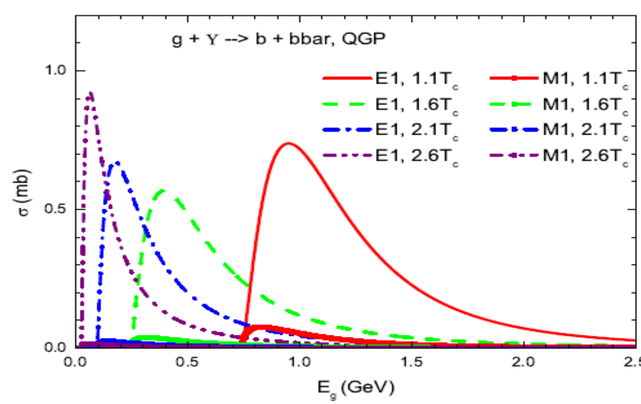


--- Coulomb wave func. + full ε_B differs from the full potential result by ~50%

--- M1 overtakes E1 at low energies ← ΔL= 0 s-wave isotropic scattering dominant

- **In-medium cross sections**





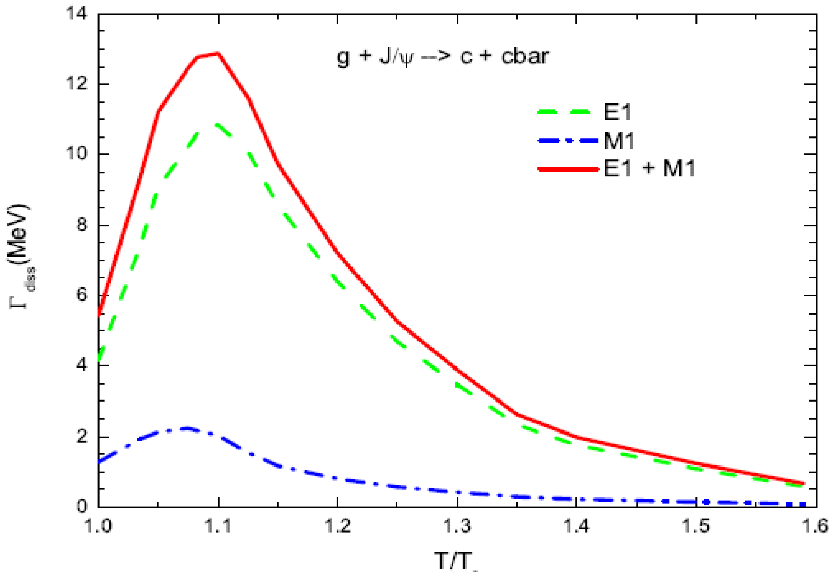
--- as T increases, dipole size grows → E1 increases, M1 most prominent at low T

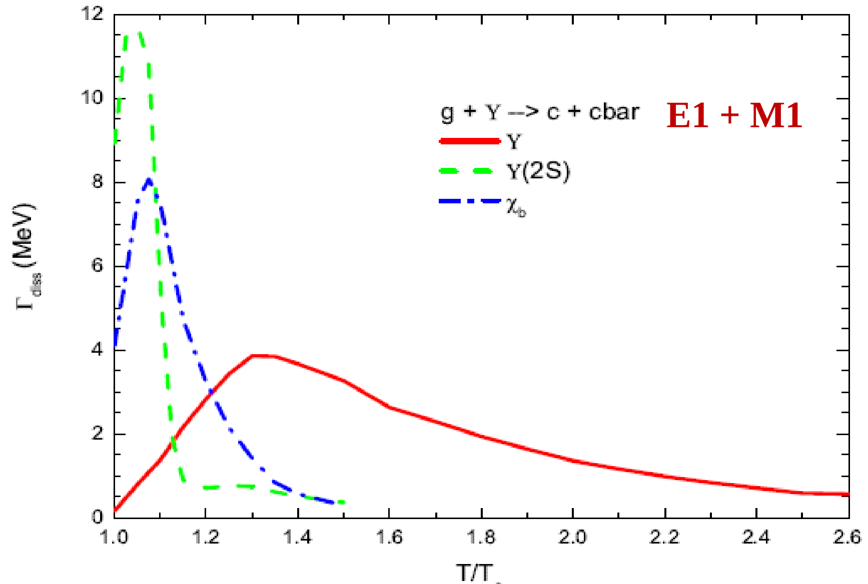
--- more tightly bound Υ(1S) much smaller cross section

