Lecture 4

The puzzle of the replicas/generations (of flavour)

Riccardo Barbieri

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Where are they? How do we see them?

How do they communicate among each others?

How do we know how many there are?

Is the proton forever?

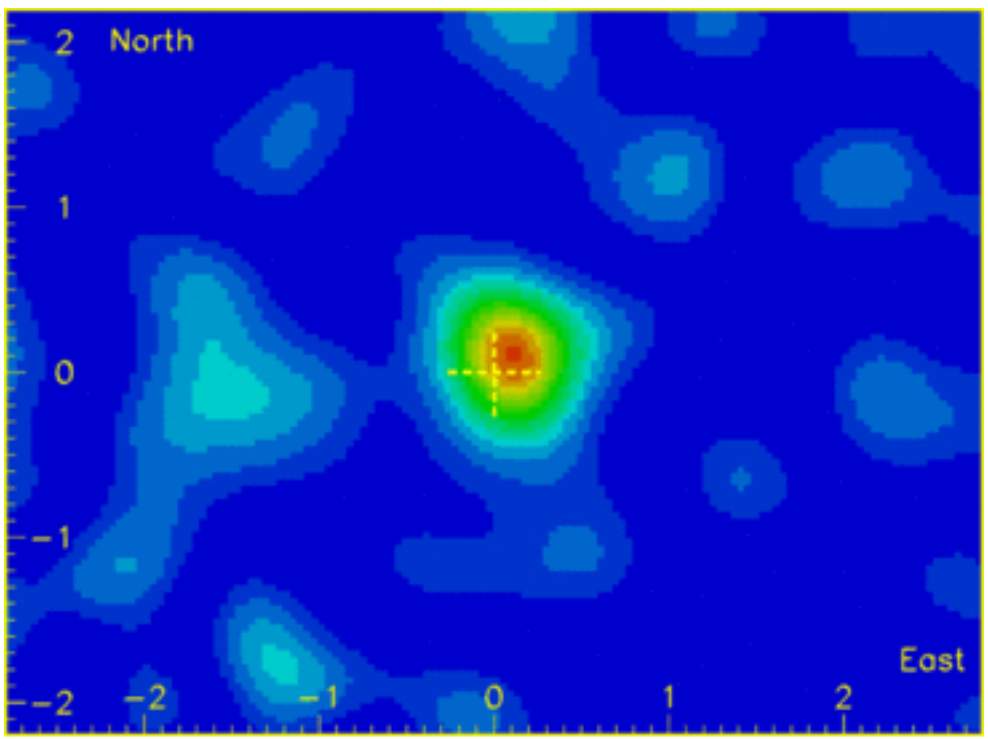
Is there an asymmetry between matter and antimatter?

Where are they?

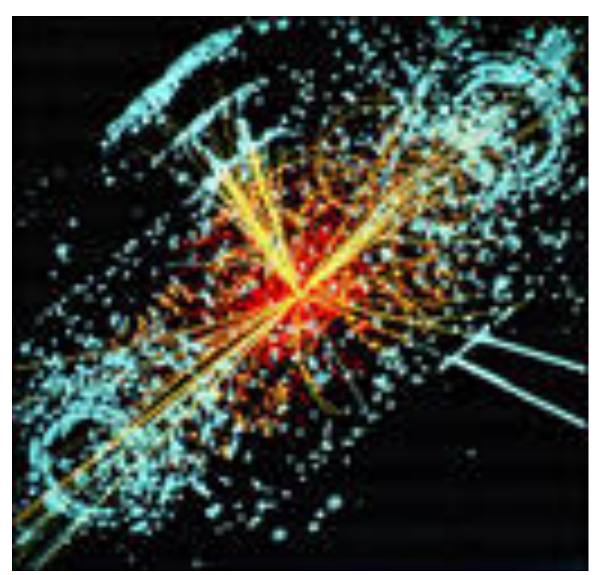
(How do we see them?)

