Lecture 3

The electroweak interactions and the Higgs phenomenon

Riccardo Barbieri

CERN Summer Student Lectures

July 9-13, 2018

Can one “INVENT” a new interaction?

What determines the range of the weak interactions?

What makes our mass budget?

What about the Lorentz symmetry?

Under Lorentz: J=0, 1/2, 1 for $h, \ \Psi, \ A_\mu$

However for a charged J=1/2 particle

$\Psi(x) = \Psi_L(x) + \Psi_R(x)$ "L-R chirality"

with Ψ_L and Ψ_R transforming independently under Lorentz

As such, Ψ_L and Ψ_R can transform differently under the "internal" symmetry group

They do not in QCD and QED (hence P and C conserved, see below)

1. massive (charged or neutral) = "Dirac" $\Psi = \Psi_L + \Psi_R, \ \bar{\Psi} = \bar{\Psi}_L + \bar{\Psi}_R$

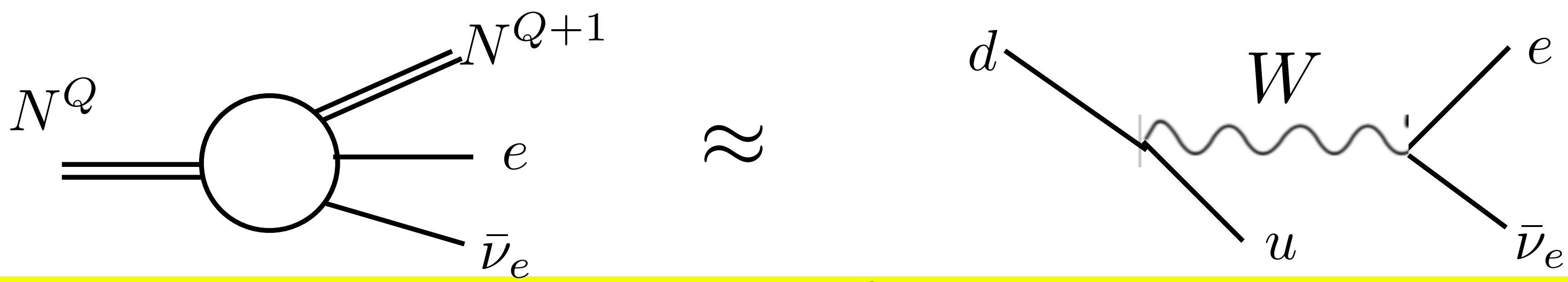
   $\Psi(\Uparrow\Downarrow) \neq \bar{\Psi}(\Uparrow\Downarrow)$

2. massless (neutral) = "Weyl" $\nu = \nu_L, \ \ \bar{\nu} = \bar{\nu}_R$ (chirality = "helicity")

   $\nu(\Leftarrow) \neq \bar{\nu}(\Rightarrow)$

How to "INVENT" a new interaction 1

Back to radioactivity (1896) in modern language



$$\mathcal{L}_F = \frac{G_F}{\sqrt{2}}(\bar{u}_L\gamma_\mu d_L)(\bar{e}_L\gamma_\mu\nu_L) \xrightarrow{\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W^2}} \mathcal{L}_W = \frac{g}{\sqrt{2}}W_\mu(\bar{u}_L\gamma_\mu d_L + \bar{\nu}_L\gamma_\mu e_L)$$

To mimic a gauge interaction $\mathcal{L}_I = gA_\mu^a\bar{\Psi}\gamma_\mu t^a\Psi, \quad [t^a, t^b] = if^{abc}t^c$

define

$$Q = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \quad L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad J_\mu^\pm = \bar{Q}\gamma_\mu\frac{\sigma^\pm}{2}Q + \bar{L}\gamma_\mu\frac{\sigma^\pm}{2}L$$

$$\sigma^\pm = \frac{1}{\sqrt{2}}(\sigma_1 \pm i\sigma_2) \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\mathcal{L}_W = \frac{g}{\sqrt{2}}(W_\mu^+J_\mu^- + W_\mu^-J_\mu^+)$$

How to "INVENT" a new interaction 2

$$\mathcal{L}_W \equiv \mathcal{L}^{(0)} = g(W^+_\mu J^-_\mu + W^-_\mu J^+_\mu) \qquad J^\pm_\mu = \bar{Q}\gamma_\mu \frac{\sigma^\pm}{2} Q + \bar{L}\gamma_\mu \frac{\sigma^\pm}{2} L$$

"To close the algebra" $[\sigma_1, \sigma_2] = 2i\sigma_3$ need to introduce a new interaction

$$\mathcal{L}^{(1)} = g W^3_\mu J^3_\mu \qquad J^3_\mu = \bar{Q}\gamma_\mu \frac{\sigma^3}{2} Q + \bar{L}\gamma_\mu \frac{\sigma^3}{2} L$$

Can we identify W^3_μ with the photon A_μ ?

$$J^3_\mu \equiv \bar{\Psi}\gamma_\mu T^3_L \Psi$$

	u_L	u_R	d_L	d_R	e_L	e_R	ν_L
T^3_L	$1/2$	0	$-1/2$	0	$-1/2$	0	$1/2$

$$\mathcal{L}^{em}_I = e A_\mu \bar{\Psi}\gamma_\mu Q_{em} \Psi$$

	u_L	u_R	d_L	d_R	e_L	e_R	ν_L
Q_{em}	$2/3$	$2/3$	$-1/3$	$-1/3$	-1	-1	0

NO! $T^3_L \neq Q_{em}$ Furthermore

$e^{i\alpha_j \sigma_j/2} = SU(2)$ matrices, BUT

$$Q_{em}\sigma^+ L \neq \sigma^+ Q_{em} L \quad \Rightarrow [Q_{em}, \sigma^+] \neq 0$$

The algebra still not closed!