Gluo-dissociation rates

- **Heavy quarkonium (at rest) gluo-dissociation rate**

$$\Gamma_{\text{diss}}(T) = d_g \int \frac{d^3k}{(2\pi)^3} f_g(E(\vec{k})) v_{\text{rel}} \sigma(|\vec{k}|, T)$$





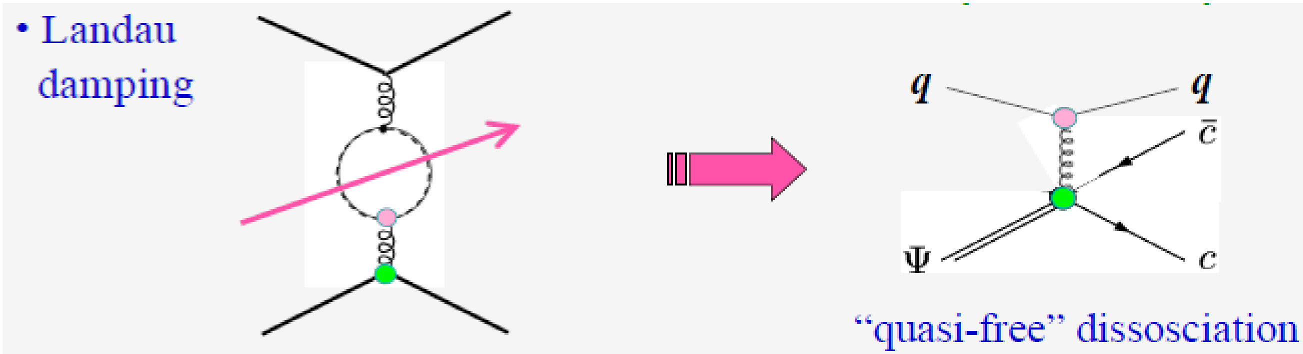
--- M1 most prominent for J/ψ at low T, accounting for ~10-25% of the total (E1+M1) rate in T_c --1.2T_c ➔ could be significant as system stays long

--- at low T, ε_B large, tightly bound J/ψ or Υ ➔ gluon sees the bound state as a whole ➔ LO gluo-dissociation sensible ➔ rate increases with T

--- at high T, ε_B decreases, σ [$k^2 * f_g(k)$] shifts toward lower [higher] energy ➔ phase space mismatched ➔ rate drops off fast ➔ calling for NLO

NLO dissociation: going beyond quasi-free

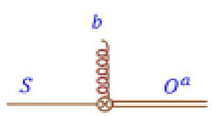
- **Quasi-free: effects of bound state wave func. completely ignored** Rapp 02



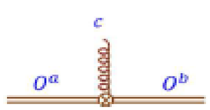
- **QCD multipole expansion/pNRQCD vertices** Yan 80, Sumino 14

$$H_{\rm eff} = H_0 + H_I,$$
$$H_0 = \frac{\vec{p}^2}{m_Q} + V_1(|\vec{r}|) + \sum_a \frac{\lambda^a}{2}\frac{\bar{\lambda}^a}{2} V_2(|\vec{r}|),$$
$$H_I = V_{SO} + V_{OO},$$