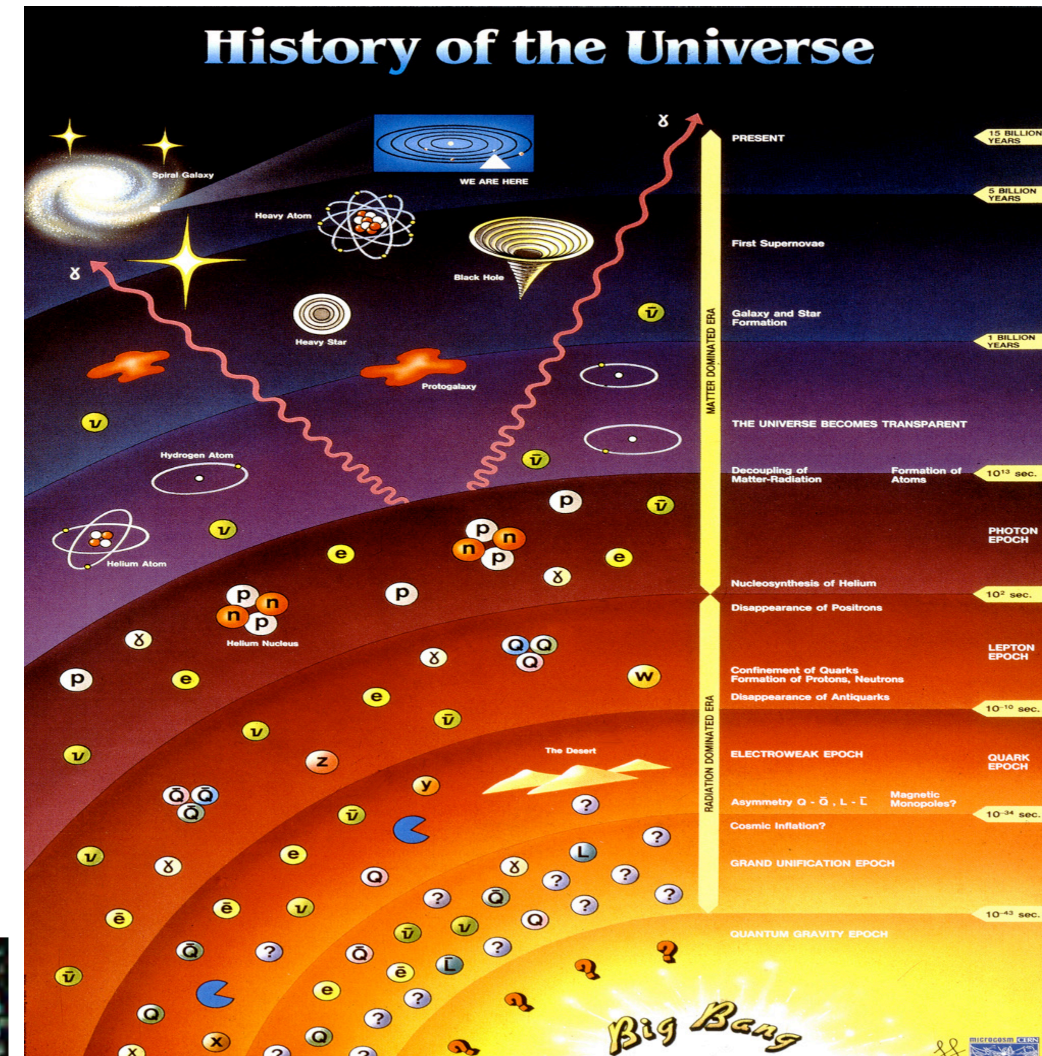
The only thing you need is enough energy

cosmic ray collisions
in the atmosphere



high energy particle
collisions





in the early
universe

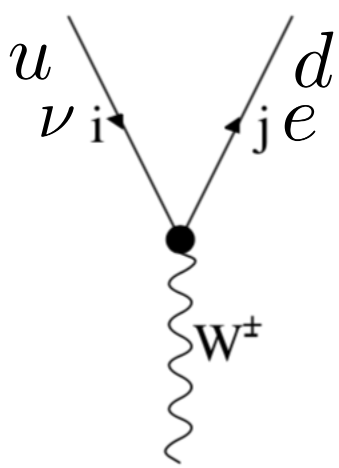
Flavour "theorems", to be proven

$$\mathcal{L}_Y = \Psi_i \lambda_{ij} \Psi_J + h.c.$$

0. In QCD + QED no communication between different families (as anticipated)

1. Baryon (B) and individual lepton (L_e, L_μ, L_τ) numbers conserved

2. The only communication between families in the weak changed current proportional to a unitary matrix



$$g V_{ij}, \quad V V^+ = 1 \qquad V(\theta_1, \theta_2, \theta_3, \delta)$$

3. "CP violation" *if and only if* at least 3 families

In QCD + QED no communication between different families

Symmetry: $SU(3) \times U(1)_{em}$

Particle content: $u = \mathbf{3}_{2/3}, d = \mathbf{3}_{1/3}, e = \mathbf{1}_{-1}$

From this (in full generality?):

$$\mathcal{L}_{QCD+QED}(u_i) = i\bar{u}_{Li} \not{D}^u u_{Li} + i\bar{u}_{Ri} \not{D}^u u_{Ri} + (\lambda^u_{ij} v \bar{u}_{Li} u_{Rj} + h.c.)$$

(and similarly for d_i and e_i)

Inter-family communication from off-diagonal λ_{ij} ?

NO!

For any matrix $\lambda v \equiv m = V_L m^{diag} V_R^+$

so that, after: $u_L \to V_L u_L, \quad u_R \to V_R u_R$ (the "physical basis")

$$\mathcal{L}_{QCD+QED}(u_i) = i\bar{u}_{Li} \not{D}^u u_{Li} + i\bar{u}_{Ri} \not{D}^u u_{Ri} + (m^u_i \bar{u}_{Li} u_{Ri} + h.c.)$$
$$= i\bar{u}_i \not{D}^u u_i + m^u_i \bar{u}_i u_i, \quad u = u_L + u_R$$

What happens with all interactions included?

Flavour in the lepton sector

(part of Theorem 1)

$$\Psi = Q(\mathbf{3},\mathbf{2})_{1/6}\; u_R(\bar{\mathbf{3}},\mathbf{1})_{2/3}\; d_R(\bar{\mathbf{3}},\mathbf{1})_{-1/3}\; L(\mathbf{1},\mathbf{2})_{-1/2}\; e_R(\mathbf{1},\mathbf{1})_{-1}$$

$$\mathcal{L} = i\bar{L}_{Li}\not{D}^L L_{Li} + i\bar{e}_{Ri}\not{D}^e e_{Ri} + (\lambda^e_{ij} h^+ \bar{L}_{Li} e_{Rj} + h.c.)$$

As before: $\lambda^e = V_L \lambda^e_{diag} V_R^+ ,\qquad L_L \to V_L L_L,\quad e_R \to V_R e_R$

$$\mathcal{L} = i\bar{L}_{Li}\not{D}^L L_{Li} + i\bar{e}_{Ri}\not{D}^e e_{Ri} + (\lambda^e_i h^+ \bar{L}_{Li} e_{Ri} + h.c.)$$