How to "INVENT" a new interaction 3

To improve the situation, consider

$$\mathcal{L}_I^B = g' B_\mu \bar{\Psi} \gamma_\mu Y \Psi$$

	u_L	u_R	d_L	d_R	e_L	e_R	ν_L
Y	$1/6$	$2/3$	$1/6$	$-1/3$	$-1/2$	-1	$-1/2$

with B_μ a new boson and its associated $U(1)_Y$

$$Q = T_L^3 + Y, \quad [Y, \sigma_i] = 0 \quad \Rightarrow SU(2) \times U(1)_Y$$

Does it work? Suppose that (to be explained in a while)

$$W_\mu^3 = sin\theta A_\mu + cos\theta Z_\mu, \quad B_\mu = cos\theta A_\mu - sin\theta Z_\mu \qquad \tan\theta \equiv \frac{g'}{g}$$

Then

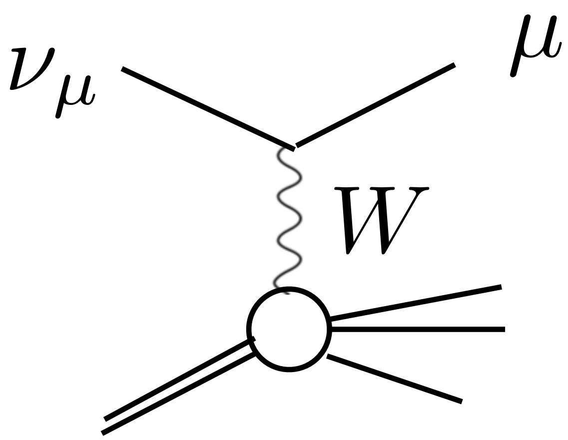
$$\mathcal{L}^{(1)} + \mathcal{L}_I^B = eA_\mu J_\mu^{em} + \frac{g}{cos\theta} Z_\mu J_\mu^{(Z)}$$

with

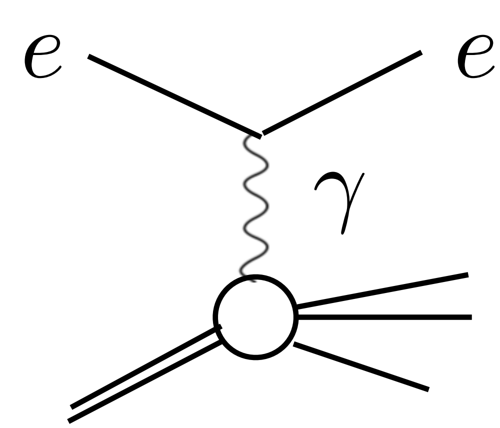
$$J_\mu^{(Z)} = \bar{\Psi}(T_L^3 - sin^2\theta Q_{em})\Psi = \frac{1}{2}\bar{\nu}_L \gamma_\mu \nu_L + \ldots$$

The discovery of "neutral currents"