Vertices



$$ig\sqrt{\frac{T_F}{N}}\,\vec{r}\cdot\vec{E}^b\,\delta^{ab}$$



$$\frac{1}{2} ig\, d^{abc}\,\vec{r}\cdot\vec{E}^c$$

$$< O,a|V_{SO}|S> = < O,a|\frac{1}{2}g_s\vec{r}(\frac{\lambda^b}{2} - \frac{\bar{\lambda}^b}{2})\cdot\vec{E}^b|S> \qquad (2)$$
$$= \frac{g_s}{\sqrt{2N_c}}\vec{E}^b\cdot < O|\vec{r}|S> \delta^{ab},$$

$$< O,a|V_{OO}|O,b> = \frac{ig_s}{2} d^{abc}\vec{E}^c\cdot < O|\vec{r}|O> .$$

With $d^{abc} = 2tr[\lambda^a/2\{\lambda^b/2, \lambda^c/2\}]$

NLO: 2^{nd} order QM perturbation

- **Transition amplitude for g + Ψ → g + c + cbar**

$$T_{fi} = \sum_m \frac{< f|H_I|m >< m|H_I|i >}{E_i - E_m + i\epsilon},$$

initial/final state: $|i >= |J/\psi, g(\vec{k}, \lambda, a) >$ $\qquad |f >= |(c\bar{c})_8(\vec{p}, b), g(\vec{\kappa}, \sigma, c) >$

- **Two kinds of possible intermediate states**

--- 1^{st}: $|m >= |(c\bar{c})_8(\vec{q}, d) >$

$$T_{fi}^{(1)} = \frac{(-ig_s^2)}{2V^2}(2\pi)^3\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}\sum_d \delta^{ad} d^{bdc}\frac{V}{(2\pi)^3}\int d^3\vec{q}\vec{\epsilon}_{\vec{k}\lambda}\cdot\frac{\vec{q}}{q}$$
$$\times \int r^3 dr j_1(qr) R_{10}(r)\frac{\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\nabla_{\vec{q}}\delta^3(\vec{q}-\vec{p})}{-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{q}^2}{m_Q} + i\epsilon}$$
$$= d^{bac}\frac{ig_s^2}{2V}\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}\vec{\epsilon}_{\vec{k}\lambda}\cdot[A(p,k)\vec{\epsilon}_{\vec{\kappa}\sigma} + (\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})\frac{\vec{p}}{p^2}B(p,k)]$$

$$A(p,k) = \frac{\int r^3 dr j_1(pr)R_{10}(r)}{p(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)},$$

$$B(p,k) = \frac{\frac{2p^2}{m_Q}\int r^3 dr j_1(pr)R_{10}(r)}{p(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)^2} - \frac{\int r^4 dr j_2(pr)R_{10}(r)}{(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)}.$$

---2^{nd} : $|m >= |(c\bar{c})_8(\vec{q}, d), g(\vec{k}_1, \lambda_1, d_1), g(\vec{k}_2, \lambda_2, d_2) >$

$$C(p,\kappa) = \frac{\int r^3 dr j_1(pr)R_{10}(r)}{p(-\epsilon_B - \omega_{\vec{\kappa}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)},$$

$$D(p,\kappa) = \frac{\frac{2p^2}{m_Q}\int r^3 dr j_1(pr)R_{10}(r)}{p(-\epsilon_B - \omega_{\vec{\kappa}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)^2} - \frac{\int r^4 dr j_2(pr)R_{10}(r)}{(-\epsilon_B - \omega_{\vec{\kappa}} - \frac{\vec{p}^2}{m_Q} + i\epsilon)}$$

$$T_{fi}^{(2)} = -d^{bac}\frac{ig_s^2}{V}\sqrt{\frac{\pi\omega_{\vec{k}}\omega_{\vec{\kappa}}}{6V}}$$
$$\times \vec{\epsilon}_{\vec{k}\lambda}\cdot[C(p,\kappa)\vec{\epsilon}_{\vec{\kappa}\sigma} + (\vec{\epsilon}_{\vec{\kappa}\sigma}\cdot\vec{p})\frac{\vec{p}}{p^2}D(p,\kappa)]$$