$\Rightarrow L_e, L_\mu, L_\tau$ individually conserved

where

	ν_{eL}	e_L	e_R
L_e	1	1	1

and -1 for the antiparticles

$$\Psi = a^- + b^+$$

(similarly for L_μ, L_τ)

YES $\quad \mu^- \to e^- + \bar{\nu}_e + \nu_\mu \qquad \tau(\mu) = 2.196\ 981\ 1(22)10^{-6}\ s$

NO $\quad \mu^\pm \nrightarrow e^\pm + \gamma \qquad \dfrac{\tau(\mu \to e + \gamma)}{\tau(\mu \to e + \bar{\nu}_e + \nu_\mu)} > 3 \cdot 10^{12}$

Flavour in the quark sector

(completing Theorem 1 and Theorem 2)

$$\Psi = Q(\mathbf{3},\mathbf{2})_{1/6}\ u_R(\bar{\mathbf{3}},\mathbf{1})_{2/3}\ d_R(\bar{\mathbf{3}},\mathbf{1})_{-1/3}\ L(\mathbf{1},\mathbf{2})_{-1/2}\ e_R(\mathbf{1},\mathbf{1})_{-1}$$

$$\mathcal{L} = i\bar{Q}_{Li}\not{D}^Q Q_{Li} + i\bar{u}_{Ri}\not{D}^u u_{Ri} + i\bar{d}_{Ri}\not{D}^d d_{Ri} + (\lambda^u_{ij} h\bar{Q}_{Li}u_{Rj} + \lambda^d_{ij} h^+\bar{Q}_{Li}d_{Rj} + h.c.)$$

Remember that, after ew symmetry breaking:

$$\lambda^u_{ij} h\bar{Q}_{Li}u_{Rj} + \lambda^d_{ij} h^+\bar{Q}_{Li}d_{Rj} \to (v + \frac{H}{\sqrt{2}})(\lambda^u_{ij}\bar{u}_{Li}u_{Rj} + \lambda^d_{ij}\bar{d}_{Li}d_{Rj})$$

Using again $\lambda^u = U_L\lambda^u_{diag}U^+_R, \quad \lambda^d = D_L\lambda^d_{diag}D^+_R$ one goes to the "physical basis" by $u_L \to U_L u_L,\ u_R \to U_R u_R,\ d_L \to D_L d_L,\ d_R \to D_R d_R$

These unitary rotations go away in every interaction with Z_μ (and G^a_μ, A_μ)

$\Rightarrow$ No "Flavour Changing Neutral Current"

but stay in

$$\frac{g}{\sqrt{2}}W^+_\mu\bar{u}_L\gamma_\mu d_L \to \frac{g}{\sqrt{2}}W^+_\mu\bar{u}_L U^+_L D_L\gamma_\mu d_L \equiv \frac{g}{\sqrt{2}}W^+_\mu\bar{u}_L V_{CKM}\gamma_\mu d_L$$

$$V_{CKM}V^+_{CKM} = 1$$

Consequences of Theorem 1

In the quark sector what is conserved are not the individual quark numbers (unlike L_e, L_μ, L_τ) but only the overall "Baryon Number"

	u_i	d_i
B	$1/3$	$1/3$

and -1/3 for the antiparticles

(with the normalisation chosen so that B(p)=B(n)=1)

The proton, the lightest particle with $B \neq 0$, is stable as is the electron stable, being the lightest charged particle

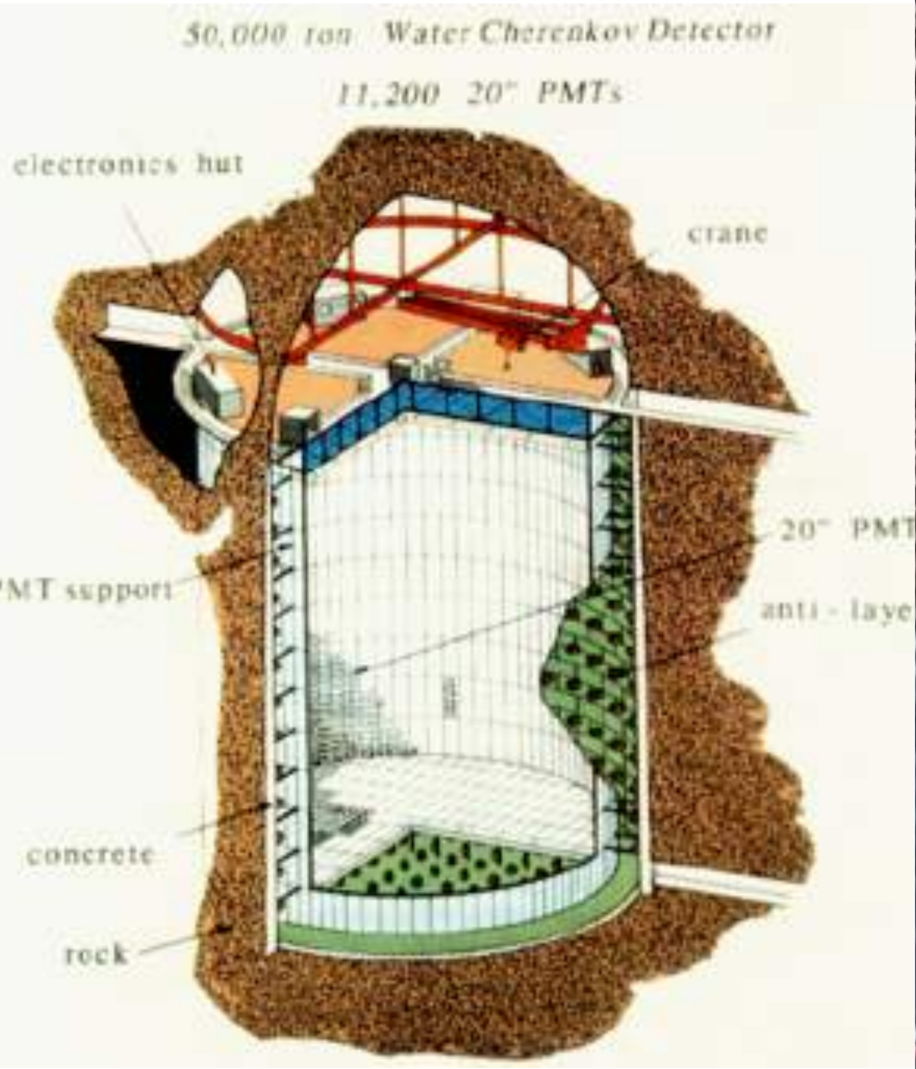
$$\tau(e) > 4.6 \cdot 10^{26} \; years$$