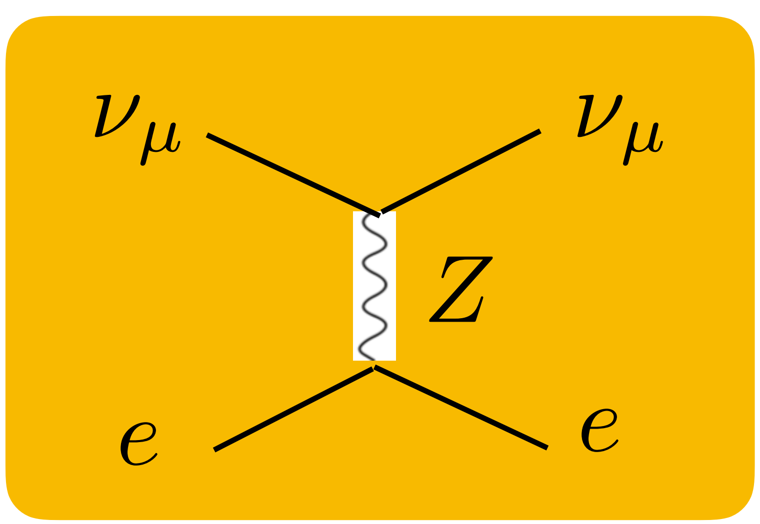
The "charged current" interaction

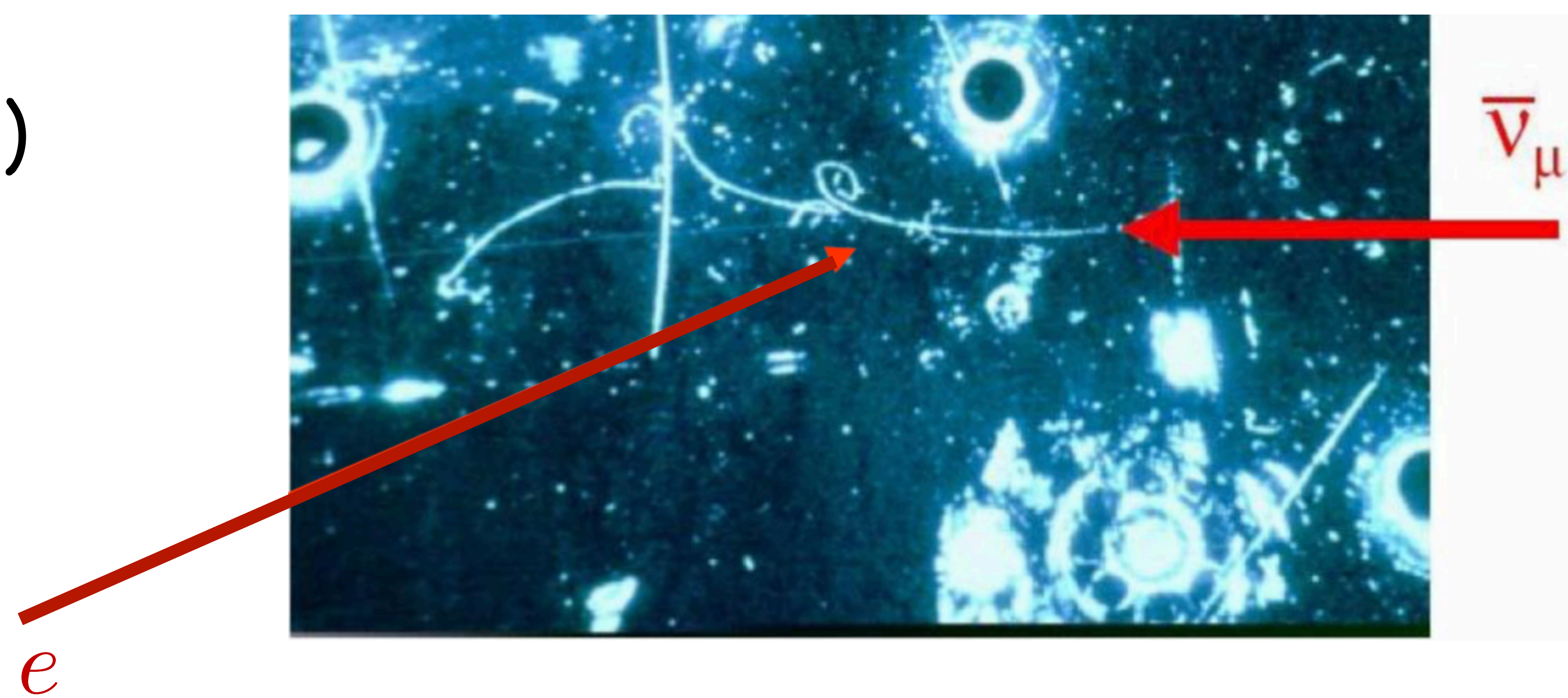


The electromagnetic interaction



The "neutral current" interaction (predicted)





CERN 1973: a "bubble chamber" event

The full gauge Lagrangian (a recap)

Gauge symmetry: $SU(3) \times SU(2) \times U(1)_Y$

$$A_\mu^A = (G_\mu^\alpha; W_\mu^a; B_\mu) \qquad \mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} \equiv G_{\mu\nu}^\alpha G_{\mu\nu}^\alpha + W_{\mu\nu}^a G_{\mu\nu}^a + B_{\mu\nu}B_{\mu\nu}$$

$$\Psi = Q(\mathbf{3},\mathbf{2})_{1/6}\ u_R(\bar{\mathbf{3}},\mathbf{1})_{2/3}\ d_R(\bar{\mathbf{3}},\mathbf{1})_{-1/3}\ L(\mathbf{1},\mathbf{2})_{-1/2}\ e_R(\mathbf{1},\mathbf{1})_{-1}$$

Most general $\mathcal{L}^{(\leq 4)}$

$$\mathcal{L} = -\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}_{\mu\nu} + i\bar{\Psi}\not{D}\Psi$$

$$\not{D} = \gamma_\mu D_\mu = \gamma_\mu(\partial_\mu + g_s G_\mu^\alpha \frac{\lambda^\alpha}{2} + g W_\mu^a \frac{\sigma^a}{2} + g' B_\mu Y)$$

Realistic?
No mass term allowed
by the gauge symmetry

fermions $m\bar{\Psi}\Psi = m\bar{\Psi}_L\Psi_R = \mathbf{2} \times \mathbf{1} \not\supset \mathbf{1}$

vectors $m^2 A_\mu A_\mu$ against g.i. $A_\mu \to A_\mu + \partial_\mu \alpha(x)$

Add a scalar $h = (\mathbf{1},\mathbf{2})_{-1/2}$ $\mathcal{L}_Y = \lambda h \bar{\Psi}_L \Psi_R = \mathbf{2} \times \mathbf{2} \times \mathbf{1} \supset \mathbf{1}$

The SM Lagrangian (finally)

$$\mathcal{L}_{\sim SM} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + i\bar{\psi}\,\not{D}\psi$$

$$+|D_\mu h|^2 - V(h)$$

$$+\psi_i\lambda_{ij}\psi_j h + h.c.$$

new!