Wave func., ΔL=2, dipole transition twice

NLO cross section: g + Ψ → g + c + cbar

- **Cross section**

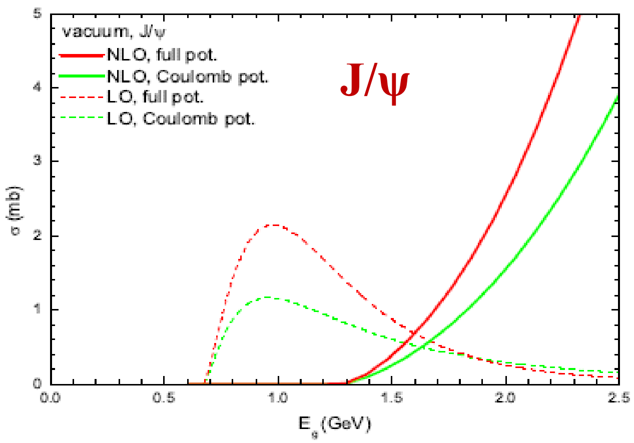
  --- 3-mom. transfered to Ψ neglected, no recoil; more justified for more massive Y

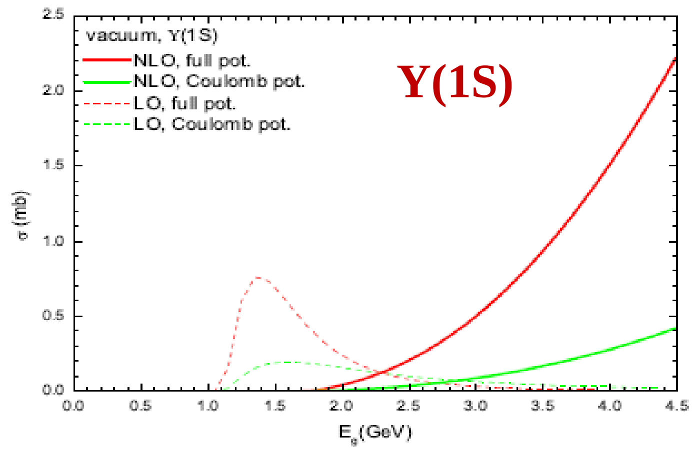
$$\sigma(E_g) = 2\pi V \frac{V}{(2\pi)^3}\int d^3\vec{p}\sum_b \frac{V}{(2\pi)^3}\int d^3\vec{\kappa}\sum_\sigma\sum_c \frac{1}{4\pi}\int d\Omega_{\vec{k}} \times \frac{1}{2}\sum_\lambda \frac{1}{8}\sum_a |T_{fi}^{(1)} + T_{fi}^{(2)}|^2 \delta(-\epsilon_B + \omega_{\vec{k}} - \frac{\vec{p}^2}{m_Q} - \omega_{\vec{\kappa}}).$$

- **Vacuum cross section**

  --- Infraredly sensitive to the outgoing gluon mass
  m_g=0, the denominator of A(p.k) & B(p,k) ~ 1/ω(κ) ~ 1/κ → divergent!