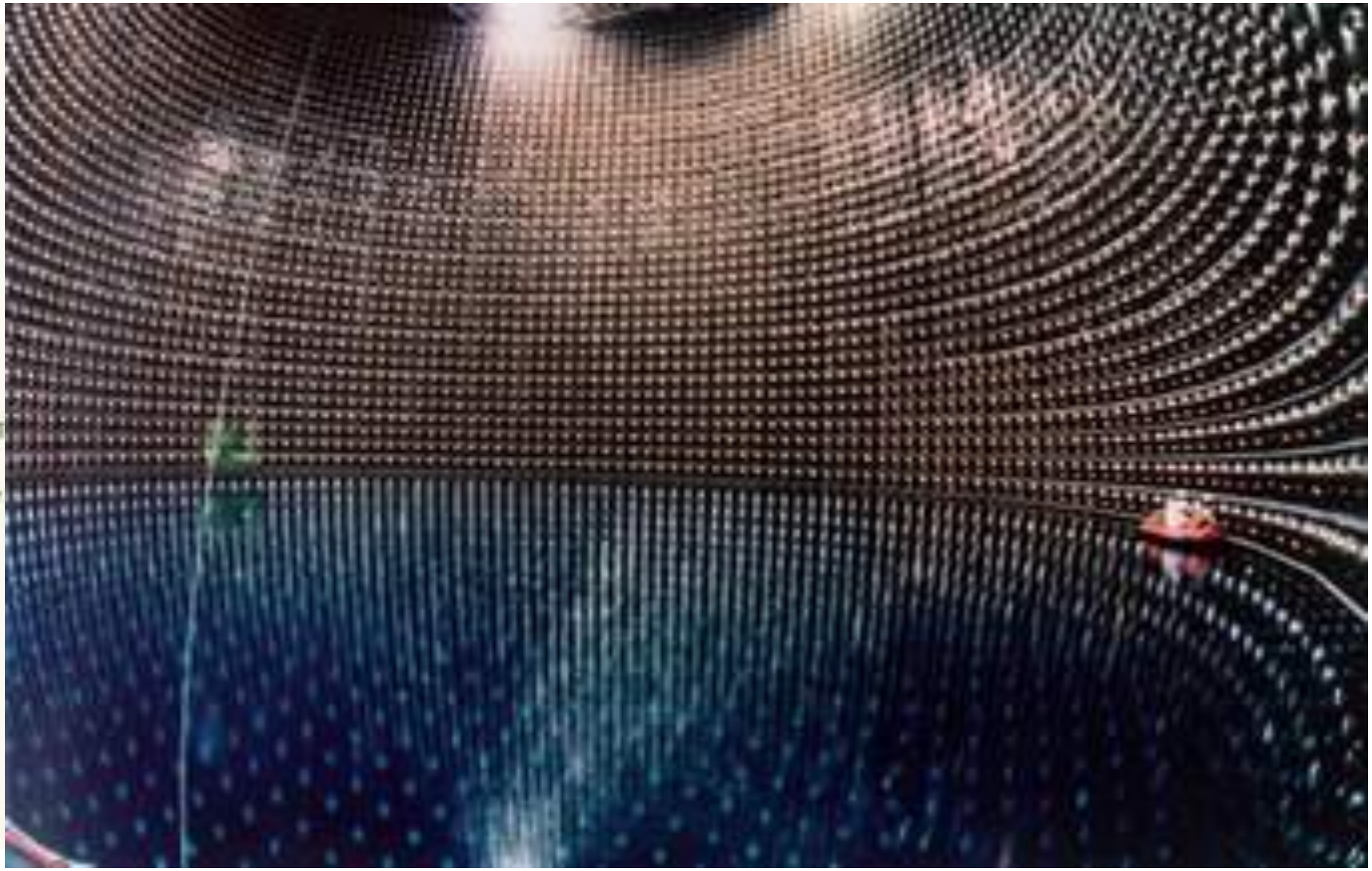
For reference $\tau(\mu) = 2.196\;981\;1(22)10^{-6}\; s$

Are the protons forever?