where

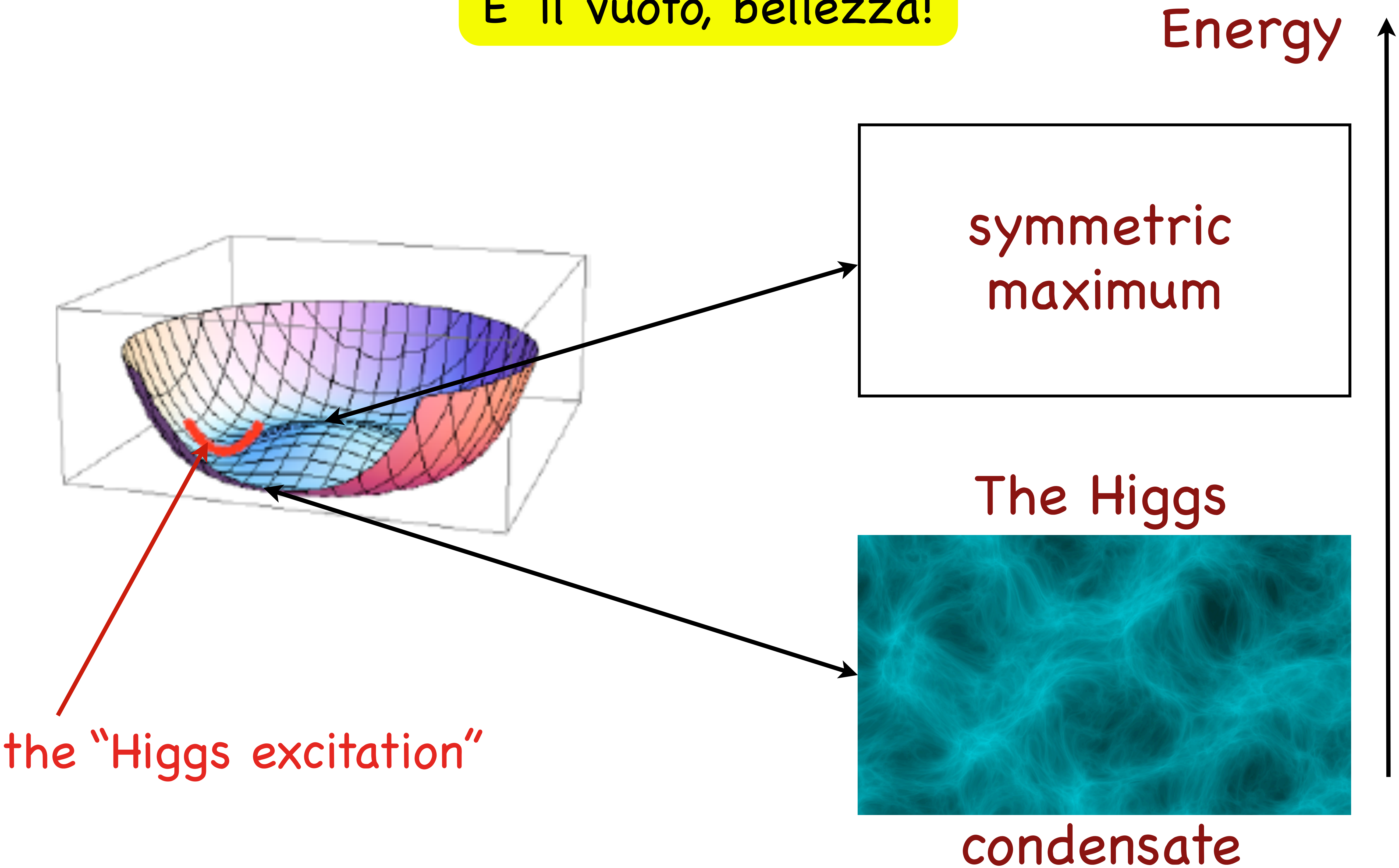
$$h = \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} \qquad D_\mu h = (\partial_\mu + gW^a_\mu\frac{\sigma^a}{2} - g'\frac{1}{2}B_\mu)h$$

$$V(h) = -\mu^2|h|^2 + \lambda(|h|^2)^2 \qquad |h|^2 = h_1^* h_1 + h_2^* h_2$$

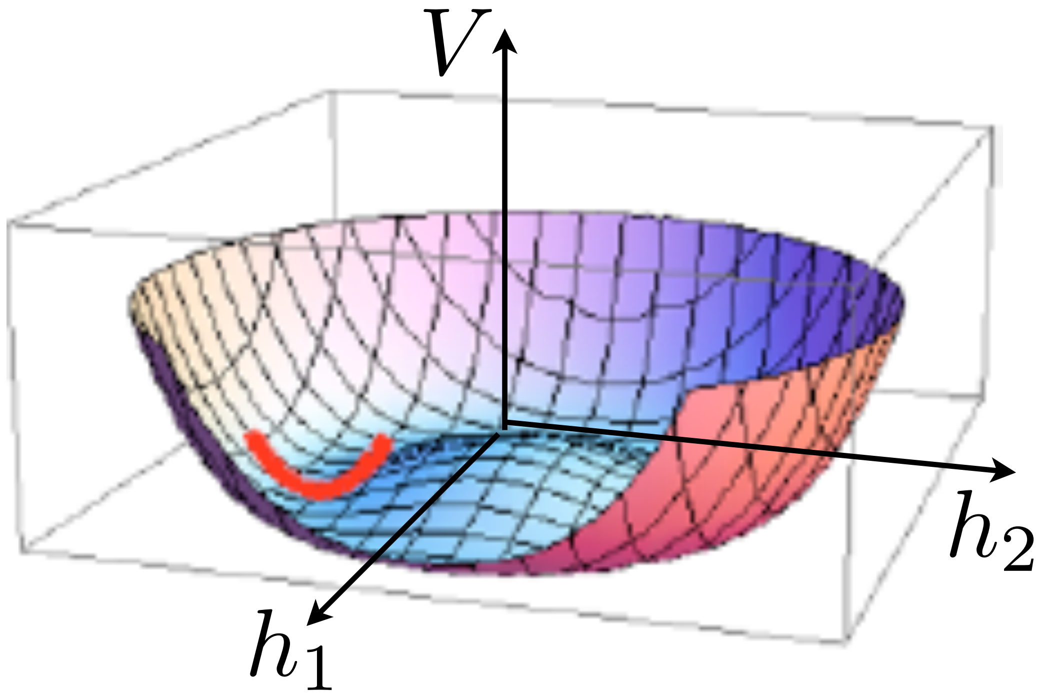
$$\Psi_i\lambda_{ij}\Psi_j = h\bar{Q}_{L_i}\lambda^u_{ij}u_{R_j} + h^+\bar{Q}_{L_i}\lambda^d_{ij}d_{R_j} + h^+\bar{L}_{L_i}\lambda^e_{ij}e_{R_j}$$