  --- If regularized by a "constituent" gluon mass m_g=600 MeV



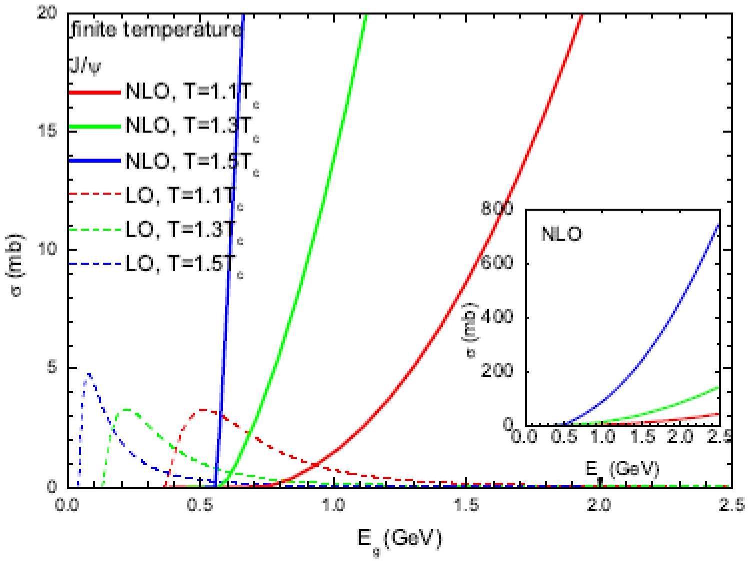


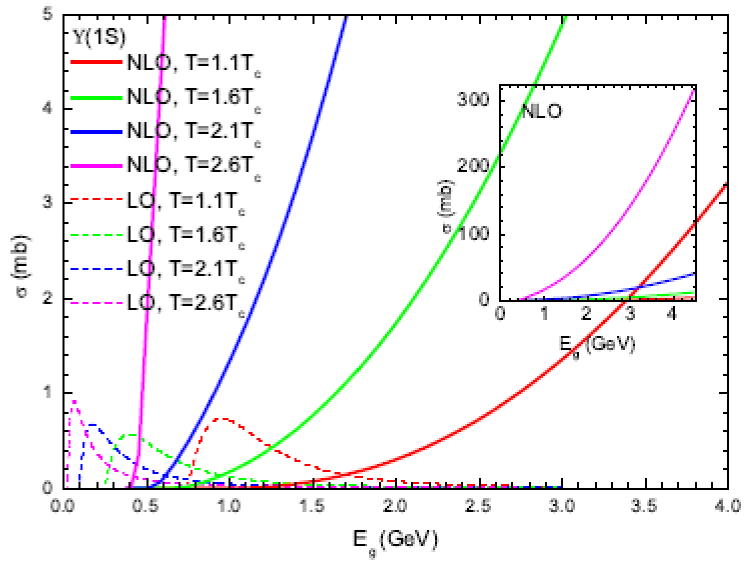
  --- NLO quickly takes over from LO; no fall-off

NLO cross section: finite temperature

- **In-medium cross section**

--- well defined as regularized by gluons' thermal mass $m_g(T) = \sqrt{3/4}g_sT$