E.g.: $p(uud) \nrightarrow e^+ + \pi^0(\bar{u}u, \bar{d}d)$

$$\tau(p \rightarrow e^+ + \pi^0) > 10^{34}\ years$$

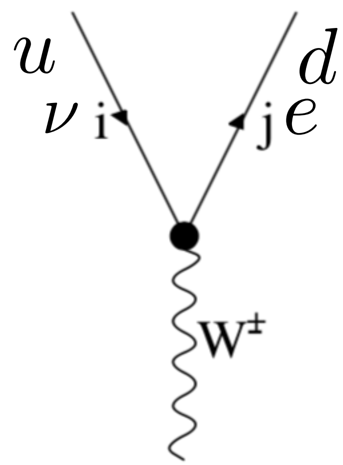




Kamioka mine - Japan

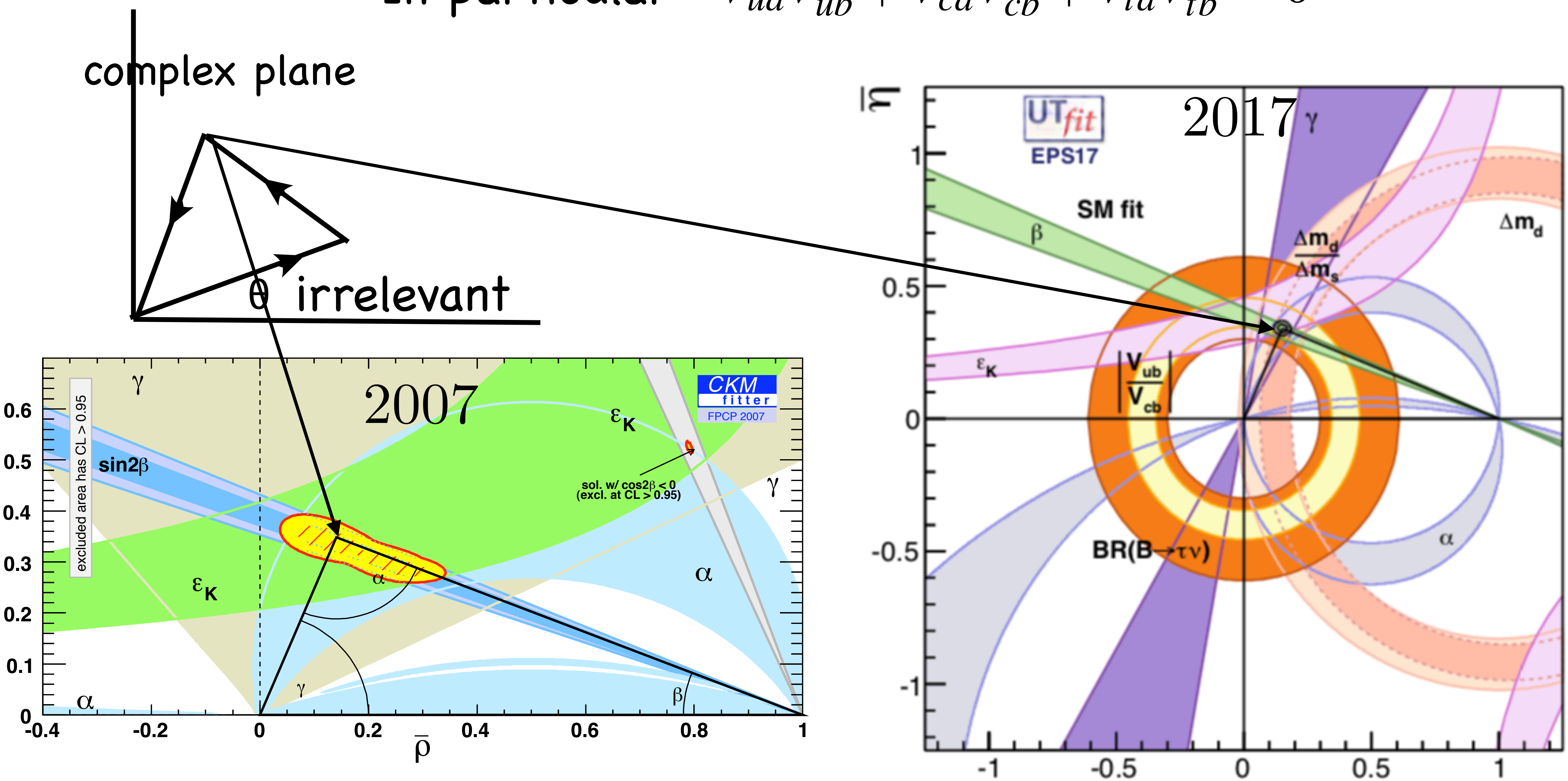
watching tens of kilotons of water for years and making sure that not a single proton has decayed