Spontaneous breaking of the EW symmetry

E' il vuoto, bellezza!



$$V(h) = -\mu^2|h|^2 + \lambda(|h|^2)^2$$



At the minimum $< h >$

$$| < h > |^2 = \frac{\nu^2}{2\lambda} \equiv v^2$$

$$< h >= \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$h \to e^{i\chi_a \sigma_a/2} \begin{pmatrix} v + \frac{H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

Consequences:

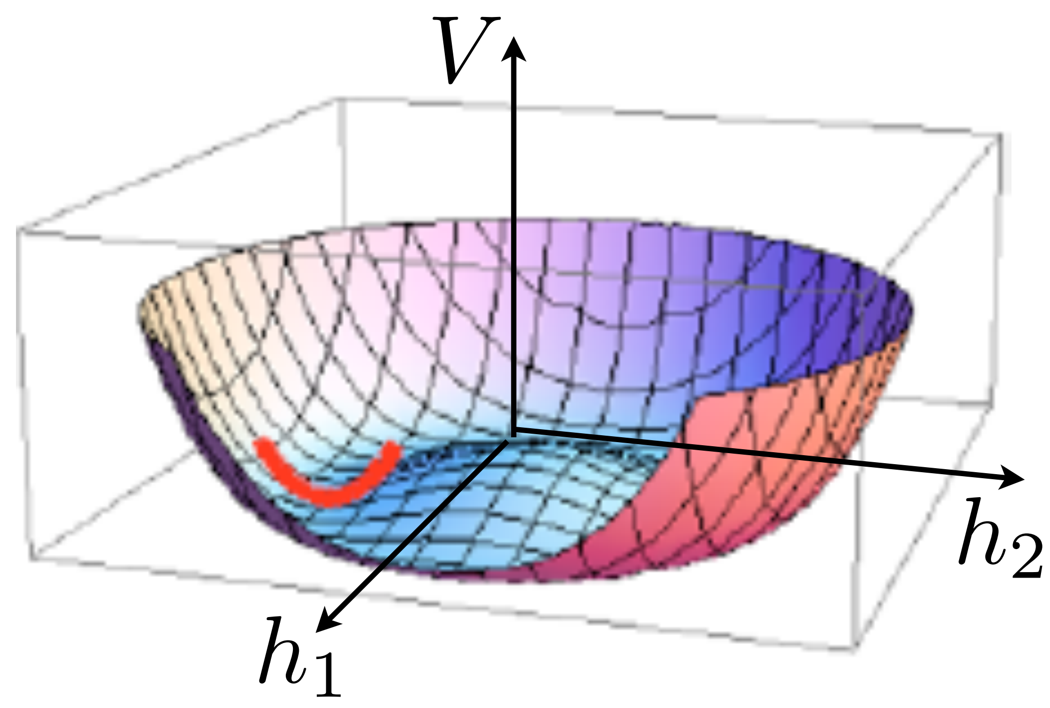
1. $Q_{em} < h >= 0 \Rightarrow SU(2) \times U(1)_Y \to U(1)_{em}$

2. The "radial excitation", the Higgs boson, gets mass $m_H = 2\sqrt{\lambda}v$

3. The Yukawa couplings give fermions a mass

E.g. $\lambda^u h\bar{Q}_L u_R \to \lambda^u(v + \frac{H}{\sqrt{2}})\bar{u}_L u_R$

Note the linear relation between the mass $m_u = \lambda^u v$ and the coupling λ^u



At the minimum $< h >$

$$| < h > |^2 = \frac{\nu^2}{2\lambda} \equiv v^2$$

$$< h > = \begin{pmatrix} v \\ 0 \end{pmatrix}$$

$$h \to e^{i\chi_a \sigma_a / 2} \begin{pmatrix} v + \frac{H}{\sqrt{2}} \\ 0 \end{pmatrix}$$

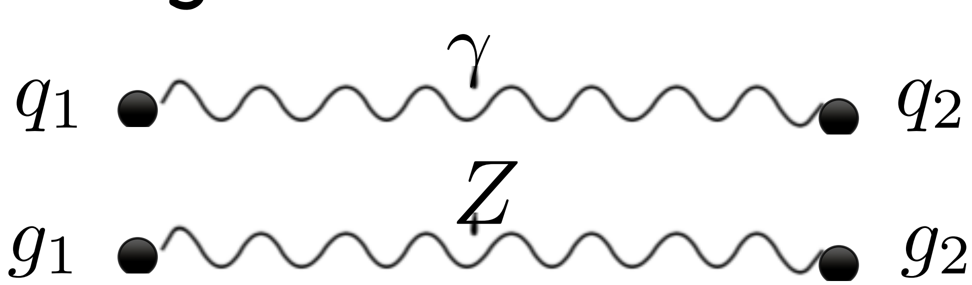
Further consequences:

4. The vectors of $SU(2) \times U(1)_Y$, all but the photon, get a mass

$$|D_\mu < h > |^2 \Rightarrow m^2_{W^\pm} = g^2 v^2 / 2, \quad m^2_Z = m^2_{W^\pm} / cos^2\theta$$

$$A_\mu = sin\theta W^3_\mu + cos\theta B_\mu, \quad Z_\mu = cos\theta W^3_\mu - sin\theta B_\mu, \quad tan\theta = g'/g$$

5. Range of the weak versus em interactions:



$$V = \frac{q_1 q_2}{r}$$

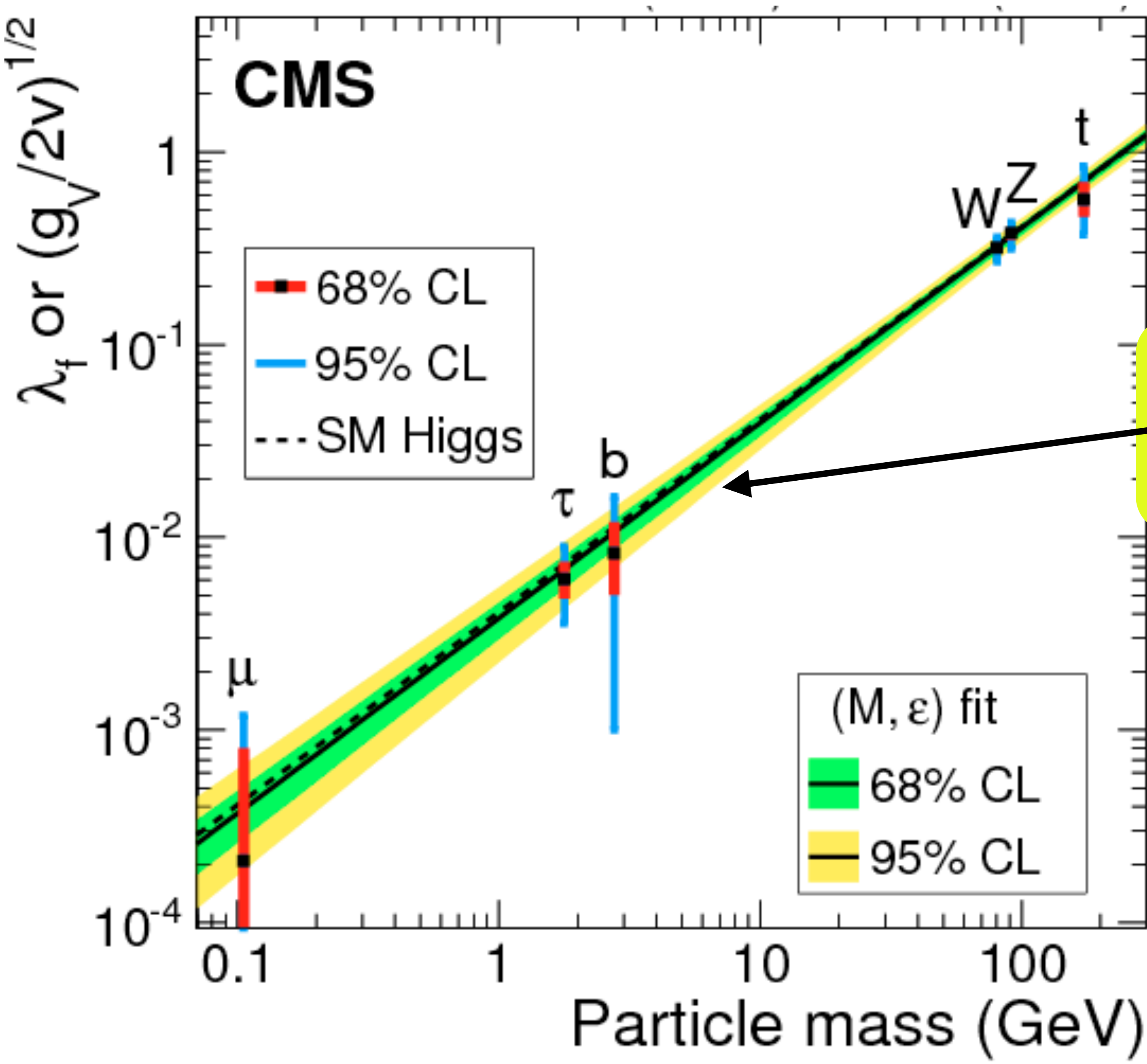
$$V = \frac{g_1 g_2}{r} e^{-m_Z r}, \quad 1/m_Z \approx 10^{-16} cm$$

6. What about the three χ_a ?

The linear relation between masses and Higgs couplings

E.g.

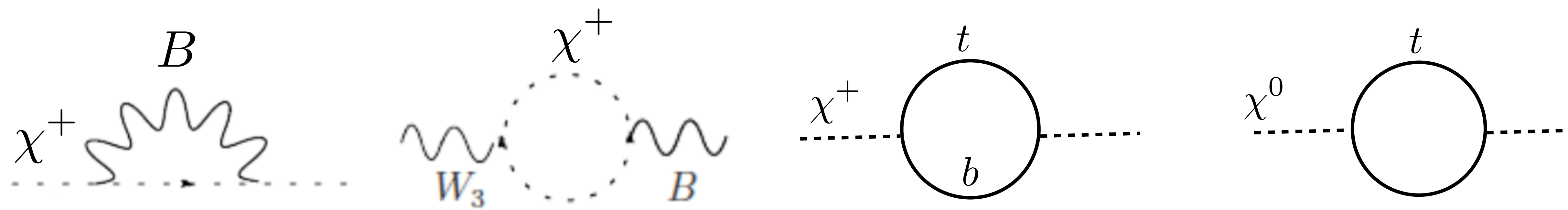
$$\lambda^u h\bar{Q}_L u_R \to \lambda^u (v + \frac{H}{\sqrt{2}})\bar{u}_L u_R$$