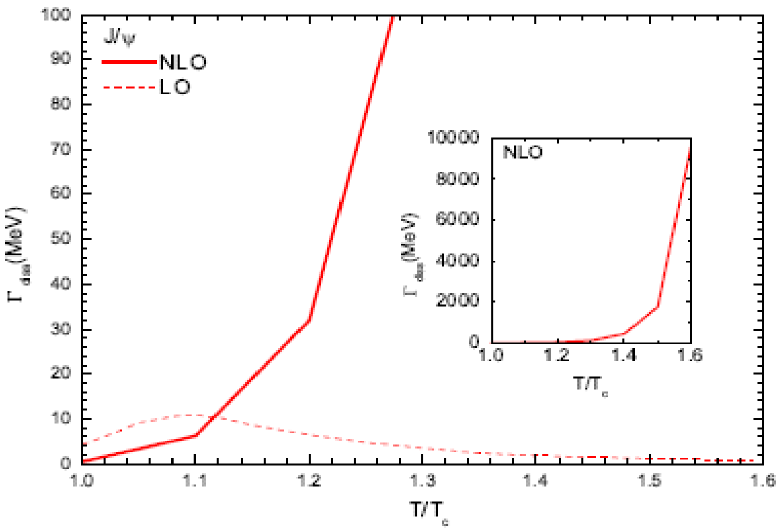


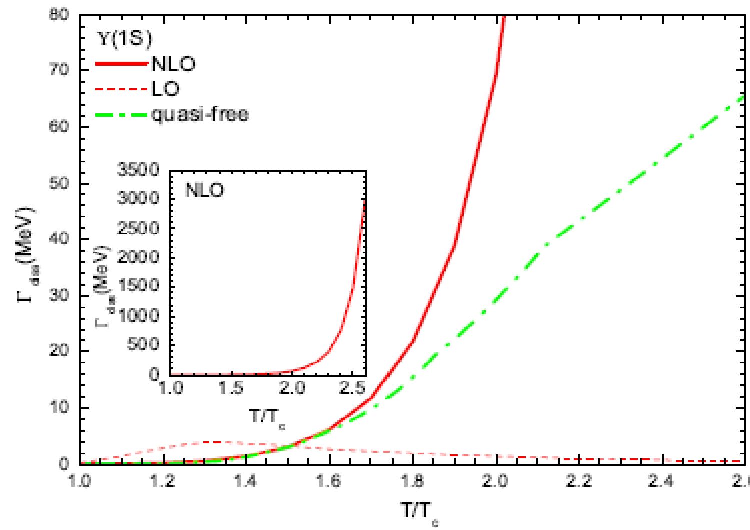
--- NLO quickly takes over from LO; no fall-off
--- NLO increases fast with T, due to expanding wave function & decreasing ε_B
--- Near T_d, dipole size blows up > gluon wave-length, dipole transition may not be a good approximation any more

NLO dissociation rate

- **Heavy quarkonium (at rest) NLO-dissociation rate**

$$\Gamma_{\rm diss}(T) = d_g \int \frac{d^3k}{(2\pi)^3} f_g(E(\vec{k})) v_{\rm rel} \sigma(|\vec{k}|, T)$$





--- The artifact of LO dropping off toward high T: replaced by NLO increase

--- Near T_d, Γ_{diss} ~ GeV: very fast break-up, conceptually consistent with static dissociation by color screening

--- Quantitatively might be questionable, but supported/needed by phenomenological transport study, e.g. **Strickland 15, Γ_{diss} > 2 GeV needed for $T>=T_d$**