Testing Theorem 2



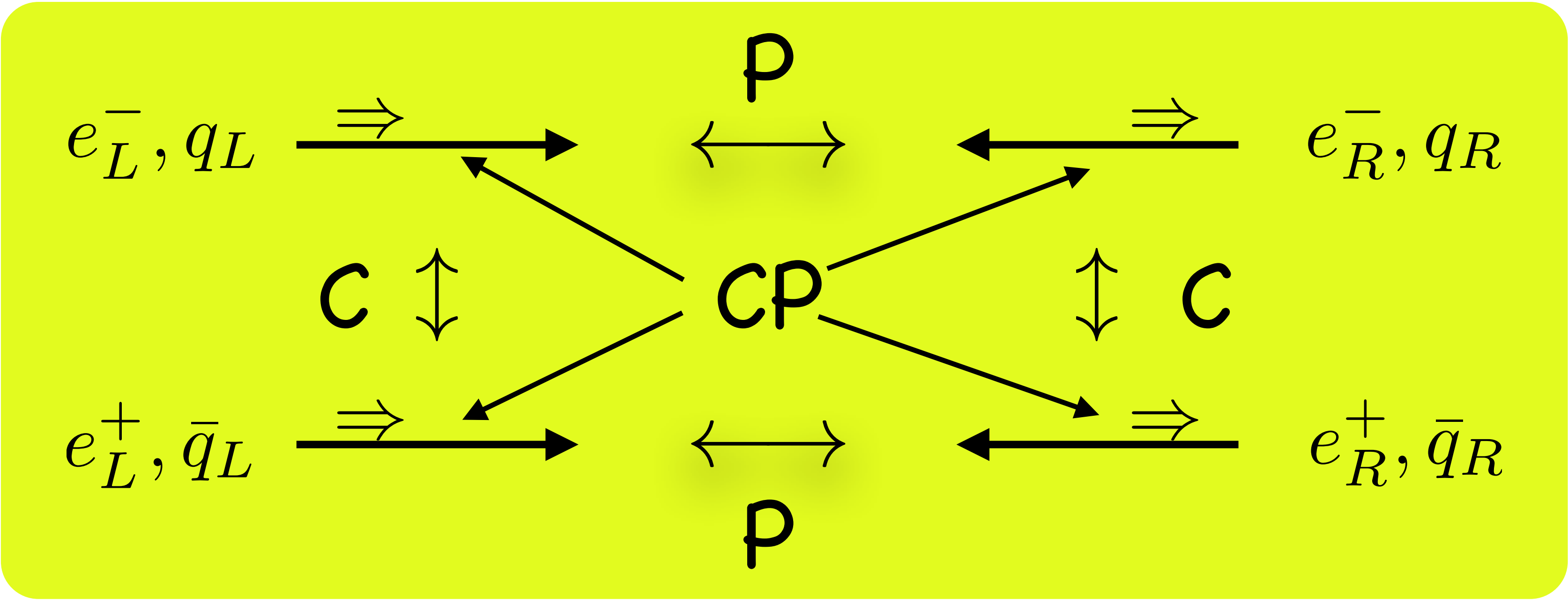
$$gV_{ij}, \quad VV^{+} = 1 \qquad V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In particular $V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$



Parity (P), Charge Conjugation (C), and CP

On fermions:



With suitable definitions of P,C on vectors as well, are the corresponding interactions invariant under P, C or CP?

A reason to care about CP violation:

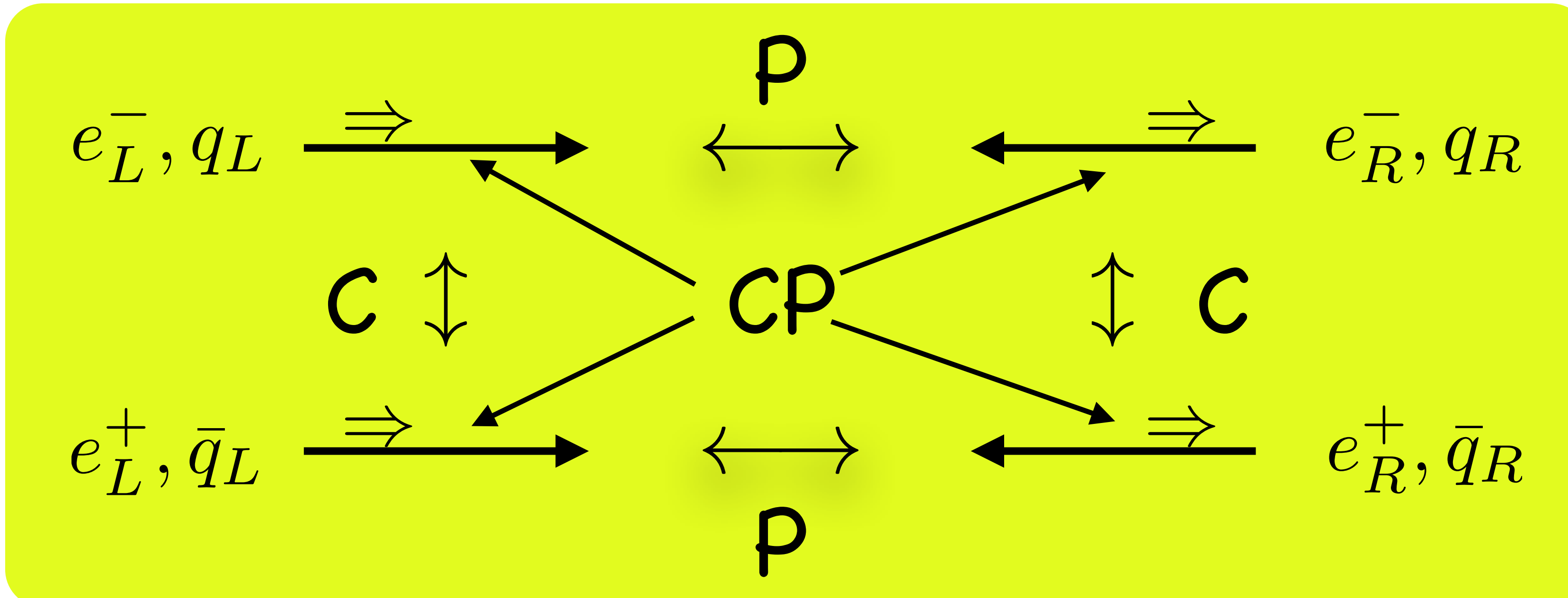
If CP were conserved, then e.g.

$\Gamma(P \to p_1 \ldots p_n) = \Gamma(\bar{P} \to \bar{p}_1 \ldots \bar{p}_n)$ for any P and $p_1 \ldots p_n$

In such a case, what would have caused the manifest matter-antimatter asymmetry in the universe if none, very likely, was there at the beginning?