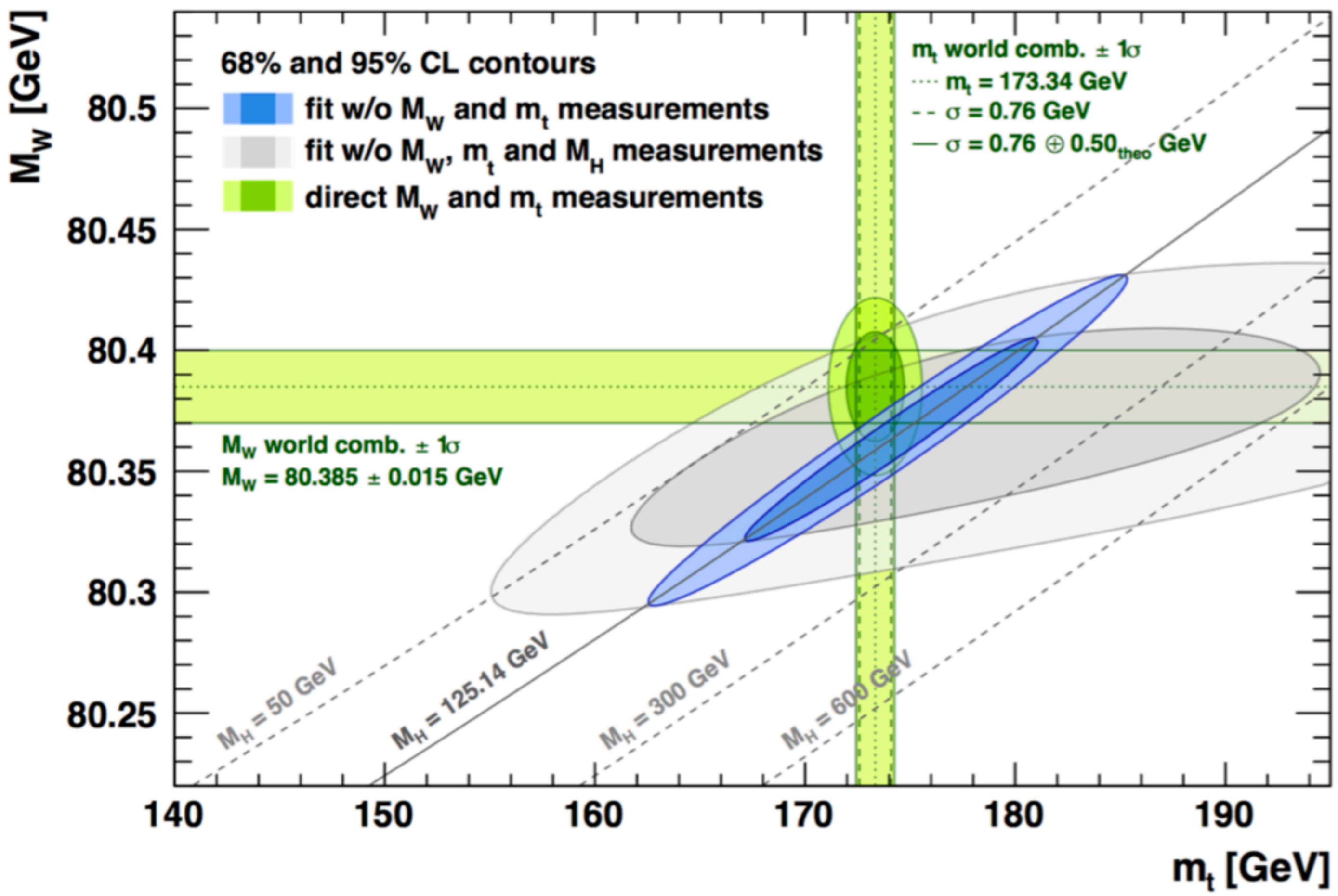


the slope of the line is the only parameter (v)

Clear evidence, although limited for the time being to the heavier particles only (note the log scale!)

Do we see virtual particle effects here as well?





Blue = prediction of m_t, M_W by fitting various ew data in the theory, with crucial inclusion of the loop effects above

Green = direct measurements of m_t, M_W

Our mass budget

We are made of

e , p (mostly uud), n (mostly odd)

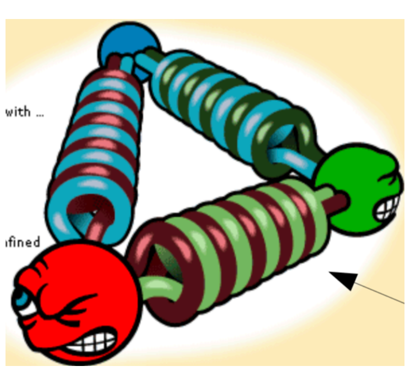
$\Delta m_{us} \approx 1\%$ from the Higgs vacuum