Summary & Outlook

- **LO & NLO dissociation of heavy quarkonium by thermal gluons**

  --- **LO M1 investigated for the first time**

  --- **Bound state wave function effects systematically included**

  --- **LO drop-off artifact of dissociation rate removed by NLO**

- **NLO dissociation of heavy quarkonium by thermal light quarks**

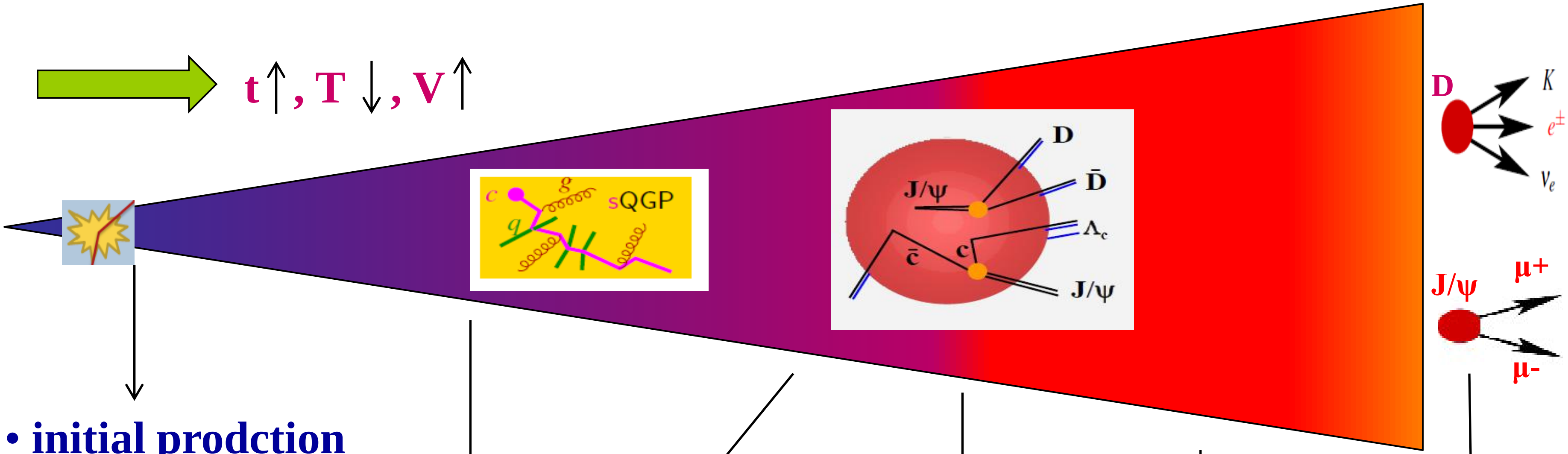
  --- **on-going work**

- **Goal**

  --- **LO and NLO dissociation by thermal gluons/light quarks computed in a single solid theoretical framework of QCD multipole expansion**

  --- **Improve theoretical input for phenomenological applications/transport**

Back-up: HQ & Heavy quarkonium probes



- **initial prodction**
  t_Q~1/m_Q , pQCD
  t_ψ ~1/ ε_B, NRQCD

- **c-quark Brownian diffusion; ψ dissolve vs regenerate**

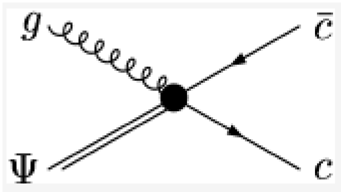
- **c & cbar hadronize coal. vs frag.**

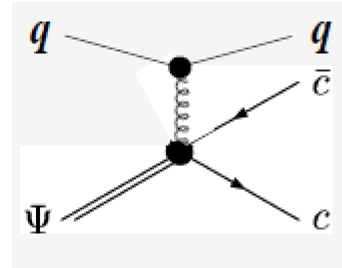
- **D & ψ hadronic kinetics**

- **EW decay outside fireball**

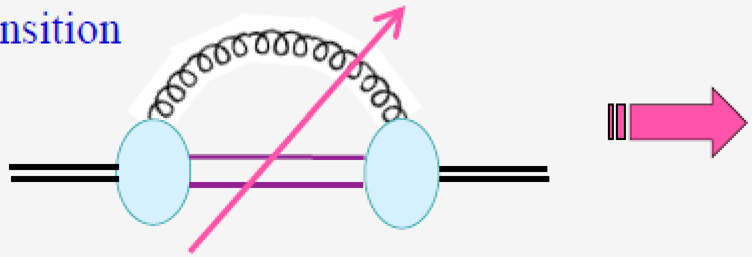
various transport coeffi., e.g. c-diff. coeffi., Ψ disso. rate, etc., to be implemented into transport eqs, e.g. Langevin, Boltzmann on top of bulk hydro ➔ Micro + Macro physics combined vs quantum effs.

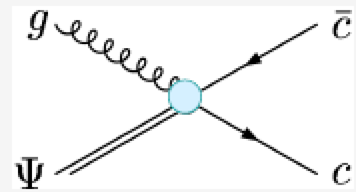
Back-up: J/ψ gluo-dissociation vs quasi-free