Parity (P), Charge Conjugation (C), and CP

On fermions:



From the Lagrangian (again and again)

$$SU(3) \times U(1)_{em}$$

$u = \mathbf{3}_{2/3}, d = \mathbf{3}_{1/3}, e = \mathbf{1}_{-1}$

	G^a_μ	A_μ	Z_μ	W_μ
P	YES	YES	NO	NO
C	YES	YES	NO	NO
CP	YES	YES	YES	?

$$SU(2) \times U(1)_Y$$

$Q(\mathbf{3},\mathbf{2})_{1/6}$ $u_R(\bar{\mathbf{3}},\mathbf{1})_{2/3}$ $d_R(\bar{\mathbf{3}},\mathbf{1})_{-1/3}$

$L(\mathbf{1},\mathbf{2})_{-1/2}$ $e_R(\mathbf{1},\mathbf{1})_{-1}$

Given the YES in the second column, the discovery of the NO in the fourth line in 1956 was a big surprise!

Theorem 3. "CP violation" *if and only if* at least 3 families

Under a CP transformation:

$$gW_\mu^+ \bar{u}\gamma_\mu V d + gW_\mu^- \bar{d}\gamma_\mu V^+ u \Rightarrow gW_\mu^- \bar{d}\gamma_\mu V^T u + gW_\mu^+ \bar{u}\gamma_\mu V^* d$$

Hence CP violation requires V "intrinsically" complex

$$N(V_{n\times n},\ VV^+ = 1) = n^2 \qquad N(O_{n\times n},\ OO^T = 1) = \frac{n(n-1)}{2}$$

$$\Rightarrow\ N(phys.phases) = n^2 - \frac{n(n-1)}{2} - (2n-1) = \frac{1}{2}(n^2 - 3n + 2)$$

2n quarks → $2n$; $U(1)_B$ → -1

n	2	3	4
angles	1	3	6
phys. phases	0	1	3

Q.E.D.

In particular, with n = 3, $V_{CKM}(\theta_1, \theta_2, \theta_3; \delta)$

Testing Theorem 3

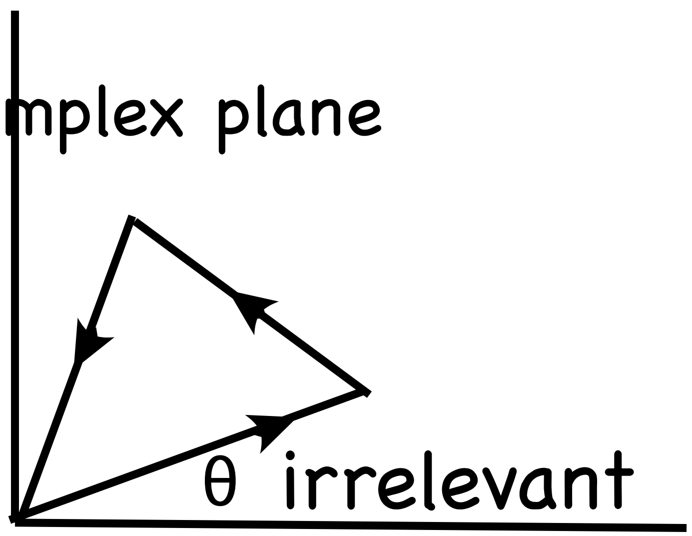
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

In particular $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

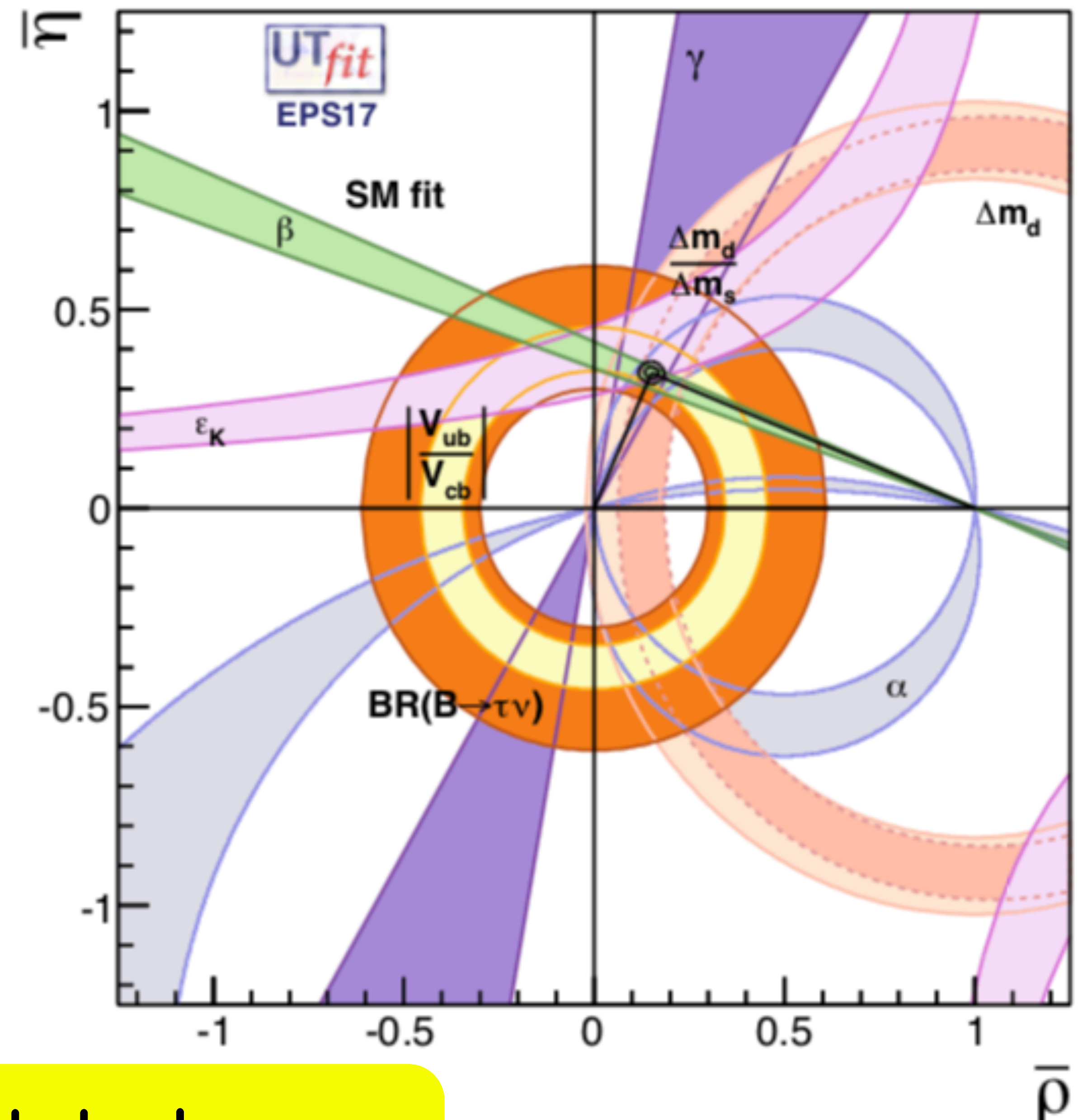
If $\delta = 0$ in $V_{CKM}(\theta_1, \theta_2, \theta_3; \delta)$