$$m_e = \lambda_e v = 0.510\ 998\ 928(11)\ MeV$$

$$m_u = \lambda_u v = 2.3 \pm 0.7\ MeV$$

$$m_d = \lambda_d v = 4.8 \pm 0.5\ MeV$$

$\Delta m_{us} \approx 99\%$ from back reaction to QCD forces



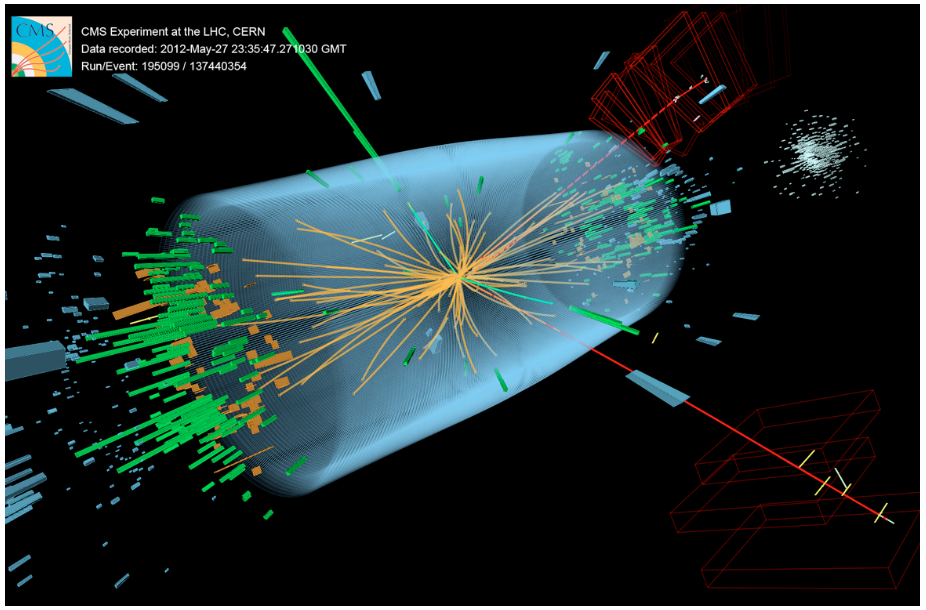
$E = mc^2$ YES!

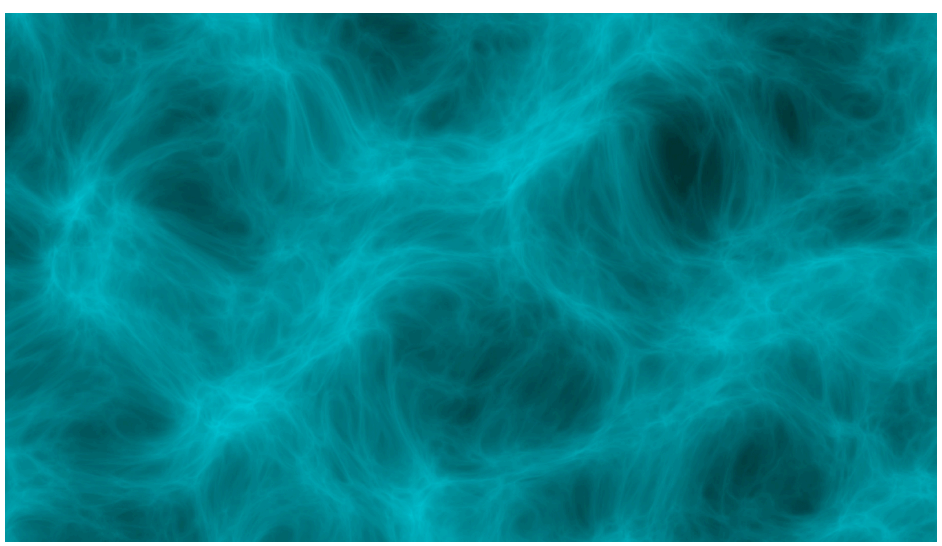
$$m_p = 938.272\ 046(21)\ MeV$$

$$m_n = 939.565\ 379(21)\ MeV$$

Puzzle 1

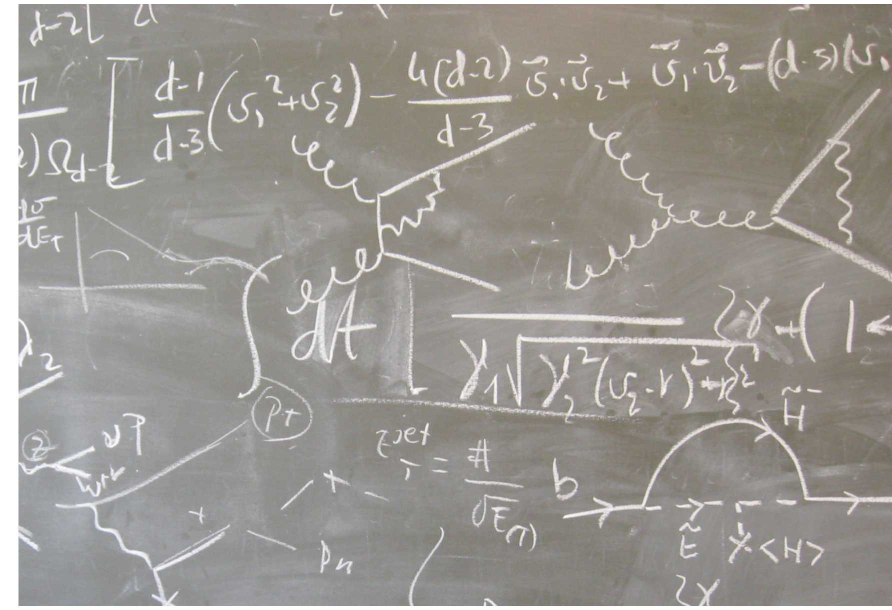
Experiment





$$\Rightarrow \quad v = 175 \ GeV$$

Theory



? But if the SM is naively extrapolated at high energies

$$\Rightarrow \quad v \approx 10^{18} \ GeV$$