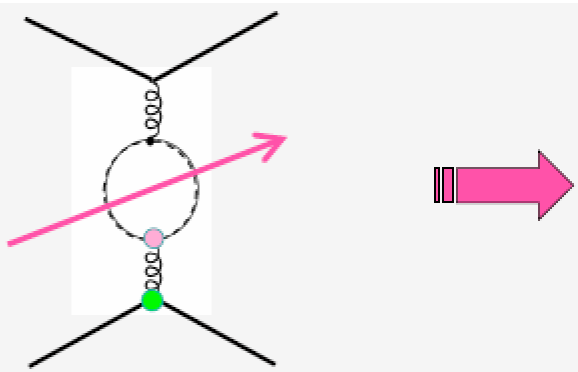
- Singlet-octet transition

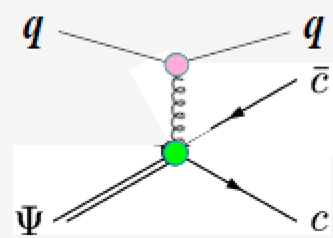




gluo-dissociation

- Landau damping





"quasi-free" dissociation

NLO: Su Houng Lee 2005

pQCD calculations

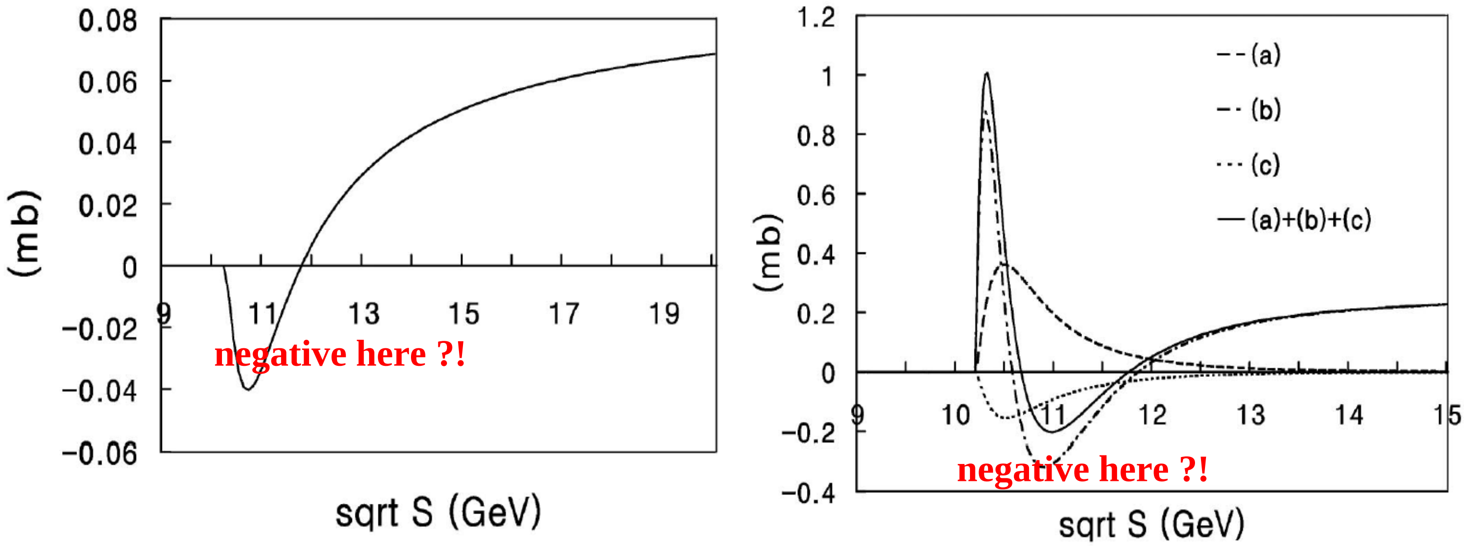


FIG. 10. The left figure is the elementary cross section for $\Phi + q \to Q + \bar{Q} + q$ and the right figure is that for $\Phi + g \to Q + \bar{Q} + (g)$. In the right figure, (a) the dashed line is the Born term given in Eq. (12). (b) The dash-dotted line is the hard gluon part, namely, the integration of the sum of Eq. (35), (37), and (43) over the hard part of the Dalitz plot plus the integration of the "$\ln\delta$" dependent part of Eq. (65) over the soft part of the Dalitz plot. (c) The dotted line is the soft plus one loop correction, namely, the integration of Eq. (65) excluding the "$\ln\delta$" dependent part over the soft part of the Dalitz plot. The solid line is the sum of (a), (b), and (c).