(no CP violation)

the triangle in



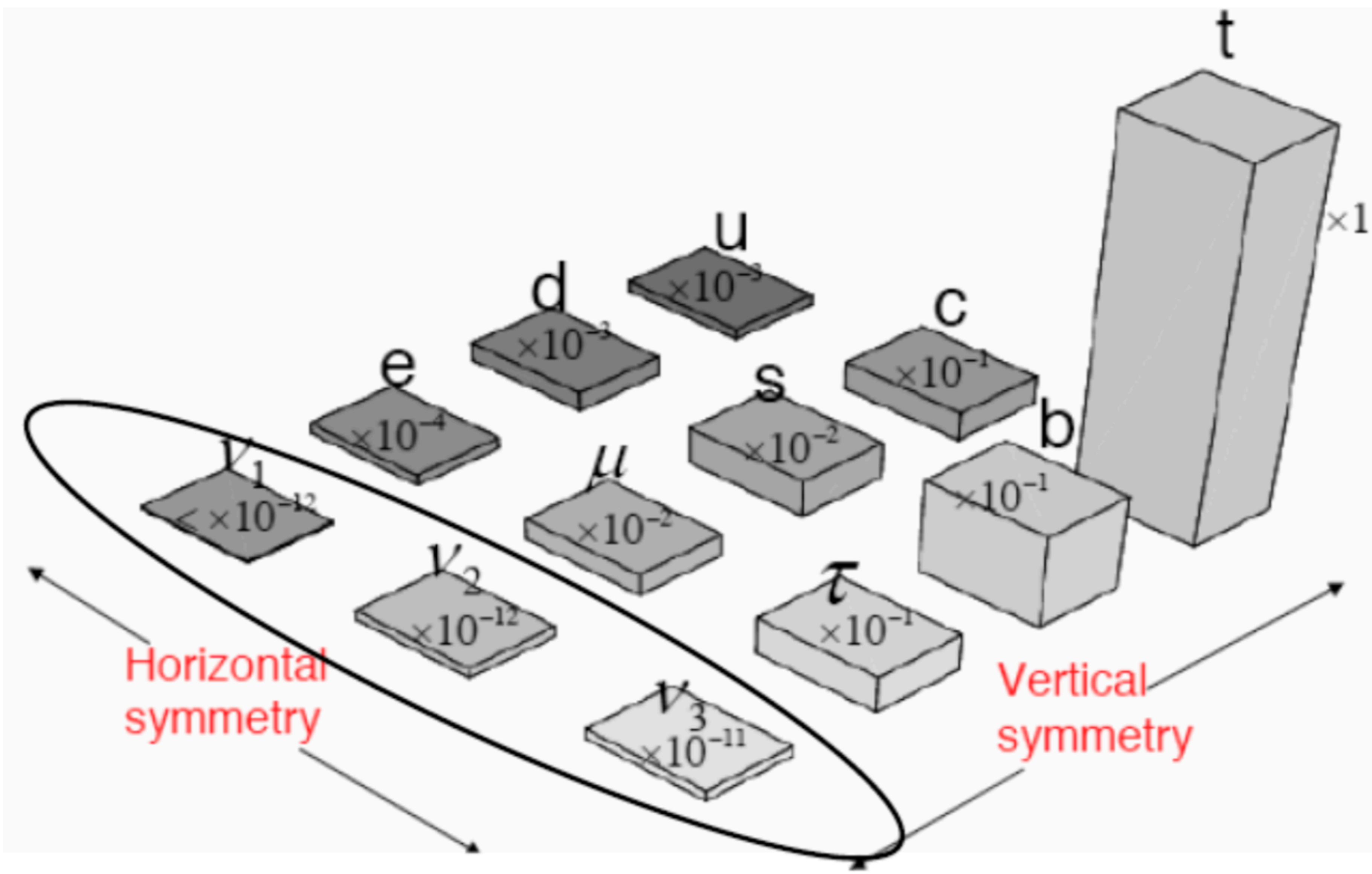
would collapse to a line



⇒ CP IS violated

Puzzle 2

Quark and lepton masses, rescaled by proper factors, as indicated



All of them, as V^{CKM} , given by pure parameters!